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IIT JEE | MEDICAL | FOUNDATION

JEE Advanced 2020

Mathematics Paper - 1_Solutions

1.(D) The expression reduce to

$a^2(c+d)-a(c^2+d^2)+b^2(c+d)-b(c^2+d^2)$

$=(a^2+b^2)(c+d)-(a+b)(c^2+d^2)=16000$

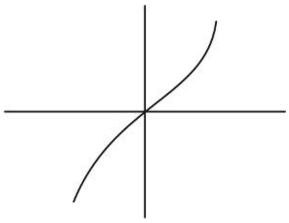
2.(C) As $f(x)$ is odd function, it is sufficient to analyse it for $x \geq 0$

$f(x)=x^2-x\sin x,\ x\geq 0$

$f'(x)=(x-\sin x)+x(1-\cos x),\ x>0$

Now $f'(x)>0\ \forall\ x>0 \Rightarrow f(x)$ is $\uparrow$

Its rough shape will be



So, $f(x)$ is both one-one & onto

3.(A) $f(x)=\begin{cases} 0 & ;\ 0\leq x\leq 1 \\ e^{x-1}-e^{1-x} & ;\ x\geq 1 \end{cases}$ ....(1)

$g(x)=\frac{1}{2}(e^{x-1}+e^{1-x})$ ....(2)

By solve (1) & (2)

$e^{x-1}-e^{1-x}-\frac{1}{2}e^{x-1}+e^{1-x}$

$e^{x-1}=3e^{1-x}$

$e^{x-1}=\sqrt{3} \Rightarrow x=1+\ln\sqrt{3}$

$\Rightarrow \quad A=\int_0^{1+\ln\sqrt{3}}|g(x)-f(x)|\,dx$

$=\int_0^1\frac{1}{2}\left(e^{x-1}+e^{1-x}\right)dx+\int_1^{1+\ln\sqrt{3}}\frac{1}{2}\left(3e^{1-x}-e^{x-1}\right)dx$

$=\frac{e-e^{-1}}{2}+2-\sqrt{3}$

4.(A) Let the end pointof latus rectum be $(\lambda, 2\lambda)$

Slope of tangent at parabola at this point = 1

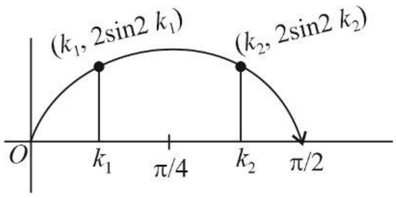
$\Rightarrow$ Slope of tangent is $= -1 \Rightarrow$ $b^2 = 2a^2$ $\Rightarrow$ $e = \frac{1}{\sqrt{2}}$

5.(B) For real & equal roots only two case are possible

α	β	
0	0	$\left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^2$
2	1	$\left(\frac{2}{3}\right)^2 \, {}^2C_1 \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)$

$\Rightarrow$ $P = \frac{20}{81}$

6.(C)



$\Rightarrow$ $\sin 2k_1 = \sin 2k_2$

$\Rightarrow$ $k_1 + k_2 = \frac{\pi}{2}$ ……………(1)

$P = 2(k_1 - k_2) + 4\sin 2k_1$

or $P = 2\left(\frac{\pi}{2} - 2k_1\right) + 4\sin 2k_1$ …………..(2)

$\frac{dp}{dk_1} = 0 \Rightarrow k_1 = \frac{\pi}{6}$

Area $= (k_2 - k_1) \times (2\sin 2k_1) = \frac{\pi}{2\sqrt{3}}$

7.(AC) Claim: if f is a differentiable function at a and $f(a) = 0$ and g is continuous at a, then fg is differentiable at a

Proof:

Let $p(x) = f(x)g(x)$

$$p'(a) = \lim_{h\to 0} \frac{p(a+h) - p(a)}{h} = \lim_{h\to 0} \frac{p(a+h)}{h}$$

$$= \lim_{h\to 0} \frac{f(a+h)g(a+h)}{h} = \lim_{h\to 0} \frac{(f(a+h) - f(a))g(a+h)}{h}$$

$$= \left(\lim_{h\to 0} \frac{(f(a+h)g(a+h))}{h}\right) \lim_{h\to 0} g(a+h)$$

Since both limits exist as f is differentiable at a and g is continuous at a. Therefore $p'(a)$ exists. That proves the validity of (A)

Now C is obviously true

Consider $g(x) = \begin{cases} 1 & ; x \neq 1 \\ 0 & ; x = 1 \end{cases}$

Clearly g is neither differentiable nor continous at 1, but the product is differentiable at 1.

