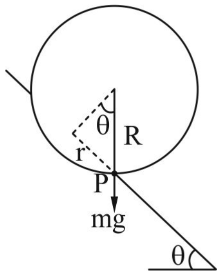
VMC Vidyamandir Classes SINCE 1986

IIT JEE | MEDICAL | FOUNDATION

JEE Advanced 2020

Physics Paper - 1_Solutions



1.(A) The sphere will not roll down if torque of mg about point P is anti-clockwise.

In limiting case, torque can be zero. So, mg should pass through point P.

$\therefore \quad \sin\theta = \dfrac{r}{R}$

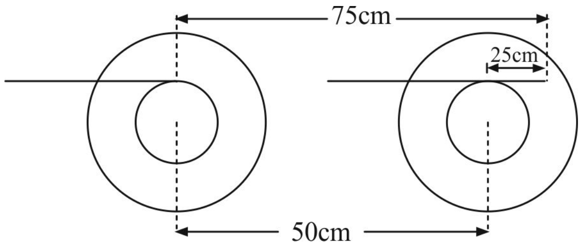
This gives maximum θ for no rolling.

2.(B) As per Lenz's law, effect will be to oppose the change in flux, so the disc will rotate in the direction of the magnet's motion.

3.(B) Let the speed of centre be $V \quad \therefore \quad V = \omega R$

Speed of point of contact of scale and roller $= V + \omega\dfrac{R}{2} = \dfrac{3V}{2}$

As $\int V dt = 50 \text{ cm}$ (given) $\quad \therefore \quad \int \dfrac{3}{2} V dt = 75 \text{ cm}$



As shown in figure, horizontal displacement of scale w.r.t. roller will be 25 cm.

4.(B) $\vec{\tau} = \vec{m} \times \vec{B}$

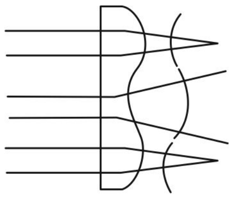
$\tau = (NIA)B_0 \sin 90° = N\dfrac{dq}{dt}\pi R^2 B_0$

$\dfrac{dL}{dt} = N\pi R^2 B_0 \dfrac{dq}{dt}$

$\int dL = N\pi R^2 B_0 \int dq$

$\Delta L = N\pi R^2 B_0 Q$

5.(A) The upper and lower part behave as converging lens while the middle part behaves as diverging lens.



6.(B) In the final situation as shown in the figure of question,

$$P_0 + (800)(10)(0.1) + (1000)(10)(h_1 - 0.1)$$
$$= P_0 + (1000)(10)(h_2)$$

$\Rightarrow \quad 0.08 + h_1 - 0.1 = h_2$

$\Rightarrow \quad h_1 - h_2 = 0.02 \qquad \text{... (1)}$

Also $(h_1 - 0.1) + h_2 = 0.58$ (length of water column)

$\Rightarrow \quad h_1 + h_2 = 0.68 \qquad \text{... (2)}$

(1) + (2) gives

$$2h_1 = 0.70$$
$$h_1 = 0.35m$$

$\therefore \quad h_2 = 0.68 - 0.35 = 0.33m$

7.(BC) $V = Fr \ \Rightarrow$ Force $= -\dfrac{dv}{dr} = -F$ (attractive force)

$$F = \frac{mv^2}{r} \qquad \text{… (1)}$$

$$mvr = n\frac{h}{2\pi} \qquad \text{… (2)}$$

Putting r from (1) into (2):

$$mv\left(\frac{mv^2}{F}\right) = n\frac{h}{2\pi}$$

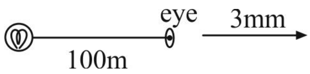
$v^3 = n\dfrac{h}{2\pi}\dfrac{F}{m^2} \ \Rightarrow \ v \propto n^{\frac{1}{3}}$; $\quad r = n\dfrac{h}{2\pi}\dfrac{1}{mv}$; $\quad$ So $r \propto n^{1-\frac{1}{3}} \propto n^{\frac{2}{3}}$

Kinetic energy $= \dfrac{1}{2}mv^2 = \dfrac{1}{2}Fr$

$\therefore$ Total energy $E = Fr + \dfrac{1}{2}Fr = \dfrac{3}{2}Fr$

$$= \frac{3}{2}F\frac{nh}{2\pi m\left(\dfrac{nhF}{2\pi m^2}\right)^{\frac{1}{3}}} = \frac{3}{2}\left(\frac{n^2h^2F^2}{4\pi^2 m}\right)^{\frac{1}{3}}$$

