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IIT JEE | MEDICAL | FOUNDATION

SOLUTIONS

Joint Entrance Exam | IITJEE-2023

30th JAN 2023 | Morning Shift

PHYSICS

SECTION - 1

1.(1) $F_m = BiL = mg$

$i = \frac{V}{R}$ $\therefore$ $V = \frac{mgR}{BL}$

Or $V = \frac{\left(10^{-3}\times 10\right)\times(10)}{\left(10^{3}\times 10^{-4}\right)\left(10^{-1}\right)} volts$

Or $V = 10 volts$

2.(3) $\vec{F} = \frac{d\vec{p}}{dt}$

$\therefore$ $\left|\vec{F}\right|_{\max}$ in region C $\therefore$ $\left|\vec{F}\right|_{\min}$ is in region b.

3.(4) Unit of $\frac{A}{x^2}$ is N/C $\therefore$ unit of A is $\frac{N.m^2}{C}$

Similarly, unit of $\frac{B}{y^3}$ is N/C $\therefore$ Unit of B is $\frac{N-m^3}{C}$

4.(1) $P_1 = \cos\theta = \frac{R}{Z} = \frac{R}{\sqrt{2}R} = \frac{1}{\sqrt{2}}$

$P_2 = \cos\theta' = \frac{R}{R} = 1$ $\therefore$ $\frac{P_1}{P_2} = \frac{1}{\sqrt{2}}$

5.(2) $V_n = \frac{2\pi kze^2}{nh}$

$V_n \propto \frac{z}{n}$

$\frac{V_3}{V_7} = \frac{7}{3}$

$\therefore$ $V_3 = \frac{3.6\times 10^6}{3}\times 7\, m/s$ Or $V_3 = 8.4\times 10^6\, m/s$

6.(1) $\sigma = \frac{3k-2\eta}{6k+2\eta}$

7.(4) $\frac{1}{V} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{V} - \frac{100}{-25} = 2$

$\frac{1}{V} = -2$

$V = -\frac{1}{2}$ or $0.5m$

$|V| = 50 cm$

8.(1) $Q_1 = 4\pi\sigma R^2$

$Q_2 = 16\pi\sigma R^2$

$Q_1^{'} + Q_2^{'} = Q_1 + Q_2$

$Q_2^{'} = 2Q_1^{'}$

$\therefore \quad Q_2^{'} = \frac{2}{3}(Q_1 + Q_2)$

$\sigma_2^{'} = \frac{40/3\pi R^2\sigma}{16\pi R^2} = \frac{40}{48}\sigma$

$\therefore \quad \frac{\sigma_2^{'}}{\sigma} = 5/6$

9.(2) $A_s = \frac{\mu A_c}{2}$

$\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} = \frac{120-80}{120+80} = \frac{1}{5}$

$A_s = \frac{1}{5} \times \frac{100}{2} = 10V$

10.(1) A-(II)

Velocity is increasing and positive.

B-(IV)

Velocity is negative and magnitude is decreasing.

C-(III)

Velocity is positive constant initially and then negative constant.

D-(I)

Velocity is a positive constant.

11.(1) Force by absorbing light pulse $'F' = \frac{P}{c}$

Momentum acquired: $p = F\Delta t$

$= \frac{P\Delta t}{c} = \frac{20\times10^{-3}\times300\times10^{-9}}{3\times10^{8}} = 2\times10^{-17}\, kg\, m/\text{sec}$

12.(2) Potential difference: $\Delta V = K\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$ as field is given as $-K/r^2$

$\Rightarrow \quad V_{@r=2} - V_{@r=3} = 6\left[\frac{1}{3} - \frac{1}{2}\right]$

$10 - V_{@r=3} = -1$

$\Rightarrow \quad V_{@r=3} = 11$

13.(4) It is OR gate.

14.(4) $h = \frac{2T\cos\theta}{R\rho g}$

Since surface tension and density of liquid B is double of liquid A, so height remain same.

15.(1) Charge flown : $Q = \alpha t - \beta t^2 + \gamma t^3$

Current = $i = \frac{dQ}{dt}$

$i = \alpha - 2\beta t + 3\gamma t^2$

For i to be minimum, $\frac{di}{dt} = 0$

$\Rightarrow \quad -2\beta + 6\gamma t = 0$

$t = \frac{\beta}{3\gamma}$

$\Rightarrow \quad i_{\min} = \alpha - 2\beta.\frac{\beta}{3\gamma} + 3\gamma.\frac{\beta^2}{\gamma^2} = \alpha - \frac{\beta^2}{3\gamma}$

16.(2) Since $PT^2 = const.$

Using Ideal gas equation, $P = \frac{nRT}{V}$

$\Rightarrow \quad \frac{nRT}{V}T^2 = const \quad \Rightarrow \quad V = CT^3$

$\frac{dV}{dT} = 3CT^2$

$\Rightarrow \quad \gamma = \frac{1}{V}\frac{dV}{dT} = \frac{3}{T}$

17.(3) $r_A = 10\,cm \qquad r_B = 20\,cm = 2r_A$

$\mu_A = \mu_B$

$\Rightarrow \quad N_A I_A A_A = N_B I_B A_B \quad \Rightarrow \quad N_A I_A = 4N_B I_B$

18.(1) Let initial velocity of bullet be v_0.

Since bullet and ball has velocity horizontally, after collision, time of fall for each : $t = \sqrt{\frac{2h}{g}}$

$t = \sqrt{\frac{2\times 20}{10}} = 2\sec$

Velocity of ball after collision, $v_b = \frac{30}{2} = 15\,m/s$

Velocity of bullet after collision, $v_t = \frac{120}{2} = 60\,m/s$

By conservation of linear momentum, $10\times10^{-3}v_0 = 10\times10^{-3}\times60 + 200\times10^{-3}\times15$

$v_0 = 60 + 20\times15 = 360\,m/s$

19.(2) Velocity of ball interchanges as masses are equal and collision is elastic and head - On.

$V_P = \sqrt{2\times gR}$ before collision

$V_Q = \sqrt{2\times10\times0.2}$

$V_Q = 2\,m/s$

20.(4) Isothermal process $\Rightarrow$ T = constant.

