VMC Vidyamandir Classes SINCE 1986
IIT JEE | MEDICAL | FOUNDATION

Solutions of Mathematics

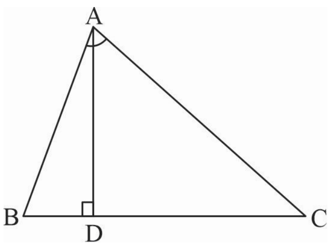
Class X

Sample Question Paper 2022-23

1.(D) In right angled ΔABC, $\angle A = 90°$

$AD \perp BC$

$\therefore\ \Delta DBA \sim \Delta ABC$ (By $AA \sim$)



$\frac{AB}{BC} = \frac{BD}{AB} \Rightarrow AB^2 = BD \times BC$ …(i)

Similarly, $\Delta ACD \sim \Delta BCA$

$DC \times BC = AC^2$ …(ii)

Dividing (ii) by (i)

$\frac{BD \times BC}{DC \times BC} = \frac{AB^2}{AC^2} \Rightarrow \frac{BD}{DC} = \frac{AB^2}{AC^2}$

Hence $\frac{BD}{DC} = \frac{AB^2}{AC^2}$

2.(A) $\alpha + \beta = -6$ & $\alpha\beta = 2$

$\therefore \left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = \frac{(\alpha+\beta)}{\alpha\beta} = \frac{-6}{2} = -3$

3.(A) Given : $a_1 = 5, a_2 = 3, b_1 = -15, b_2 = -9, c_1 = 8$ and $c_2 = \frac{24}{5}$ here

$\frac{a_1}{a_2} = \frac{5}{3}, \frac{b_1}{b_2} = \frac{-15}{-9} = \frac{5}{3}, \frac{c_1}{c_2} = \frac{8}{\frac{24}{5}} = \frac{5}{3} \because \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Since all have the same answer $\frac{5}{3}$

Therefore, the pair of given linear equation has infinitely many solutions

4.(B) Let larger of the two supplementary angle be x and smaller by y

According to question, $x + y = 180°$ ...(i)

and $x = y + 180°$

$\Rightarrow \; x - y = 18°$ ...(ii)

Subtracting equation (ii) from equation (i)

We get $2y = 162°$

$\Rightarrow \; y = 81°$

Therefore, the smaller angle is 81°

Putting the value of y in equation 1

$x + 81° = 180°$

$x = 180° - 81°$

$x = 99°$, which is a larger angle.

5.(C) Here, $\angle CAD = 180° - (130° + 25°) = 25°$

Now, since $\angle CAD = \angle DAB$, therefore, the AD is the bisector of $\angle BAC$

Since the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

$$\therefore \; \frac{BD}{DC} = \frac{AB}{AC} \Rightarrow \frac{x}{6} = \frac{15}{9}$$

$$\Rightarrow \; x = \frac{15 \times 6}{9} = 10\,cm$$

6.(B) Probability of guessing the correct answer $= \frac{x}{12}$

and probability of not guessing the correct answer $= \frac{2}{3}$

$$\frac{x}{12} + \frac{2}{3} = 1 \quad \because \; (P(A) + P(\bar{A}) = 1)$$

$$\Rightarrow \; \frac{x}{12} = 1 - \frac{2}{3} = \frac{1}{3} \Rightarrow x = \frac{12}{3} = 4$$

$\therefore \; x = 4$

7.(C) We have, $\frac{\sin\theta}{1+\cos\theta} = \frac{\sin\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$$= \frac{\sin\theta(1-\cos\theta)}{1-\cos^2\theta} = \frac{\sin\theta(1-\cos\theta)}{\sin^2\theta}$$

$$= \frac{1-\cos\theta}{\sin\theta}$$

8.(D) Mean of observation $x_1, x_2, ..., x_n$ is $\overline{x}$

$$\overline{x} = \frac{x_1 + x_2 + x_3 + ... + x_n}{n}$$

Now, $(x_1 + a) + (x_2 + a) + (x_3 + a) + ... + (x_n + a)$

$= x_1 + x_2 + x_3 + ... + x_n + na$

$\therefore$ Mean of $(x_1 + a), (x_2 + a), (x_3 + a), ..., (x_n + a)$

$$= \frac{(x_1 + x_2 + x_3 + ... + x_n) + na}{n}$$

$$= \frac{(x_1 + x_2 + x_3 + ... + x_n)}{n} + \frac{na}{n}$$

$$= \overline{x} + \frac{na}{n} = \overline{x} + a$$

9.(B) In ΔABC & ΔDEF,

$\angle B = \angle E, \angle F = \angle C$ & $AB = 3DE$

The triangles are similar as two angles are qual but including sides are not equal.

10.(D) $\left(2 + \sqrt{2}\right)$ is irrational number

If it is rational, then the difference of two rational is rational

$\therefore \left(2 + \sqrt{2}\right) - 2 = \sqrt{2} =$ irrational, which is a contradiction

Hence, $\left(2 + \sqrt{2}\right)$, is an irrational number.

