SOLUTIONS

Joint Entrance Exam | IITJEE-2019

9th APRIL 2019 | Morning Session
1.(3) Given \( m = 5 \Rightarrow -\frac{v}{u} = 5 \Rightarrow v = -5u \\
\therefore \frac{1}{-5u} + \frac{1}{u} = -\frac{1}{40} \Rightarrow u = 32 \text{ cm}

2.(3) \[ B_{\text{net}} = \sqrt{B_0^2 + B_1^2} = 10^{-6} \times \sqrt{904} \]
\[ \therefore E = C \times B_{\text{net}} = 3 \times 10^8 \times 10^{-6} \times \sqrt{904} \]
\[ \therefore F = QE = 10^{-4} \times 3 \times 10^8 \times 10^{-6} \times \sqrt{904} = 0.9 \text{ N.} \]
\[ \therefore F_{\text{rms}} = \frac{F}{\sqrt{2}} = 0.6 \text{ N} \]

3.(1) \[ R_{eq} = 6 + \frac{12 \times 4}{12 + 4} = 9 \Omega \quad \therefore i_{\text{main}} = \frac{72}{9} = 8 \text{ amp.} \]
\[ \therefore i_{100} = -\frac{4}{12 + 4} \times 8 = 2 \text{ amp.} \]
\[ \therefore Q = CV = 10 \times 10^{-6} \times (10 \times 2) = 200 \mu \text{C} \]

4.(3) Clearly \( \frac{2\pi}{\lambda} = 0.157 \quad \therefore \lambda = 40 \text{ m} \)

Hence for 4\(^\text{th}\) harmonic, \( l = 4 \times \frac{\lambda}{2} = 80 \text{ m} \)

5.(4)
\[ \begin{array}{c|c|c}
2 \text{ kg} & \text{m} & 2 \text{ kg} \\
\hline
v_0 & & v_0/4 \\
\hline
\end{array} \]

COM gives: \( 2v_0 = 2 \times \frac{v_0}{4} + mv \quad \ldots(1) \)

For elastic collision \( v - \frac{v_0}{4} = v_0 \quad \ldots(2) \)

From (1) and (2) \( m = 1.2 \text{ kg} \)

6.(3) \[ U_{\text{system}} = \frac{1}{4\pi e_0} \left[ \frac{(Q \times q)}{D + d/2} + \frac{Q(-q)}{(D - d/2)} + \frac{2 \times (-q)}{d} \right] \]
\[ = \frac{1}{4\pi e_0} \left[ \frac{q^2}{d} - \frac{2qd}{D^2} \right] \quad \therefore D^2 - \frac{d^2}{4} = D^2 \]

7.(3) We have \( V_{\text{r.m.s.}} \propto \sqrt{I} \)
\[ \therefore \frac{200}{V} = \sqrt{\frac{273 + 127}{273 + 227}} \Rightarrow V = 100\sqrt{5} \text{ m/s} \]

8.(3) Conservation of energy gives \( \frac{1}{2}I_{\text{A}}\omega^2 = mg \)
9. (3) Net repulsive force
\[ F = F_1 - F_2 = \frac{\mu_0}{2\pi} \frac{i_1 x a}{a} - \frac{\mu_0 i_2}{2\pi} \frac{2a}{2a} \]
\[ = \frac{\mu_0 i_1 i_2}{4\pi} \]

10. (1) Theoretical

11. (3) For no drifting, \( \sin \theta = \frac{u}{v_m} = \frac{2}{4} = \frac{1}{2} \)
\[ \therefore \theta = 30^\circ \]
Hence direction with flow of rivers is
\[ \alpha = 180^\circ - \theta = 120^\circ \]

12. (1) Work-done will be equal to rise in potential energy of suspended section
\[ \therefore W = mg \times \frac{L}{n} \times \frac{L}{2n} = \frac{MgL}{2n^2} \]

13. (4) H-Cl is diatomic molecule hence degree of freedom is
\[ f = 3 \text{ (translational)} + 2 \text{ (rotational)} + 1 \text{ (vibration)} = 6 \]
\[ \therefore \frac{1}{2} m\bar{v}^2 = \frac{6}{2} K_b T \quad \therefore T = \frac{m\bar{v}^2}{6K_b} \]

14. (3) \( V^2 = u^2 - 2gh \Rightarrow m^2\bar{v}^2 = m^2u^2 - 2mgh \)
\[ \Rightarrow p^2 = A - Bh \quad \text{(where} A = m^2u^2, b = 2mg) \]
Clearly it is parabolic functions.
Starting from ground momentum first decreases and then becomes zero at highest position. Therefore it increases in –ve direction. Hence 3rd option is correct.

