# SOLUTIONS

## Joint Entrance Exam | IITJEE-2023

### $31^{st}$ JAN 2023 | Morning Shift

# PHYSICS

## SECTION - 1

**1.(2)** $\because N\times\frac{4}{3}\pi r^3=\frac{4}{3}\pi R^3$

$\Rightarrow R=N^{1/3}.r$

Loss of surface energy $=$ Energy released.

Energy released $=N\sigma 4\pi r^2-\sigma 4\pi R^2=4\pi\sigma\left(Nr^2-R^2\right)=4\pi\sigma\left(Nr^2-N^{2/3}r^2\right)$

$=4\pi\sigma r^2\left(N-N^{2/3}\right)=4\times 3.14\times 0.07\times 10^{-6}(1000-100)\cong 7.9\times 10^{-4}J$

**2.(4)** Work done $=U_f-U_i=(-MB\cos 180°)-(-MB\cos 0°)=2MB=2\times 5\times 0.4=45$

**3.(4)** Drift velocity $=V_d=\frac{eE}{m}\tau=\left(\frac{e\Delta V}{l.m}\right)\tau$

So, drift velocity doesn't depend on area.

**4.(1)** $\Delta W=-\frac{1}{2}\times(50+10)\times 150\times 10^3\times 10^{-6}\,j=-4.5\,joule$

$\Delta Q=0 \qquad \because \Delta Q=\Delta U+\Delta W \qquad \Rightarrow \Delta U=-\Delta W=+4.5J$

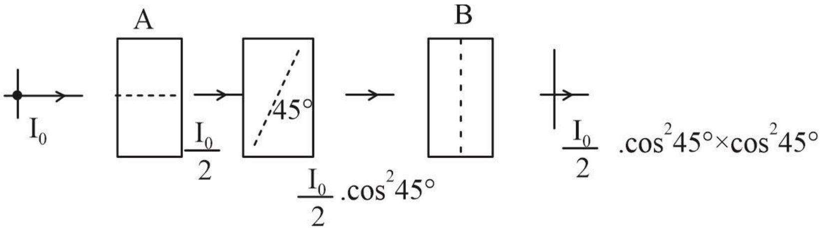
**5.(4)** $V(r)=\begin{cases}v_0,\ 0\le r\le R\\ \frac{kQ}{r},\ r\ge R\end{cases}=\begin{cases}v_0,\ 0\le r\le R\\ \frac{v_0R}{r},\ r\ge R\end{cases}$

Where $\frac{kQ}{R}=v_0 \qquad \Rightarrow kQ=v_0R$

**6.(4)** Unit of $x_L$ and $x_C$ is ohm.

So, option 4

**7.(2)**




So, Intensity (final) $=\frac{I_0}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{I_0}{8}$

**8.(4)** $\because g_d=4g_{h=3R}$

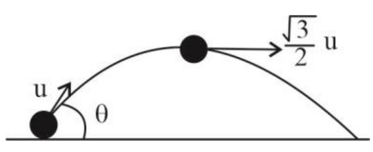
$\Rightarrow g_0\left(1-\frac{d}{R}\right)=4.\frac{GM}{(4R)^2}=\frac{GM}{R^2}\times\frac{1}{4}=\frac{g_0}{4}$

$\Rightarrow 1-\frac{d}{R}=\frac{1}{4}\Rightarrow\frac{d}{R}=\frac{3}{4}\Rightarrow d=\frac{3}{4}R=\frac{3}{4}\times 6400km=4800km$

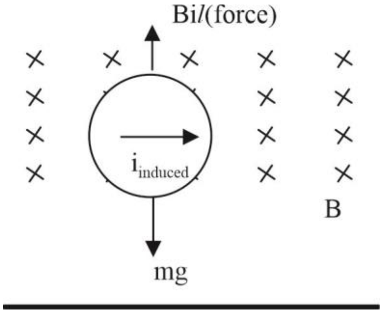
**9.(1)** $\because u\cos\theta=\frac{\sqrt{3}}{2}\times u$

$\Rightarrow\theta=30°$

$\Rightarrow T=\frac{2u\sin\theta}{g}=\frac{2u}{g}\times\frac{1}{2}=\frac{u}{g}$



**10.(3)** Due top earth's magnetic field effect, conducting ball will accelerate with acceleration less than g.




**11.(2)** Power $=\left(\frac{N}{\sec}\right)\times hf$

$\Rightarrow hf=\frac{15\times10^{3}}{10^{16}}J=15\times10^{-13}J \qquad \Rightarrow f=\frac{15\times10^{-13}}{6\times10^{-34}}=\frac{5}{2}\times10^{-21}Hz$

$\Rightarrow \lambda=\frac{3\times10^{8}}{\frac{5}{2}\times10^{21}}m=\frac{6}{5}\times10^{-13}m=1.2\times10^{-13}m$

(gamma rays)

**12.(4)** Both Assertion (A) and Reason (R) are true independently but reason statement (R) only validates the statement (A) experimentally.

**13.(4)** $y=\left(A_C+A_m\sin(\omega_c t)\right)\sin(\omega_c t)=A_c\sin(\omega_c t)+\frac{A_m}{2}.2.\sin(\omega_c t).\sin(\omega_m t)$

$=A_c\sin(\omega_c t)+\frac{A_m}{2}\cos(\omega_c-\omega_m)t-\frac{A_m}{2}\cos(\omega_m+\omega_c)t$

$f_1=f_c=500Hz$

$f_2=\frac{\omega_c-\omega_m}{2\pi}=\frac{1000\pi-4\pi}{2\pi}=498Hz$

$f_3=\frac{\omega_m+\omega_c}{2\pi}=502Hz$

**14.(2)** $n=\frac{N}{L};\ B=\mu_r\mu_0 ni$

Total flux $=N.B.A=N.\mu_r\mu_0 ni.A$

$=\left(\frac{N^2}{L}\right)\mu_r\mu_0 Ai\ =\frac{400^2}{0.4}\times\mu_r\times4\pi\times10^{-7}\times2\times10^{-4}\times0.4=4\pi\times10^{-6}$

$16\times10^{4}\times10^{-11}\times2\mu_r=10^{-6}\ \Rightarrow\mu_r=\frac{10^{-6}}{32\times10^{-7}}=\frac{10}{32}=\frac{5}{16}$

**15.(3)** Free neutron is radioactive outside stable nucleus. It decays to proton as it has higher mass.

**16.(1)** As temperature increases, more hole-electron pairs are created and thus conductivity increased. These free electrons populate conduction bond.

