SOLUTIONS

Joint Entrance Exam | IITJEE-2019

08th APRIL 2019 | Morning Session

Joint Entrance Exam | JEE Mains 2019

PART-A	PHYSICS

1.(3) Refer NCERT

2.(1) Let us assume x grams of water vaporizer.

$\Rightarrow$ amount of water frozen $=(150-x)$

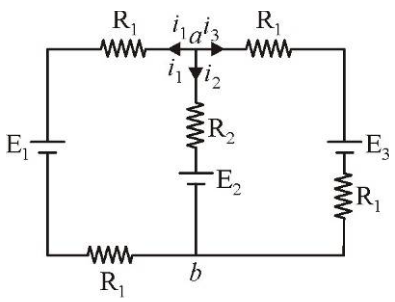
Heat gained by vaporized water = Heat lost by frozen water

$\Rightarrow \quad x \times L_V = (150-x)\times L_f$

$x \times 21 \times 10^5 = (150-x)\times 3.36\times 10^5$

$\Rightarrow \quad 21x = 504 - 3.36x \qquad \Rightarrow \qquad x = \dfrac{504}{24.36} = 20.7g \approx 20g$

3.(3) Let us assume $V_b = 0V$ and $V_a = xV$



$i_1 + i_2 + i_3 = 0$

$\Rightarrow \quad \dfrac{x-E_1}{2R_1} + \dfrac{x=E_2}{R_2} + \dfrac{x-E_3}{2R_1} = 0$

$\dfrac{x-2}{2} + \dfrac{x-4}{2} + \dfrac{x-4}{2} = 0$

$\Rightarrow \quad 3x - 10 = 0 \qquad \Rightarrow \qquad x = 3.3V$

$V_a - V_b = x - 0 = 3.3V$

4.(1) When catapult is released, elastic potential energy stored in cord gets converted to KE of stone.

EPE lost by cord = KE gained by stone

$\dfrac{1}{2}\left(\dfrac{YA}{\ell}\right)\Delta\ell^2 = \dfrac{1}{2}mv^2$

$\Rightarrow \quad Y \times \dfrac{\pi\left(3\times 10^{-3}\right)^2}{0.42} \times (0.2)^2 = 0.02\times(20)^2 \quad \Rightarrow \quad Y = \dfrac{0.02\times 400\times 0.42}{\pi\times 9\times 10^{-6}\times 0.04} \quad \approx 3\times 10^6$ SI units

5.(3) $\dfrac{\mathrm{I}_{\max}}{I_{\min}} = \left(\dfrac{a_2+a_1}{a_2-a_1}\right)^2$

Given $\dfrac{a_1}{a_2} = \dfrac{1}{3}$

$\Rightarrow \quad \dfrac{a_2+a_1}{a_2-a_1} = \dfrac{3+1}{3-1} = 2 \qquad \Rightarrow \qquad \dfrac{I_{\max}}{I_{\min}} = 4$

6.(4) $R = \dfrac{\rho V d}{n}$

$Q = 100l/\min = 0.1m^3/\min$

$= \dfrac{0.1}{60}m^3/s$

$V = \dfrac{Q}{A} = \dfrac{0.1}{60}\times\dfrac{1}{\pi\times(0.05)^2}$

$\Rightarrow \quad R = 10^3 \times \dfrac{\dfrac{0.1}{60\times\pi\times(0.05)^2}}{10^{-3}} \times 0.1 \quad = \dfrac{10^4}{60\times\pi\times 25\times 10^{-4}} \quad \approx 2\times 10^4$

7.(1) Stress $= \dfrac{\text{load}}{\text{area}} = \dfrac{mg}{\pi r^2}$

$$= \frac{4 \times 3.1\pi}{\pi \times (2 \times 10^{-3})^2}$$

$$= 3.1 \times 10^6 \ N/m^2$$

8.(1) $\bar{\tau} = \bar{m} \times \bar{B}$

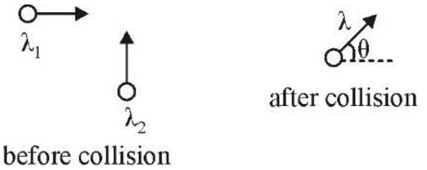
$\bar{B} = B\hat{i}$

$\bar{m} = [N(\pi r^2)i]\hat{j}$

$\Rightarrow \quad \bar{\tau} = -[N\pi r^2 i.B]\hat{R}$

$\Rightarrow \quad |\bar{\tau}| = \pi r^2 NiB$

9.(3)



Conserver momentum,

along x : $\dfrac{h}{\lambda_1} + 0 = \dfrac{h}{\lambda}\cos\theta$

along y : $\dfrac{h}{\lambda_2} + 0 = \dfrac{h}{\lambda}\sin\theta$

Squaring and adding,

$$h^2\left(\frac{1}{\lambda_1^2 + \lambda_2^2}\right) = \frac{h^2}{\lambda_2} \qquad \Rightarrow \qquad \frac{1}{\lambda_2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$$

10.(2) Rate of heat dissipation $= i^2 R$

Rate of energy storage $= iL\dfrac{di}{dt}$ in inductor

$$i^2 R = iL\frac{di}{dt} \qquad \Rightarrow \qquad \frac{di}{dt} = \frac{R}{L}i$$

Assuming initially current was zero, after time t,

$$i = \frac{E}{R}\left(1 - e^{\frac{-Rt}{L}}\right), \qquad \frac{di}{dt} = \frac{E}{L}e^{-\frac{Rt}{L}}$$

$$\Rightarrow \qquad \frac{E}{L}e^{\frac{-Rt}{L}} = \frac{R}{L} \times \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

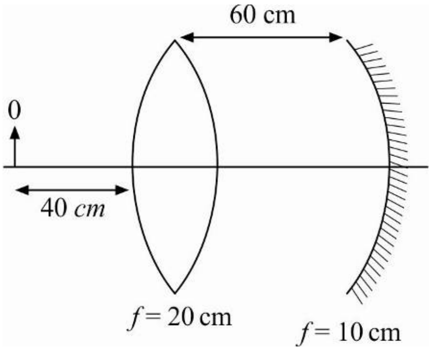
$$\Rightarrow \qquad e^{-\frac{Rt}{L}} = \frac{1}{2} \qquad \Rightarrow \qquad \frac{Rt}{L} = \ln 2$$

$$t = \frac{L}{R}ln2 = \frac{20}{10}ln2$$

$$= 2\,ln\,2$$

11.(3) KE of particle = work done by force = area under F–s curve $= 2 \times 2 + \dfrac{1}{2} \times (2+3) \times 1 = 6.5\text{J}$

12.(4)



(1) For first refraction by lens

$4 = -40\ cm = -2f$

$\Rightarrow \quad v = +2f = +40\ cm$

$$m = \frac{v}{u} = -1$$

(2) For reflection by mirror

$u = -(60-20) = -40\ cm = -2f$

$\Rightarrow \quad v = -2f = -40\ cm$

$$m = -\frac{v}{u} = -1$$

(3) For second refraction by lens

$u = -(60-20) = -40cm = -2f$

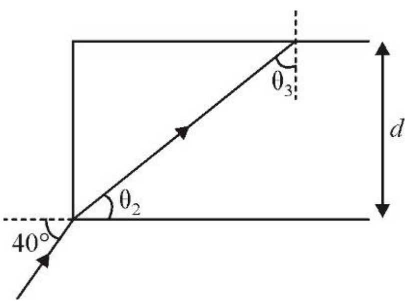
$\Rightarrow \quad v = 2f = 40cm$

$\Rightarrow \quad m = \frac{v}{u} = -1$

$\Rightarrow$ image is at same position as object $m_{net} = (-1)\cdot(-1)\cdot(-1) = -1$

$\Rightarrow$ image is of same size and inverted.