11.(B) $4x^2 - 6x + 3 = 0$

$a = 4, b = -6, c = 3$

$D = b^2 - 4ac$

$= (-6)^2 - 4(4)(3)$

$= 36 - 48 = -12$

12.(A) Distance between $(\sin\theta, \cos\theta)$ & $(\cos\theta, -\sin\theta)$

$= \sqrt{(\cos\theta - \sin\theta)^2 + (-\sin\theta - \cos\theta)^2}$

$= \sqrt{\cos^2\theta + \sin^2\theta - 2\cos\theta\sin\theta + \cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta}$

$= \sqrt{2\cos^2\theta + 2\sin^2\theta}$

$= \sqrt{2(\cos^2\theta + \sin^2\theta)}$

$[\because \cos^2\theta + \sin^2\theta = 1]$

$= \sqrt{2}$ units

13.(A) $\overline{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$

$= 225 + \frac{-7}{25} \times 50$

$= 225 - 14 = 211$

14.(C) Given : $\cot^2\theta - \frac{1}{\sin^2\theta}$

$= \frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta}$

$= \frac{\cos^2\theta - 1}{\sin^2\theta}$

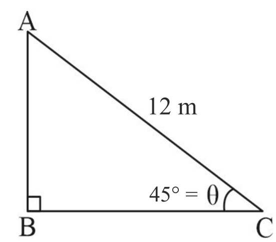
$= \frac{-\sin^2\theta}{\sin^2\theta} = -1$

$[\because 1 - \cos^2\theta = \sin^2\theta]$

15.(D) Let the height of the top of the ladder reaches to a vertical wall = AB

The length of the ladder = AC = 12 m

The angle of elevation $= \theta = 45°$



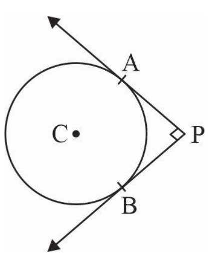
$\therefore \sin 45° = \frac{AB}{AC}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{AB}{12}$

$\Rightarrow AB = \frac{12}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$\Rightarrow AB = 6\sqrt{2}m$

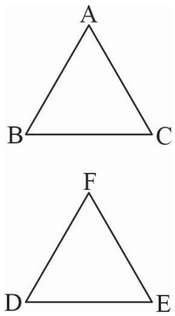
16.(C)



Construction : Joined *AC* and *BC*. Here $CA \perp AP$ & $BP \perp PB$ Also *AP* = *PB*

Therefore, *BPAC* is a square $\Rightarrow AP = PB = BC = 4cm$

17.(A) In ΔABC & ΔDEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then if, $\angle B = \angle D$ (the included angles) are equal then the triangles are similar



18.(B) $3x^2 + (k-1)x + 9 = 0$

$x = 3$ is a solution of the equation means if satisfies the equation

Put $x = 3$, we get

$3(3)^2 + (k-1)3 + 9 = 0$

$27 + 3k - 3 + 9 = 0$

$27 + 3k + 6 = 0$

$3k = -33$

$k = -11$

19.(C) An irrational roots zeros always occurs in pairs if the coefficients are rational, therefore, when one zero is $\left(2-\sqrt{3}\right)$ then other will be $\left(2+\sqrt{3}\right)$.

20.(D) **(**A) is false but (R) is true.

21. Total number of cards = 18

$$\text{Probability} = \frac{\text{favourable outcome}}{\text{total outcome}}$$

(I) Prime numbers less than 15 = 3, 5, 7, 11, 13

P(a prime number less than 15) $= \frac{5}{35}$

(II) Number divisible by 3 and 5 = 15

P(a number divisible by 3 and 5) $= \frac{1}{35}$

22. Given equations are

$9x + 3y + 12 = 0$

$18x + 6y + 24 = 0$

Comparing equation $9x + 3y + 12 = 0$ with $a_1x + b_1y + c_1 = 0$

and $18x + 6y + 24 = 0$ with

$a_2x + b_2y + c_2 = 0$

We get, $a_1 = 9, b_1 = 3, c_1 = 12, a_2 = 18, b_2 = 6, c_2 = 24$

We have $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ because $\frac{9}{18} = \frac{3}{6} = \frac{12}{24} \Rightarrow \frac{1}{2} = \frac{1}{2} = \frac{1}{2}$

Hence, lines are coincident

23. α, β are zeroes of $ax^2 + bx + c$

Then $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$ ...(1)

$\alpha^2\beta + \alpha\beta^2$

$= \alpha\beta(\alpha + \beta)$

$= \frac{c}{a}\left(\frac{-b}{a}\right)$ [By (1)]

$= \frac{-bc}{a^2}$

24. $P\left(\sqrt{2}, \sqrt{2}\right), Q\left(-\sqrt{2} - \sqrt{2}\right)$ & $R\left(-\sqrt{6}, \sqrt{6}\right)$ are the vertices of ΔPQR

Now,

$PQ = \sqrt{\left(-\sqrt{2} - \sqrt{2}\right)^2 + \left(-\sqrt{2} - \sqrt{2}\right)^2} = \sqrt{\left(-2\sqrt{2}\right)^2 + \left(-2\sqrt{2}\right)^2} = \sqrt{8+8} = \sqrt{16} = 4$ units