$\Delta U = 0$

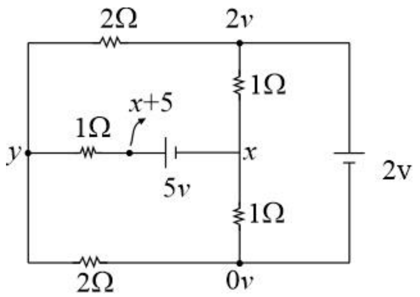
$\Rightarrow$ Internal energy is constant.

$\Rightarrow \quad Q = W = nRT_0 \ell n\left(\frac{V_2}{V_1}\right) \qquad \Rightarrow \quad Q = + \Rightarrow V_2 > V_1$

Gas will do positive work.

SECTION – 2

21.(2)



Net current at node 'x'

(A) $\frac{2-x}{1} + \frac{0-x}{1} + \frac{y-(x+5)}{1} = 0$

Net current at node 'y'

(B) $\frac{2-y}{2} + \frac{(x+5)-y}{1} + \frac{0-y}{2} = 0$

(A): $2 - x + 0 - x + y - x - 5 = 0$

$y - 3x - 3 = 0$ … (i)

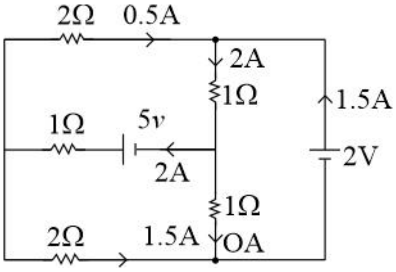
(B): $2 - \overline{y} + 2\overline{x} + 10 - 2y + 0 - \overline{y} = 0$

$2x - 4y + 12 = 0$

Or $x - 2y + 6 = 0$ … (ii)

Solving $x = 0$; $y = 3$

So current distribution will be like:



22.(220)

$L.C. = \left(\frac{0.5}{100}\right) mm$

$V.S.R = (46-6)\left(\frac{0.5}{100}\right) mm$

$M.S.R = (4 \times 0.5) mm$

$\therefore$ Measured value,

$M = (2 + 0.2) mm$

Or $M = 220 \times 10^{-2}\ mm$

23.(32) $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ $\quad f=\frac{uv}{u+v}$

$\therefore \quad \frac{dv}{v^2}+\frac{du}{u^2}=\frac{df}{f^2}$

$df=\left(\frac{u}{u+v}\right)^2 dv+\left(\frac{v}{u+v}\right)^2 du$

$df=\left(\frac{40}{160}\right)^2 \frac{1}{20}+\left(\frac{120}{160}\right)^2 \frac{1}{20}=\frac{1}{32}$

$\therefore \quad K=32\,cm$

24.(50) Total distance is S

$V_{av}=\frac{S}{t_1+t_2+t_3}$

$t_2=t_3=t_0$

Then $10t_0+15t_0=\frac{S}{2}$

Or $t_0=\frac{S}{50}$

$t_1=\frac{S}{10}$

$V_{av}=\frac{S}{\frac{S}{10}+\frac{S}{50}+\frac{S}{50}}=\frac{50}{7}m/s$

25.(4) $dF=\frac{2W}{\pi r^2}\left(\frac{dA}{c}\right)$

$dF\cos\theta=\frac{2W}{4\pi r^2 c}.(\cos\theta)$

$\therefore \quad$ Net force $F=\frac{W}{2c}$

$F=\frac{24}{2\times 3\times 10^8}N=4\times 10^{-8}N$

26.(3) Energy of Rod, $E=\frac{1}{12}I\omega^2$

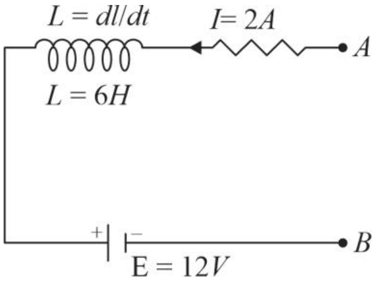
$=\frac{1}{24}(A\ell d)\ell^2\omega^2=\frac{1}{24}Ad(2)^3\omega^2$

$E=\frac{1}{3}Ad\omega^2$

$\Rightarrow \quad \omega=\sqrt{\frac{3E}{Ad}}$

$\alpha=3$

27.(30)



$$V_{AB} = V_R + V_L + V_E = +24 - 6 + 12 \quad = 30V$$

28.(10) Shift due to introduction of sheets $\Delta y = \frac{t[\mu_1 - \mu_2]D}{d}$

Fringe width, $W = \frac{\lambda D}{d}$

$\Rightarrow$ Shift by no. of fringes. $\frac{\Delta y}{W} = \frac{t[\mu_1 - \mu_2]D \times d}{d\lambda D}$

$$= \frac{t[\mu_1 - \mu_2]}{\lambda} = \frac{0.1\times10^{-3}\times(1.55-1.51)}{4000\times10^{-10}} = \frac{0.1\times10^{-3}\times0.04}{4000\times10^{-10}} = 10$$

29.(8) $x = A\sin\omega t$

$\Rightarrow$ $P.E.\ \ U = \frac{1}{2}m\omega^2x^2$

$$U = \frac{1}{2}m\omega^2A^2\sin^2\omega t$$

$$\frac{dU}{dt} = \frac{1}{2}m\omega^2A^2.2\sin\omega t\cos\omega t$$

Slope $\frac{dU}{dt} = \frac{1}{2}m\omega^2A^2\sin 2\omega t$

$\Rightarrow$ $2\omega t = \frac{\pi}{2}$ for slope to be maximum

$\Rightarrow$ $2\times\frac{2\pi}{T}\times t = \frac{\pi}{2}$

$$t = \frac{T}{8} \Rightarrow \beta = 8$$

30.(225)

$$U_i = \frac{1}{2}CV^2$$

After charged capacitor is connected to another uncharged capacitor; charge is equally shared as capacitors are identical.

$$U_f = \left[\frac{2}{2C}\left(\frac{CV}{2}\right)^2\right]2 = \frac{1}{4}CV^2$$

Loss in energy = $U_i - U_f = \frac{1}{4}CV^2 = \frac{1}{4}\times900\times10^{-6}\times10^4 = 225\times10^{-2}J$

$x = 225$

CHEMISTRY

SECTION - 1

1.(4) It converts FeO to $FeSiO_3$

2.(1) Aliphatic aldehydes give positive Fehling test.
In case, nitrogen and sulphur both are present in an organic compound, sodium thiocyanate is formed. It gives blood red colour and no Prussian blue since there are no free cyanide ions.

$$Na + C + N + S \longrightarrow NaSCN$$

$$Fe^{3+} + SCN^- \longrightarrow \underset{\text{Blood Red}}{[Fe(SCN)]^{2+}}$$

3.(1) Caprolactam on heating at high temperature gives Nylon-6 polymer.