15. (4) \( V = i_e [R_g + R] \)
\[ = 4 \times 10^{-3} [50 + 5000] = 20.2 \text{ volt} \]
\[ \approx 20 \text{ volt}. \]

16. (2) \( W = |\Delta U| = \frac{Q^2}{2C_1} - \frac{Q^2}{2C_2} = 3.75 \times 10^{-6} J \)

17. (4) At \( 0^\circ C, v = \frac{\omega}{K} = \frac{1000}{3} \text{ m/s} \quad \therefore \frac{1000}{3} \times \frac{3}{336} = \sqrt{\frac{273}{273 + T}} \quad (\because V \propto \sqrt{T}) \quad \therefore \quad T \approx 277 K = 4^\circ C \]
18. (4) \[ \frac{1}{2} I \omega^2 = K \theta^2 \Rightarrow \frac{1}{2} I \times 2 \phi \times \frac{d\phi}{dt} = K \times 2 \theta \cdot \frac{d\theta}{dt} \] \[ \therefore \alpha = \frac{d\omega}{dt} = \frac{2k\theta}{I} \]

19. (3) \[ R_{AB} = R_{BC} = R_{CD} = \frac{R}{4} \quad \& \quad R_{DE} = R_{EC} = \frac{R}{8} \] \[ \therefore \quad R_{eq} = \left( \frac{3R}{4} + \frac{R}{8} \right) \times \frac{R}{8} = \frac{7R}{64} \]

20. (1) \[ \frac{\lambda_2}{660} = \frac{1}{2} \cdot \frac{\lambda_1}{3} \cdot \frac{\lambda_1}{3} \Rightarrow \lambda_2 = 488.88 \approx 488.9 \text{Å} \]

21. (3) if \( \theta_c \) is contact angle, then for equilibrium of rises water in tube, we have \( T \cos \theta \times 2\pi r = Mg \) hence if \( r \) is doubled, rises mass will also doubled.

22. (3) \[ E_p = \frac{GM}{(3a)^2} + \frac{G.2M}{(3a)^2} = \frac{GM}{3a^2} \]

23. (1) \[ \tau = \mu B \sin \theta = (niA)B \sin \theta = 100 \times 3 \times \frac{5 \times 2.5}{100 \times 100} \times 1 \times \frac{1}{\sqrt{2}} = 0.265 \text{N} \approx 0.27 \text{N} \]

24. (1) According to question shift = width of \( n \) fringe pattern
\[ \Rightarrow (\mu - 1)t = n \times \frac{D}{\lambda} \cdot \lambda \quad \therefore \quad t = \frac{nD\lambda}{a[\mu - 1]} \]

25. (2) \[ T = 2\pi \left[ \frac{l}{g} \right] \quad \text{&} \quad T' = 2\pi \left[ \frac{l}{g - (g / 16)} \right] \quad \text{(:: Buoyancy = \( mg / 16 \))} \]
\[ \therefore \quad T' = T \times \frac{1}{\sqrt{1 - \frac{1}{16}}} = 4T \times \frac{1}{\sqrt{15}} \]

26. (4) For a solenoid, self inductance is given as
\[ L_{self} = N \times A \times \mu_0 \left( \frac{N}{L} \right) \quad \therefore \quad L_{self} \propto \frac{1}{L} \]

27. (3) input resistance = \( r_1 = \frac{10 \text{mV}}{15 \times 10^{-6}} \approx 0.67 \times 10^3 \Omega = 6.0 \times 10^2 \Omega = 0.67 \text{ KΩ} \)
Voltage gain = \( +B \frac{R_c}{r} = +\frac{3 \text{mA}}{15 \mu\text{A}} \times \frac{1 \text{KΩ}}{0.67 \text{KΩ}} = +300 \text{ Volt} \)

28. (3) Clearly \( \frac{2\pi}{\lambda} = \frac{2\pi}{5 \times 10^{-7}} \quad \therefore \quad \lambda = 5000 \text{Å} \)
\[ \therefore \quad E = eV + \phi \quad \Rightarrow \quad V = \frac{12375}{5000} - 2 = 0.475 \text{ V} \approx 0.48 \text{ volt} \]

29. (1) Area under curve in process `A’ on P-V diagram is more. Hence
\[ W_A > W_B \therefore \Delta Q_A > \Delta Q_B \quad \& \quad \Delta U_A = \Delta U_B \]

30. (4) \[ \rho = \frac{10}{(0.1)^3} = 10^4 \text{ kg/m}^3 \]
Also \[ \rho = \frac{M}{l^3} \quad \therefore \quad \Delta \rho = \frac{\Delta M}{M} = \frac{3\Delta l}{l} = \frac{0.1}{10} + \frac{0.01}{0.1} \times 3 = \frac{1}{100} + \frac{3}{10} = \frac{31}{100} = 0.31 \text{ kg/m}^3 \]
1.(2) from given eq:

\[ \text{N}_2(g) + 3\text{H}_2(g) \rightarrow 2\text{NH}_3(g) \]

\[ \text{1mole} \quad \text{3mole} \]

\[ = 28\text{gm} \quad = 6\text{gm} \]

\[ \text{for option -1} \quad \therefore 6\text{ gm} \text{ H}_2 \text{ reacts } 28\text{ gm} \text{ N}_2 \text{ i.e. both reactants utilized completely.} \]

\[ \text{for option -2} \quad \therefore 6\text{ gm} \text{ H}_2 \text{ reacts } 28\text{ gm} \text{ N}_2 \]

\[ \therefore 10\text{ gm} \text{ H}_2 \text{ reacts } \frac{28}{6} \times 10\text{ gm} \text{ N}_2 \]