13.(3) Using Snell's law, $1 \cdot \sin 40° = 1.31 \sin \theta_2$



$\Rightarrow \quad \sin\theta_2 \approx 0.5 \quad \Rightarrow \quad \theta_2 = 30°$

Length of fibre transverse in one refraction $= d \cdot \cot\theta_2 = 20 \times 10^{-6} \times \sqrt{3} \approx 3.46 \times 10^{-5} m$

Length of wire = 2m

$\Rightarrow$ Number of refractions $= \dfrac{2}{3.4 \times 10^{-5}} \approx 57000$

14.(3) $\dfrac{E}{B} = c \qquad \Rightarrow \qquad B = \dfrac{6}{3 \times 108} = 2 \times 10^{-8} T$

Direction of magnetic field will be perpendicular to both electric field and direction of propagation of wave

15.(1) $\bar{a}_{cM} = \dfrac{m_1\bar{a}_1 + m_2\bar{a}_2 + m_3\bar{a}_3 + m_4\bar{a}_4}{m_1 + m_2 + m_3 + m_4} = \dfrac{m(-a\hat{i}) + 2m(a\hat{j}) + 3m(a\hat{i}) + 4m(-a\hat{j})}{m + 2m + 3m + 4m} = \dfrac{2a\hat{i} - 2a\hat{j}}{10} = \dfrac{a}{5}(\hat{i} - \hat{j})$

16.(2) $Mass_A = V \times 8 \times 10^2$ $\qquad mass_B = V \times 10^3$

$\therefore \quad mass_B > mass_A$

$\left|\dfrac{msd\theta}{dt}\right| = k\left|(\theta - \theta_0)\right| \quad \Rightarrow \quad \left|\dfrac{d\theta}{dt}\right| = \dfrac{k}{ms}|\theta - \theta_0|$

$m_A s_A = V \times 8 \times 10^2 \times 2000 \qquad m_B s_B = V \times 10^3 \times 4000$

$\therefore \quad m_B s_B > m_A s_A \quad \therefore$ slope of A will greater in magnitude at $t = 0$

17.(4) We know $c = \dfrac{1}{\sqrt{\mu_0 \in_0}} \quad \Rightarrow \quad c \in_0 = \sqrt{\dfrac{\in_0}{\mu_0}}$

$F = \dfrac{1}{4\pi \in_0} \dfrac{q_1 q_2}{r^2} \qquad [\in_0] = \dfrac{[q_1\, q_2]}{[f][r^2]} = \dfrac{[A^2T^2]}{[MLT^{-2}[L^2]} = [M^{-1}L^{-3}T^4A^2]$

$\therefore \quad [c \in_0] = [LT^{-1}][M^{-1}L^{-3}T^4A^2] = [M^{-1}L^{-2}T^3A^2]$

18. BONUS

Force per molecule = $2mv = 2 \times 10^{-26} \times 10^4 = 2 \times 10^{-22}$

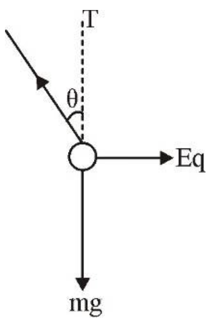
Total force $= 10^{22} \times 2 \times 10^{-22} = 2N/m^2$

19.(3) Energy from radiation $= 13.6 \times (1)\left(\dfrac{1}{1^2} - \dfrac{1}{2^2}\right] = 10.2\, eV$

For helium (2 to n)

$10.2 = 13.6 \times (4)\left(\dfrac{1}{2^2} - \dfrac{1}{n^2}\right) \quad \Rightarrow \quad n = 4 \quad \therefore \qquad$ 2 to 4

20.(2)



$T \cos\theta = mg\,, \quad \tan\theta = \dfrac{Eq}{mg} = \dfrac{200 \times 5 \times 10^{-6}}{2 \times 10^{-3} \times 10} = \dfrac{1}{2}$

$T \sin\theta = Eq \quad \Rightarrow \quad \theta = \tan^{-1} 0.5$

21.(4) Assuming no diode potential drop across 800Ω resistor = 7.2 V

$\Rightarrow$ Zener breakdown has occurred $\Rightarrow$ Current through 800Ω resistor $= \dfrac{5.6}{800} A$

$\Rightarrow$ Current through 200Ω resistor $= \dfrac{9 - 5.6}{200} = \dfrac{3.4}{200}$

$\Rightarrow$ Current through Zener diode $= \dfrac{3.4}{200} - \dfrac{5.6}{800} = \dfrac{8}{800} \Rightarrow 10mA$

22.(1) 200Ω means 20×10^1 means Red, Black, Brown. If Red replaced by green $50\times10^1 = 500\ \Omega$

23.(4) Frequency will be same in both

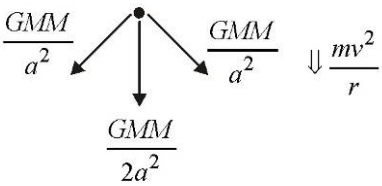
$$\frac{pc_1}{2l} = \frac{qc_2}{2l} \Rightarrow pc_1 = qc_2$$

Tension will be same in both but mass per unit length μ will be 4 times in 2 (as radius is twice)

$\therefore \quad \mu_2 = 4\mu_1 \quad \therefore \quad p\sqrt{\frac{T}{\mu_1}} = q\sqrt{\frac{T}{\mu_2}} \quad \therefore \quad \frac{p}{q} = \sqrt{\frac{\mu_1}{\mu_2}} = \frac{1}{2}$

24.(3) radius of the circle $= \frac{a}{\sqrt{2}} = r$

FBD of any one particle



$$\frac{GM^2}{2a^2} + \frac{2GM^2}{a^2}\frac{1}{\sqrt{2}} = \frac{Mv^2}{a/\sqrt{2}} \quad \Rightarrow \quad \sqrt{\frac{GM}{a}}\sqrt{\frac{1}{2\sqrt{2}}+1} = 1.16\sqrt{\frac{GM}{a}}$$

25.(4) If velocity of strip is v emf in it is Bvl

Current in it is $\frac{Bvl}{R}$

$\therefore$ Resistive force exerted by $B = Bil = \frac{B^2vl^2}{R}$

Amplitude reduces as $A(t) = A_0 e^{-\frac{b}{2m}t}$

If amplitude reduces by factor of e

$$\frac{bt}{2m} = 1 \qquad t = \frac{2m}{b} = \frac{2mR}{B^2l^2} = \frac{2\times50\times10^{-3}\times10}{(0.1)^2(0.1)^2} = 10^4 s$$

The new frequency $\omega' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} = \sqrt{\frac{0.5}{50\times10^{-3}} - \left(\frac{1}{10^4}\right)^2} = \sqrt{10 - \left(\frac{1}{10^4}\right)^2} \simeq \sqrt{10}$

New time period $= T' = \frac{2\pi}{\omega'} \approx \frac{2\pi}{\sqrt{10}} \approx 2 \quad \therefore \quad$ Number of oscillations $= \frac{10^4}{2} = 5000$