4.(3) Ranitidine is used as the antacid.

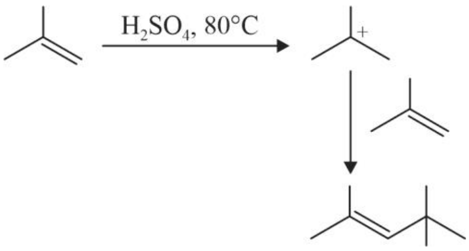
5.(1) Seliwanoff's test – It is used to differentiate between aldose and ketose, i.e., it helps to detect the presence of aldo sugar and keto sugar. Ketose gives deep cherry red colour with seliwanoff's reagent (mixture of resorcinol and concentrated HCl) and aldose gives fainty pink colour with seliwanoff's reagent.

6.(4) Solubility of sulphates of group-2 elements decreases down the group. $BeSO_4$ and $MgSO_4$ are appreciably soluble in water. $CaSO_4, SrSO_4$ and $BaSO_4$ are practically insoluble in water.

7.(2)

Atomic No.	Element	Block
37	Rb	s-block
78	Pt	d-block
52	Te	p-block
65	Tb	f-block

8.(2) Cold sulphuric acid reacts with alkene and forms alkyl hydrogen sulphate by electrophilic addition reaction. But in case of 80°C temperature sulphuric acid forms stable carbocation. Which attacks on another molecule of alkene.



9.(4) Filed strength order : $S^{2-} < C_2O_4^{2-} < NH_3 < en < CO$

10.(1)

Species	No. of lone pairs on central atom
IF_7	0
ICl_4^-	2
XeF_6	1
XeF_2	3

11.(3) (i) $NO_2 + \text{Sunlight} \longrightarrow NO + O$ (ii) $O + O_2 \longrightarrow O_3$ (iii) $NO + O_3 \longrightarrow NO_2 + O_2$

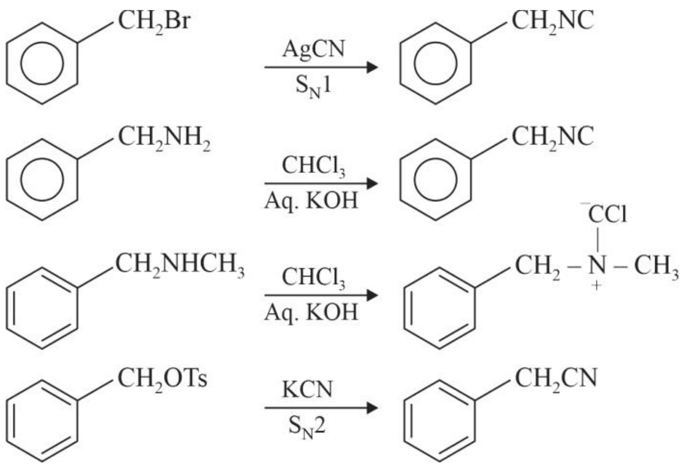
$A - NO$

$B - O$

$C - O_3$

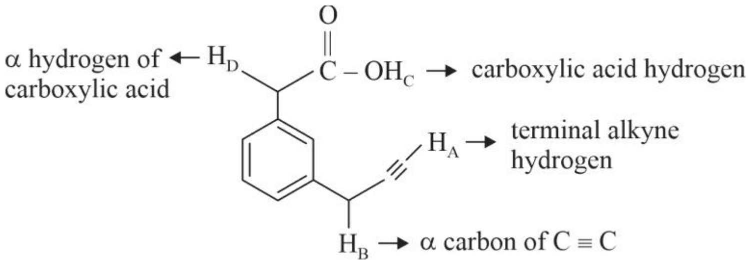
12.(1) Factual

13.(1)



14.(1) $4LiH + AlCl_3 \longrightarrow LiAlH_4 + 3LiCl$

15.(2) $H_C > H_D > H_A > H_B$



16.(2) $SO_3^{2-} + 2H^+ \longrightarrow SO_2 \uparrow + H_2O$

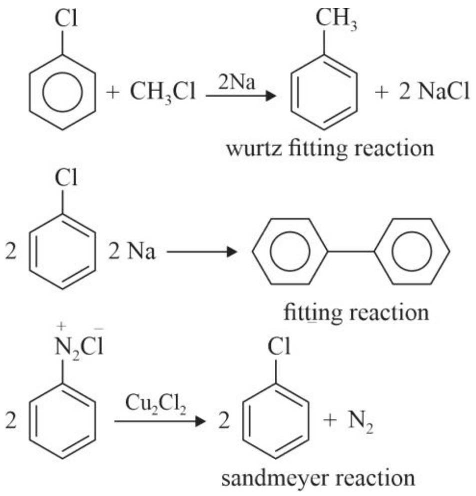
$$\underset{\text{orange}}{SO_2 + K_2Cr_2O_7} \xrightarrow{H^+} \underset{\text{green}}{Cr_2(SO_4)_3} + K_2SO_4 + H_2O$$

17.(2) Two bond pair and two lone pair : Hence geometry is tetrahedral and shape is ‘V’
No. of lone pair of electrons – 2
Repulsion order $\ell p - \ell p > \ell p - bp > bp - bp$
Bond angle decreases and shape becomes bent or ‘V’

18.(4) Group IV contains are $Ni^{+2}, Co^{+2}, Mn^{+2}, Zn^{+2}$

19.(3) Factual

20.(4) $A - II, B - I, C - IV, D - III$



$$\underset{}{C_2H_5Cl + NaI} \xrightarrow{\text{acetone}} C_2H_5I + NaCl$$
Finkelstein reaction

SECTION – 2

21.(1.86)

$$HCl \rightarrow M_1 = 0.01M$$
$$n = 1$$
$$N_1 = nM_1 = 1 \times 0.01 = 0.01N$$
$$V_1 = 600\,ml$$
$$H_2SO_4 \rightarrow n = 2$$
$$M_2 = 0.01$$
$$N_2 = 2 \times 0.01 = 0.02$$
$$V_2 = 400\,ml$$
$$V_f = V_1 + V_2 = 600 + 400 = 1000\,ml$$
$$N_f = \frac{N_1V_1 + N_2V_2}{V_f} = \frac{0.01 \times 600 + 0.02 \times 400}{1000}$$
$$\left[H^+\right] = N_f = \frac{6+8}{1000} = 14 \times 10^{-3}$$
$$pH = -\log\left[H^+\right] = -\log 14 \times 10^{-3} = -\log 14 - \log 10^{-3}$$
$$pH = -\log 7 - \log 2 + 3$$
$$= -0.84 - 0.30 + 3$$
$$= 1.86$$