$QR = \sqrt{\left(-\sqrt{6} + \sqrt{2}\right)^2 + \left(\sqrt{6} + \sqrt{2}\right)^2} = \sqrt{6 + 2 - 2\sqrt{12} + 6 + 2 + 2\sqrt{12}} = \sqrt{16} = 4$ units

$PR = \sqrt{\left(-\sqrt{6} - \sqrt{2}\right)^2 + \left(\sqrt{6} - \sqrt{2}\right)^2} = \sqrt{6 + 2 + 2\sqrt{12} + 6 + 2 - 2\sqrt{12}} = \sqrt{16} = 4$ units

Since $PQ = QR = PR$, ΔPQR is an equilateral triangle

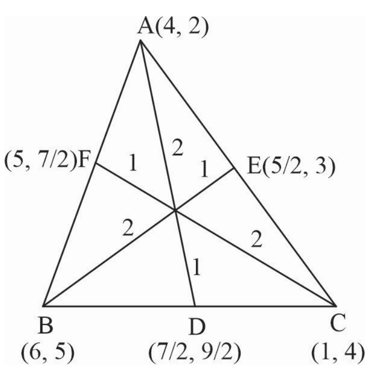
OR

The median from A meets BC at D

$\therefore$ D is the mid-point at BC

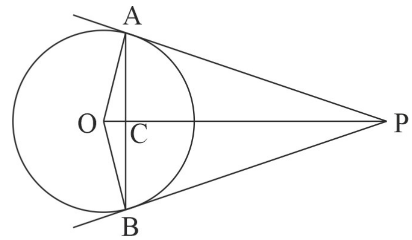
$\therefore D \to \left(\frac{6+1}{2}, \frac{5+4}{2}\right)$ [Using mid-point formula]

$\Rightarrow D \to \left(\frac{7}{2}, \frac{9}{2}\right)$



25. Suppose OP intersects AB at C

In triangles PAC and PBC we have



PA = PB [$\because$ tangents from an external point are equal]

$\angle APC = \angle BPC$ [$\because$ PA and PB are equally inclined to OP]

And PC = PC [common]

So, by SAS-criterion of similarity, we obtain

$\Delta PAC \cong \Delta PBC$

$\Rightarrow \ AC = BC$ and $\angle ACP = \angle BCP$

But, $\angle ACP + \angle BCP = 180°$ (Linear Pair)

$\therefore \ \angle ACP = \angle BCP = 90°$

Hence, $OP \perp AB$

OR

$\because \ PT = PQ$ [$\because$ tangents from an external point are equal]

$\therefore \ PQ = 7\,cm$

Also, SR = QR [$\because$ tangents from an external point are equal]

$\therefore \ QR = 4$ cm

Now $RP = PQ - QR = 7 - 4 = 3\,cm$

26. Let the total number of swans be x.

Number of swans playing on the shore of the tank $= \frac{7}{2}\sqrt{x}$

It is given that there are two remaining swans playing in the water. Hence, total no. of swans $= \frac{7}{2}\sqrt{x} + 2,$ which is equal to x

Clearly ; $x = \frac{7}{2}\sqrt{x} + 2$

$\Rightarrow \ x - \frac{7}{2}\sqrt{x} - 2 = 0$

$\Rightarrow \ y^2 - \frac{7}{2}y - 2 = 0,$ where $y = \sqrt{x} \Rightarrow \ y^2 = x$

$\Rightarrow \ 2y^2 - 7y - 4 = 0$

$\Rightarrow \ 2y^2 - 8y + y - 4 = 0$

$\Rightarrow \ 2y(y-4) + 1(y-4) = 0$

$\Rightarrow \ (y-4)(2y+1) = 0$

$\Rightarrow \ y = 4 \ or, \ y = -\frac{1}{2}$

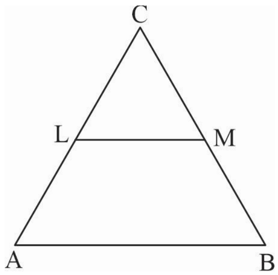
$\Rightarrow \ y = 4 [\because y = -\frac{1}{2}$ is not possible as y is square root of x]

$\Rightarrow \ x = y^2 \Rightarrow x = 4^2 = 16$

Hence, the total number of swans $= x = 16$

27. We have, $AL = x-3$, $AC = 2x$, $BM = x-2$ and $BC = 2x+3$, and we need to find the value of x

In ΔABC we have



$LM \parallel AB$

$\therefore \frac{AL}{LC} = \frac{BM}{MC}$ [By Thale's Theorem]

$\Rightarrow \frac{AL}{AC-AL} = \frac{BM}{BC-BM}$

$\Rightarrow \frac{x-3}{2x-(x-3)} = \frac{x-2}{(2x+3)-(x-2)}$

$\Rightarrow \frac{x-3}{x+3} = \frac{x-2}{x+5}$

$\Rightarrow (x-3)(x+5) = (x-2)(x+3)$

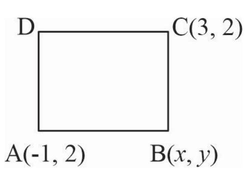
$\Rightarrow x^2 + 2x - 15 = x^2 + x - 6$