\[ = 46.67\text{ gm} \text{ N}_2 \text{ required, however amount of } \text{N}_2 \text{ given = 56 gm} \]

\[ \text{i.e. N}_2 \text{ present in excess.} \]

\[ \text{So, H}_2 \text{ utilizes completely & hence limiting reagent.} \]

\[ \text{for option -3} \quad \therefore 6\text{ gm} \text{ H}_2 \text{ reacts } 28\text{ gm} \text{ N}_2 \]

\[ \therefore 4\text{ gm} \text{ H}_2 \text{ reacts } \frac{28 \times 4}{6} \text{ gm} \text{ N}_2 \]

\[ = 18.67\text{ gm} \text{ N}_2 \text{ required. But } \text{N}_2 \text{ given = 14 gm.} \]

\[ \therefore \text{N}_2 \text{ is L. R.} \]

\[ \text{for option - 4} \quad \therefore 6\text{gm} \text{ H}_2 \text{ reacts } 28\text{ gm} \text{ N}_2 \]

\[ \therefore 6\text{ gm} \text{ H}_2 \text{ reacts } \frac{28 \times 8}{6} \text{ gm} \text{ N}_2 = 37.33\text{gm} \text{ N}_2 \text{ required but } \text{N}_2 \text{ given = 35 gm.} \]

2.(2)

\[
\begin{array}{c}
\text{NH}_2 \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\end{array}
\xrightarrow{\text{NaH}_2 \text{O}, \text{HCl, 0°C}}
\begin{array}{c}
\text{O} \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\text{ } \\
\end{array}
\xrightarrow{T} \begin{array}{c}
\text{O} \\
\text{N} = \text{N} \\
\text{ } \\
\text{H}_2 \\
\end{array}
\]

\[ \text{Added to equimolar mix of phenol aniline in acidic medium promotes coupling reaction in case of aniline} \]

\[ \therefore \text{ major product should be} \]

\[ \begin{array}{c}
\text{O} \\
\text{N} = \text{N} \\
\text{O} \\
\text{H}_2 \\
\end{array} \]

3.(3) \( q \) & \( w \) are path functions.

Option – 3 is correct
4. (4) Aerosol is a colloidal solution of solid in air. Eg. Smoke, dust etc. are considered as aerosol.
   Option – 4 is correct

5. (1) \( \text{Mg} \xrightarrow{\text{air, } \Delta} \text{MgO} + \text{Mg}_3\text{N}_2 \)
   Option – 1 is correct

6. (2) \[
\begin{align*}
\text{Cl} & \quad \text{alc KOH} \quad \text{Cl} \\
\text{Cl} & \quad \text{free radical polymerisation} \\
\text{Cl} & \quad \text{Cl}
\end{align*}
\]

7. (4) from Raoult’s law: \[
P_M = x_M \cdot P_M^0 \quad \text{from Dalton’s law:} \quad P_M = Y_M \cdot P_T
\]
   \[& \quad \text{& } \quad P_N = x_N \cdot P_N^0 \quad P_N = Y_N \cdot P_T
\]
   \[\therefore \quad x_M \cdot P_M^0 = Y_M \cdot P_T \quad \text{...... (i)}
\]
   \[& \quad x_N \cdot P_N^0 = Y_N \cdot P_T \quad \text{...... (ii)}
\]
   \[\text{eq. (i)} \div \text{eq. (ii)}
\]
   \[
x_M \cdot P_M^0 \quad x_N \cdot P_N^0 = \frac{x_M \cdot P_M^0}{x_N \cdot P_N^0} = \frac{Y_M \cdot P_T}{Y_N \cdot P_T}
\]
   \[
\frac{x_M}{x_N} \times \frac{450}{700} = \frac{Y_M}{Y_N}
\]
   \[
x_M \times 14 = 9 \times \frac{Y_M}{Y_N}
\]
   \[\therefore \quad \frac{x_M}{x_N} > \frac{Y_M}{Y_N}
\]

8. (1) \( \text{CuSO}_4 \cdot 5\text{H}_2\text{O} \)
   \[
   \left[\text{Cu(H}_2\text{O)}_4\right]\text{SO}_4 \cdot \text{H}_2\text{O}
   \]
   One \( \text{H}_2\text{O} \) molecule is present as water of crystallization.

9. (4) for EMR \( C = \nu \lambda \) \[\therefore \quad \nu = \frac{C}{\lambda}
\]
   From Planck’s formula,
\[ E = hv \quad \therefore \nu = \frac{E}{h} \quad \therefore \frac{C}{\lambda} = \frac{E}{h} \]

So, \[
\frac{\Delta \nu_{\text{Lyman}}}{\Delta \nu_{\text{Balmer}}} = \frac{(\nu_{\text{max}} - \nu_{\text{min}})_{\text{Lyman}}}{(\nu_{\text{max}} - \nu_{\text{min}})_{\text{Balmer}}} = \frac{(E_{\text{max}} - E_{\text{min}})_{\text{Lyman}}}{(E_{\text{max}} - E_{\text{min}})_{\text{Balmer}}}
\]

\[
= \frac{13.6 \left(\frac{1}{1} \frac{1}{1} \frac{1}{\alpha}\right) - 13.6 \left(\frac{1}{1} \frac{1}{4} \frac{1}{1}\right)}{13.6 \left(\frac{1}{4} \frac{1}{1} \frac{1}{\alpha}\right) - 13.6 \left(\frac{1}{4} \frac{1}{1} \frac{1}{9}\right)} = \frac{1 - 0 - 1 + \frac{1}{4}}{1 - 4 - 4 - \frac{1}{4} - \frac{1}{9} + \frac{2}{9}} = \frac{1}{4} = \frac{9}{4}
\]

\[ \therefore 9 : 4 \]