22.(3)

$$\underset{+7}{\underset{\downarrow}{MnO_4^{-1}}} \longrightarrow \underset{+4}{\underset{\downarrow}{MnO_2}}$$

Change in oxidation No. = 3

23.(623)

$$k = 2.011 \times 10^{-3}\,s^{-1}$$
$$Ao = 7gm$$
$$At = 2gm$$
$$t = \frac{2.303}{k}\log\frac{Ao}{At} \quad \text{for first order kinetics}$$
$$t = \frac{2.303}{2.011 \times 10^{-3}}\log\frac{7}{2}$$
$$t = 1.145 \times 10^3(\log 7 - \log 2)$$
$$t = 1.145 \times 10^3 \times 0.544$$
$$t = 622.88\,sec$$
$$t = 623\,sec$$

24.(0) For ideal gas, U is function of temperature only.

25.(1362)

Moles of $CO_2 = M \times V \rightarrow \frac{0.2 \times 300}{1000} \rightarrow 0.06$ moles

1 mole of CO_2 have a total volume of = 22.7 L

0.06 ………………….. = $22.7 \times 0.06\,L$

$= 22.7 \times 0.06 \times 1000\,ml = 1362\,ml$

26.(100)

$$\Delta T_b = k_b m$$

$$0.52 = 0.52 \times \frac{\frac{W_B}{M.W_B}}{W_A \times 10^{-3}}$$

$$0.52 = \frac{0.52 \times 2}{W_A \times 20 \times 10^{-3}}$$

$$M.W_B = 100$$

27.(148)

$$\text{wt of DCM} = M \times V \times M.wt$$

$$= 2.6 \times 10^{-3} \times 671.14 \times 85 = 148.3\,mg$$

Mass of solution = wt of DCM + wt of $CHCl_3$

$$= 148.3 + (671.14 \times 1.49) \times 1000 = 148.3 + 1000 \times 1000 \simeq 10^6$$

(conc.) in ppm = $\frac{\text{wt of DCM}}{\text{wt of solution}} \times 10^6 = \frac{148.3}{10^6} \times 10^6 = 148.3\,ppm$

28.(798)

$$E = nh\nu$$

$$= 6.022 \times 10^{23} \times 6.626 \times 10^{-34} \times 2 \times 10^{12} = 798\,\text{Joules}$$

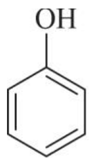
29.(4) $DU \rightarrow (C+1) - \frac{(H+X-N)}{2} = (10+1) - \frac{12}{2} = 5$

Neutral $FeCl_3$ test confirms the presence of phenolic group.

Rex'n with $NaOH/CH_3Br$ confirms the presence of phenolic / alcoholic group.

Rex'n wih HI and gives CH_3I confirms the presence of $-O-CH_3$.

Decolorization of alkaline $KMnO_4$ confirms the presence of alkene.

Expected structure contains $\rightarrow$  , $-OCH_3$ & $-C=C-$

Thus no. of π-bonds $\rightarrow (3+1) = 4$

30.(10) $H_2 + 2Fe^{3+} \rightleftharpoons 2Fe^{+2} + 2H^+$

$$E_{cell} = E^0_{cell} - \frac{2.303RT}{nF} \log \frac{[H^+]^2 [Fe^{2+}]^2}{P_{H_2} [Fe^{3+}]^2}$$

$$0.712 = 0.771 - \frac{0.06}{2} \log \left(\frac{[Fe^{2+}]}{[Fe^{3+}]} \right)^2$$

$$\log \frac{[Fe^{2+}]}{[Fe^{3+}]} = 1 \Rightarrow \frac{[Fe^{2+}]}{[Fe^{3+}]} = 10$$

MATHEMATICS

SECTION - 1

1.(4) $\tan 15° = 2-\sqrt{3}$

$\frac{1}{\tan 75°} = \cot 75° = 2-\sqrt{3}$

$\frac{1}{\tan 105°} = \cot(105°) = -\cot 75° = \sqrt{3}-2$

$\tan 195° = \tan 15° = 2-\sqrt{3}$

$\therefore \quad 2\left(2-\sqrt{3}\right) = 2a \Rightarrow a = 2-\sqrt{3} \quad \Rightarrow \quad a+\frac{1}{a} = 4$

2.(4) Either all outcomes are positive or any two are negative.

Now, $p = P$ (positive) $= \frac{3}{6} = \frac{1}{2}$

$q = p$ (negative) $= \frac{2}{6} = \frac{1}{3}$

Required probability

$= {}^5C_5\left(\frac{1}{2}\right)^5 + {}^5C_2\left(\frac{1}{3}\right)^2\left(\frac{1}{2}\right)^3 + {}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{1}{2}\right)^1 = \frac{521}{2592}$

3.(3) $\log_{\cos x}\cot x + 4\log_{\sin x}\tan x = 1$

$\Rightarrow \quad \frac{\ln\cos x - \ln\sin x}{\ln\cos x} + 4\frac{\ln\sin x - \ln\cos x}{\ln\sin x} = 1 \quad \Rightarrow \quad \ln\sin x = 2\ln\cos x$

$\Rightarrow \quad \sin^2 x + \sin x - 1 = 0 \Rightarrow \sin x = \frac{-1+\sqrt{5}}{2} \quad \therefore \quad \alpha+\beta = 4$

4.(1) $5f(x+y) = f(x).f(y)$

$5f(0) = f(0)^2 \Rightarrow f(0) = 5$

$5f(x+1) = f(x).f(1)$

$\Rightarrow \quad \frac{f(x+1)}{f(x)} = \frac{f(1)}{5}$

$\Rightarrow \quad \frac{f(1)}{f(0)}.\frac{f(2)}{f(1)}.\frac{f(3)}{f(2)} = \left(\frac{f(1)}{5}\right)^3$