$\Rightarrow x = 9$

28. Let $ABCD$ be a square and $B(x, y)$ be the unknown vertex

$AB = BC$

$\Rightarrow AB^2 = BC^2$



$\Rightarrow (x+1)^2 + (y-2)^2 = (x-3)^2 + (y-2)^2$

$\Rightarrow x^2 + 1 + 2x + y^2 + 4 - 4x = x^2 - 6x + 9 + y^2 + 4 - 4x$

$\Rightarrow 2x + 1 = -6x + 9$

$\Rightarrow 8x = 8$

$\Rightarrow x = 1$ ...(i)

In ΔABC, $AB^2 + BC^2 = AC^2$

$\Rightarrow (x+1)^2 + (y-2)^2 + (x-3)^2 + (y-2)^2 = (3+1)^2 + (2-2)^2$

$\Rightarrow x^2 + 1 + 2x + y^2 - 4x + 4 + x^2 + 9 - 6x + y^2 + 4 - 2y = 16 + 0$

$\Rightarrow\ 2x^2 + 2y^2 + 2x - 4y - 6x - 4y + 1 + 4 + 9 + 4 = 16$

$\Rightarrow\ 2x^2 + 2y^2 - 4x - 8y + 2 = 0$

$\Rightarrow\ x^2 + y^2 - 2x - 4y + 1 = 0 \qquad \ldots\text{(ii)}$

Putting the value of x in equation (ii).

$1 + y^2 - 2 - 4y + 1 = 0$

$\Rightarrow\ y^2 - 4y = 0$

$\Rightarrow\ y(y-4) = 0$

$\Rightarrow\ y = 0 \text{ or } 4$

Hence the other vertices are (1, 0) and (1, 4).

OR

Let the coordinates of A be (x, y) which lies on line joining $P(6,-6)$ & $Q(-4,-1)$

Such that $\dfrac{PA}{PQ} = \dfrac{2}{5}$

$\Rightarrow\ \dfrac{PA}{PQ - PA} = \dfrac{2}{5-2}$

$\Rightarrow\ \dfrac{PA}{AQ} = \dfrac{2}{3}$

$\Rightarrow\ PA : AQ = 2 : 3$

Now by section formula x and y becomes as shown below

Since $P(6,-6)$ and $Q(-4,-1)$

$\therefore\ x = \dfrac{mx_2 + nx_1}{m+n} = \dfrac{2(-4) + 3\times 6}{2+3}$

$= \dfrac{-8+18}{5} = \dfrac{10}{5} = 2$

$y = \dfrac{my_2 + my_1}{m+n} = \dfrac{2\times(-1) + 3(-6)}{2+3}$

$= \dfrac{-2-18}{5} = \dfrac{-20}{5} = -4$

Coordinates of A are (2, -4). As A lies on line segment joining the points P and Q so it must satisfy equation of line segment.

Therefore, substituting the value of x and y value of $A(2,-4)$ in $3x + k(y+1) = 0$

$\Rightarrow\ 3\times 2 + k(-4+1) = 0 \Rightarrow 6 - 3k = 0$

$\Rightarrow\ 3k = 6 \Rightarrow k = \dfrac{6}{3} = 2$

29. Let first we consider, $\dfrac{\sqrt{2}}{3}$ be rational. We can write $\dfrac{\sqrt{2}}{3}$ as

$\frac{1}{3}\times\sqrt{2}$

We know that product of two rational number is always a rational number.

$\frac{\sqrt{2}}{3}\times 3=\sqrt{2}$

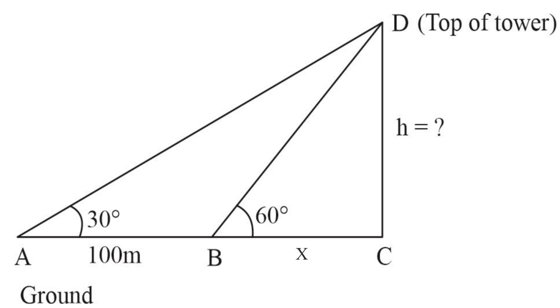
But $\sqrt{2}$ is irrational

$\therefore$ Here the Contradiction arises by assuming that $\frac{\sqrt{2}}{3}$ is rational. Actually it is irrational

Hence, $\frac{\sqrt{2}}{3}$ is irrational

30. In ΔBCD, $\frac{h}{x}=\tan 60^\circ=\sqrt{3}(BC=x)$

$h=\sqrt{3}x$ ...(i)



In ΔACD, $\frac{h}{100+x}=\tan 30^\circ=\frac{1}{\sqrt{3}}$

$\Rightarrow h\sqrt{3}=100+x$

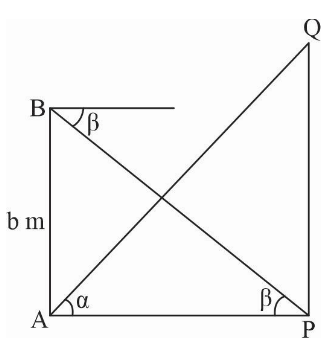
$\Rightarrow h\sqrt{3}=100+\frac{h}{\sqrt{3}}$