8.(BCD) (A) $P = e\sigma AT^4 = 1 \times 5.67 \times 10^{-8} \times 64 \times (10^{-3})^2 \times (2500)^4 = 141.75W$

(B) 

Ioitride eye $= \dfrac{141.75}{4\pi(100)^2} = \dfrac{141.75}{4x \times 10^4}$

Radiated power entering eye

$$= \frac{141.75}{4\pi \times 10^4} \times \pi(3 \times 10^{-3})^2$$

$$= \frac{141.75}{4 \times 10^4} \times 9 \times 10^{-6} \qquad = 318.9375 \times 10^{-10}$$

$$= 3.19 \times 10^{-8} W$$

(C) $\lambda_m T = b \qquad \Rightarrow \lambda_m \times 2500 = 2.90 \times 10^{-3}$

$$\Rightarrow \lambda_m = 1.16 \times 10^{-6} m = 1160\ nm$$

(D) $n \times \dfrac{hc}{\lambda} =$ Radiated power entering eye

$$n = \frac{3.19 \times 10^{-8} \times 1740 \times 10^{-9}}{6.63 \times 10^{-34} \times 3 \times 10^8} = 279 \times 10^9 \ = 2.79 \times 10^{11}$$

9.(AB) $L = x^{\alpha}$

$LT^{-1} = x^{\beta}$

$LT^{-2} = x^{p}$

$MLT^{-1} = x^{q}$

$MLT^{-2} = x^{r}$

(A) $L \times LT^{-2} = x^{\alpha} x^{p}$

$L^2T^{-2} = x^{\alpha + p}$

$(x^{\beta})^2 = x^{\alpha+p} \quad \Rightarrow \quad \alpha + p = 2\beta$

(B) $\dfrac{LT^{-2} \times MLT^{-1}}{MLT^{-2}} = x^{p+q-r}$

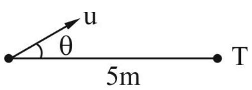
$x^{\beta} = x^{p+q-r} \quad \Rightarrow \quad p + q - r = \beta$

(C) $\dfrac{LT^{-2} \times MLT^{-2}}{MLT^{-1}} = x^{p+r-q}$

$LT^{-3} = x^{p+r-q}$

(D) $LT^{-2} \times MLT^{-1} \times MLT^{-2} = x^{p+q+r}$

$M^2L^3T^{-5} = x^{p+q+r}$

10.(BC) 

$u = 2\sqrt{10} \times 10^6$ m/s

$$\Rightarrow \qquad a = \frac{qE}{m} = 10^{10} \times 400\sqrt{3} = 4\sqrt{3} \times 10^{12}\ \text{m/s}^2$$

$$R = \frac{u^2 \sin 2\theta}{a}$$

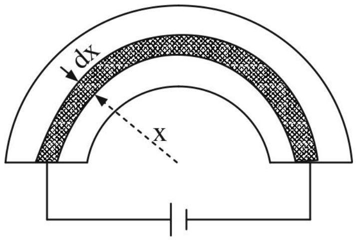
$$\Rightarrow \qquad 5 = \frac{(2\sqrt{10}\times 10^6)^2 \sin 2\theta}{4\sqrt{3}\times 10^{12}}$$

$$\Rightarrow \qquad 5 = \frac{4\times 10\times 10^{12}}{4\sqrt{3}\times 10^{12}}\sin 2\theta \quad \Rightarrow \quad \sin 2\theta = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \theta = 30^\circ \text{ or } \theta = 60^\circ$$

$$T = \frac{2u\sin\theta}{a} \qquad \text{when } \theta = 30^\circ,\ T = \frac{2\times 2\sqrt{10}\times 10^6}{4\sqrt{3}\times 10^{12}}\times\frac{1s}{2} = \sqrt{\frac{10}{3\times 4}}\ \mu s = \sqrt{\frac{5}{6}}\mu s$$

$$\text{when } \theta = 60^\circ,\ T = \frac{2\times 2\sqrt{10}\times 10^6}{4\sqrt{3}\times 10^{12}}\times\frac{\sqrt{3}}{2}s = \sqrt{\frac{5}{2}}\mu s$$

11.(ACD)



Current flowing through the ring of radius x and width dx (its thickness is t)

$$dI = \frac{V_0}{dR} = \frac{V_0}{\left[\rho\frac{\pi x}{t(dx)}\right]} = \frac{V_0 t}{\pi\rho}\frac{dx}{x}$$

$\therefore$ Total current

$$I = \int dI = \frac{V_0 t}{\pi\rho}\int_{R_1}^{R_2}\frac{dx}{x} = \frac{V_0 t}{\pi\rho}\ln\frac{R_2}{R_1}$$

As electrons are moving on circular path, a force must exist to provide the required centripetal acceleration.