**17.(4)** $\gamma=\frac{C_p}{C_v}=$ is independent of temperature for ideal gas it depends only on degree of freedom.

**18.(2)** $\because\frac{1}{2}KA^2=25 \qquad \because\frac{1}{2}KA^2=\frac{1}{2}Kx^2+\frac{1}{2}mv^2$

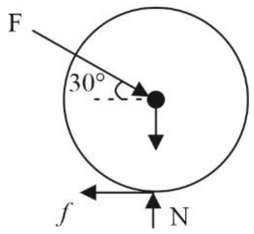
When $x=\frac{A}{2}$

$$\frac{1}{2}KA^2 = \frac{1}{2}.K\left(\frac{A}{2}\right)^2 + \frac{1}{2}mv^2$$

$$\Rightarrow \frac{1}{2}KA^2\left(1-\frac{1}{4}\right) = \frac{3}{2}\times\frac{1}{2}KA^2 = \frac{1}{2}mv^2 = K.E \qquad \Rightarrow K.E = \frac{3}{4}\times 25 = 18.75J$$

**19.(2)** Force $= \dfrac{Change\ in\ momentum}{Change\ in\ time} = \dfrac{100(2v\times 2)}{t} = \dfrac{200mv}{t}$

**20.(3)** $N = F\sin 30° + mg = 200\times\frac{1}{2} + 700 = 800$




**SECTION – 2**

**21.(5)** $C = \dfrac{C_0}{\sqrt{\mu_r \in_r}} \Rightarrow$ Refractive index $= n = \dfrac{C_0}{C} = \sqrt{\mu_r \in_r}$

$\because n = \dfrac{C_0}{C} = \sqrt{\mu_r \in_r} = \dfrac{C_0}{0.2C_0} = 5$

$\Rightarrow \mu_r \in_r = 25$

$\because \mu_r = 1 \Rightarrow \in_r = 25$

So, $\dfrac{\in_r}{n} = \dfrac{25}{5} = 5$

**22.(3)** Time to cross $= \dfrac{W}{speed\ of\ swimmer} = \dfrac{1km}{4km/hr} = 0.25hr$

$\because$ Drift = time to cross $\times$ speed of river

$= 0.25hr \times V_{river} = 0.75km$

$V_{river} = 3km/hr$

**23.(7)** $u = 2m/s$

$a = 2m/s^2$

$s = 6m$

$\Rightarrow v^2 = u^2 + 2as = 4 + 2\times 2\times 6 = 4 + 24 = 28$

So, $K.E = \frac{1}{2}\times 500\times v^2 = 14\times 500J = 7KJ$

**24.(20)** Both springs are in parallel

So, $k_{eq} = 2k$

$\because T = 2\pi\sqrt{\dfrac{m}{k_{eq}}} = 2\pi\sqrt{\dfrac{0.49}{4}} = \dfrac{2\pi\times 7}{20} = \dfrac{7\pi}{10}\sec$

No. of oscillation in 14 $\pi$ sec $= \dfrac{14\pi}{7\pi/10} = 20$

**25.(60)** $\in_T = \alpha\Delta\theta$

$$\in_m = \frac{\sigma_m}{\gamma} = \frac{Mg/A}{\gamma}$$

$\because \in_m = \in_T$

$$\Rightarrow \frac{Mg}{A\gamma} = \alpha\Delta\theta \Rightarrow M = \frac{A\gamma\alpha\Delta\theta}{g} = \frac{3\times10^{-6}\times2\times10^{11}\times2\times10^{-5}\times50}{10} = 60kg$$

**26.(242)** $I_{rms} = \frac{220}{R} = \frac{220}{20} = 11A$

$$I_{\max} = \sqrt{2}.I_{rms} = 11\sqrt{2} = \sqrt{121\times2} = \sqrt{242}$$

$x = 242$

**27.(640)** $\vec{E} = \left(4000x^2\right)\hat{i}$

$$\phi_{left\ face} = 0;\ \phi_{right\ face} = 4000\times(0.2)^2\times4000\times10^{-4} = 0.64\times10 = 6.4$$

$\phi_{other\ face} = 0$

$\phi_{total} = 6.4$ V-meter = 640 V-cm

**28.(27)** $\frac{hc}{\lambda_1} = 13.6\left(1-\frac{1}{9}\right) = 13.6\times\frac{8}{9}eV$

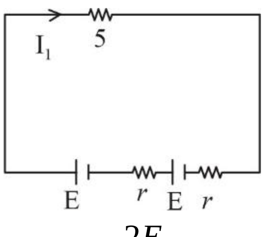
$$\frac{hc}{\lambda_2} = 13.6\left(1-\frac{1}{4}\right) = 13.6\times\frac{3}{4}eV$$

$$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{3/4}{8/9} = \frac{3}{4}\times\frac{9}{8} = \frac{27}{32} \Rightarrow x = 27$$

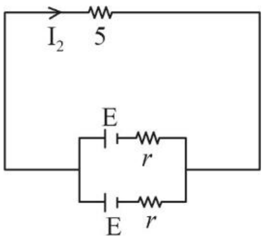
**29.(10)** $\frac{1}{2}mv^2 + \frac{1}{2}.\frac{2}{5}mr^2\omega^2 = k \qquad \Rightarrow \frac{7}{10}mv^2 = k = 7\times10^{-3}$

$$\Rightarrow v^2 = \frac{7\times10^{-3}\times10}{7m} = \frac{10^{-2}}{1} = \frac{1}{100} \qquad \Rightarrow v = 0.1m/s = 10cm/s$$

**30.(5)**




$$\Rightarrow I_1 = \frac{2E}{5+2r}$$



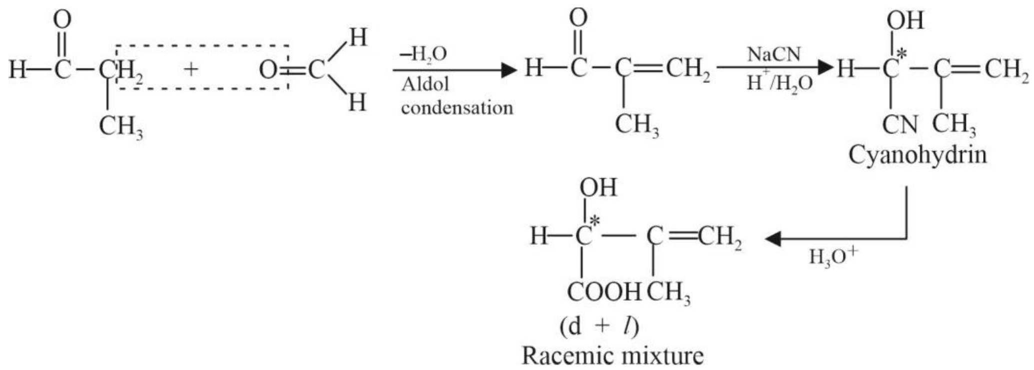

$$\Rightarrow I_2 = \frac{E}{5+\frac{r}{2}} \quad \because I_2 = I_1 \Rightarrow \frac{2E}{10+r} = \frac{2E}{5+2r} \Rightarrow r = 5\Omega$$

# CHEMISTRY

## SECTION - 1

**1.(4)**

$$CH_3-CH_2CHO+H-\overset{\overset{\displaystyle O}{||}}{C}-H$$




Carboxylic acids give $CO_2$ with $NaHCO_3$ solution.