$\Rightarrow h\left[\sqrt{3}-\frac{1}{\sqrt{3}}\right]=100$

$\Rightarrow h\left[\frac{3-1}{\sqrt{3}}\right]=100$

$\Rightarrow h=\frac{100\sqrt{3}}{2}=50\sqrt{3}=50\times 1.732=86.6m$

OR



Let height of tower = QP = h m

In ΔBAP

$\tan\beta=\frac{BA}{AP}$

$\Rightarrow \tan\beta = \frac{b}{AP}$

$\Rightarrow AP = \frac{b}{\tan\beta}$

$\Rightarrow AP = b \times \cot\beta$ ...(i)

In ΔQPA

$\tan\alpha = \frac{QP}{AP} = \frac{h}{AP}$

$\Rightarrow h = AP \times \tan\alpha$

$\Rightarrow h = b\cot\beta \times \tan\alpha$ from (i)

31. First, we will convert the graph given into tabular form as shown below:

Class interval	Frequency (f_i)	Mid value (x_i)	$f_i x_i$	Cumulative frequency
1 – 4	6	2.5	15	6
4 – 7	30	5.5	165	36
7 – 10	40	8.5	340	76
10 – 13	16	11.5	184	92
13 – 16	4	14.5	58	96
16 – 19	4	17.5	70	100
	$N = \Sigma f_i = 100$		$\Sigma f_i x_i = 832$	

(I) $N = 100$

Mean $= \frac{\Sigma f_i x_i}{N} = \frac{832}{100} = 8.32$

(II) $\frac{N}{2} = \frac{100}{2} = 50$

The cumulative frequency just greater than $\frac{N}{2}$ is 76, then the median class is 7 – 10 such that

$l = 7,\ h = 10 - 7 = 3,\ f = 40,\ F = 36$

Median $= l + \frac{\frac{N}{2} - F}{f} \times h$

$= 7 + \frac{50 - 36}{40} \times 3$

$= 7 + \frac{42}{40} = 7 + 1.05 = 8.05$

(III) Mode = 3 Median – 2 Mean

$= 3 \times 8.05 - 2 \times 8.32 = 7.51$

32. Let x and y be the numerator and the denominator of the fraction

According to the question

$x+y=8$ …(i)

Also, we have

$$\frac{x+3}{y+3}=\frac{3}{4}$$

$\Rightarrow\ 4(x+3)=3(y+3)$

$\Rightarrow\ 4x+12=3y+9$

$\Rightarrow\ 4x-3y=-3$ …(ii)

Multiplying equation (i) by 4, we get

$4x+4y=32$ …(iii)

Subtracting equation (ii) from equation (iii) we get,

$7y=35$

Thus, $y=5$

Substitute the value of y in equation (i), we get, $x=3$

Thus, we have $x=3$ and $y=5$

Hence, the required fraction is $\frac{3}{5}$.

OR

Suppose, speed of the train be x km/hr and the speed of taxi be y km/hr

time taken to cover 300 km by the train $=\frac{300}{x}$ hours

time taken to cover 200 km by the taxi $=\frac{200}{y}$ hours

Total time taken $=5\frac{30}{60}$ hours $=5\frac{1}{2}$ hours $=\frac{11}{2}$ hours

$$\therefore\ \frac{300}{x}+\frac{200}{y}=\frac{11}{2}$$

$$\Rightarrow\ \frac{600}{x}+\frac{400}{y}=11$$

Put $\frac{1}{x}=u$ & $\frac{1}{y}=v$

$\Rightarrow\ 600u+400v=11$ …(i)

Time taken to cover 260 km by the train $=\frac{260}{x}$ hours

Time taken to cover 240 km by the taxi $=\frac{240}{y}$ hours

Total time taken $= 5\frac{36}{60}$ hours = 5.6 hours

$\Rightarrow \; 1300u + 1200v = 28 \quad \ldots(ii)$

Multiplying (i) by 3 and subtracting (ii) from it,

$\Rightarrow \; 500u = 5 \Rightarrow u = \frac{5}{500} \Rightarrow u = \frac{1}{100}$

Substituting $u = \frac{1}{100}$ in (i) $\Rightarrow \; v = \frac{1}{80}$

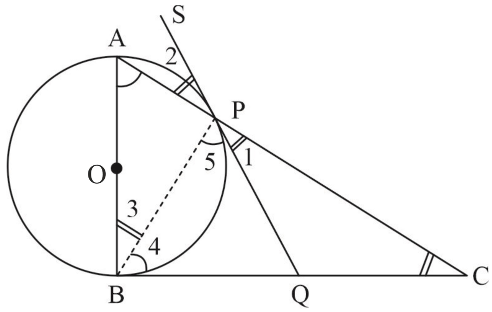
$\therefore u = \frac{1}{100} \;\Rightarrow\; \frac{1}{x} = \frac{1}{100} \;\Rightarrow\; x = 100$

$v = \frac{1}{80} \;\Rightarrow\; \frac{1}{y} = \frac{1}{80} \;\Rightarrow\; y = 80$