$\Rightarrow \quad \frac{320}{5} = \frac{(f(1))^3}{5^3} \Rightarrow f(1) = 20$

$\therefore \quad 5f(x+1) = 20.f(x) \Rightarrow f(x+1) = 4f(x)$

$\sum_{n=0}^{5} f(n) = 5+5.4+5.4^2+5.4^3+5.4^4+5.4^5$

$= \frac{5\left[4^6-1\right]}{3} = 6825$

5.(1) $\int_1^2 x^2 e^{\left[x^3\right]+1} dx$

$x^3 = t$

$3x^2 dx = dt$

$= \frac{e}{3}\int_1^8 e^{[t]} dt = \frac{e}{3}\left\{\int_1^2 e\,dt + \int_2^3 e^2 dt + ... + \int_7^8 e^7 dt\right\}$

$= \frac{e}{3}\left(e + e^2 + ... + e^7\right) = \frac{e^2}{3}\left(1 + e + ... + e^6\right) = \frac{e^2}{3}\frac{\left(e^7 - 1\right)}{(e-1)}$

$\frac{3(e-1)}{e}\int_1^2 x^2 \times e^{[x]+\left[x^3\right]} dx = \frac{3}{e}(e-1) \times \frac{e^2}{3}\frac{\left(e^7-1\right)}{(e-1)} = e\left(e^7 - 1\right) = e^8 - e$

6.(3) Normal of line is parallel to line $x + 90y + 2 = 0$

$m_N = -\frac{1}{90}$

$-\left(\frac{dx}{dy}\right)_{(x_1 y_1)} = -\frac{1}{90} \Rightarrow \left(\frac{dy}{dx}\right)_{(x_1 y_1)} = 90$

Now, $\frac{dy}{dx} = 270x^4 - 540x^3 - 210x^2 + 360x + 210 = 90 \quad \Rightarrow \quad x = 1, 2, \frac{-2}{3}, \frac{-1}{3}$

(4) normals

7.(2) $A = \begin{bmatrix} m & n \\ p & q \end{bmatrix}, |A - d(adj\ A)| = 0$

$\Rightarrow \quad |A - d(adj\ A)| = \left|\begin{bmatrix} m & n \\ p & q \end{bmatrix} - d\begin{bmatrix} q & -n \\ -p & m \end{bmatrix}\right|$

$\begin{vmatrix} m - qd & n(1+d) \\ p(1+d) & q - md \end{vmatrix} = 0$

$\Rightarrow \quad (m - qd)(q - md) - np(1+d)^2 = 0$

$\Rightarrow \quad mq - m^2 d - q^2 d + mqd^2 - np(1+d)^2 = 0$

$\Rightarrow \quad (mq - np) + d^2(mq - np) - d\left(m^2 + q^2 + 2np\right) = 0$

$\Rightarrow \quad d + d^3 - d\left((m+q)^2 - 2d\right) = 0 \Rightarrow 1 + d^2 = (m+q)^2 - 2d \quad \Rightarrow \quad (1+d)^2 = (m+q)^2$

8.(3) $\begin{vmatrix} 1 & 1 & k \\ 2 & 3 & -1 \\ 3 & 4 & 2 \end{vmatrix} = 0$

$\Rightarrow \quad 1(10) - 1(7) + k(-1) - 0 \quad \Rightarrow \quad k = 3$

For $k = 3, 2^{nd}$ system is $4x + 5y = 7$ ...(1)

And $7x + 87 = 10$ ...(2)

Clearly, they have a unique solution $(2) - (1) \Rightarrow 3x + 3y = 3 \quad \Rightarrow \quad x + y = 1$

9.(4) $S_1 \equiv ((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$

p	q	r	$p \vee q$	$(p \vee q) \Rightarrow r$	$p \Rightarrow r$	$((p \vee q) \Rightarrow r) \Leftrightarrow (p \Rightarrow r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	T	T	T	T
F	T	T	T	T	T	T
T	F	F	T	F	F	T
F	T	F	T	F	T	F
F	F	F	F	T	T	T
F	F	F	F	T	T	T

$S1 \rightarrow$ not a tautology

$S_2 \equiv (p \vee q) \Rightarrow r \Leftrightarrow ((p \Rightarrow r) \vee (q \Rightarrow r))$

p	q	r	$(p \vee q) \Rightarrow r$	$p \Rightarrow r$	$q \Rightarrow r$	$(p \Rightarrow r \vee (q \Rightarrow r))$	$S2$
T	T	T	T	T	T	T	T
T	T	F	F	F	F	F	T
T	F	T	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	F	F	T	T	T	F
F	T	F	F	F	F	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

$S2 \rightarrow$ not a tautology

10.(2) $L_1 : y = x + 2, L_2 : 4y = 3x + 6, L_3 : 3y = 4x + 1$

Bisector of lines L_2 & L_3

$$\frac{4x-3y+1}{5} = \pm\left(\frac{3x-4y+6}{5}\right)$$

$$(+)4x-3y+1=3x-4y+6$$

$$x+y=5$$

Centre lies on Bisector of $4x-3y+1=0$ and $(0)\ 3x-4y+6=0 \quad \Rightarrow \quad h+k=5$

11.(2) $$\frac{dy}{dx}+\left(\frac{-3x^5\tan^{-1}x^3}{\left(1+x^6\right)^{3/2}}\right)y = 2xe^{\left\{\frac{x^3-\tan^{-1}x^3}{\sqrt{1+x^6}}\right\}}$$

$$I.F. = e^{\int\frac{-3x^5\tan^{-1}x^3}{\left(1+x^6\right)^{3/2}}dx} = e^{\frac{\tan^{-1}x^3-x^3}{\sqrt{1+x^6}}}$$

Solution of differential equation

$$y.e^{\frac{\tan^{-1}x^3-x^3}{\sqrt{1+x^6}}} = \int 2xe^{\left(\frac{x^3-\tan^{-1}x^3}{\sqrt{1+x^6}}\right)}.e^{\left(\frac{\tan^{-1}\left(x^3\right)-x^3}{\sqrt{1+x^6}}\right)}dx = \int 2xdx + c$$

$$y.e^{\frac{\tan^{-1}x^3-x^3}{\sqrt{1+x^6}}}=x^2+c$$

Also, it passes through origin

$c=0$

$$y(1).e^{\frac{\tan^{-1}(1)-1}{\sqrt{2}}}=1$$

$$y(1).e^{\frac{\frac{\pi}{4}-1}{\sqrt{2}}}=1$$

$$y(1).e^{\frac{\pi-4}{4\sqrt{2}}}=1$$

$$y(1)=\frac{1}{e^{\frac{\pi-4}{4\sqrt{2}}}}=e^{\frac{4-\pi}{4\sqrt{2}}}$$

12.(1) Equation of plane

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 1 & 1 \\ 0 & 3 & 4 \end{vmatrix}=0$$