10.(4)

\[
\text{OH} \xrightarrow{\text{dil. HCl}} \text{NR}
\]

\[
\text{NaOH} \xrightarrow{\text{H}_2\text{O}} \text{ONa} + \text{H}_2\text{O}
\]

\[
\text{Br}_2, \text{H}_2\text{O} \xrightarrow{\text{Br}} \text{Salt}
\]

11.(4)

\[
\text{NO}_2
\]

2-chloro-1-methyl-4-nitrobenzene

12.(1) Increase in CO\(_2\) induces Global warming.
13. (4) Sucrose is formed due to bonding between.

\[ \alpha - \text{Glucose from C}_1 \text{ position and} \ \beta - \text{fructose from C}_2 \text{ position.} \]

\[ \begin{array}{c}
\text{O} \\
\end{array} \]

14. (4) \[ \text{CH}_3 - \text{CH} = \text{CH} - \text{C} - \text{O} - \text{CH}_3 \xrightarrow{\text{LiAlH}_4} \text{CH}_3 - \text{CH} = \text{CH} - \text{CH}_2 - \text{OH} \]

15. (4)

\[ \left[ \text{Cr(H}_2\text{O)}_6\right]^{3+} \]

\[ \begin{array}{c}
\text{d}^3 \text{sp}^3 \quad \text{degenerate orbitals} \\
\text{dx}^2 - \text{y}^2 \text{d}z^2 \text{degenerate orbitals}
\end{array} \]

16. (2)

\[ \begin{array}{c}
\text{CH}_3 - \text{C} \equiv \text{CH} \xrightarrow{\text{DCl}} \text{CH}_3 - \text{C} = \text{CHD} \\
\downarrow \text{DI} \\
\text{Cl} \\
\text{CH}_3 - \text{C} - \text{CHD}_2
\end{array} \]

17. (4) \[ K = 1s^2 2s^2 2p^6 3s^1 \]

\[ \text{I.P.} = \text{I.P.} \]

\[ K^+ = 1s^2 2s^2 2p^6 \]

inert gas configuration

18. (3) \[ \xrightarrow{-\text{CN} \ -\text{Cl} \ -\text{Me} \ -\text{OMe}} \]

i.e. \[ D < A < C < B \]

19. (4) Cryolite = \[ \text{Na}_3[\text{AlF}_6] \]
20.(2) \( \text{Cu}^{2+} + \text{Zn} \rightarrow \text{Zn}^{2+} + \text{Cu} \)

\[ n = 2 \]

\[ \Delta G^\circ = - n f E_\text{cell}^\circ = -2 \times 96500 \times 2 = -386 \text{ KJ} \]

21.(1)

\((M(AA)A_2B_2)\text{type}\)

No symmetry

22.(3) \( C_{60} \) contains 12 Pentagons of 20 hexagons

23.(3) \( V_2O_5 \) \( \rightarrow \) \( H_2SO_4 \)

\( TiCl_4 / Al(Me)_3 \) \( \rightarrow \) Polyethylene

\( PdCl_2 \) \( \rightarrow \) Ethanal

Iron oxide \( \rightarrow \) \( NH_3 \)

24.(4)

\[
\begin{align*}
&\text{O} &\text{OH} &\rightarrow &\text{Br} &\text{al.KOH} &\rightarrow
\end{align*}
\]

\[ R \rightarrow P \]

25.(1)

\[ a \hspace{1cm} x \]

\[ a-x \hspace{1cm} x \]

For 1st order reaction

\[
\frac{dx}{dt} = k(a-x)^1
\]

\[
\int \frac{dx}{a-x} = \int kdt
\]

\[-\ln(a-x) = kt + c\]

If \( t = 0 \)

\[ c = -\ln a \]

\[ \therefore -\ln(a-x) = kt - \ln a \]

or

\[ y = \frac{m}{x} \]

\[ C \]

Similarly

\[ R \rightarrow P \]

\[ ^{(A_0)} \rightarrow ^{(A)} \]
For complete dissociation takes place.