The speed of the train = 100 km/hr

The speed of the taxi = 80 km/hr

33. Given : ΔABC in which $\angle B = 90°$



Circle with diameter AB intersect the hypotenuse AC at P

A tangent SPQ at P is drawn to meet BC at Q

To prove : Q is mid-point of BC

Construction : Join PB

Proof : SPQ is tangent and AP is chord at contact point P

$\therefore \; \angle 2 = \angle 3$ [since angles in alternate segment of circle are equal]

$\angle 2 = \angle 1$ [Vertically opposite angles]

$\angle 3 = \angle 1 \quad \ldots$(i) [From above two relations]

$\angle ABC = 90°$ [Given]

OB is radius, therefor BC will be tangent at B

$\therefore \; \angle 3 = 90° - \angle 4 \quad \ldots(ii)$

In $\Delta BPC, \angle BPC = 90°$

$\therefore \angle C = 90° - \angle 4 \quad \ldots(iii)$

From (ii) and (iii), $\angle C = \angle 3$

Using (i), $\angle C = \angle 1$

$\Rightarrow \; CQ = QP \quad \ldots(iv)$ [Sides opposite to equal angles are equal]

$\angle 4 = 90° - \angle 3$

$\angle 5 = 90° - \angle 1$

$\angle 3 = \angle 1$

$\therefore\ \angle 4 = \angle 5$

$\Rightarrow\ PQ = BQ$ ...(v) [Sides opposite to equal angles are equal]

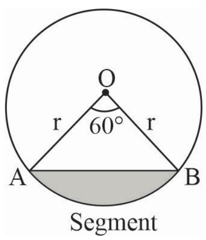
From (iv) and (v)

$BQ = CQ$

Therefore, Q is mid-point of BC. Hence, proved

34. Area of minor segment = Area of sector – Area of ΔOAB

In ΔOAB



$\theta = 60°$

$OA = OB = r = 12\,cm$

$\angle B = \angle A = x$ [$\angle s$ opposite to equal sies are equal]

$\Rightarrow\ \angle A + \angle B + \angle O = 180°$

$\Rightarrow\ x + x + 60° = 180°$

$\Rightarrow\ 2x = 180° - 60°$

$\Rightarrow\ x = \dfrac{120°}{2} = 60°$

$\therefore\ \Delta OAB$ is equilateral Δ with each side $(a) = 12\,cm$

Area of the equilateral $\Delta = \dfrac{\sqrt{3}}{4}a^2$

Area of minor segment = Area of the sector – Area of ΔOAB

$= \dfrac{\pi r^2 \theta}{360°} - \dfrac{\sqrt{3}}{4}a^2$

$= \dfrac{3.14 \times 12 \times 12 \times 60°}{360°} - \dfrac{\sqrt{3}}{4} \times 12 \times 12$

$= 6.28 \times 12 - 36\sqrt{3}$

$\therefore$ Area of minor segment $= (75.36 - 36\sqrt{3})\,cm^2$

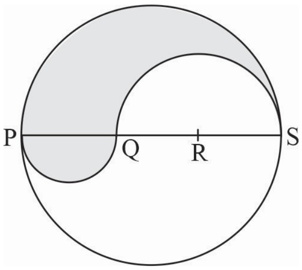
OR

PS = Diameter of a circle of radius 6 cm = 12 cm

$\therefore PQ = QR = RS = \frac{12}{3} = 4cm,\ QS = QR + RS = (4+4)cm = 8cm$

Let P be the perimeter and A be the area of the shaded region.

P = Arc of semi-circle of radius 6 cm + Arc of semi-circle of radius 4 cm + Arc of semi-circle of radius 2 cm



$\Rightarrow \ P = (\pi \times 6 + \pi \times 4 + \pi \times 2)cm = 12\pi cm$

and, A = Area of semi-circle with PS a s diameter + Area of semi-circle with PQ as diameter – Area of semi-circle with QS as diameter

$\Rightarrow A = \frac{1}{2} \times \frac{22}{7} \times (6)^2 + \frac{1}{2} \times \frac{22}{7} \times 2^2 - \frac{1}{2} \times \frac{22}{7} \times (4)^2$

$\Rightarrow A = \frac{1}{2} \times \frac{22}{7} \times (6^2 + 2^2 - 4^2) = \frac{1}{2} \times \frac{22}{7} \times 24 = \frac{264}{7} cm^2 = 37.71 cm^2$

35. Total number of outcomes = (1,1), (1,4), (1,9), (2,1), (2,4), (2,9), (2,16), (3,1), (3,4), (3,9), (3,16), (4,1), (4,4), (4,9), (4,16)

Total possible outcomes = 16

Total favorable event having product less than 16 = (1,1), (1,4), (1,9), (1,16), (2,1), (2,4), (3,1), (3,4), (4,1)