26.(3) $I_{CM} = \int dm\ r^2 = \int_0^R (2\pi r\, dr)(\rho_0 r)r^2 = \frac{2\pi\rho_0 R^5}{5}$

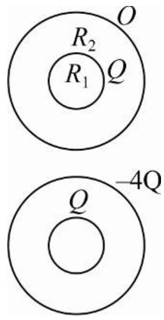
$$M = \int dm = \int_0^R 2\pi r dr \rho_0 r = \frac{2\pi\rho_0 R^3}{3}$$

Using parallel axis theorem $I = I_{CM} + MR^2$

$$= \frac{2}{5}\pi\rho_0 R^5 + \frac{2}{3}\pi\rho_0 R^5 = \frac{16}{15}\pi\rho_0 R^5$$

$$= \frac{3}{2}\times\frac{16}{15}\times\frac{2}{3}\pi\rho_0 R^5 = \frac{8}{5}MR^2$$

27.(4) Initial $pd = \frac{KQ}{R_1} - \frac{KQ}{R_2} = V$



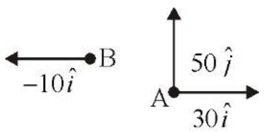
Final $pd = \left(\frac{KQ}{R_1} - \frac{4KQ}{R_2}\right) - \left(\frac{KQ}{R_2} - \frac{4KQ}{R_2}\right)$

$= \frac{KQ}{R_1} - \frac{KQ}{R_2} = V$

28.(1) Time period $= \frac{2\pi}{\omega} = \frac{2\pi}{100\pi} = \frac{1}{50} = T$

Time to go from $\frac{A}{2}$ *to* $A = \frac{T}{4} - \frac{T}{12} = \frac{T}{6} = \frac{1}{300} s = 3.3\ ms$

29.(4)



Position of B as function of time $= 80\hat{i} + 150\hat{j} - 10t\hat{i}$

Position of A as function of time $30t\,\hat{i} + 50t\,\hat{j}$

$\therefore$ Relative position of B as fraction of time $= 80\hat{i} + 150\hat{j} - 40t\,\hat{i} - 50t\,\hat{j}$

Taking magnitude square $= (80 - 40t)^2 + (150 - 50t)^2$

Time derivative $= 2(80 - 40t)(-40) + 2(150 - 50t)(-50) = 0$

$\Rightarrow \quad (8 - 4t)(-4) + (15 - 5t)(-5) = 0 \qquad \Rightarrow \quad 16t + 25t = 32 + 75$

$\Rightarrow \quad 41t = 107 \qquad \Rightarrow \quad t = \frac{107}{41} = 2.6\ hrs$

30.(3) $E_{\max} d = V_{\max}$

$d = \frac{500}{106} = 5 \times 10^{-4} m$

$C = \frac{k \in_0 A}{d} \Rightarrow k = \frac{Cd}{\in_0 A} = \frac{15 \times 10^{-12} \times 5 \times 10^{-4}}{8.86 \times 10^{-12} \times 10^{-4}} \approx 8.5$

PART-B	CHEMISTRY

1.(1) Ellingham diagram tells us about ΔG values (feasibility) of thermal reduction of an ore using suitable reducing agents.

2.(4) From given data

Rate $= k[A]^a[B]^b$

Now, $0.045 = k[0.05]^{a+b}$ … (i)

$0.090 = k[0.05]^{a+b} 2^a$ … (ii)

$0.72 = k[0.05]^{a+b} \cdot 2^{2a+b}$ … (iii)

From equation (i) and (ii) we get,

$\Rightarrow \quad \frac{1}{2} = \frac{1}{2^a} \Rightarrow a = 1$

And from equation (i) and (iii) we get

$\Rightarrow \quad \frac{0.045}{0.72} = \frac{1}{2^{2+b}} \quad \Rightarrow \quad \frac{45}{720} = \frac{1}{2^{2+b}}$

$\Rightarrow \quad \frac{720}{45} = 2^{(2+b)} \quad \Rightarrow \quad 2^4 = 2^{2+b} \quad \Rightarrow \quad B = 2$

3.(3) $[V(CN)_6]^{4-}, [Fe(CN)_6]^{4-}, [Ru(NH_3)_6]^{3+}, [Cr(NH_3)_6]^{2+}$

$\Rightarrow$	V^{2+}	Fe^{2+}	Ru^{3+}	Cr^{2+}
$\Rightarrow$	$s^\circ d^3$	$s^\circ d^6$	$s^\circ d^5$	$s^\circ d^4$
No. of Unpaired	3	0	1	2

electrons (all complex are inner orbital complex because ligands are strong) as, Magnetic moment $\mu = \sqrt{n(n+2)}$, n = no. of unpaired electrons

More the number of unpaired electron more will the value of spin only magnetic moment

4.(1) For iso-electronic species size is governed by proton to electron ratio i.e. $\left(\frac{P}{e^-}\right)$ ratio

More the value of $\left(\frac{P}{e^-}\right)$ ratio, smaller will be its size as more number of proton will have an ability to hold electrons more strongly resulting in decrement in ionic size.

Hence, size is affected by nuclear charge i.e. no. of proton.

5.(4) In acidic medium $KMnO_4$ reduces to Mn^{2+}

$C_2O_4^{2-}$ oxidizes to CO_2

Fe^{2+} oxides to Fe^{3+}

For

FeC_2O_4	$1+(1\times 2)$	$Fe^{+2} \to Fe^{+3}, C^{+3} \to C^{+4}$
$Fe_2(C_2O_4)_3$	$2\times 3 = 6$	$C^{+3} \to C^{+4}$
$FeSO_4$	1	$Fe^{+2} \to Fe^{+2}$
$Fe(SO_4)_3$	0	No oxidation

gm equivalents of $KMnO_4$ = gm equivalents of $FeC_2O_4 +$ gm equivalents of $Fe_2(C_2O_4)_3 +$ gm equivalent of $FeSO_4 +$ gm of $Fe_2(SO_4)_3$.

$(\text{moles}\times n_f)_{KMnO_4} = (\text{moles}\times n_f)_{FeC_2O_4} + (\text{moles}\times n_f)_{Fe_2(C_2O_4)_3} + (\text{moles}\times n_f)_{FeSO_4} + (\text{moles}\times n_f)_{Fe_2(SO_4)_3}$

$(\text{moles})_{KMnO_4} \times 5 = (1\times 3) + (1\times 6) + (1\times 1) + (1\times 0)$

$(\text{moles})_{KMnO_4} = 2$

6.(1) $g_{Ca(HCO_3)_2} = 0.81g,\ n_{Ca(HCO_3)_2} = \frac{0.81}{162} = \frac{1}{200};\ g_{Mg(HCO_3)_2} = 0.73g,\ n_{Mg(HCO_3)_2} = \frac{1}{200}$ moles

$n_T = \frac{1}{200} + \frac{1}{200} = 0.01$

0.01moles in 100 ml water.