$\Rightarrow \quad (x-1)-4(y-2)+3(z-3)=0$

$\Rightarrow \quad x-4y+3z=2$

D.R's of normal of plane $<1,-4,3>$

D.C's of $\left\langle \pm\frac{1}{\sqrt{26}},\pm\frac{4}{\sqrt{26}},\pm\frac{3}{\sqrt{26}}\right\rangle$

$$\cos\beta=\frac{4}{\sqrt{26}}$$

$$\cos\alpha=\frac{-1}{\sqrt{26}} \quad \frac{\pi}{2}<\alpha<\pi$$

$$\cos\gamma=\frac{-3}{\sqrt{26}} \quad \frac{\pi}{2}<\gamma<\pi$$

13.(3) If $a_n=\dfrac{-2}{4n^2-16n+15}$ then $a_1+a_2+......a_{25}$

$$\Rightarrow \quad \sum_{n=1}^{25}a_n=\sum\frac{-2}{4n^2-16n+15}=\sum_{n=1}^{25}\frac{-2}{4n^2-6n-10n+15}$$

$$=\sum_{n=1}^{25}\frac{-2}{2n(2n-3)-5(2n-3)}=\sum_{n=1}^{25}\frac{-2}{(2n-3)(2n-5)}$$

$$=\sum_{n=1}^{25}\frac{1}{2n-3}-\frac{1}{2n-5}=\frac{1}{47}-\frac{1}{(-3)}$$

$$=\frac{50}{141}$$

14.(3) Equation of normal

$y = -tx + 2at + at^3$

$y = -tx + \frac{2}{16}t + \frac{1}{16}t^3$

If passes through (0, 33)

$33 = \frac{t}{8} + \frac{t^3}{16}$

$t^3 + 2t - 528 = 0$

$(t-8)(t^2 + 8t + 66) = 0$

$t = 8$

$P(at^2, 2at) = \left(\frac{1}{16} \times 64, 2 \times \frac{1}{16} \times 8\right) = (4, 1)$

Parabola:

$y^2 = 4(x+y)$

$\Rightarrow \quad y^2 - 4y = 4x \quad \Rightarrow \quad (y-2)^2 = 4(x+1)$

Equation of directrix:

$x + 1 = -1$

$x = -2$

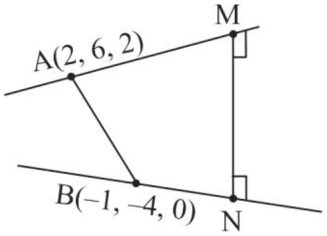
Distance of point = 6

15.(1) Line ℓ, is given by

$L_1 : \frac{x-2}{2} = \frac{y-6}{1} = \frac{z-2}{-2}$

Given,

$L_2 : \frac{x+1}{2} = \frac{y+4}{-3} = \frac{z}{2}$



Shortest distance = $\left|\frac{\overrightarrow{AB}.\overrightarrow{MN}}{|\overrightarrow{MN}|}\right|$

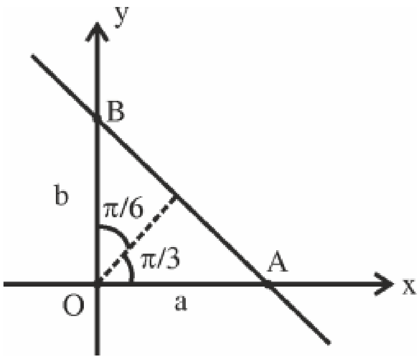
$\overrightarrow{AB} = 3\hat{i} + 10\hat{j} + 2\hat{k}$

$\overrightarrow{MN} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 2 & -3 & 2 \end{vmatrix} = -4\hat{i} - 8\hat{j} - 8\hat{k}$

$|\overrightarrow{MN}| = \sqrt{16 + 64 + 64} = 12$

$\therefore$ Shortest distance = $\left|\frac{-12 - 80 - 16}{12}\right| = 9$

16.(1)



Equation of straight line: $\frac{x}{a}+\frac{y}{b}=1$

Or $x\cos\frac{\pi}{3}+y\sin\frac{\pi}{3}=p$

$\frac{x}{2}+\frac{y\sqrt{3}}{2}=p$

$\frac{x}{3p}+\frac{y}{2p}=1$

Comparing both: $a=2p, b=\frac{2p}{\sqrt{3}}$

Now area of $\Delta OAB=\frac{1}{2}.ab=\frac{98}{3}.\sqrt{3}$

$\frac{1}{2}.2p.\frac{2p}{\sqrt{3}}=\frac{98}{3}.\sqrt{3}$

$p^2=49$

$a^2-b^2=4p^2-\frac{4p^2}{3}=\frac{2}{3}4p^2=\frac{8}{3}.49=\frac{392}{3}$

17.(4) Coefficient of x^{15} in $\left(ax^3+\frac{1}{bx^{1/3}}\right)^{15}$

$T_{r+1}={}^{15}C_r\left(ax^3\right)^{15-r}\left(\frac{1}{bx^{1/3}}\right)^r$

$45-3r-\frac{r}{3}=15$

$30=\frac{10r}{3}$

$r=9$

Coefficient of $x^{15}={}^{15}C_9\,a^6b^{-9}$

Coefficient of x^{-15} in $\left(ax^{1/3}-\frac{1}{bx^3}\right)^{15}$

$T_{r+1}={}^{15}C_r\left(ax^{1/3}\right)^{15-r}\left(-\frac{1}{bx^3}\right)^r$

$5-\frac{r}{3}-3r=-15$

$\frac{10r}{3}=20$

$r=6$

Coefficient $={}^{15}C_6\,a^9\times b^{-6}$

$\Rightarrow \quad \frac{a^9}{b^6}=\frac{a^6}{b^9} \quad \Rightarrow \quad a^3b^3=1 \Rightarrow ab=1$