Probability $= \frac{\text{Favourable event outcome}}{\text{Total event}}$

$P(E) = \frac{8}{16} = \frac{1}{2}$

36. Given, $(2p+1)x^2 - (7p+2)x + (7p-3) = 0 \ \ldots$(i)

$\because$ Roots are equal $\quad \therefore D = 0$

$\Rightarrow (7p+2)^2 - 4(2p+1)(7p-3) = 0$

$\Rightarrow 49p^2 + 4 + 28p - 4(14p^2 + 7p - 6p - 3) = 0$

$\Rightarrow 49p^2 + 28p + 4 - 56p^2 - 4p + 12 = 0$

$\Rightarrow 7p^2 - 24p - 16 = 0 \Rightarrow 7p^2 + 4p - 28p - 16 = 0$

$\Rightarrow p(7p+4) - 4(7p+4) = 0 \Rightarrow (p-4)(7p+4) = 0$

$\Rightarrow p = 4 \ or \ p = \frac{-4}{7}$

When $p = 4$, (i) becomes $9x^2 - 30x + 25 = 0$

$\Rightarrow (3x)^2 - 2(3x)(5) + (5)^2 = 0$

$\Rightarrow (3x-5)^2 = 0 \Rightarrow x = \frac{5}{3}, \frac{5}{3}$

When $p = \frac{-4}{7}$, (1) becomes

$\frac{-x^2}{7} + 2x - 7 = 0 \Rightarrow x^2 - 14x + 49 = 0$

$\Rightarrow (x-7)^2 = 0 \Rightarrow x = 7, 7$

Thus, equal roots of given equation are either $\frac{5}{3}$ *or* 7

37. (I) Since the production increases uniformly by a fixed number every year, the number of Cars manufactured in 1st, 2nd, 3rd, …., years will form an AP

So, $a + 3d = 1800$ & $a + 7d = 2600$

So, $d = 200$ & $a = 1200$

(II) $t_{12} = a + 11d \Rightarrow t_{12} = 1200 + 11 \times 200$

$\Rightarrow t_{12} = 3400$

(III) $S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow S_{10} = \frac{10}{2}[2 \times 1200 + (10-1)200]$

$\Rightarrow S_{10} = \frac{10}{2}(2 \times 1200 + 9 \times 200]$

$\Rightarrow S_{10} = 5 \times [2400 + 1800]$

$\Rightarrow S_{10} = 5 \times 4200 = 21000$

OR

Let in n years the production will reach to 31200

$S_n = \frac{n}{2}[2a + (n-1)d] = 31200 \Rightarrow \frac{n}{2}[2 \times 1200 + (n-1)200] = 31200$

$\Rightarrow \frac{n}{2}[2 \times 1200 + (n-1)\,200] = 31200 \Rightarrow n[12 + (n-1)] = 312$

$\Rightarrow n^2 + 11n - 312 = 0$

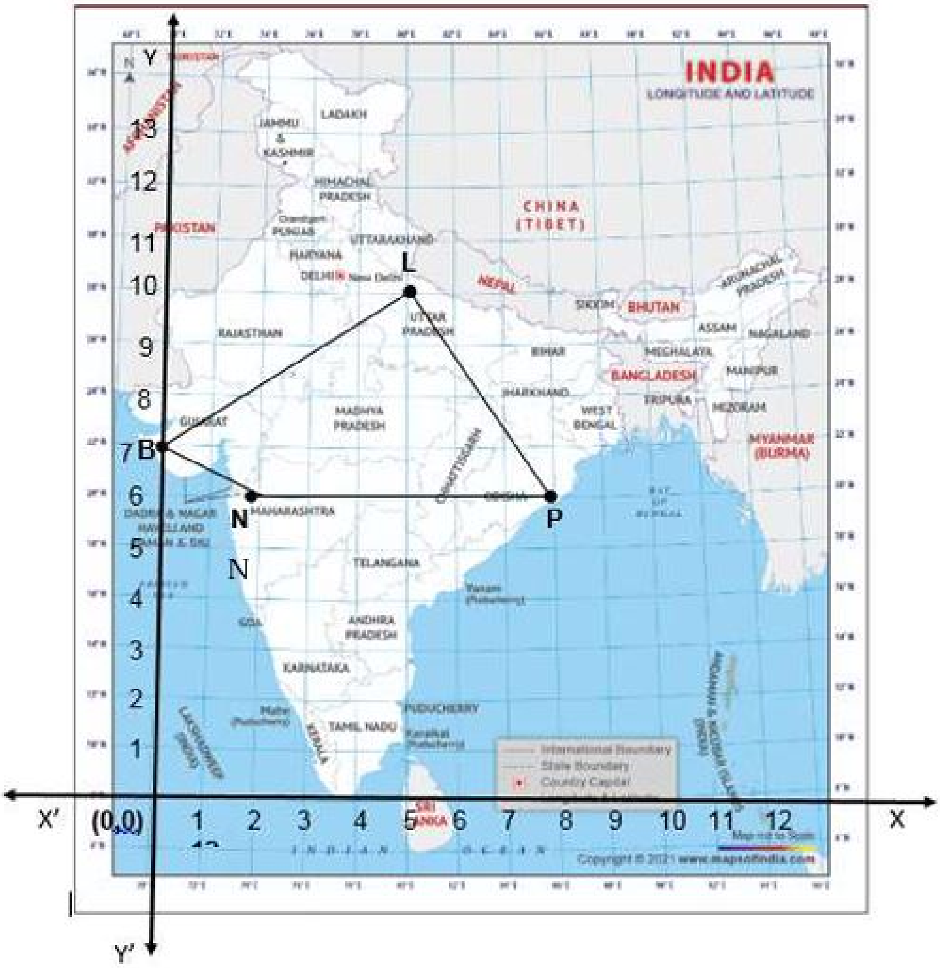
$\Rightarrow n^2 + 24n - 13n - 312 = 0$

$\Rightarrow (n+24)(n-13) = 0$

$\Rightarrow n = 13 \; or \; -24.$

As n can't be negative. So, $n = 13$

38.