(0.01×2) equivalent in 100 ml water $\quad \therefore \quad$ 0.02 equivalent of $CaCO_3$ in 100 ml water

$\therefore \quad$ 0.01 moles of $CaCO_3$ in 100 ml water

0.01×100g of $CaCO_3$

Hardness $\Rightarrow \quad \frac{1}{100\,L}\times 10^3\,mg \times 1000 = 10{,}000\,ppm$

7.(3) Given : $P_A^o = 400$ mm Hg

$P_B^o = 600$ mm Hg

As, $V_A + V_B = V_{solution}$ [hence solution formed is ideal solution]

And $x_B = 0.5,\ x_A = 0.5$

Using Raoult's law:

$P_T = x_A P_A^o + x_B P_B^o,\qquad P_T = 200 + 300,\qquad P_T = 500$ mm Hg

And in vapor phase:

$P_A = x_A P_A^o$ [from Raoult's law]…(i)

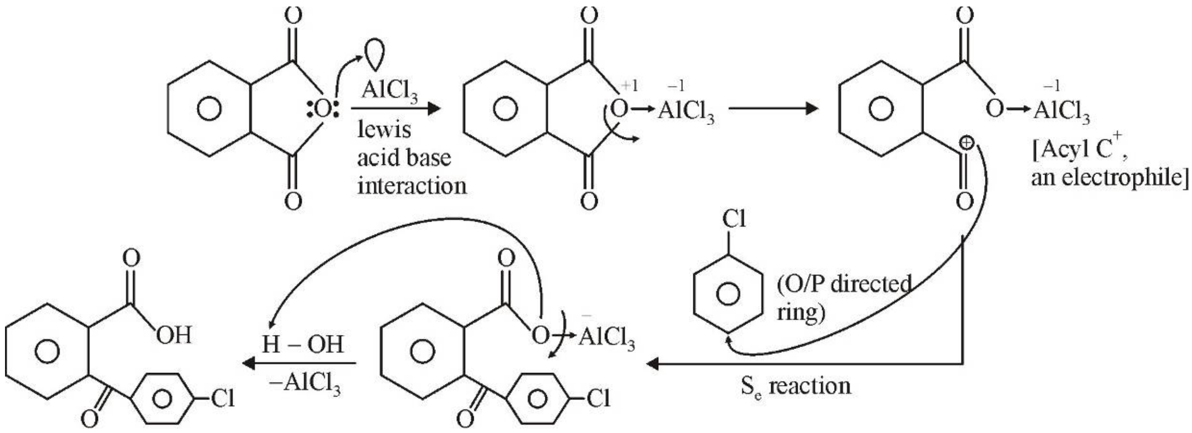
$P_A = (x_A)_v P_T$ [from Dalton's law of partial pressure]

Equating (i) and (ii), we get

$$(x_A)_v = \frac{0.5 \times 400}{500} = 0.5 \times \frac{4}{5} = \frac{2}{10} = 0.4$$

$$(x_B)_v = 1 - 0.4 = 0.6$$

8.(1)



9.(1) $\underset{a-s}{Zr_3(PO_4)_4} \qquad \underset{3s}{3Zr^{4+}} + \underset{4s}{4PO_4^{3-}}$

$K_{sp} = [Zr^{4+}]^3 [PO_4^{3-}]^4$

$= [3s]^3 [4s]^4$

$= 27s^3 \times 256s^4$

$= 6912\ s^7$

$$S = \left(\frac{K_{sp}}{6912}\right)^{1/7}$$

10.(3) $E \propto (n + l)$ value

E : Energy of electron in a particular subshell

n : principal quantum number

l : azimuthal quantum number

(I) $n + l = 6$ (II) $n + l = 5\ (n = 3)$

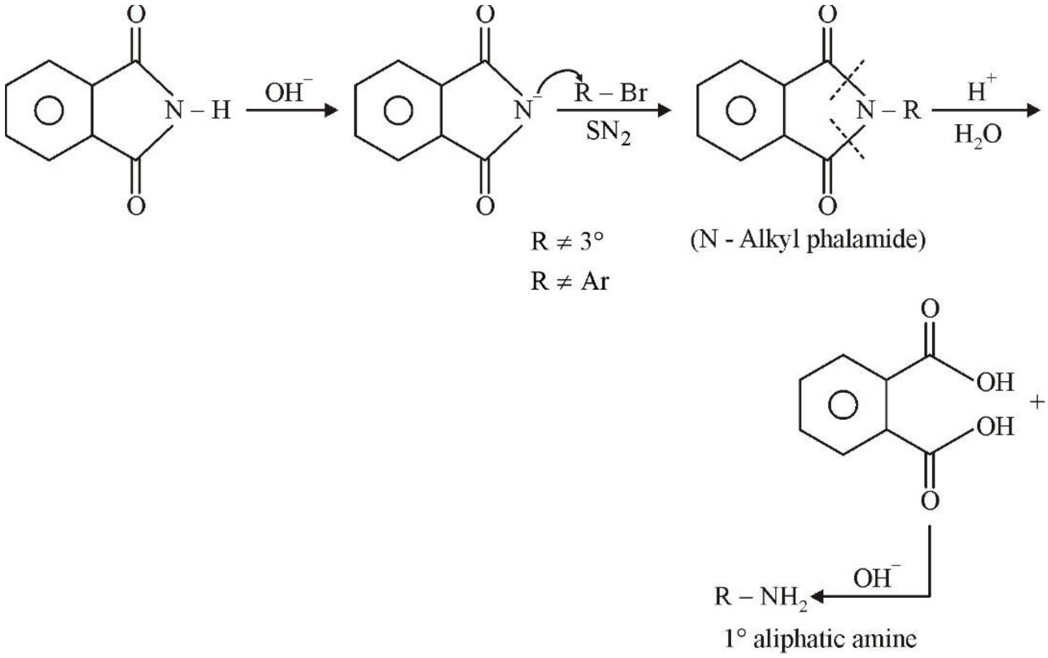
(III) $n + l = 5\ (n = 4)$ (IV) $n + l = 4$

According to Aufbau's rule, lower the value of (n + l), lower will be its energy. In case if (n + l) value are same for two different subshell then subshell having lower value of n will have lower energy.

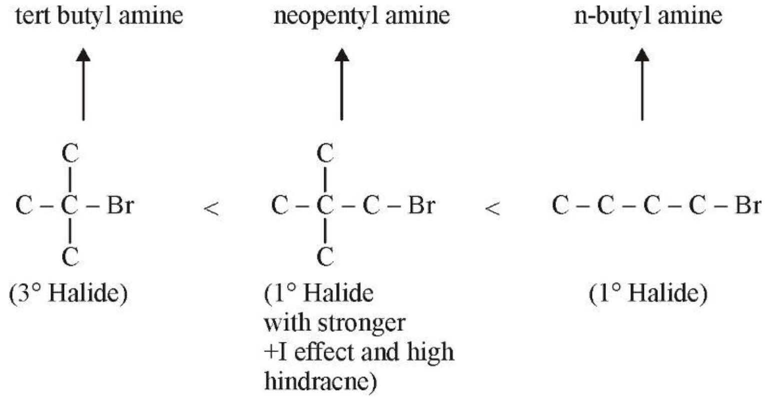
So, correct increasing order of energy is: (IV) < (II) < (III) < (I)

11.(3) As alkylation of nitrogen increases, Basicity of amines increase due to (+I) effect of Alkyl groups which results in more electron cloud density over nitrogen atom (available toward donation).
Hence correct order is $(C_2H_5)_2NH > C_2H_5NH_2 > NH_3$ [Gaseous phase]

12.(3) Gabriel pthalimide reaction is used to prepare only 1° aliphatic amines.