So, a electric field in radically outward direction is developed.

For radius x,

$$eE = \frac{mv^2}{x} \qquad \Rightarrow \quad E = \frac{m}{e}\frac{v^2}{x} \qquad \dots (1)$$

Where v is drift velocity of electron which is related by

$$dI = ne(dA)v$$

$$\frac{V_0 t}{\pi\rho}\frac{dx}{x} = net(dx)v$$

$$\Rightarrow \qquad v = \frac{V_0}{\pi\rho ne}\frac{1}{x}$$

Putting in (1): $E = \frac{m}{ex}\left(\frac{V_0}{\pi\rho nex}\right)^2$

$V_1 - V_2 = \int_{R_1}^{R_2} E\,dx \ \propto V_0^2 \propto I^2$; Outer surface will be at lower voltage

12.(ABC)

As collection of molecules behave as ideal gas, so pressure is related as

$$PV = \mu RT;\ \mu: \text{ no of moles}$$

$PV = \frac{N}{N_A} RT \quad \Rightarrow \quad P = \frac{N}{V} k_B T$

$\therefore \qquad P = nk_B T$; n: molecules per unit volume

Force causing molecules to move

$= (P_1 - P_2)s$

$= (n_1 - n_2)k_B Ts$

$= \Delta n k_B Ts$

For steady flow, driving force = opposing force

$\Delta n\, k_B Ts = \beta v(n_1 \ell s)$

where $(n_1 \ell s)$ is the total no. of molecules in the pipe.

$\therefore \qquad n_1 \beta v \ell = \Delta n k_B T$

$v = \frac{\Delta n k_B T}{n_1 \beta \ell}$

Number of molecules flowing through the tube per second

$= \frac{n_1 s dx}{dt} = n_1 s \left(\frac{dx}{dt}\right)$

$= n_1 sv = n_1 s \frac{\Delta n k_B T}{n_1 \beta \ell}$

$= \left(\frac{\Delta n}{\ell}\right)\left(\frac{k_B T}{\beta}\right) s$

As molecules flow from left to right vessel, n_1 decreases while n_2 increases

$\Rightarrow \qquad \Delta n$ decreases

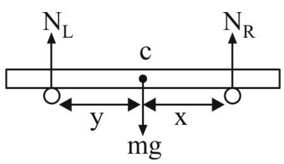
$\therefore \qquad$ rate decreases with time

13.(25.60)

Let us consider mass of rod is m & left finger is at a distance of y and right finger is at a distance of x from the centre we need to calculate the value of friction according to the above data.

$N_L + N_R = mg$; $\qquad N_L y = N_R x$

$\therefore \qquad N_R = \frac{mg\, y}{x + y}$



& $\qquad N_L = \frac{mg\, x}{x + y}$

For 1st move $x = 40, \; y = 50$

So $\qquad N_L = \frac{4mg}{9} \qquad \& \qquad N_R = \frac{5mg}{9}$

$\Rightarrow \qquad$ Limiting static friction will be less for left finger and its slips

The left finger will move till the value of kinetic friction on left finger becomes equal to the value of static friction on right finger

So $\qquad .32 \frac{40\, mg}{y + 40} = 0.4 \frac{y\; mg}{y + 40} \;\Rightarrow\; y = 32\text{cm}$

Now the right finger will start slipping and it will stop slipping till x_r. At this condition static friction of left will be equal to kinetic friction of right so

$$\frac{.4\,mg\,x_r}{32+x_r}=\frac{.32\,mg\,32}{32+x_r}$$

$$x_r=\frac{.32\times 32}{.4}$$

$$x_r=25.6\text{ cm}$$

14.(3.74)

$$P_0+\rho gh=P_0+\frac{\sigma}{r}=P_0+\frac{\sigma}{(h/2)}$$

$$\Rightarrow \qquad \rho gh^2=2\sigma$$

$$h=\sqrt{\frac{\sigma^2}{\rho g}}=\sqrt{\frac{2(0.07)}{(1000)(10)}}$$

$$=\frac{\sqrt{14}}{1000}m=\sqrt{14}\,mm=3.74\text{ mm}\ \ \text{(Note : a range will be given)}$$

15.(3.14)