**2.(4)** For isoelectronic species,

$$\text{ionic radius} \propto \frac{1}{\text{Nuclear charge}}$$

$\therefore$ Order : $Ca^{2+} < K^+ < Cl^- < S^{2-}$

**3.(1)** $2NaOCl + H_2O_2 \rightarrow 2NaCl + H_2O + O_2$

$$NaO\overset{+1}{Cl} \longrightarrow Na\overset{-1}{Cl} \ : \ \text{Reduction}$$

$$H_2O_2^{-1} \longrightarrow \overset{0}{O_2} \ : \text{Oxidation}$$

$H_2O_2$ acts as reducing agent in above reaction

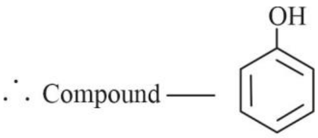
**4.(3)** Proteins are polymers of $\alpha-$Amino acids

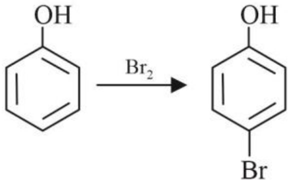
$$CH_3-\underset{}{\overset{\overset{\displaystyle NH_2}{|}}{CH}}-CH_2-\underset{\underset{\displaystyle NH_2}{|}}{\overset{\alpha}{CH}}-COOH$$

$\alpha-$Amino acid

**5.(2)** $Cl\dot{O} + NO_2 \longrightarrow ClO\underset{x}{N}O_2 \xrightarrow{H_2O} H\underset{y}{O}Cl + H\underset{z}{N}O_3$

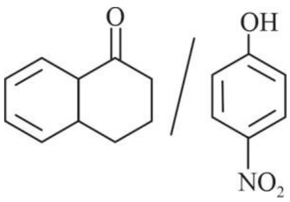
**6.(2)** Aromatic compounds burn with a sooty flame**.**



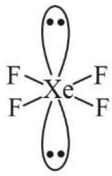




In low polarity solvent.

**7.(1)**

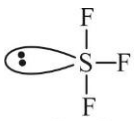
| | Mixture | Separation Technique |
|---|---|---|
| **(A)** | $H_2O/CH_2Cl_2$ | Differential solvent extraction |
| **(B)** |  | Column chromatography |
| **(C)** | Kerosene/Naphthalene | Fractional Distillation |
| **(D)** | $C_6H_{12}O_6$ / NaCl | Crystallization |

**8.(3)** $XeF_4$



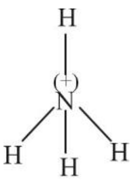

Square planar

$SF_4$



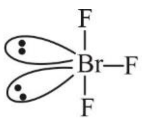

See-Saw

$NH_4^+$




Tetrahedral

$BrF_3$



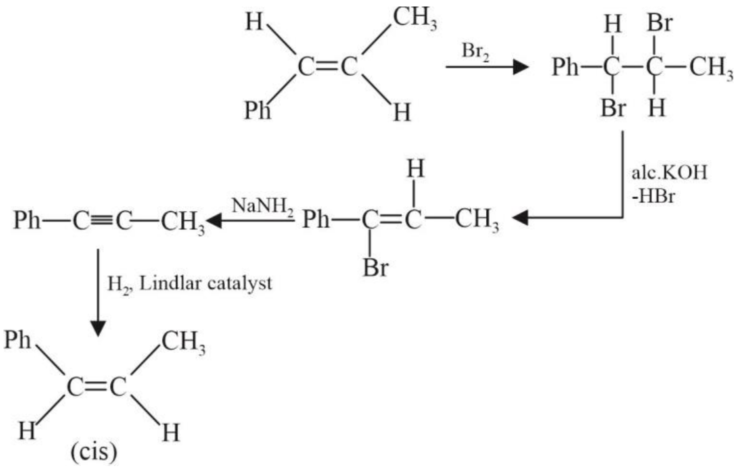

Bent T shaped

**9.(3)** Electrolyte: Brine solution (dil NaCl)

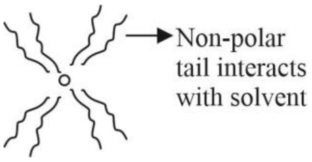
At anode: $2Cl^- \rightarrow Cl_2 + 2e^-$

At cathode: $H_2O + 2e^- \rightarrow H_2 + 2OH^-$

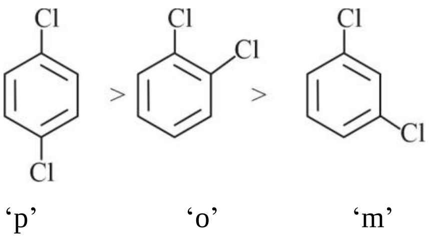
**10.(4)**




**11.(2)** In non-polar solvent surfactant:




**12.(1)** Order of melting point:




**13.(4)** $Cu^{2+} + KI \longrightarrow \underset{(X)}{Cu_2I_2} \xrightarrow{Na_2S_2O_3} \underset{\text{Sodium Tetrathionate}}{Na_2S_4O_6}$

**14.(2)** For $He^+ : E_n = \frac{-13.6 \times 4}{n^2}$; $\left[\because E_n = -13.6\frac{z^2}{n^2}\right]$

$$\Delta E = E_4 - E_2 = -13.6 \times 4\left[\frac{1}{16} - \frac{1}{4}\right] = -13.6 \times 4 \times \frac{3}{16} = 10.2\,eV$$

In hydrogen , this energy difference corresponds to transition : $n_2 \rightarrow n_1$

$E_2 \rightarrow E_1$

$$\Delta E = E_2 - E_1 = -13.6\left[\frac{1}{4} - 1\right] = 10.2eV$$

**15.(1)** Higher the oxidation state, more is the acidic nature

Order of basicity: $V_2O_3 > V_2O_4 > V_2O_5$

**16.(1)**

| Artificial sweetner | Sweetness value compared to cane sugar |
|---|---|
| Aspartame | 100 |
| Saccharin | 550 |
| Sucrose | 600 |
| Autane | 2000 |

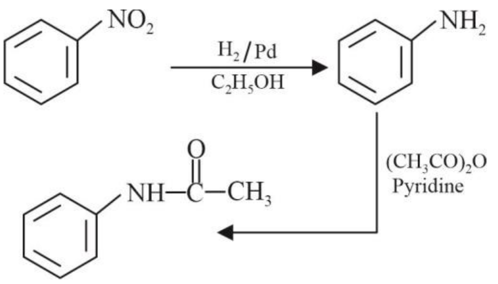
**17.(2)** $CoCl_2 \xrightarrow{H_2O} \underset{\substack{\text{Pink} \\ (X)}}{\left[Co(H_2O)_6\right]^{2+}} \xrightarrow{\text{conc HCl}} \underset{\substack{\text{Blue}(Y) \\ \text{Tetrahedral}(Z)}}{[CoCl_4]^{2-}}$