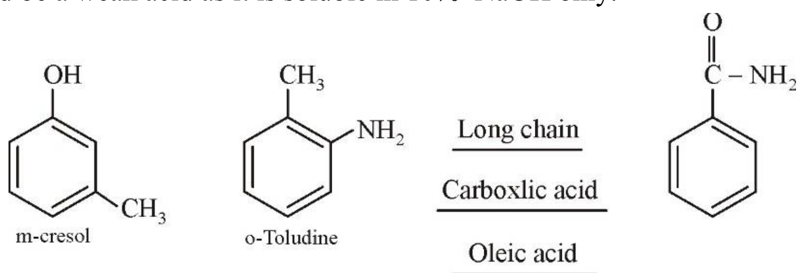


In above reaction, R – Br undergoes SN_2 reaction, Hence electrophilicity of α-carbon should be high as well as its hindrance should be low for greater ease of reaction.



Hence, ease of formation of n-butyl amine is higher. Therefore, it is most probable answer

13.(1) x should be a weak acid as it is soluble in 10% NaOH only.



Oleic acid > Benzamide > o-toluidine > m-cresol
Order of decreasing acidic strength
∴ x is m-cresol (weakest of all)

14.(4) Electronic configuration of following lanthanide ions are given below.

$1u^3 : 4f^{14}$ (colorless, no unpaired electron)

$1a^{3+} : 4f^0$ (colorless, no unpaired electron)

$Gd^{3+} : 4f^7$ colourless

$Sm^{3+} : 4f^5$ (yellow, 5 unpaired electrons)

Ions having f electrons show colour. of these ions may be attributed to presence of f electrons.

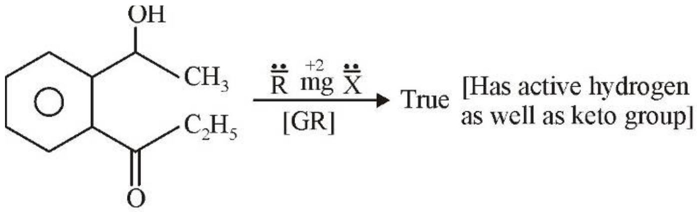
15.(3)

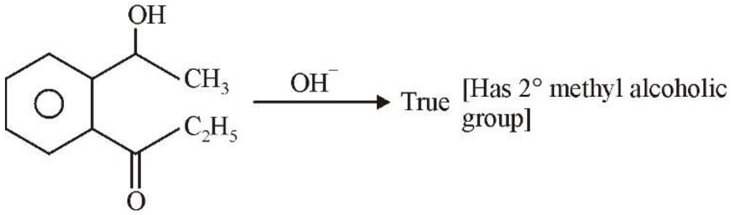
$$\overset{5}{H_3C}-\overset{\overset{CH_3}{|}}{\underset{4}{CH}}-\overset{\overset{OH}{|}}{\underset{3}{CH}}-\overset{2}{CH_2}-\overset{1}{COOH}$$

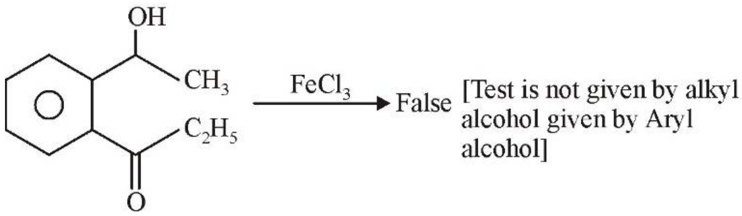
Carboxylic group has high priority than hydroxyl group so numbering starts from carbon of carboxylic group.

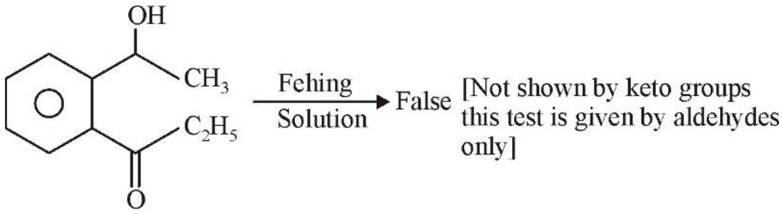
3-hydroxy-4-methylpentanoic acid.

16.(1)









17.(1) Benzene diazonium chloride react with that ring of 1-naphthol which contain $-OH$ group as it is activating and also it will undergo coupling at p-position w.r.t. $-OH$ group of 1-napthol.

18.(2) $\frac{x}{m} = k(P)^{\frac{1}{n}}$ … (i)

$\log\frac{x}{m} = \log k + \frac{1}{n}\log P$

Slope of $\log\frac{x}{m}$ v/s $\log P$ graph gives value of 1/n

$\therefore$ From graph $\frac{1}{n} = \frac{2}{3}$

Putting $\frac{1}{n}$ in (i), $\frac{x}{m} \propto (P)^{2/3}$

19.(3) Four donor atoms are present in it hence it act as a tetradentate.

20.(2) According to first law of thermodynamics: $\Delta U = q + w$

For adiabatic process, q = 0

Hence, $\Delta U = w$

21.(2) Consider the following reactions:

$$B_2H_6 + \frac{3}{2}O_2 \xrightarrow{\text{Oxidation}} B_2O_3 + 3H_2 \uparrow$$

$$B_2H_6 + 6H_2O \xrightarrow{\text{Hydrolysis}} 2H_3BO_3 + 6H_2 \uparrow$$

22.(4) $\Delta H = \int nC_P dT$

$$= 3\int_{300}^{1000} (23 + 0.01T)dT$$

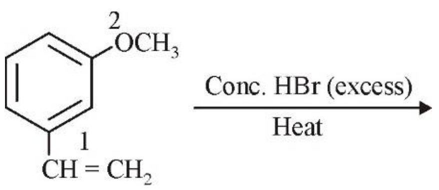
$$= 3\left[23T + \frac{0.01T^2}{2}\right]_{300}^{1000}$$

$$= 3\left[23000 + \frac{0.01}{2}(1000)^2 - 23(300) - \frac{0.01(300)^2}{2}\right]$$

$$= 3[23000 + 5000 - 6900 - 450] = 61950\,J$$

$$\simeq 62\,kJ$$

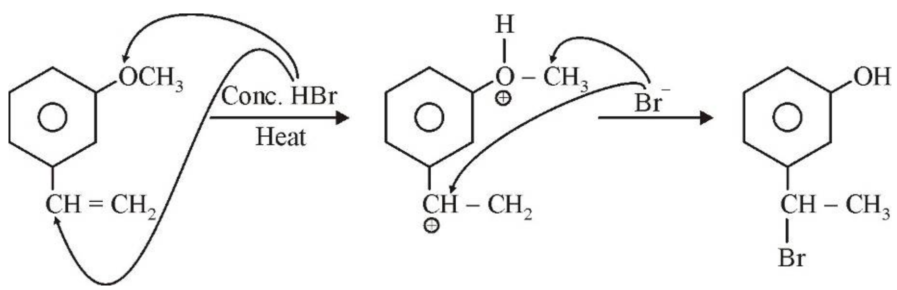
23.(4) Maltose is a disaccharide made up two D-glucose units. On treatment with dil. HCl is undergoes hydrolysis to give two D-glucose units. (Monosaccharide)