18.(4) $(1+x)^{500}+x(1+x)^{499}+x^2(1+x)^{498}+...+x^{500}$

$$=(1+x)^{500}.\left\{\frac{1-\left(\frac{x}{1+x}\right)^{501}}{1-\frac{x}{1+x}}\right\}=(1+x)^{500}\frac{\left((1+x)^{501}-x^{501}\right)}{(1+x)^{501}}.(1+x)=(1+x)^{501}-x^{501}$$

Coefficient of x^{301} in $(1+x)^{501}-x^{501}$ is given by ${}^{501}C_{301}={}^{501}C_{200}$

19.(1) For symmetric $(a,b),(b,c)\in R$

$\Rightarrow \quad (b,a),(c,b)\in R$

For Transitive $(a,b),(b,c)\in R$

$\Rightarrow \quad (a,c)\in R$

Now

1. Symmetric

$\therefore \quad (a,c)\in R \Rightarrow (c,a)\in R$

2. Transitive

$\therefore \quad (a,b),(b,a)\in R$

$\Rightarrow \quad (a,a)\in R \,\&\, (b,c),(c,b)\in R \quad \Rightarrow \quad (b,b)\,\&\,(c,c)\in R$

$\therefore$ Elements to be added $\{(b,a),(c,b),(a,c),(c,a),(a,a),(b,b),(c,c)\}$

Number of elements to be added = 7

20.(4) $\hat{n}\perp\vec{c} \qquad \vec{a}=\alpha\vec{b}-\hat{n}$

$\vec{b}.\vec{c}=12$

$\vec{a}.\vec{c}=\alpha(\vec{b}.\vec{c})-\hat{n}.\vec{c}$

$\vec{a}.\vec{c}=\alpha(\vec{b}.\vec{c})$

$\left|\vec{c}\times(\vec{a}\times\vec{b})\right|=\left|(\vec{c}.\vec{b})\vec{a}-(\vec{c}.\vec{a})\vec{b}\right|=\left|(\vec{c}.\vec{b})\vec{a}-\alpha(\vec{b}.\vec{c})\vec{b}\right|$

$=\left|(\vec{c}.\vec{b})\right|\left|\vec{a}-\alpha\vec{b}\right|=12\times(|\hat{n}|)$

$=12\times1$

= 12

SECTION – 2

21.(37) $\frac{x_1+x_2+...+x_7}{7}=8$

$\frac{x_1+x_2+x_3...+x_6+14}{7}=8$

$\Rightarrow \quad x_1+x_2+...+x_6=42$

$\therefore \quad \frac{x_1+x_2...+x_6}{6}=\frac{42}{6}=7=a$

$\frac{\sum x_i^2}{7}-8^2=16$

$\sum x_i^2=560$

$\Rightarrow \quad x_1^2+x_2^2+...+x_6^2=364$

$b=\frac{x_1^2+x_2^2+...+x_6^2}{6}-7^2=\frac{364}{6}-49$

$b=\frac{70}{6}$

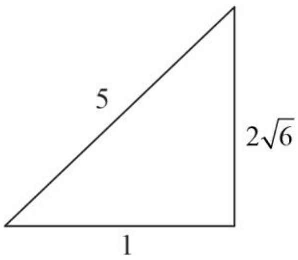
$a+3b-5=7+3\times\frac{70}{6}-5=37$

22.(315) $P_1=\vec{r}.\left(3\hat{i}-5\hat{j}+\hat{k}\right)=7$

$P_2=\vec{r}.\left(\lambda\hat{i}+\hat{j}-3\hat{k}\right)=9$

$\theta=\sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$

$\Rightarrow \quad \sin\theta=\frac{2\sqrt{6}}{5}$



$\therefore \quad |\cos\theta|=\frac{1}{5}$

$|\cos\theta|=\left|\frac{\vec{r}_1.\vec{r}_2}{|\vec{r}_1||\vec{r}_2|}\right|=\left|\frac{(3i-5j+K)(\lambda i+j-3K)}{\sqrt{35}.\sqrt{\lambda^2+10}}\right|$

$\frac{1}{5}=\left|\frac{3\lambda-8}{\sqrt{35}.\sqrt{\lambda^2+10}}\right|$

Square $\Rightarrow \quad \frac{1}{25}=\frac{9\lambda^2+64-48\lambda}{35\left(\lambda^2+10\right)}$

$\Rightarrow \quad 19\lambda^2 - 120\lambda + 125 = 0 \Rightarrow \quad 19\lambda^2 - 95\lambda - 25\lambda + 125 = 0$

$\Rightarrow \quad \lambda = 5, \frac{25}{19}$ so, $\lambda_1 = \frac{25}{19}, \lambda_2 = 5$

Perpendicular distance of point

$(38\lambda_1, 10\lambda_2, 2) \equiv (50, 50, 2)$ from plane P_1

$$= \frac{|3\times 50 - 5\times 50 + 2 - 7|}{\sqrt{35}} = \frac{105}{\sqrt{35}}$$

Square = $\frac{105\times 105}{35} = 315$

23.(12) $48 \lim_{x\to 0} \frac{\int_0^x \frac{t^3}{t^6+1}dt}{x^4} \left(\frac{0}{0}\right)$

Applying L' Hospitals Rule

$48 \lim_{x\to 0} \frac{x^3}{x^6+1} \times \frac{1}{4x^3} = 12$

24.(9) $z = 1 + i$

$$z_1 = \frac{1 + i\bar{z}}{\bar{z}(1-z) + \frac{1}{z}}$$

$$z_1 = \frac{1 + i(1-i)}{(1-i)(1-1-i) + \frac{1}{1+i}} = \frac{1 + i - i^2}{(1-i)(-i) + \frac{1-i}{2}} = \frac{2+i}{\frac{-3i-1}{2}} = \frac{4+2i}{-3i-1} = \frac{-(4+2i)(3i-1)}{(3i)^2 - (1)^2}$$

$Arg(z_1) = \frac{3\pi}{4} \quad \therefore \quad \frac{12}{\pi}\arg(z_1) = \frac{12}{\pi} \times \frac{3\pi}{4} = 9$

25.(3240)

Let $S = \{1, 2, 3, 4, 5, 6\}$ then the number of one-one functions, $f : S \to P(S)$, where $P(S)$ denotes the power set of S, such that $f(n) \subset f(m)$ where $n < m$ is:

$n(S) = 6$

$P(S) = \{\phi, \{1\}, ...\{6\}, \{1, 2\}, ..., \{5, 6\}, ..., \{1, 2, 3, 4, 5, 6\}\}$