For BaCl₂

As \( \pi_{xy} = 4 \pi_{BaCl₂} \)

\[
\frac{\pi_{xy}}{\pi_{BaCl₂}} = \frac{2 \times C \times RT}{0.03RT}
\]

\[
\frac{4 \times \pi_{BaCl₂}}{\pi_{BaCl₂}} = \frac{2 \times C}{0.03}
\]

\[
C = \frac{4 \times 0.03}{2 \times 100} = 6 \times 10^{-2}
\]

For Kr the value of \( \left( \frac{a}{b} \right) \) is highest

Thus \( T_C \) is also highest

28.

\[
\text{CH}_2-\text{CH}_3 \xrightarrow{\text{KMnO}_4} \text{COO}^\ominus \xrightarrow{\text{H}_3\text{O}^+} \text{COOH}
\]

29.(3) \( \text{NO, N}_2\text{O, NO}_2, \text{N}_2\text{O}_3 \)

30.(1) For \( C_2, Z_2 = 12 \)

BO = 2.5

\[
\therefore \text{For } \text{C}_2^-, Z_2 = 13
\]

BO \( _{C_2^-} \) = 2.5 \hspace{1cm} \text{(filling in BMO)}

BO \( _{O_2} \) = 2.5 \hspace{1cm} \text{(filling in ABMO)}
BO\text{NO} = 2.0

BO\text{F}_2 = 0.5

**PART-C | MATHEMATICS**

1.(1) The points (1, \( f(1) \)) = (1, -2) and (-1, \( f(-1) \)) = (-1, 0)

So, slope of line joining the two pts = \( \frac{-2}{2} = -1 \)

Again, slope of tangent to \( y = x^3 - x^2 - 2x \) is \( \frac{dy}{dx} = 3x^2 - 2x - 2 \)

\[ 3x^2 - 2x - 2 = -1 \quad \Rightarrow \quad 3x^2 - 2x - 1 = 0 \quad \Rightarrow \quad x = \frac{2 \pm \sqrt{4 + 12}}{6} = -\frac{1}{3}, 1 \]

2.(2) Since S.D. = \( \sqrt{\frac{\sum xi^2}{x} - \left( \frac{\sum xi}{x} \right)^2} \) = \( \sqrt{\frac{(-1)^2 + 0^2 + (1)^2 + K^2}{4} - \left( \frac{(-1) + 0 + (1) + k}{4} \right)^2} \)

\[ = \sqrt{\frac{K^2 + 2}{4} - \frac{K^2}{16}} \quad \Rightarrow \quad 3K^2 + 8 = 16 \times 5 = 80 \]

\[ = \sqrt{\frac{3K^2 + 8}{4}} \quad \Rightarrow \quad K^2 = \frac{72}{3} = 24 \]

Now, \( \sqrt{\frac{3K^2 + 8}{4}} = \sqrt{5} \quad \Rightarrow \quad K = 2\sqrt{6} \)

3.(3) \( \int_{\pi/2}^{2} \frac{\sin^3 x}{\sin x + \cos x} \, dx = I \) (say) \( \quad \Rightarrow \quad I = \int_{0}^{\pi/2} \frac{\cos^3 x}{\sin x + \cos x} \, dx \)

\[ \Rightarrow \quad 2I = \int_{0}^{\pi/2} \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} \, dx \quad \Rightarrow \quad 2I = \int_{0}^{\pi/2} (\sin^2 x + \cos^2 x - \sin x \cos x) \, dx \]

\[ \Rightarrow \quad 2I = \int_{0}^{\pi/2} \left[ \frac{\pi}{2} - \frac{1}{2} \sin 2x \right] \, dx \quad \Rightarrow \quad 2I = x + \frac{\cos 2x}{4} \bigg|_{0}^{\pi/2} \]

\[ \Rightarrow \quad 2I = \left( \frac{\pi}{2} - \frac{1}{4} \right) - \left( \frac{1}{4} \right) = \frac{\pi}{2} - \frac{1}{2} \quad \Rightarrow \quad I = \frac{\pi - 1}{4} \]

4.(1) \( \cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ \)

\[ = \frac{1}{2} \left[ 2\cos^2 10^\circ - 2\cos 10^\circ \cos 50^\circ + 2\cos^2 50^\circ \right] = \frac{1}{2} \left[ 1 + \cos 20^\circ - \cos 40^\circ - \cos 60^\circ + 1 + \cos 100^\circ \right] \]

\[ = \frac{1}{2} \left[ \frac{3}{2} + (\cos 20^\circ - \cos 40^\circ) + \cos (90^\circ + 10^\circ) \right] = \frac{1}{2} \left[ \frac{3}{2} + 2 \cdot \sin 30^\circ \sin 10^\circ - \sin 10^\circ \right] = \frac{3}{4} \]
5.(2) \[ 2 \cos^2 \theta + 3 \sin \theta = 0 \quad \Rightarrow \quad 2 - 2 \sin^2 \theta + 3 \sin \theta = 0 \]

\[ \Rightarrow \quad 2 \sin^2 \theta + 3 \sin \theta - 2 = 0 \quad \Rightarrow \quad 2 \sin^2 \theta - 4 \sin^2 \theta + \sin \theta - 2 = 0 \]

\[ (2 \sin \theta + 1) (\sin \theta - 2) = 0 \]

\[ \Rightarrow \quad \sin \theta = \frac{-1}{\sqrt{2}} \quad \Rightarrow \quad \theta = \frac{7\pi}{6}, \frac{11\pi}{6}, -\frac{5\pi}{6}, -\frac{\pi}{6} \Rightarrow \quad \frac{7\pi + 11\pi - 5\pi - \pi}{6} = \frac{12\pi}{6} = 2\pi \]