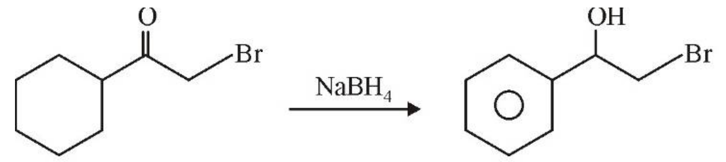
24.(2) 

Here 1 and 2 are conc. HBr sensitive regions

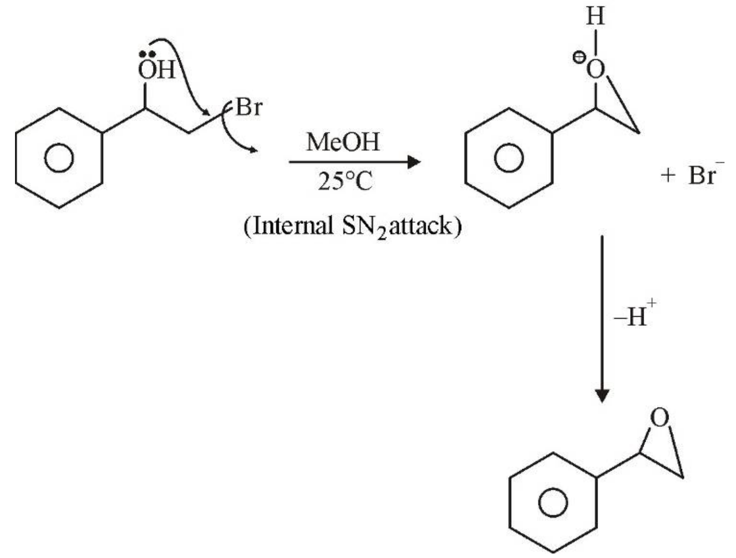
$1 \equiv$ Alkene, will undergo electrophilic addition reaction [Markonikov's addtion]

$2 \equiv$ Ether, will undergo forced [Acid catalysed] nucleophilic substitution reaction [SN_2 mechanism]



25.(2) 

SBH reduces keto group to 2° ol group.



26.(3) Let number of B atoms = N

Number of atoms of $A = \frac{N}{2}$

Number of oxygen atoms = 2N

A : B : O

$\frac{N}{2} : N : 2N$

$\frac{1}{2} : 1 : 2$

$1 : 2 : 4$

AB_2O_4 is the formula

27.(1) Greater is the reduction potential, stronger is the oxidizing agent.

28.(2) Fact (refer of NCERT) Chemistry ; Class XI, Page No – 405 & 406.

29.(3) Hydration enthalpy $\propto$ charge on an ion

$$\propto \frac{1}{\text{size of an ion}}$$

Hence, correct order of hydration enthalpy is: $Li^+ > Na^+ > K^+ > Rb^+ > Cs^+$

30.(3) Using plastic bags is wrong as plastic bags cause environmental pollution.

PART-C	MATHEMATICS

1.(2) $(x^2+1)^2\dfrac{dy}{dx}+2x(x^2+1)y=1$

Liner differential equation $\Rightarrow$ $\dfrac{dy}{dx}+\left(\dfrac{2x}{x^2+1}\right)y=\dfrac{1}{(x^2+1)^2}$

Integrating factor $=e^{\int\frac{2x}{1+x^2}dx}$

Let $x^2+1=t \Rightarrow 2x\,dx=dt$

$=e^{\int\frac{dt}{t}}=e^{\ln t}=t=x^2+1$

$\Rightarrow$ $y(x^2+1)=\int\dfrac{dx}{x^2+1}=\tan^{-1}x+C$ $\Rightarrow$ $y(x^2+1)=\tan^{-1}x+C$

$\because$ $x=0$ $\Rightarrow$ $y=0$

$0=\tan^{-1}0+C \Rightarrow C=0$

$y(x^2+1)=\tan^{-1}x \Rightarrow y=\dfrac{\tan^{-1}x}{x^2+1}$

$y(1)=\dfrac{\pi}{8}$, $\sqrt{a}\,y(1)=\dfrac{\pi}{32}$

$\Rightarrow$ $\sqrt{a}\times\dfrac{\pi}{8}=\dfrac{\pi}{32}$ $\Rightarrow$ $\sqrt{a}=\dfrac{1}{4} \Rightarrow a=\dfrac{1}{16}$

2.(1) $x-cy-cz=0$

$cx-y+cz=0$

$cx+cy-z=0$

$\because$ $D_x=D_y=D_z=0$ $\therefore$ For non-trivial solutions

D = 0

$$D=\begin{vmatrix}1 & -c & -c\\ c & -1 & c\\ c & c & -1\end{vmatrix}=1(1-c^2)+c(-c-c^2)-c(c^2+c)$$

$=1-c^2-c^2-c^3-c^3-c^2$

$=-2c^3-3c^2+1=0$

$2c^3+3c^2-1=0$

$\Rightarrow$ $(2c-1)(c+1)^2=0$, $c=-1,-1,1/2$

$\therefore$ Greatest value of 'c' is 1/2.

3.(4) $f\left(\dfrac{2x}{1+x^2}\right)=\ln\left(\dfrac{1-\dfrac{2x}{1+x^2}}{1+\dfrac{2x}{1+x^2}}\right)$

$=\ln\left(\dfrac{(x-1)^2}{(x+1)^2}\right)=2\ln\left|\dfrac{x-1}{x+1}\right|$

$=2\ln\left(\dfrac{1-x}{1+x}\right)=2f(x)$

4.(2) **Case I**

$\sqrt{x} \geq 2$

$\Rightarrow \quad x \geq 4$

$\sqrt{x} - 2 + x - 4\sqrt{x} + 2 = 0$

$\Rightarrow \quad \sqrt{x} = 3$

$x = 9$

Case II

$\sqrt{x} < 2$

$\Rightarrow \quad x < 4$

$2 - \sqrt{x} + x - 4\sqrt{x} + 2 = 0$

$4 + x - 5\sqrt{x} = 0$

$\left(\sqrt{x} - 4\right)\left(\sqrt{x} - 1\right) = 0$

$\sqrt{x} = 1$ or $\sqrt{x} = 4$ (Not possible since $\sqrt{x} < 2$)

$\Rightarrow \quad x = 1 \quad \Rightarrow \quad \text{Sum} = 9 + 1 = 10$

5.(1) $\left(x + \sqrt{x^3 - 1}\right)^6 + \left(x - \sqrt{x^3 - 1}\right)^6$

$$= 2\left[C_0 x^6 + C_2 x^4 (x^3 - 1) + C_4 x^2 (x^3 - 1)^2 + C_6 (x^3 - 1)^3\right]$$

Sum of coefficients of all even degree term is

$2\left[C_0 - C_2 + C_4 + C_4 - C_6 - 3C_6\right]$

$2[1 - 15 + 15 + 15 - 4] = 24$

6.(4) $\tan(\alpha + \beta) = \dfrac{4}{3} \qquad \tan(\alpha - \beta) = \dfrac{5}{12}$

$\Rightarrow \quad (\alpha + \beta) = \tan^{-1}\left(\dfrac{4}{3}\right) \quad \ldots \text{(i)}$

$\Rightarrow \quad (\alpha - \beta) = \tan^{-1}\left(\dfrac{5}{12}\right) \quad \ldots \text{(ii)}$