Total no. of elements in $P(S)$ = 64

Case – 1

$f(6)$ = S i.e. 1 option,

$f(5)$ = any 5 element subset A of S i.e. 6 options,

$f(4)$ = any 4 element subset B of A i.e. 5 options,

$f(3)$ = any 3 element subset C of B i.e. 4 options,

$f(2)$ = any 2 element subset D of C i.e. 3 options,

$f(1)$ = any 1 element subset E of D or empty subset i.e. 3 options,

Total functions = 1080

Case – 2

$f(6)$ = any 5 element subset A of S i.e. 6 options,

$f(5)$ = any 4 element subset B of A i.e. 5 options,

$f(4)$ = any 3 element subset C of B i.e. 4 options,

$f(3)$ = any 2 element subset D of C i.e. 3 options,

$f(2)$ = any 1 element subset E of D i.e. 2 options,

$f(1)$ = empty subset i.e. 1 option

Total functions = 720

Case – 3

$f(6)$ = S

$f(5)$ = any 4 element subset A of S i.e. 15 options,

$f(4)$ = any 3 element subset B of A i.e. 4 options,

$f(3)$ = any 2 element subset C of B i.e. 3 options,

$f(2)$ = any 1 element subset D of C i.e. 2 options,

$f(1)$ = empty subset i.e. 1 option

Total functions = 360

Case – 4

$f(6)$ = S

$f(5)$ = any 5 element subset A of S i.e. 6 options,

$f(4)$ = any 3 element subset B of A i.e. 10 options,

$f(3)$ = any 2 element subset C of B i.e. 3 options,

$f(2)$ = any 1 element subset D of C i.e. 2 options,

$f(1)$ = empty subset i.e. 1 option

Total functions = 360

Case – 5

$f(6)$ = S

$f(5)$ = any 5 element subset A of S i.e. 6 options,

$f(4)$ = any 4 element subset B of A i.e. 5 options,

$f(3)$ = any 2 element subset C of B i.e. 6 options,

$f(2)$ = any 1 element subset D of C i.e. 2 options,

$f(1)$ = empty subset i.e. 1 option

Total functions = 360

Case – 6

$f(6)$ = S

$f(5)$ = any 5 element subset A of S i.e. 6 options,

$f(4)$ = any 4 element subset B of A i.e. 5 options,

$f(3)$ = any 3 element subset C of B i.e. 4 options,

$f(2)$ = any 1 element subset D of C i.e. 3 options,

$f(1)$ = empty subset i.e. 1 option

Total functions = 360

$\therefore$ Number of such functions = 3240

26.(21) For number to be divisible by 15, last digit should be 5 and sum of digits must be divisible by 3.

Possible combinations are

1	2	1	5

Numbers = 3

2	2	3	5

Numbers = 3

3	3	1	5

Numbers = 3

1	1	5	5

Numbers = 3

2	3	5	5

Numbers = 6

3	5	5	5

Numbers = 3

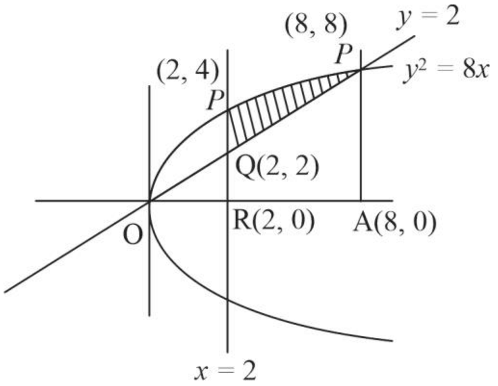
Total Numbers = 21

27.(26) $$\sum_{n=0}^{\infty}\frac{n^3((2n)!)+(2n-1)(n!)}{(n!)((2n)!)}=\sum_{n=3}^{\infty}\frac{1}{(n-3)!}+\sum_{n=2}^{\infty}\frac{3}{(n-2)!}+\sum_{n=1}^{\infty}\frac{1}{(n-1)!}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)!}-\sum_{n=0}^{\infty}\frac{1}{(2n)!}$$

$$=e+3e+e+\frac{1}{2}\left(e-\frac{1}{e}\right)-\frac{1}{2}\left(e+\frac{1}{e}\right)=5e-\frac{1}{e}$$

$\therefore \quad a^2-b+c=26$

28.(22)



$y=x \,\&\, y^2=8x$

Solving it $x^2=8x$

$\therefore \quad x=0,8 \qquad \therefore \quad y=0,8$

$x=2$ will intersect occur at $y^2=16 \Rightarrow y=\pm 4$

$\therefore$ Area of shaded

$$=\int_2^8\left(\sqrt{8x}-x\right)dx=\int_2^8\left(2\sqrt{2}\sqrt{x}-x\right)dx=\left[2\sqrt{2}.\frac{x^{3/2}}{3/2}-\frac{x^2}{2}\right]_2^8$$

$$=\left(\frac{4\sqrt{2}}{3}.2^{9/2}-32\right)-\left(\frac{4\sqrt{2}}{3}.2^{3/2}-2\right)$$

$$=\frac{128}{3}-32-\frac{16}{3}+2=\frac{112-90}{3}=\frac{22}{3}=\alpha$$

$\therefore \quad 3\alpha=22$

29.(15) $x-3y+2z-1=0$

$4x-y+z=0$

$$\vec{n}_1\times\vec{n}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -3 & 2\\ 4 & -1 & 1\end{vmatrix}=-\hat{i}+7\hat{j}+11\hat{k}$$

Dr^{s} of normal to the plane is $-1,7,11$

Equation of plane:

$$-1(x-1)+7(y-1)11(z-2)=0$$

$$-x+7y+11z=28$$

$$\frac{-1}{28}x+\frac{7y}{28}+\frac{11z}{28}=1$$

$$Ax+By+Cz=1$$

$$140(C-B+A)=140\left(\frac{11}{28}-\frac{7}{28}-\frac{1}{28}\right)=140\times\frac{3}{28}=15$$

30.(3125)

$$f^1(x)=\frac{3x+2}{2x+3}$$

$$\Rightarrow \quad f^2(x)=\frac{13x+12}{12x+3} \quad \Rightarrow \quad f^3(x)=\frac{63x+62}{62x+63}$$

$$\therefore \quad f^5(x)=\frac{1563x+1562}{1562x+1563}$$

$$a+b=3125$$