**18.(3)** Liquation and hydraulic washing are not involved in concentration of ore.

**19.(4)** $\underset{(60)}{Nd} \longrightarrow \underset{4f^4 6s^2}{\text{Neodymium}}$

$Nd^{2+} : 4f^4$

**20.(2)**




## SECTION – 2

**21.(6)** $Pd^{2+} + 4Cl^- \rightleftharpoons PdCl_4^{2-}$

$Pd^{2+}_{(aq)} + 2e^- \rightleftharpoons Pd_{(s)} \quad E° = 0.83V$

$Pd_{(s)} + 4Cl^-_{(aq)} \rightleftharpoons PdCl_4^{2-} + 2e^- \quad E^\circ = -0.65V$

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$Pd^{2+} + 4Cl^- \rightleftharpoons PdCl_4^{2-}; \quad E^\circ = 0.83 - 0.65 = 0.18V$

$\Delta G° = -nFE^\circ = -RT \ln K_{eq}$

$2 \times F \times 0.18 = 2.303RT \log K_{eq}$

$2 \times 0.18 = \frac{2.303RT}{F} \log K_{eq}$

$2 \times 0.18 = 0.06 \log K_{eq}$

$\log K_{eq} = 6$

**22.(555)** $n_x = \frac{0.6}{20} = 0.03$

$$n_y = \frac{0.45}{20} = 0.01$$

Mole fraction of X $= \frac{0.03}{0.04} = 0.75$

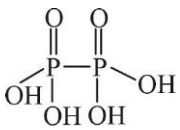
Mole fraction of Y $= \frac{0.01}{0.04} = 0.25$

$$P_x = 0.75 \times 740 = 555 \text{ mm of Hg}$$

**23.(62250)** $\pi = \frac{n}{V} RT$

$$400 \times 10^{-5} = \frac{2.5 \times 1000 \times 0.083 \times 300}{M \times 250} = 62,250$$

**24.(4)**




Oxidation state of phosphorus $= +4$

**25.(2520)**

$$\log \frac{k_2}{k_1} = \frac{E_a}{2.3}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\log\left(\frac{0.05}{0.03}\right) = \frac{E_a}{2.3}\left[\frac{1}{200} - \frac{1}{300}\right]$$

$$\log\left(\frac{5}{3}\right) = \frac{E_a}{2.3} \times \frac{1}{600}$$

$$\log 5 - \log 3 = \frac{E_a}{2.3 \times 600}$$

$$E_a = 2520$$

**26.(44)** Organic compound + $O_2 \longrightarrow CO_2$

0.492g 0.792g

Moles of $CO_2 = \frac{0.792}{44} = 0.018$

Moles of 'C' = 0.018 mole

Mass of 'C' $= 0.018 \times 12$

% of carbon $= \frac{0.018 \times 12}{0.492} \times 100 \simeq 44\%$

**27.(4)** $Zn + 2HCl \longrightarrow ZnCl_2 + H_2$

11.5 g

= 0.175 moles

$$\frac{n_{Zn}}{1} = \frac{n_{H_2}}{1}$$

Moles of $H_2 = 0.175$

Volume of $H_2$ at S.T.P = 0.175 $\times 22.7 \simeq 4$

**28.(1)** $SO_{2(g)} + \frac{1}{2}O_{2(g)} \rightleftharpoons SO_{3(g)}$

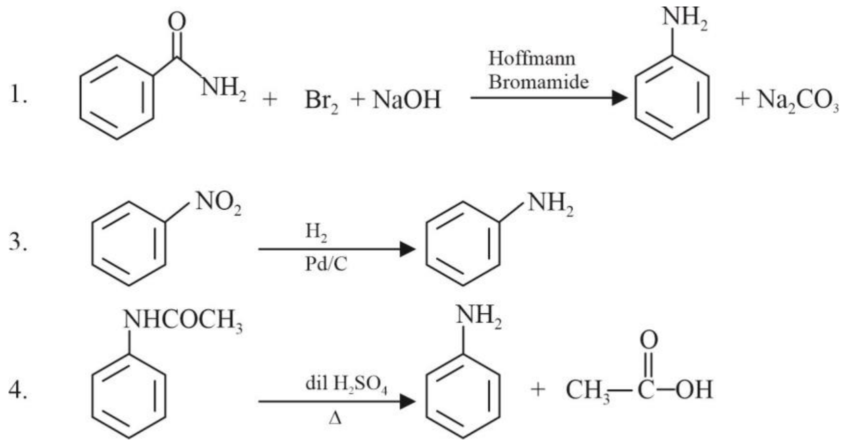
$$\Delta n_g = -\frac{1}{2}$$

$$K_p = K_c (RT)^{\Delta n_g}$$

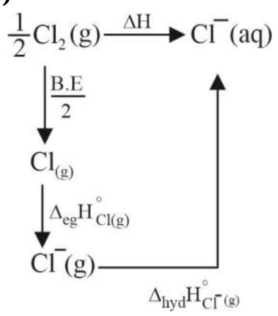
$$2 \times 10^{12} = K_c (0.082 \times 300)^{-1/2}$$

$$K_c = 1 \times 10^{13}$$

**29.(3)**




**30.(610)**




$$\Delta H = \frac{240}{2} - 350 - 380 = 610\,kJ$$

# MATHEMATICS

## SECTION - 1

**1.(1)** $\vec{a} = 2\hat{i} + j + k$ , $|\vec{b}| \neq 0, |\vec{c}| \neq 0$

$|\vec{a}+\vec{b}+\vec{c}| = |\vec{a}+\vec{b}-\vec{c}|$, $\vec{b}.\vec{c} = 0$

$2\vec{a}.\vec{b}\hat{i} + \vec{b}.\vec{c}\hat{i} + \vec{a}.\vec{c}\hat{i} = 2\vec{a}.\vec{b} - 2\vec{b}.\vec{c} - 2\vec{a}.\vec{c}$

$4\vec{b}.\vec{c} + 4\vec{a}.\vec{c} = 0$

$\vec{a}.\vec{c} = 0$

So $\vec{a}$ and $\vec{c}$ are perpendicular to each other.