8.(BCD)

As $adj(adjA) = |A|^{n-2} A$

$\Rightarrow \quad M^{-1} = |M| M$ ……………(1)

$\Rightarrow \quad \frac{1}{|M|} = |M|^4$

$\Rightarrow \quad |M|^5 = 1 \Rightarrow |M| = 1$ …………….(2)

Also pre multiply (1) by M & placing $|M| = 1$

$M^2 = I$ …………….(3)

Also from (1)

$AdjM = M$

$\Rightarrow \quad (AdjM)^2 = M^2 = I$

9.(BC) Given $|z^2 + z + 1| = 1$ ………(1)

$\Rightarrow \quad \left|\left(z + \frac{1}{2}\right)^2 + \frac{3}{4}\right| = 1$

$\Rightarrow \quad \left|z + \frac{1}{2}\right|^2 + \frac{3}{4} \geq 1$

$\left|\left(z + \frac{1}{2}\right)\right|^2 \geq \frac{1}{4}$

$\Rightarrow \quad \left|z + \frac{1}{2}\right| \geq \frac{1}{2}$ …………..(2)

Again from (1)

$\left|\left(z^2 + z\right) - |-1|\right| \leq 1$

$-1 \leq |z^2 + z| - 1 \leq 1$

$|z^2 + z| \leq 2$

$\left||z|^2 - |z|\right| \leq 2$

$-2 \leq |z|^2 - |z| \leq 2$

$|z|^2 - |z| + 2 \geq 0$ always true

$|z|^2 - |2| - 2 \leq 0$

$\Rightarrow \quad |z| \leq 2$

10.(BC) Given

$$\frac{\Delta}{S(S-x)}+\frac{\Delta}{S(S-z)}=\frac{y}{S}$$

$\Rightarrow \quad \Delta=(S-x)(S-z) \quad \Rightarrow \quad S(S-x)(S-y)(S-z)=(S-x)^2(S-z)^2$

$\Rightarrow \quad y^2=x^2+z^2 \;\Rightarrow \angle y=\frac{\pi}{2}$

$\Rightarrow \quad \angle X+\angle Z=\frac{\pi}{2}$

Now $\dfrac{x}{y+z}=\dfrac{\sin X}{\sin y+\sin z}=\dfrac{\sin X}{1+\sin\left(\dfrac{\pi}{2}-X\right)}=\tan\dfrac{X}{2}$

11.(AB) As the intersection point of L_1 & L_2 is $(1,\ 0,\ 1)$

$\Rightarrow \quad \dfrac{1-2}{l}=\dfrac{-1}{m}=\dfrac{1-y}{-2}$ …….(1)

The direction ratio of acute angle bisector of L_1 & L_2 is $(-2,-2,4)$

$\Rightarrow \quad \dfrac{l}{-2}=\dfrac{m}{-2}=\dfrac{-2}{4}=l=m=1$

From (1) $\quad \alpha=2,\ y=-1$

12.(ABD)

$$\cos x=1-\frac{x^2}{\underline{|2}}+\frac{x^4}{\underline{|4}}$$

$\Rightarrow \quad \cos x\ge 1-\dfrac{x^2}{2}$ (As x lies between $(0,\ 1)$)

$\Rightarrow \quad \displaystyle\int_0^1 x\cos x\ge\int_0^1 x-\frac{x^3}{2}$

$\displaystyle \ge \left.\frac{x^2}{2}-\frac{x^4}{8}\right|_0^1$

$\ge\dfrac{1}{2}-\dfrac{1}{8}\ge\dfrac{3}{8}$ ………..(1)

Like wise

$$\int_0^1 x^2\cos x\ge\int_0^1 x-\frac{x^4}{2}\ge\frac{1}{3}-\frac{1}{10}\ge\frac{7}{30}$$

In the similarly

$$\int_0^1 x\sin x\ge\int_0^1 x-\frac{x^4}{6}\ge\left.\frac{x^3}{3}-\frac{x^5}{30}\right|_0^1$$

$\ge\dfrac{3}{10}$

And $\displaystyle\int_0^1 x^2\sin x\ge\int_0^1 x^3-\frac{x^5}{6}\ge\frac{1}{4}-\frac{1}{36}$