Adding (i) and (ii) we get,

$\Rightarrow \quad 2\alpha = \tan^{-1}\left(\dfrac{4}{3}\right) + \tan^{-1}\left(\dfrac{5}{12}\right)$

$$\tan 2\alpha = \frac{\frac{4}{3} + \frac{5}{12}}{1 - \frac{20}{36}} = \frac{63}{16}$$

7.(3) $x^2 - 2x + 2 = 0 \quad \alpha \,\&\, \beta$ are roots

$\alpha = \dfrac{2 + \sqrt{-4}}{2} = 1 + i, \qquad \beta = \dfrac{2 - \sqrt{-4}}{2} = 1 - i$

$\dfrac{\alpha}{\beta} = \dfrac{1 + i}{1 - i} = i$

$\left(\dfrac{\alpha}{\beta}\right)^n = (i)^n = 1$

Least value of $n = 4$

8.(4) $\sum_{r=0}^{20}(3r+2)\,{}^{20}C_r$

$= 3\sum_{r=0}^{20} r\,{}^{20}C_r + 2\sum_{r=0}^{20} C_r$

$= 3\sum_{r=0}^{20} 20\times {}^{19}C_{r-1} + 2\times 2^{20}$

$= 3\times 20(2)^{19} + 2\times(2)^{20}$

$= 2^{21}(15+1) = 2^{25}$

9.(4) $\overrightarrow{AP} = 5\hat{i} - 3\hat{j} + 4\hat{k}$

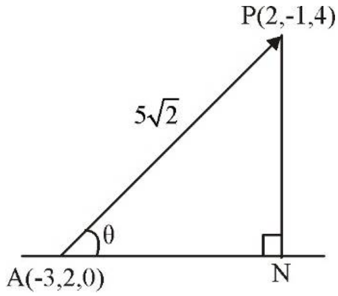
Parallel vector to line is $10\hat{i} - 7\hat{j} + \hat{k}$

$$\cos\theta = \frac{(5\hat{i} - 3\hat{j} + 4\hat{k})\cdot(10\hat{i} - 7\hat{j} + \hat{k})}{\sqrt{150}}$$

$\Rightarrow \quad \cos\theta = \frac{75}{50\sqrt{3}} = \frac{\sqrt{3}}{2} \Rightarrow \sin\theta = \frac{1}{2}$

$PN = 5\sqrt{2}\sin\theta$

$= \frac{5\sqrt{2}\times 1}{2} = \frac{5}{\sqrt{2}}$



10.(1) Vector perpendicular to plane containing vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$ is parallel to vector

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \hat{i} - 2\hat{j} + \hat{k}$$

$\therefore$ Required magnitude of projection is $\left|\frac{(2\hat{i} + 3\hat{j} + \hat{k})\cdot(\hat{i} - 2\hat{j} + \hat{k})}{\sqrt{6}}\right| = \sqrt{\frac{3}{2}}$

11.(4) $p \Rightarrow q$

Then contrapositive of this is $\sim q \Rightarrow \sim p$

If you are not a citizen of India, then you are not born in India.

12.(3) $y^2 = x - 2$; $\quad y = x$

$y^2 = (x-2)$; $\quad y^2 = 4\times\frac{1}{4}(x-2)$

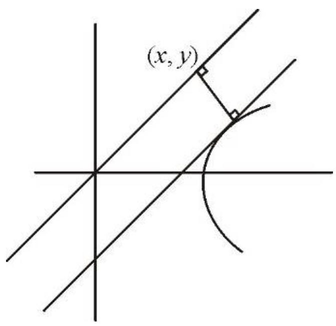
Vertex = (2,0) ; $a = \frac{1}{4}$

Tangent of slope 1 on parabola

$y = 1\times(x-2) + \frac{1}{4}$; $\quad y = x - \frac{7}{4}$

Distance of this line from $y = x$

$$d = \frac{7}{4\times\sqrt{1+1}} = \frac{7}{4\sqrt{2}}$$



13.(2) $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx$

$\Rightarrow \quad \int \frac{2\cdot\sin\frac{5x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}} \quad \Rightarrow \quad \int \frac{2\cdot\sin\frac{5x}{2}\cos\frac{x}{2}}{\sin x}$

$\Rightarrow \quad \int \frac{\sin 3x+\sin 2x}{\sin x} \quad \Rightarrow \quad \int \frac{3\sin x-4\sin^3 x+2\sin x\cos x}{\sin x}$

$\Rightarrow \quad \int (3-4\sin^2 x+2\cos x)\,dx$

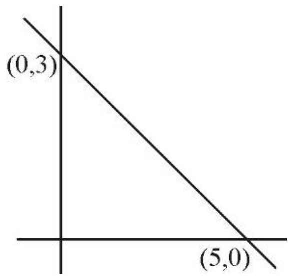
$= \int (3-2(1-\cos 2x)+2\cos x)\,dx$

$= \int (2\cos 2x+2\cos x+1)\,dx$

$= \sin 2x+2\sin x+x+C$

14.(1) For point to be equidistant from axes it must lie on the $y=x$ or $y=-x$

line $3x+5y=15$



$y=x \Rightarrow 8x=15 \qquad \left.\begin{array}{l} x=\frac{15}{8} \\ y=\frac{15}{8}\end{array}\right\} Pt\left(\frac{15}{8},\frac{15}{8}\right)$

$y=-x \quad 3x+5y=15$

$3x+5(-x)=15$

$-2x=15$

$x=\frac{-15}{2},\ y=\frac{15}{2}$

2^{nd} quadrant $\left(\frac{-15}{2},\frac{15}{2}\right)$

15.(4) Odd digits are 1, 1, 3

Even places 2^{nd}, 4^{th}, 6^{th}, 8^{th}

No. of ways $=4_{C_3}\times\frac{3!}{2!}\times\frac{6!}{4!2!}=180$

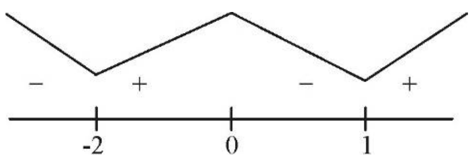
16.(2) $f'(x)=36x^3+36x^2-72x$

$=36(x^3+x^2-2x)$

$=36x(x^2+x-2)$

$=36x(x+2)(x-1)$

Sign scheme of $f'(x)$ is

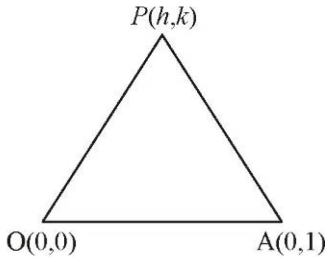


Local minimum at –2 and 1

Local maximum at 0

$S_1=\{-2,1\},\ S_2=\{0\}$

17.(1) Let the point P be (h,k)



$\therefore \quad 1+\sqrt{h^2+k^2}+\sqrt{h^2+(k-1)^2}=4$

$\Rightarrow \quad \sqrt{h^2+(k-1)^2}=3-\sqrt{h^2+k^2}$

$\Rightarrow \quad h^2+k^2-2k+1=9+h^2+k^2-6\sqrt{h^2+k^2}$

$\Rightarrow \quad 3\sqrt{h^2+k^2}=4+k$

$\Rightarrow \quad 8k^2+9h^2-8k-16=0$

$\Rightarrow \quad 9x^2+8y^2-8y=16$ (required locus)