Since $\vec{a}$ and $\vec{c}$ are perpendicular to each other so $|\vec{a}+\lambda\vec{c}| \geq |a|$ *for all* $\lambda \in R$

**2.(2)** $\beta = \log_e(1-\alpha)$. $P_n(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + ...... + \frac{x^n}{n}$, $x \in (0,1)$

$$\int_0^\alpha \frac{t^{50}}{1-t}dt = \int_0^\alpha \frac{t^{50}-1+1}{1-t} = \int_0^\alpha \left(\frac{t^{50}-1}{1-t} + \frac{1}{1-t}\right)dt$$

$$\int_0^\alpha \left\{-\left(1+t+t^2+.....+t^{49}\right) + \frac{1}{1-t}\right\}dt$$

$$\left[-\left(t + \frac{t^2}{2} + \frac{t^3}{3} + ..... + \frac{t^{50}}{50}\right) - \log_e(1-t)\right]_0^\alpha$$

$$-\left(\alpha + \frac{\alpha^2}{2} + \frac{\alpha^3}{3} + ..... + \frac{\alpha^{50}}{50}\right) - \log_e(1-\alpha) = -(\beta + P_{50}\alpha)$$

**3.(2)** $|z| = 4 \Rightarrow x = 4\cos\theta, y = 4\sin\theta$

$$u + iv = (4\cos\theta + 4i\sin\theta) + \frac{1}{4(\cos\theta + i\sin\theta)}$$

$$u + iv = (4\cos\theta + i\sin\theta) + \frac{1}{4}(\cos\theta - i\sin\theta)$$

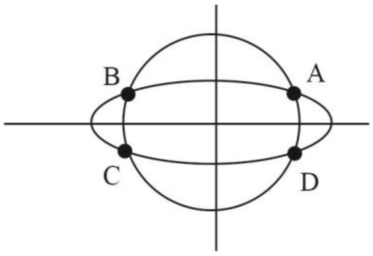
$$u + iv = \left(4 + \frac{1}{4}\right)\cos\theta + i\sin\theta\left(4 - \frac{1}{4}\right)$$

$$u + iv = \frac{17}{4}\cos\theta + \frac{15}{4}i\sin\theta$$

$$u = \frac{17}{4}\cos\theta, v = \frac{15}{4}\sin\theta$$

$$\frac{4u}{17} = \cos\theta, \frac{4v}{15} = \sin\theta$$

$$\frac{16u^2}{17^2}+\frac{16v^2}{15^2}=1$$




$$\frac{u^2}{\frac{17^2}{16}}+\frac{v^2}{\frac{15^2}{16}}=1$$

So the locus of $z+\frac{1}{z}$ is $\frac{x^2}{\frac{17^2}{16}}+\frac{y^2}{\frac{15^2}{16}}=1 \Rightarrow \frac{x^2}{\left(\frac{17}{4}\right)^2}+\frac{y^2}{\left(\frac{15}{4}\right)^2}=1$

So the two curves intersect at a four distinct point

**4.(4)** $\sin^{-1}\frac{\alpha}{17}=\tan^{-1}\frac{77}{36}-\cos^{-1}\frac{4}{5}$, Let $\tan^{-1}\frac{77}{36}=a,\ area\cos^{-1}\frac{4}{5}=b$

$$\sin^{-1}\frac{\alpha}{17}=(a-b)$$

$$\frac{\alpha}{17}=\sin(a-b)$$

$$\frac{\alpha}{17}=\sin a.\cos b-\cos a\sin b$$

$$\frac{\alpha}{17}=\frac{77}{65}\times\frac{4}{5}-\frac{36}{65}\times\frac{3}{5}$$

$$\alpha=\frac{3.8}{25}-\frac{1.8}{25}$$

$$\alpha=8$$

Now $\sin^{-1}(\sin 8)+\cos^{-1}(\cos 8)$

$$3\pi-8+8-2\pi=\pi$$

**5.(4)** $(p\rightarrow q)\vee(p\wedge(\sim q))$

$(\sim p\vee q)\vee(p\wedge(\sim q))$ $\quad\because\ p\rightarrow q\equiv\ \sim p\vee q$

$(\sim p\vee q\vee p)\wedge(\sim p\vee q\vee\sim q)$

$(\sim p\vee p\vee q)\wedge(\sim p\vee q\vee\sim q)$

$(T\vee q)\wedge(\sim p\vee T)$

$T\wedge T\equiv T$ (Tautology)

$S2:(\sim p\rightarrow\sim q)\wedge((\sim p)\vee q)$

$(\sim(\sim p)\vee\sim q)\wedge((\sim p)\vee q)$

$(p\vee\sim q)\wedge((\sim p)\vee q)$

$(\sim q\vee p)\wedge(\sim p\vee q)$

$(q\to p)\wedge(p\to q)$

$p\Leftrightarrow q$ (Not a contradiction)

So S1 is correct and S2 is false.

**6.(3)** Event A = Two drawn balls are black.

Event B = At least five balls in the bag are black.

$$P(B/A)=\frac{P(A\cap B)}{P(A)}$$

$$P(B)=\frac{{}^{2}C_2}{{}^{6}C_2}+\frac{{}^{3}C_2}{{}^{6}C_2}+\frac{{}^{4}C_2}{{}^{6}C_2}+\frac{{}^{5}C_2}{{}^{6}C_2}+\frac{{}^{6}C_2}{{}^{6}C_2}=\frac{{}^{7}C_3}{{}^{6}C_2}$$

$$P(A\cap B)=\frac{{}^{5}C_2}{{}^{6}C_2}+\frac{{}^{6}C_2}{{}^{6}C_2}=\frac{25}{15}$$

Required point $=\frac{25}{35}=\frac{5}{7}$

**7.(1)** $$A=\begin{bmatrix}1&0&0\\0&4&-1\\0&12&-3\end{bmatrix}$$

$$A^2=\begin{bmatrix}1&0&0\\0&4&-1\\0&12&-3\end{bmatrix}\begin{bmatrix}1&0&0\\0&4&-1\\0&12&-3\end{bmatrix}=\begin{bmatrix}1&0&0\\0&4&-1\\0&12&-3\end{bmatrix}=A$$

$$A^3=A,\ A^4=A.........A^{11}=A$$

$$(A+I)^{11}=A\left({}^{11}C_0+{}^{11}C_1+{}^{11}C_2+......+{}^{11}C_{10}\right)+{}^{11}C_{11}I\ =A\left(2^{11}-1\right)+I$$

$$=1.\left(2^{11}-1\right)+4\left(2^{11}-1\right)-3\left(2^{11}-1\right)+3=2\times2^{11}+1=4097$$

**8.(1)**


Let $l_1$ be the perimeter of square and $l_2$ be the perimeter of circle.