At equilibrium :

$$k\ell=q\frac{2kp}{\ell^3}$$

When stretched further by x

$$F_{net}=q\frac{2kP}{(\ell+x)^3}-K(\ell+x)=q\frac{2kP}{\ell^3\left(1+\frac{x}{\ell}\right)^3}-k\ell-kx=q\frac{2kp}{\ell^3}\left(1-\frac{3x}{\ell}\right)-k\ell-kx$$

$$=k\ell\left(1-\frac{3x}{\ell}\right)-k\ell-kx=(k\ell)\left(-\frac{3x}{\ell}\right)-kx=-4kx$$

$$\therefore \qquad ma=-4kx$$

$$a=-\frac{4k}{m}x$$

$$\therefore \qquad \omega^2=\frac{4k}{m}$$

$$T=\frac{2\pi}{\omega}=\frac{2\pi}{\sqrt{\frac{4k}{m}}}=\pi\sqrt{\frac{m}{k}} \quad \therefore \qquad f=\frac{1}{T}=\frac{1}{\pi}\sqrt{\frac{k}{m}}$$

16.(1.77 or 1.78)

$$\underset{P_1V_1}{\text{He gas}}\xrightarrow[\text{Isothermal}]{}\underset{\frac{P_1}{4},\,4V_1}{\text{He gas}}\xrightarrow[\text{Adiabatic}]{}\underset{P_2,\,32V_1}{\text{He gas}}$$

For adiabatic process,

$$\frac{P_1}{4}.(4V_1)^{\gamma}=P_2(32V_1)^{\gamma} \quad \Rightarrow \quad P_2=\frac{P_1}{4}\left(\frac{4V_1}{32V_1}\right)^{5/3} \qquad \left\{\gamma=\frac{5}{3}\text{ for He}\right\}$$

$$P_2=\frac{P_1}{2^2}\left[\left(\frac{1}{2}\right)^3\right]^{5/3}; \qquad P_2=\frac{P_1}{2^7}$$

So $\quad W_{Iso} = P_1V_1 \ell n\left(\dfrac{4V_1}{V_1}\right) = 2P_1V_1\ell n 2$

$$W_{Adia} = \frac{P_iV_i - P_fV_f}{\gamma - 1} = \frac{(P_1/4)(4V_1) - \left(\frac{P_1}{2^7}\right)\left(2^5 V_1\right)}{\frac{5}{3} - 1} = \frac{\left(P_1V_1 - \frac{P_1V_1}{4}\right)}{\frac{2}{3}} = \frac{3}{2}\left[\frac{3}{4}P_1V_1\right] \quad = \frac{9}{8}P_1V_1$$

$$\frac{W_{iso}}{W_{adia}} = \frac{2P_1V_1\ell n 2}{9/8\ P_1V_1} = \frac{16}{9}\ell n 2 \ \Rightarrow \ \boxed{f = 1.77}$$

17.(0.62 or 0.63)

$$f = \frac{v}{4\ell},\ f' = \frac{v}{4\ell'}$$

$$f' = \frac{320}{320-2}f \quad \Rightarrow \quad \frac{f'}{f} = \frac{320}{318}$$

$$\frac{f}{f'} = \frac{\ell'}{\ell} \quad \Rightarrow \quad \frac{318}{320} = \frac{\ell'}{\ell}$$

$$1 - \frac{318}{320} = 1 - \frac{\ell'}{\ell} = \frac{\ell - \ell'}{\ell} = \frac{\Delta\ell}{\ell}; \qquad \frac{\Delta\ell}{\ell} = \frac{2}{320}$$

Percentage change $= \dfrac{2}{320} \times 100 = 0.625$

18.(6.40) Given for disc of radius 'R' $\quad \sigma = \sigma_0(1 - r/R)$

If total charge on disc is Q

For large spherical surface by gauss law $\phi_0 = \dfrac{Q}{\epsilon_0}$.

If charge on disc upto radius $R/4$ is q by Gauss Law $\quad \phi = \dfrac{q}{\epsilon_0}$

$$\Rightarrow \qquad \frac{\phi_0}{\phi} = \frac{Q}{q} = \frac{\int_0^R 2\pi r dr \sigma}{\int_0^{R/4} 2\pi r dr \sigma} = \frac{\int_0^R \left(r - \frac{r^2}{R}\right)dr}{\int_0^{R/4}\left(r - \frac{r^2}{R}\right)dr} = \frac{R^2/6}{R^2\left(\frac{5}{64\times 3}\right)}$$

$$\frac{\phi_0}{\phi} = \frac{64\times 3}{5\times 6} = \frac{32}{5} = 6.4; \qquad \boxed{\frac{\phi_0}{\phi} = 6.4}$$