6.(1) \[ \frac{x-\frac{1}{2}}{2} = \frac{y+\frac{1}{3}}{3} = \frac{z-\frac{2}{4}}{4} = \lambda \quad \Rightarrow \quad \text{Point on the line } = \left(2\lambda + 1, 3\lambda - 1, 4\lambda + 2\right) \]

Now for \( p \)

\[ (2\lambda + 1) + 2(3\lambda - 1) + 3(4\lambda + 2) = 15 \quad \Rightarrow \quad 20\lambda + 5 = 15 \quad \Rightarrow \quad \lambda = \frac{1}{2} \]

\[ \Rightarrow \quad p = \left(2, \frac{1}{2}, 4\right) \quad \Rightarrow \quad OP = \sqrt{4 + \frac{1}{4} + 16} = \sqrt{20 + \frac{1}{4}} = \frac{9}{2} \]

7.(2) \[ f(x) = \frac{x^2}{1-x^2} = \frac{1}{1-x^2} - 1 \quad \Rightarrow \quad A = R \setminus \left(-1, 0\right) \quad \Rightarrow \quad f(x) \in (-\infty, -1) \cup [0, \infty) \]

8.(3) \[ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \ldots \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix} \]

\[ \Rightarrow \quad \Sigma(n-1) = 78 = \frac{n(n-1)}{2} = 78 \quad \Rightarrow \quad n(n-1) = 156 = 12 \times 13 \quad \Rightarrow \quad n = 13 \]

\[ \Rightarrow \quad \begin{bmatrix} 1 & 13 \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix} \]

9.(3) \[ f'(x) = \lambda x(x-1)(x+1) \quad \Rightarrow \quad f''(x) = \lambda x^3 - \lambda x \quad \Rightarrow \quad f(x) = \frac{\lambda x^4}{4} - \frac{\lambda x^2}{2} + \mu \]

For \( f(x) = f(0) \)

\[ \Rightarrow \quad \frac{\lambda x^4}{4} - \frac{\lambda x^2}{2} = 0 \quad \Rightarrow \quad \frac{\lambda x^2}{4}(x^2 - 2) = 0 \quad \Rightarrow \quad x = 0, \sqrt{2}, -\sqrt{2} \]

10.(1) \[ \frac{M(24+18)}{\sqrt{24-18}M^2} = 7\sqrt{3} \quad \Rightarrow \quad 6M = \sqrt{3} \sqrt{24-8M^2} \]

\[ \Rightarrow \quad 36M^2 = 72 - 54M^2 \quad \Rightarrow \quad 90M^2 = 72 \quad \Rightarrow \quad M = \frac{2}{\sqrt{5}} \]

11.(2) Let equation of the plane be \( a(x-0) + b(y+1) + c(z-0) = 0 \)

\[ \Rightarrow \quad ax + by + cz = -b \quad \ldots \ldots (1) \]
Now \( a(0-0) + b(0+1) + c(1-0) = 0 \)

\[ b + c = 0 \quad \text{......(2)} \]

and \( \frac{a \times 0 + b \times 1 + c \times (-1)}{\sqrt{a^2 + b^2 + c^2}} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} \)

\[ \Rightarrow \quad b-c = \sqrt{a^2 + 2b^2} \]

\[ 2b = \sqrt{a^2 + 2b^2} \]

\[ 4b^2 = a^2 + 2b^2 \]

\[ \Rightarrow \quad a^2 = 2b^2 \quad \Rightarrow \quad a = \pm \sqrt{2}b \]

So, equation of plane \( \pm \sqrt{2}x + y - z = -1 \)

\((\sqrt{2},+4)\) satisfy.

12. \( \vec{\beta}_1 = \lambda \vec{a} = 3\lambda \hat{i} + \lambda \hat{j} \quad \Rightarrow \quad 2\hat{i} - \hat{j} + 3\hat{k} = (3\lambda - \mu)\hat{i} + (\lambda + 3\mu)\hat{j} + 3\hat{k}. \)

\( \vec{\beta}_2 = \mu \hat{i} - 3\mu \hat{j} + \mu \hat{k} \)

Also, \( \vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2 \quad \Rightarrow \quad 3\lambda - \mu = 2 \text{ and } \delta = -3 \)

\( \lambda + 3\mu = -1 \)

So, \( \lambda = \frac{1}{2} \text{ and } \mu = -\frac{1}{2} \)

\[ \Rightarrow \quad \vec{\beta}_1 = \frac{1}{2}(3\hat{i} + \hat{j}) \text{ and } \vec{\beta}_2 = -\frac{1}{2} \hat{i} + \frac{3}{2} \hat{j} + 3\hat{k} \]

\[ \vec{\beta}_1 \times \vec{\beta}_2 = \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
\frac{3}{2} & \frac{1}{2} & 0 \\
\frac{1}{2} & \frac{3}{2} & -3
\end{vmatrix} = \hat{i}\left(-\frac{3}{2}\right) - \hat{j}\left(-\frac{9}{2}\right) + \hat{k}\left(\frac{9}{4} + \frac{1}{4}\right) = \frac{1}{2}(-3\hat{i} + 9\hat{j} + 3\hat{k}) \]

13. (3) \( S_n = 50n + \frac{n(n-7)}{2} A \quad \Rightarrow \quad T_n = S_n - S_{n-1} \)

\[ \Rightarrow \quad T_n = 50(n-n+1) + \frac{A}{2}[n(n-7) - (n-1)(n-8)] \]