$$l_1+l_2=20\Rightarrow l_1=20-l_2\Rightarrow\frac{dl_1}{dl_2}=-1$$

Let $S=2A_1+3A_2$

$$S=2\left(\frac{l_1}{4}\right)^2+3(\pi)\left(\frac{l_2}{2\pi}\right)^2$$

$$S=\left(\frac{l_1}{8}\right)^2+\frac{3}{4\pi}(l_2)^2$$

Now $\frac{dS}{dl_2}=\frac{2l_1}{8}.\frac{dl_1}{dl_2}+\frac{3}{4\pi}.2l_2$

For critical point $\frac{dS}{dl_2}=0$

$$\frac{l_1}{4}=\frac{3}{2\pi}l_2$$

$$\frac{l_1}{l_2}=\frac{6}{\pi}$$

$$\frac{l_1\pi}{l_2}=6:1$$

**9.(3)** $\vec{b}_1=-2\hat{i}+0j+k$

$\vec{b}_2=\hat{i}+j-k$

$\vec{a}_1=5\hat{i}+\lambda j-\lambda k$

$\vec{a}_2=-\hat{i}+j+4k$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix} i & j & k \\ -2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}=-\hat{i}-j-2k$$

Now $\vec{a}_1-\vec{a}_2=6\hat{i}+(\lambda-1)j-(\lambda+4)k$

S.D = $\frac{(\vec{a}_1-\vec{a}_2).(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}$

$$2\sqrt{6}=\left|\frac{-6-\lambda+1+2\lambda+8}{\sqrt{1+1+4}}\right|$$

$\lambda=-15,9$

$$\frac{\alpha-5}{-2}=\frac{\beta-\lambda}{0}=\frac{\gamma+\lambda}{1}=l$$

$\alpha=-2l+5$

$\beta=\lambda,\ \gamma=l-\lambda$

$\alpha+2\gamma=5-2\lambda$

$\alpha+2\gamma=35\ or\ -13$

**10.(4)** $a^4 = 1296, a = 6$

$$\frac{6}{r^3} + \frac{6}{r} + 6r + 6r^3 = 126$$

$$\frac{1}{r^3} + \frac{1}{r} + r + r^3 = 21$$

$$\left(\frac{1}{r^3} + r^3\right) + \left(\frac{1}{r} + r\right) = 21$$

$$\left(\frac{1}{r} + r\right) + \left(\frac{1}{r^2} + r^2 + 1\right) + \left(\frac{1}{r} + r\right) = 21$$

$$\left(\frac{1}{r} + r\right) + \left(\frac{1}{r^2} + r^2\right) = 21$$

$$\left(\frac{1}{r} + r\right) + \left(\frac{1}{r^2} + r^2 + 2 - 2\right) = 21$$

$$\left(\frac{1}{r} + r\right) + \left(\left(r + \frac{1}{r}\right)^2 - 2\right) = 21$$

Let $r + \frac{1}{r} = t$

$t^3 - 2t - 21 = 0$

$t = 3$

Other two values of $t$ will be imaginary.

$$r + \frac{1}{r} = 3$$

So, the value of $r_1^2 + r_2^2 = (r_1 + r_2)^2 - 2r_1r_2 = 9 - 2 = 7$

$r^3 - 3r + 1 = 0$

$r_1 + r_2 = 3$

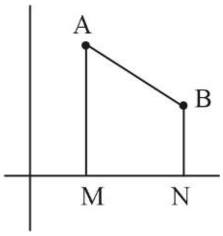
$r_1r_2 = 1$

**11.(4)** A be the centre of the rolled circle $C_1$ , B be the centre of the image of the circle $C_1$ in the tangent

Given Circle is $x^2 + y^2 - 4x - 6y + 11 = 0$, centre E $(2,3)$

Tangent at $(3,2)$ is $x - y - 1 = 0$

By using the parametric form, A $\left(2 + 2\sqrt{2}, 3 + 2\sqrt{2}\right)$ and $B\left(4 + 2\sqrt{2}, 1 + 2\sqrt{2}\right)$




AM $=3+2\sqrt{2}$, $BM=1+2\sqrt{2}$, $MN=2$

$\therefore$ Area of trapezium:

$$\frac{1}{2}\left(4+4\sqrt{2}\right)2=4\left(1+\sqrt{2}\right)$$

**12.(3)** Domain $[2,6)$

$$f(x)=\frac{2}{1+x^2}\ for\ x\in[2,3]\rightarrow\text{Decreasing}\rightarrow\text{Range}\left(\frac{1}{5},\frac{2}{5}\right]$$

$$f(x)=\frac{3}{1+x^2}\ for\ x\in[3,4]\rightarrow\text{Decreasing}\rightarrow\left(\frac{3}{17},\frac{3}{10}\right]$$

$$f(x)=\frac{4}{1+x^2}\ for\ x\in[4,5]\rightarrow\text{Decreasing}\rightarrow\left(\frac{4}{25},\frac{4}{17}\right]$$

$$f(x)=\frac{5}{1+x^2}\ for\ x\in[5,6]\rightarrow\text{Decreasing}\rightarrow\left(\frac{5}{37},\frac{5}{26}\right]$$

So range is $\rightarrow\left(\frac{5}{37},\frac{2}{5}\right]$

**13.(2)** Equation of normal can be written as $2x\sec\theta-by\cos ec\theta=4-b^2$

Let P be the distance from the origin

$$P=\left|\frac{b^2-4}{\sqrt{4\sec^2\theta+b^2\cos ec^2\theta}}\right|$$

We have to maximise P, so we will make

$4\sec^2\theta+b^2\cos ec^2\theta$ as minimum

Let $l=4\sec^2\theta+b^2\cos ec^2\theta$

$$\frac{dl}{d\theta}=8\sec^2\theta.\tan\theta-2b^2\cos ec^2\theta.\cot\theta=0$$

$$\tan^2\theta=\frac{b}{2}$$

So the maximum value is

$$\left|\frac{4-b^2}{\sqrt{4\left(1+\frac{b}{2}\right)+b^2\left(1+\frac{2}{b}\right)}}\right|$$

$$\left|\frac{4-b^2}{\sqrt{2(b+2)+b(2+b)}}\right|=1$$

$$b=1$$

So ellipse is $\frac{x^2}{4}+\frac{y^2}{1}=1,\ e=\frac{\sqrt{3}}{2}$ by using $b^2=a^2\left(1-e^2\right)$