$\ge\dfrac{2}{9}$

13.(8) $\dfrac{3^{y_1}+3^{y_2}+3^{y_3}}{3} \geq 3^{\frac{y_1+y_2+y_3}{3}} \geq 3^3$

$\Rightarrow \quad m = 4$

$\dfrac{x_1+x_2+x_3}{3} \geq (x_1x_2x_3)^{1/3}$

$\Rightarrow \quad x_1x_2x_3 \leq 27$

$\Rightarrow \quad M = 3$

14.(1) Given that

$2n[c+(n-1)] = c(2^n - 1)$

$\Rightarrow \quad c = \dfrac{2n^2-2n}{2^n-2n-1}$

As c is the integer

$\Rightarrow \quad 2n^2 - 2n \geq 2^n - 2n - 1$

$\Rightarrow \quad 2n^2 + 1 \geq 2^n \quad \Rightarrow 2 < n < 6$

For only $n = 3$, c is integer

15.(1) Let us change the value for simplify it

Let $\pi x = y + \dfrac{\pi}{4}$ ....(1)

$\Rightarrow \quad (3-\cos 2y)\sin y + \sin 3y \geq 0$

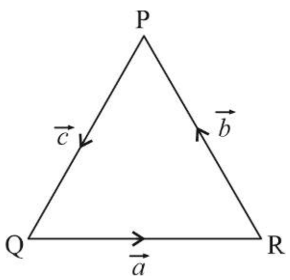
$\Rightarrow \quad (3-\cos 2y)\sin y - \sin^3 y + 3(\sin y + \sin^3 y) \geq 0$

Which is +ve when $\sin y \geq 0$

$\Rightarrow \quad y \in [0, \pi]$

$\Rightarrow \quad x \in \left[\dfrac{1}{4}, \dfrac{5}{4}\right]$

16.(108)



$\vec{a} + \vec{b} + \vec{c} = 0$ ....(1)

$\dfrac{\vec{a}\cdot(\vec{c}-\vec{b})}{\vec{c}\cdot(\vec{a}-\vec{b})} = \dfrac{3}{7}$

$\dfrac{|\vec{b}|^2 - |\vec{c}|^2}{|\vec{b}|^2 - |\vec{a}|^2} = \dfrac{3}{7} \Rightarrow |c|^2 = 13$ (using(1))

$\Rightarrow \quad |c| = \sqrt{13}$ ....(2)

Again from (1)

$\vec{a} + \vec{b} = -\vec{c}$

$\Rightarrow \quad (\vec{a}+\vec{b})^2 = -\vec{c}$

$\Rightarrow \quad \vec{a}\cdot\vec{b} = -6 \Rightarrow \cos\theta = -\frac{1}{2}$

$\Rightarrow \quad \left|\vec{a}\times\vec{b}\right|^2 = |a|^2|b|^2\sin^2\theta = 9\times 16\times\frac{3}{4} = 108$

17.(5) Given

$f(x) = (x^2-1)^2(a_0 + a_1x + a_2x^2 + a_3x^3)$ (Assuming not all a_i are 0)

Now as $f(x)$ have atleast four real roots $x = 1, 1, -1, 1$

So $f'(x)$ will have atleast 3 distinct roots -1, 1 & 1 root between $(-1, 1)$

$\Rightarrow \quad f''(x)$ will have atleast two distinct roots.

18.(1) $\lim\limits_{x\to 0^+} \dfrac{(1-x)^{1/x} - e^{-1}}{x^a}$

$\lim\limits_{x\to 0^+} \dfrac{1}{e}\dfrac{\left[e^{\ln(1-x)^{1/x}+1} - 1\right]}{x^a}$

$\lim\limits_{x\to 0^+} \dfrac{1}{e}\dfrac{e^{\left(\frac{\ln(1-x)+x}{x}\right)} - 1}{x^a}$

$\lim\limits_{x\to 0^+} \dfrac{1}{e}\left[\dfrac{\ln(1-x)+x}{x^a}\right]$

$\lim\limits_{x\to 0^+} \dfrac{1}{e}\left[\dfrac{-\frac{x^2}{2} - \frac{x^3}{3}}{x^{a+1}}\right]$

$\Rightarrow \quad a+1 = 2 \Rightarrow a = 1$