(I) $LB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \Rightarrow LB=\sqrt{(0-5)^2+(7-10)^2}$

$LB=\sqrt{(5)^2+(3)^2} \Rightarrow LB=\sqrt{25+9} \Rightarrow LB=\sqrt{34}$

Hence the distance is $150\sqrt{34}\ km$

(II) Coordinate of Kota (K) is $\left(\frac{3\times5+2\times0}{3+2}, \frac{3\times7+2\times10}{3+2}\right)$

$=\left(\frac{15+0}{5}, \frac{21+20}{5}\right)=\left(3, \frac{41}{5}\right)$

(III) $L(5,10),\ N(2,6),\ P(7,6)$

$LN=\sqrt{(2-5)^2+(6-10)^2}=\sqrt{(3)^2+(4)^2}=\sqrt{9+16}=\sqrt{25}=5$

$NP=\sqrt{(8-2)^2+(6-6)^2}=\sqrt{(4)^2+(0)^2}=4$

$PL=\sqrt{(8-5)^2+(6-10)^2}=\sqrt{(3)^2+(4)^2} \Rightarrow LB=\sqrt{9+16}=\sqrt{25}=5$

As $LN=PL\neq NP$, So, ΔLNP is an isosceles triangle

OR

Let $A(0,b)$ be a point on the y-axis then $AL=AP$

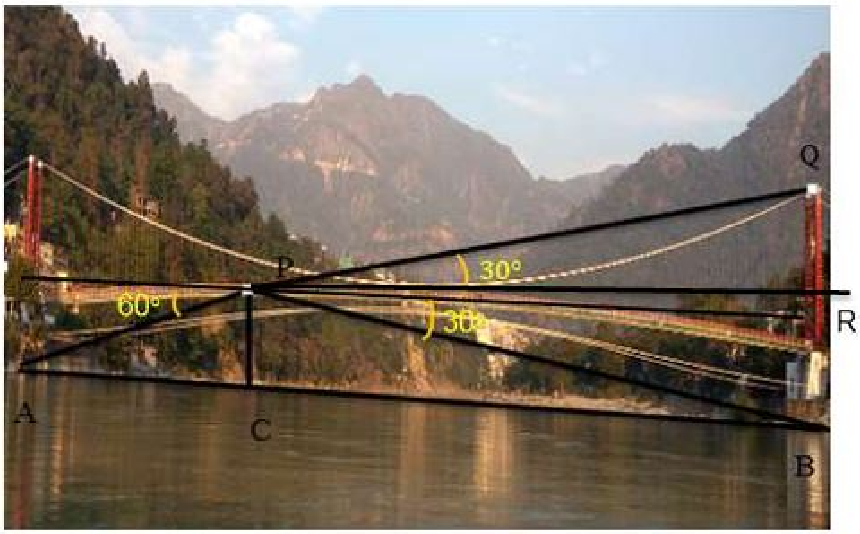
$\Rightarrow \sqrt{(5-0)^2+(10-b)^2} = \sqrt{(8-0)^2+(6-b)^2}$

$\Rightarrow (5)^2+(10-b)^2 = (8)^2+(6-b)^2$

$\Rightarrow 25+100-20b+b^2 = 64+36-12b+b^2 \Rightarrow 8b = 25 \Rightarrow b = \dfrac{25}{8}$

So, the coordinate on y axis is $\left(0, \dfrac{25}{8}\right)$

39.



(I) $\sin 60° = \dfrac{PC}{PA}$

$\Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{18}{PA} \Rightarrow PA = 12\sqrt{3}m$

(II) $\sin 30° = \dfrac{PC}{PB}$

$\Rightarrow \dfrac{1}{2} = \dfrac{18}{PB} \Rightarrow PB = 36m$

(III) $\tan 60° = \dfrac{PC}{AC} \Rightarrow \sqrt{3} = \dfrac{18}{AC} \Rightarrow AC = 6\sqrt{3}m$

$\tan 30° = \dfrac{PC}{CB} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{18}{CB} \Rightarrow CB = 18\sqrt{3}m$

Width $AB = AC + CB = 6\sqrt{3} + 18\sqrt{3} = 24\sqrt{3}m$

OR

$RB = PC = 18m$ & $PR = CB = 18\sqrt{3}m$

$\tan 30° = \dfrac{QR}{PR} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{QR}{18\sqrt{3}} \Rightarrow QR = 18m$

$QB = QR + RB = 18 + 18 = 36m.$

Hence height BQ is 36m