**14.(2)** $y=\sin^3\left(\pi/3\cos g(x)\right)$

. $$p(x)=\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{3/2}$$

$$y'(x)=3\sin^2\left(\frac{\pi}{3}\cos p(x)\right)\times\cos\left(\frac{\pi}{3}\cos p(x)\right)\times\frac{\pi}{3}\left(-\sin p(x)\right)p'(x) \qquad g(1)=\frac{2\pi}{3}$$

$$p(1)=\frac{2\pi}{3}$$

$$y'(1)=3\sin^2\left(\frac{\pi}{3}\cos p(1)\right)\times\cos\left(\frac{\pi}{3}\cos p(1)\right)\times\frac{\pi}{3}\left(-\sin p(1)\right)p'(1)$$

$$y'(1)=3\sin^2\left(-\frac{\pi}{6}\right)\times\cos\left(-\frac{\pi}{6}\right)\times\frac{\pi}{3}\left(-\sin\frac{2\pi}{3}\right)p'(1)$$

$$p'(x)=\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{1/2}\left(-12x^2+10x\right)$$

$$p'(1)=\frac{\pi}{2\sqrt{2}}\left(\sqrt{2}\right)(-2)=-\pi$$

$$y'(1)=\frac{3}{4}\cdot\frac{\sqrt{3}}{2}\cdot\frac{\pi}{3}\left(\frac{-\sqrt{3}}{2}\right)(-\pi)=\frac{3\pi^2}{16}$$

$$y(1)=\sin^3\left(\pi/3\cos 2\pi/3\right)=-\frac{1}{8}$$

$$2y'(1)+3\pi^2y(1)=0$$

**15.(1)** $$\int_{\pi/3}^{\pi/2}\frac{2+3\sin x}{\sin x(1+\cos x)}dx=\underbrace{\int_{\pi/3}^{\pi/2}\frac{2}{\sin x(1+\cos x)}dx}_{I_1}+\underbrace{\int_{\pi/3}^{\pi/2}\frac{3}{(1+\cos x)}dx}_{I_2}$$

$$I_1=2\int_{\pi/3}^{\pi/2}\frac{dx}{\sin x(1+\cos x)}=\int_{\pi/3}^{\pi/2}\frac{\left(1+\tan^2\frac{x}{2}\right).\sec^2\frac{x}{2}dx}{2\tan\frac{x}{2}},\ \text{Let } \tan\frac{x}{2}=t,\sec^2\frac{x}{2}.\frac{dx}{2}=dt$$

$$I_1=\int_{1/\sqrt{3}}^{1}\frac{1-t^2}{t}dt=\left[\ln t+\frac{t^2}{2}\right]_{1/\sqrt{3}}^{1}\rightarrow\left(\frac{1}{3}+\ln\sqrt{3}\right)$$

$$I_2=\int_{\pi/3}^{\pi/2}\frac{3}{1+\cos x}dx=\int_{\pi/2}^{\pi/2}\frac{3(1-\cos x)\,dx}{\sin^2 x}=3\int_{\pi/3}^{\pi/2}\left(\cos ec^2 x-\cot x\times\cos ecx\right)dx$$

$$3\left[-\cot x+\cos ecx\right]_{\pi/3}^{\pi/2}=3\left[1-\frac{1}{\sqrt{3}}\right]$$

$$I_1+I_2=\frac{10}{3}-\sqrt{3}+\ln\sqrt{3}$$

**16.(2)** $ad(b-c)=bc(a-d)$

$$\frac{b-c}{bc}=\frac{a-d}{a.d}\Rightarrow\frac{1}{c}-\frac{1}{b}=\frac{1}{d}-\frac{1}{a}\qquad\Rightarrow\frac{1}{a}-\frac{1}{b}=\frac{1}{d}-\frac{1}{c}$$

$(a,b)R(a,b)\Rightarrow\frac{1}{a}-\frac{1}{b}=\frac{1}{b}-\frac{1}{a}$ is false $\qquad\Rightarrow R$ is not reflexive

$(a,b)R(c,d)\Rightarrow(c,d)R(a,b)$

$\Rightarrow\frac{1}{a}-\frac{1}{b}=\frac{1}{d}-\frac{1}{c}\Rightarrow\frac{1}{c}-\frac{1}{d}=\frac{1}{b}-\frac{1}{a}$ is True $\qquad\Rightarrow$R is symmetric

$(a,b)R(c,d)$ and $(c,d)R(e,f)$

i.e., $\frac{1}{a}-\frac{1}{b}=\frac{1}{d}-\frac{1}{c}$ and $\frac{1}{c}-\frac{1}{d}=\frac{1}{f}-\frac{1}{e}$

R is symmetric but neither reflexive nor transitive.

**17.(2)** $\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$

$$\sqrt{(x-3)(x-1)}+\sqrt{(x-3)(x+3)}=\sqrt{(x-3)(4x-2)}$$

$$x-3=0\Rightarrow x=3 or \sqrt{x-1}+\sqrt{x+3}=\sqrt{4x-2}$$

$$\Rightarrow x-1+x+3+2\sqrt{x^2+2x-3}=(4x-2)\qquad\Rightarrow 2\sqrt{x^2+2x-3}=2x-4$$

$$\Rightarrow\sqrt{x^2+2x-3}=x-2\qquad\Rightarrow x^2+2x-3=x^2-4x+4\quad\Rightarrow 6x=7\Rightarrow x=\frac{7}{6}$$

Which is not valid.

$\Rightarrow x=3$ is the only solution.

**18.(4)** $\Delta=\begin{vmatrix}1&1&1\\\alpha&\beta&7\\1&2&3\end{vmatrix}=2\beta-\alpha-7$

$$\Delta_x=\begin{vmatrix}6&1&1\\3&\beta&7\\14&2&3\end{vmatrix}=4\beta+11$$

$$\Delta_y = \begin{vmatrix} 1 & 6 & 1 \\ \alpha & 3 & 7 \\ 1 & 14 & 3 \end{vmatrix} = -4\alpha - 50$$

$$\Delta_z = \begin{vmatrix} 1 & 1 & 6 \\ \alpha & \beta & 3 \\ 1 & 2 & 14 \end{vmatrix} = 8\beta - 2\alpha - 3$$

If $\alpha = \beta = 7 \Rightarrow \Delta = 0$ and $\Delta_x, \Delta_y, \Delta_z \neq 0$

$\Rightarrow$ No solution

If $\alpha = -\frac{50}{4}$ and $\beta = -\frac{11}{4} \Rightarrow \Delta, \Delta_x, \Delta_y$ and $\Delta_z = 0 \Rightarrow \infty$ solution

**19.(3)** The parabola $\left(x + \frac{1}{2}\right)^2 + y^2 = \left(y + \frac{1}{y}\right)^2$

$\Rightarrow y = x^2 + x \Rightarrow f(x) = x^2 + x$ and $\tan^{-1}\sqrt{f(x)} + \sin^{-1}\sqrt{f(x) + 1} = \frac{\pi}{2}$

$\Rightarrow f(x) = 0 \Rightarrow x^2 + x = 0 \qquad \Rightarrow x = 0, x = -1$ two solution