\( T_n = 50 + \frac{A}{2}[-7n + 9n - 8] \)

\[ \Rightarrow \quad T_n = 50 + A(n-4) \text{ and } d = T_5 - T_4 = (50 + A) - (50) = A \]

\[ \Rightarrow \quad a_{50} = 50 + 46A \]
(d, 9_{50}) = (A, 50 + 46.4)

14.4: \[ k = \lim_{x \to \frac{\pi}{4}} \frac{\sqrt{2} \cos x - 1}{\cot x - 1} = \lim_{x \to \frac{\pi}{4}} \frac{1 - \sqrt{2} \cos x}{\cosec^2 x} = \lim_{x \to \frac{\pi}{4}} \sqrt{2} \sin^2 x = \sqrt{2} \left( \frac{1}{\sqrt{2}} \right)^3 = \frac{1}{2} \]

15.2: \[ z = \frac{\alpha + i}{\alpha - i} \Rightarrow |z| = \frac{|\alpha + i|}{|\alpha - i|} \Rightarrow |z| = 1 \]
So, circle with radius 1.

16.3: \[ T_4 = 6 \binom{2}{1} \left( x^{\log_8 x} \right)^{\frac{2}{3}} = 20 \times 8^7 \Rightarrow x^{\log_8 x} = 64 \]
\[ \Rightarrow 20 \times 8 \times \left( x^{\log_8 x} \right)^{\frac{3}{4}} = 20 \times 9^7 \Rightarrow (\log_8 x)^2 = \log_6 64 + \log_8 x \]
\[ \Rightarrow \frac{x^{\log_8 x}}{x} = 64 \Rightarrow (\log_8 x - 2) (\log_8 x + 1) = 0 \Rightarrow x = 64, \frac{1}{8} \]

17.2: \[ g(x) = f(f(x)) = 15 - |f(x) - 10| \]
\[ = 15 - |15 - |x - 10|| - 10| \]
\[ \Rightarrow g(x) = 15 - |5 - |x - 10|| \Rightarrow g(x) = 15 - |15 - x| ; x \geq 10 \]
\[ 15 - |x - 5| \}<10 \Rightarrow g(x) = 10 + x ; \ x < 5 \]
\[ 20 - x ; 5 \leq x < 10 \]
\[ x ; 10 \leq x < 15 \]
\[ 30 - x ; 15 \leq x \]
So, g(x) is not differentiable at x = 5, 10, 15

18.\( p, q \in R \) The other root is \( 2 + \sqrt{3} \)
\[ p, q \in R \Rightarrow \) The other root is \( 2 + \sqrt{3} \)
So, \( p = -4 \ q = 1 \)
\[ p^2 - 4q = 16 - 4 = 12 \]
\[ p^2 - 4q = 12 = 0 \]

19.4: \[ I = \int \sec^{2/3} x \cos ec x^{4/3} x \ dx = \int \frac{dx}{\sin^{4/5} x \cos^{2/5} x} = \int \frac{sec^2 x \ dx}{(\tan x)^{4/3}} \]
\[ I = \int \frac{d(\tan x)}{\tan^{4/3}} = \frac{(\tan x)^{1/3}}{1 - \frac{4}{3}} + C = -3(\tan x)^{-1/3} + C \]

20.(4) \quad y = x^3 + ax - b \quad \Rightarrow \quad \frac{dy}{dx} = 3x^2 + a

Now \quad \frac{dy}{dx}_{(1, -5)} = -1

\Rightarrow \quad a + 3 = -1 \quad \Rightarrow \quad a = -4

(1, -5) lies on the curve

\Rightarrow \quad -5 = 1 - 4 - b

\quad b = 5 - 3 = 2

So, curve is \quad y = x^3 - 4x - 2

21.(4) \quad \Rightarrow \quad \text{Mid-point } A, B = P = \left( \frac{h}{2}, \frac{k}{2} \right)

\Rightarrow \quad h = 2x, k = 2y

Now line as: \quad \frac{x}{h} + \frac{y}{k} = 1 \text{ is layout}

\Rightarrow \quad \left| \frac{0 + 0 - 1}{h - k} \right| = 1 \quad \Rightarrow \quad \frac{1}{h^2} + \frac{1}{k^2} = 1

So, locus: \quad \frac{1}{4x^2} + \frac{1}{4y^2} = 1 \quad \Rightarrow \quad x^2 + y^2 = 4x^2y^2 \quad \Rightarrow \quad x^2 + y^2 - 4x^2y^2 = 0

22.(3) \quad \text{Path } = 1 - \left( 1 - \frac{1}{2} \right) \left( 1 - \frac{1}{3} \right) \left( 1 - \frac{1}{4} \right) \left( 1 - \frac{1}{3} \right) = 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{7}{8} = 1 - \frac{7}{32} = \frac{25}{32}