18.(1) Mean $=\dfrac{a+b+2+4+10+12+14}{7}=8 \Rightarrow \dfrac{a+b+42}{7}=8$

Variance = 16 $\quad a+b+42=56$

$V=\dfrac{\sum x^2}{N}-\mu^2 \quad a+b=14$

$V=\dfrac{a^2+b^2+2^2+4^2+10^2+12^2+14^2}{7}-8^2$

$16=\dfrac{a^2+b^2+460}{7}-64$

$80=\dfrac{a^2+b^2+460}{7}$

$560=a^2+b^2+460$

$a^2+b^2=100$

$a^2+b^2=100$

$(a+b)^2-2ab=100$

$(14)^2-2ab=100$

$196-2ab=100$

$196-100=2ab$

$\dfrac{96}{2}=ab \Rightarrow ab=48$

19.(3) Slope of the tangent at (x_1, y_1) for ellipse $\dfrac{x^2}{2}+\dfrac{y^2}{8}=1$ is $-\dfrac{8}{2}\times\dfrac{x_1}{y_1}=-4\dfrac{x_1}{y_1}$

$\therefore$ Slope of tangent at (1,2) is –2

Perpendicular tangents slope will be 1/2

Slope of tangent at $(a\cos\theta, b\sin\theta)$

For $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ is $\dfrac{-b}{a}\cot\theta=-2\cot\theta$

$\Rightarrow \quad -2\cot\theta=\dfrac{1}{2}; \quad \cot\theta=\dfrac{-1}{4} \Rightarrow \cos\theta=\pm\dfrac{1}{\sqrt{17}}$

$\Rightarrow \quad a^2=\left(\sqrt{2}\cos\theta\right)^2=2\cos^2\theta=\dfrac{2}{17}$

20.(2) $\cos\alpha=\dfrac{3}{5} \quad \sin\alpha=\dfrac{4}{5}$

$\cos\beta = \frac{3}{\sqrt{10}}$ $\sin\beta = \frac{1}{\sqrt{10}}$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\frac{12}{5\sqrt{10}} - \frac{3}{5\sqrt{10}} \quad = \frac{9}{5\sqrt{10}}$

$\alpha - \beta = \sin^{-1}\left(\frac{9}{5\sqrt{10}}\right)$

21.(2) n is between 100 & 200

HCF (91, n) > 1

(Sum of natural no divisible by 7) + (Sum of natural no divisible by 13) – (Sum of no divisible by 91)

$\sum_{r=1}^{14}(98+7r) + \sum_{r=1}^{8}(91+13r) - (182)$

$1372 + 735 + 728 + 468 - (182) = 3121$

22.(3) $2y = \left(\cot^{-1}\left(\frac{\cos\left(\frac{\pi}{6}-x\right)}{\cos\left(\frac{\pi}{3}+x\right)}\right)\right)^2$

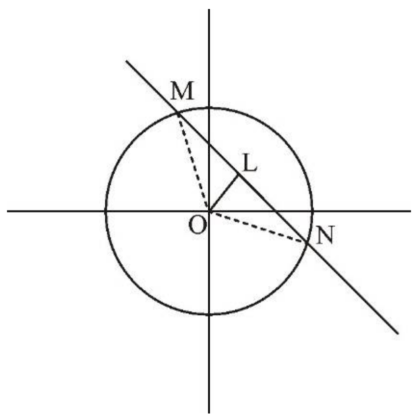
$2y = \left(\cot^{-1}\left(\frac{\cos\left(\frac{\pi}{6}-x\right)}{\sin\left(\frac{\pi}{6}-x\right)}\right)\right)^2$

$2y = \left(\cot^{-1}\left(\cot\left(\frac{\pi}{6}-x\right)\right)\right)^2$

$2y = \left(\frac{\pi}{6}-x\right)^2$; $2y' = -2\left(\frac{\pi}{6}-x\right)$; $y' = x - \frac{\pi}{6}$

23.(4) $x + y = n$ will be intersecting

$x^2 + y^2 = 16$ if $n = 1,2,3,4,5$ $(n \in N)$



Length $OL = \frac{n}{\sqrt{2}}$ (Distance of $x + y = n$ from origin)

$OM = 4$ (radius)

$MN = 2\sqrt{16 - \frac{n^2}{2}}$

$(MN)^2 = 4\left(16 - \frac{n^2}{2}\right)$

$\sum_{n=1}^{5} 4\left(16 - \frac{n^2}{2}\right) = 4\left[16\times5 - \frac{5\times6\times11}{2\times6}\right] = 320 - 110 = 210$

24.(4) $A = \begin{bmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{bmatrix}$

$A^2 = \begin{bmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{bmatrix}$

$$A^2 = \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$A^{32} = \begin{bmatrix} \cos 32\alpha & -\sin 32\alpha \\ \sin 32\alpha & \cos 32\alpha \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$32\alpha = \frac{\pi}{2} \qquad \Rightarrow \qquad \alpha = \frac{\pi}{64}$$

25.(2) $f''(x) > 0 \ \forall \ x \in (0,2)$

$\Rightarrow \quad f'(x)$ is $\uparrow$ function

$\phi'(x) = f'(x) - f'(2-x)$

If $x \in (0,1)$

$\phi'(x) < 0 \Rightarrow \phi(x)$ is decreasing

If $x \in (1,2)$

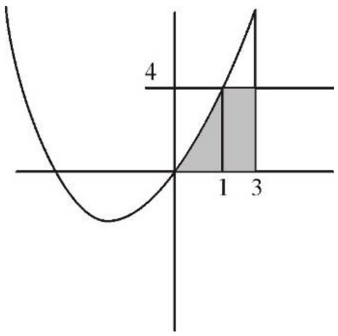
$\phi'(x) > 0 \Rightarrow \phi(x)$ is increasing

26.(3) $\lim\limits_{x\to 0} \dfrac{\sin^2 x}{\sqrt{2}\left(1-\cos\dfrac{x}{2}\right)}$

$$= \lim_{x\to 0} \frac{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}}{2\sqrt{2}\sin^2\frac{x}{4}}$$

$$= \lim_{x\to 0} \frac{4\frac{x^2}{4}}{2\sqrt{2}\frac{x^2}{16}}\cos^2\frac{x}{2} \qquad \left[\lim_{x\to 0}\frac{\sin x}{x} = 1\right] \qquad = 4\sqrt{2}$$

27.(1)



Required area $= \int_0^1 (x^2+3x)\,dx + 4\times 2 = \dfrac{11}{6} + 8 = \dfrac{59}{6}$

28.(4) $P(A/B) = \dfrac{P(A\cap B)}{P(B)} = \dfrac{P(A)}{P(B)}$

$\dfrac{P(A)}{P(B)} \geq P(A)$

29.(2) Plane passing through given two planes can be written as

$(2x-y-4)+\lambda(y+2z-4)=0$

$-3+\lambda(-3)=0$

$\lambda = -1$

$$2x - y - 4 - 1(y + 2z - 4) = 0$$

$$\Rightarrow \quad x - y - z = 0$$

30.(3) $g(f(x)) = \ln\left(\dfrac{2 - x\cos x}{2 + x\cos x}\right)$

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ln\left(\frac{2 - x\cos x}{2 + x\cos x}\right) dx$$

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ln\left(\frac{2 + x\cos x}{2 - x\cos x}\right) dx$$

$$\Rightarrow \qquad 2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ln(1)\, dx = 0 \qquad \Rightarrow \qquad I = 0 \quad \Rightarrow \qquad \log_e 1$$