**20.(2)** $f(x) + \int_3^x \frac{f(t)}{t} dt = \sqrt{x+1} \Rightarrow f(3) = 2$

Applying Leibnitz theorem

$$\Rightarrow f'(x) + \frac{f(x)}{x} = \frac{1}{2}\frac{1}{\sqrt{x+1}}$$

$$xf'(x) + f(x) = \frac{1}{2}\frac{x}{\sqrt{x+1}}$$

$$\frac{d}{dx}(x.f(x)) = \frac{1}{2}\left((x+1)^{1/2} - (x+1)^{-1/2}\right)$$

$$\Rightarrow x.f(x) = \frac{1}{3}(x+1)^{3/2} - (x+1)^{1/2} + c$$

Put $x = 3 \Rightarrow 3.f(x) = \frac{1}{3} \times 8 - 2 + c \Rightarrow c = \frac{16}{3}$

Now put $x = 8$

$$8.f(8) = \frac{1}{3} \times 27 - 3 + c = 6 + \frac{16}{3} = \frac{34}{3}$$

$$\Rightarrow 12 f(8) = 12 \times \frac{34}{3} \times \frac{1}{8} = 17$$

## SECTION - 2

**21.(9)** $5^5 = 3125 = 11 \times 284 + 1$

$5^{99} = 5^4\left(5^5\right)^{19} = 625[11 \times 284 + 1]^{19}$

$= 625$ (multiple of 11 $+1^{19}$) = multiple of $11 + 625$ = multiple of $11 + 9$

$\Rightarrow$ Remainder $= 9$

**22.(2)** General term $= {}^{30}C_r\left(x^{2/3}\right)^{30-r}\left(\frac{2}{x^3}\right)^r \Rightarrow {}^{30}C_r\, 2^r\, x^{\left(20-\frac{11}{3}r\right)}$

At $r = 6;\ 20 - \frac{11}{3}r = -2 \Rightarrow \alpha = 2$

**23.(2997)** Number starting with 2 $= 1 \times 6^4 = 1296$

Number starting with 3 $= 1 \times 6^4 = 1296$

Number starting with 40 $= 6^3 = 216$

Number starting with 420 or 422 or 423 or 424 or 427 $= 5 \times 6^2 = 180$

Number starting with $4290 = 6$

Then appears 42920, 42922, 42923

So the position of 42923 is $= 1296 + 1296 + 216 + 180 + 6 + 1 + 1 + 1 = 2997$

**24.(710)** $A = \{1002, 1005, .......2799\} = 600$ numbers

$B = \{1001, 1012, ......, 2794\} = 164$ numbers

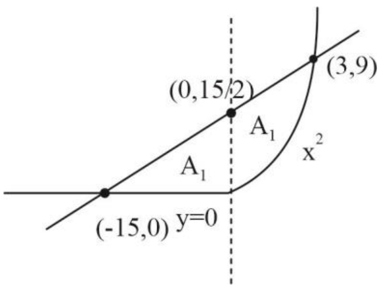
$C = \{1023, 1056, ......, 2772\} = 54$ numbers

Common multiples of 3 and $11 = 600 + 164 - 54 = 710$

**25.(72)** $f(x)\frac{x+|x|}{2},\ g(x) = \begin{cases} x & x < 0 \\ x^2 & x \ge 0 \end{cases}$

$x \ge 0, \quad f(g(x)) = \frac{x^2 + |x^2|}{2} = x^2$

$x < 0, \quad f(g(x)) = \frac{x+|x|}{2} = 0$




Required Area $A_1 + A_2 = \frac{1}{2} \times 15 \times \frac{15}{2} + \int_0^3 \left(\frac{x}{2} + \frac{15}{2} - x^2\right) dx = 72$

**26.(36)** $|a \times b|^2 + \left(\vec{a}.\vec{b}\right)^2 = |a|^2.|b|^2$

$48 + (a.b)^2 = 14 \times 6$

$(a.b)^2 = 84 - 48 = 36$

**27.(8)** $a_5 = 2a_7 \Rightarrow a + 4d = 2(a+6d)$

$\Rightarrow a = -8d$

And $a_{11} = a + 10d = 2d = 18 \Rightarrow d = 9$

Now $S = 12\left(\frac{1}{\sqrt{a_{10}}+\sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}}+\sqrt{a_{12}}} .........\frac{1}{\sqrt{a_{17}}+\sqrt{a_{18}}}\right)$

$$S = 12\left(\frac{\sqrt{a_{11}}-\sqrt{a_{10}}}{d} + \frac{\sqrt{a_{12}}-\sqrt{a_{11}}}{d} ......\frac{\sqrt{a_{18}}-\sqrt{a_{17}}}{d}\right)$$

$$S = \frac{12}{d}\left(\sqrt{9d}-\sqrt{d}\right) = \frac{24}{\sqrt{d}}$$

**28.(5)** $\sigma^2 = \frac{1}{n}\sum f_i (x_i)^2 - (\bar{x})^2 \; = \frac{12+54+256+25\alpha+324+245+384}{45+\alpha}$

$$-\left(\frac{6+18+64+5\alpha+54+35+48}{45+\alpha}\right)^2$$

$$3 = \left(\frac{1275+25\alpha}{45+\alpha}\right) - \left(\frac{225+5\alpha}{45+\alpha}\right)^2$$

$$3 = \frac{1275+25\alpha}{45+\alpha} - 25$$

$$28\alpha + 1260 = 25\alpha + 1275$$

$$3\alpha = 15 \Rightarrow \alpha = 5$$

**29.(9)** $\cos\theta = \frac{2-1+2}{\sqrt{6}\sqrt{6}} = \frac{1}{2}$

$\theta = 60°$ *and* $\alpha = 30° \Rightarrow \tan^2\theta.\cot^2\theta = 3\times 3 = 9$

**30.(180)** The line L $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1}$

Point P $(2\lambda+1, -\lambda-1, \lambda+3)$ of the line L

Lies in the plane $2x + y + 3z = 16$

$\Rightarrow 4\lambda + 2 - \lambda - 1 + 3\lambda + 9 = 16 \;\Rightarrow \lambda = 1 \Rightarrow P = (3,-2,4)$

Let the point $Q = (2\mu+1, -\mu-1, \mu+3)$ in the foot of perpendicular from $R(1,-1,-3)$

Direction ratio of RQ in $(2\mu, -\mu, \mu+6)$ is perpendicular to the direction ratio of line $(2,-1,1)$

$\Rightarrow 4\mu + \mu + \mu + 6 = 0,\; \mu = -1 \quad \Rightarrow Q = (-1,0,2)$

Now the Area of triangle PQR is $= \frac{1}{2}\left|\overrightarrow{RP}\times\overrightarrow{RQ}\right|$

$$\alpha = \frac{1}{2}\left|\begin{vmatrix} i & j & k \\ 2 & -1 & 7 \\ -2 & 1 & 5 \end{vmatrix}\right|$$

$$\alpha = \frac{1}{2}\left|-12\hat{i} + 24\hat{j}\right| = \left|-6\hat{i} + 12\hat{j}\right| = 6\sqrt{5} \;\Rightarrow \alpha^2 = 180$$