23.(2) \quad A = \left\{ (x, 4); x^2 \leq y \leq x + 2 \right\}

\quad y \geq x^2 \quad \text{and} \quad y - x - 2 \leq 0

Point of intersection

\Rightarrow \quad x^2 = x + 2 \quad \Rightarrow \quad x^2 - x - 2 = 0

\quad x = -1, 2
\[ A = \int_{-1}^{2} \left( (x+2) - x^2 \right) dx = \frac{x^2}{2} + 2x - \frac{x^3}{3} \bigg|_{-1}^{2} = \left( 2 + 4 - \frac{8}{3} \right) - \left( \frac{1}{2} - 2 + \frac{1}{3} \right) \]

\[ = 8 - 3 - \frac{1}{2} = 5 - \frac{1}{2} = \frac{9}{2} \]

24.(2) \hspace{1cm} f(x + y) = f(x) \cdot f(y) \Rightarrow f(x) = a^x

\[ f(1) = 2 \Rightarrow f(x) = 2^x \]

\[ \sum_{k=1}^{10} f(a + k) = 2^a \left[ 2 + 2^2 + \ldots + 2^{10} \right] = \frac{2^a(2^{10} - 1)}{2-1} \]

Also, \( 2^{a+1}(2^{10} - 1) = 16(2^{10} - 1) \)

\[ \Rightarrow a + 1 = 4 \Rightarrow a = 3 \]

25.(2) \hspace{1cm} \text{Roots of} \ x^2 + x + 1 = 0 \ \text{are} \ \omega, \omega^2

\[ \Rightarrow \alpha = \omega, \beta = \omega^2 \Rightarrow 1 + \alpha + \beta = 0 \]

\[
\begin{vmatrix}
\alpha + \beta & \beta \\
\alpha & y + \beta \\
\beta & 1 + \alpha \\
\end{vmatrix} = \begin{vmatrix}
\alpha & \beta \\
y & y + \beta \\
y & 1 + \alpha \\
\end{vmatrix} = \begin{vmatrix}
1 & \alpha & \beta \\
y & y + \alpha & 1 - \beta \\
0 & 1 - \alpha & y + \alpha - \beta \\
\end{vmatrix}
\]

(C₁ → C₁ + C₂ + C₃) \hspace{1cm} (R₂ → R₂ - R₁) \hspace{1cm} (R₃ → R₃ - R₁)

\[ = y \begin{vmatrix}
1 & \beta \\
y - i\sqrt{3} & 1 - \beta \\
0 & 1 - \alpha \\
\end{vmatrix} \]

\[ = y \{(y^2 + 3) - (1 - \alpha - \beta + \alpha \beta)\} \]

\[ = y\{y^2 + 3 - 2 \alpha - 2 \beta\} = y^3 \]

26.(3) \hspace{1cm} x \frac{dy}{dx} + 2y = x^2 \Rightarrow \frac{dy}{dx} + \left( \frac{2}{x} \right)y = x

I.F. = e \int \frac{2}{x} dx = x^2

So, \ yx^2 = \int x^3 dx \Rightarrow yx^2 = \frac{x^4}{4} + c \]

\[ y(1) = 1 \Rightarrow 1 = \frac{1}{4} + C \Rightarrow C = \frac{3}{4} \]

So, set \ y = \frac{x^2}{4} + \frac{3}{4x^2} \]
27.(2) \( y^2 = 16x \) if point is \((1, 4) \Rightarrow t_1 = \frac{1}{2} = (at_1^2, 2t_1) \)

Again, for focal chord \( t_1t_2 = -1 \Rightarrow t_2 = -2 \)

So, length of focal chord = \( a(t_1 - t_2) = 4 \left( \frac{1}{2} + 2 \right)^2 = 25 \)

28.(4) Let the equation of line be \( x = 2 + r \cos \theta, y = 3 + r \sin \theta \)

For, \( r = 24 \)

\( (2 + 4 \cos \theta) + (3 + 4 \sin \theta) = 7 \)

\( \Rightarrow 4(\cos \theta + \sin \theta) = 2 \quad \Rightarrow 4(\cos^2 \theta + \sin^2 \theta + 2 \sin \theta \cos \theta) = 1 \)

\( \Rightarrow 4 \tan^2 \theta + 8 \tan \theta + 4 = \sec^2 \theta \quad \Rightarrow 3 \tan 2\theta + 8 \tan \theta + 3 = 0 \)

\( \Rightarrow \tan \theta = \frac{-8 \pm \sqrt{64 - 36}}{6} \)

\( \Rightarrow \tan \theta = \frac{-8 \pm 2\sqrt{7}}{6} \quad \Rightarrow \tan \theta = \frac{2\sqrt{7} - 8}{6} \)

\( \tan \theta = \frac{2\sqrt{7} - 8}{6} = \frac{-\left(\sqrt{7} - 1\right)^2}{(7 - 1)} = -\frac{\sqrt{7} - 1}{\sqrt{7} + 1} \)

\( \tan \theta = \frac{1 - \sqrt{7}}{1 + \sqrt{7}} \)

29.(3) \( pV(\sim p \land q) \equiv (pV \sim p) \land (pVq) \equiv pVq \)

So, \( \sim (pVq) \equiv (\sim p) \land (\sim q) \)

30.(3) \( M = 8C_6 \times 5C_5 + 8C_7 \times 5C_4 + 8C_8 \times 5C_3 = n \Rightarrow M = n = 78 \)