VMC Vidyamandir Classes SINCE 1986

IIT JEE | MEDICAL | FOUNDATION

SOLUTIONS

Joint Entrance Exam | IITJEE-2023

24^{th} JAN 2023 | Morning Shift

PHYSICS

SECTION - 1

1.(4) In medium, $F = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{d^2}$ . . . (i)

In air, $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{x^2}$ . . . .(ii)

From (i) and (ii) we get $x = d\sqrt{k}$

2.(4) $W = P\Delta V \quad \Rightarrow \quad W = (3\times10^5 Pa)(1600\times10^{-6} m^3) = 480 J$

Given $W = \frac{10}{100}\times\Delta Q$

$\Rightarrow \quad \Delta Q = 4800\,J$

So, $\Delta U = \Delta Q - W$ = 4320 J

3.(3) For constant velocity of lift

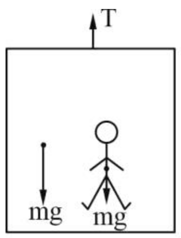
$T = Mg + mg$

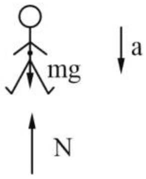
For lift accelerating down,

$mg - N = ma$

$\Rightarrow \quad N = mg - ma$

$\Rightarrow \quad N < mg$





4.(2) $[h] = [M L^2 T^{-1}]$

$[V_s] = [M L^2 T^{-3} A^{-1}]$

$[\phi] = [M L^2 T^{-2}]$

$[P] = [M L T^{-1}]$

5.(3) Lets, $B = -Kt + B_0$.

At $t = 0s, B = 0.5T \quad \Rightarrow \quad B_0 = 0.5T$

and at t = 0.5 sec, B = 0T

$\Rightarrow$ K = 1 $\Rightarrow \quad B = (-t + 0.5)T$

$\varepsilon = A\left|\frac{dB}{dt}\right|\cos 0^\circ$

$\Rightarrow \quad \varepsilon = \pi\left(\frac{10}{100\sqrt{\pi}}\right)^2\left|\frac{d}{dt}(-t+0.5)\right|$

$\Rightarrow \quad \varepsilon = \frac{1}{100}V = 10mV$

6.(3) $_{84}^{218}A \xrightarrow{\alpha} {}_{82}^{214}A_1 \xrightarrow{\beta^-} {}_{83}^{214}A_2$

$_{83}^{214}A_2 \xrightarrow{\gamma} {}_{83}^{214}A_3 \xrightarrow{\alpha} {}_{81}^{210}A_4$

$_{81}^{210} \xrightarrow{\beta^+} {}_{80}^{210}A_5 \xrightarrow{\gamma} {}_{80}^{210}A_6$

7.(3) Photo diode a p-n junction diode that operates under reverse bias.

For a p-n junction diode, current is more in forward bias than reverse bias.

8.(1) For vertical projection,

$$y_{\max} = \frac{u^2}{2g}$$

$$\Rightarrow \quad u^2 = (y_{\max})(2g) \quad \Rightarrow \quad u^2 = (y_{\max})(2g) = (136)(2\times 9.8)$$

For horizontal projection

$$x_{\max} = \frac{2u^2 \sin 45^\circ \cos 45^\circ}{g}$$

$$x_{\max} = \frac{136\times 2\times 9.8}{9.8} = 272m$$

9.(1) $V_{rms} = \sqrt{\frac{3RT}{M}} \quad \Rightarrow \quad \frac{(V_{rms})_2}{(V_{rms})_1} = \sqrt{\frac{T_2}{T_1}} \quad \Rightarrow \quad \frac{(V_{rms})_2}{(V_{rms})_1} = \sqrt{\frac{(527+273)K}{(-73+273)K}} = \frac{2}{1}$

$KE_{Translation} = \frac{3}{2}nRT$ and $PV = nRT$

So, $KE_{Translation} \neq PV$

10.(2) Modulation Index $(\mu) = \frac{A_m}{A_C}$

From the given graph, $A_m = 1V$

and $c(t) = 2\sin(8\pi t)V$

so, $A_C = 2V \quad \Rightarrow \quad \mu = \frac{1}{2}$

11.(4) Due to a current carrying circular loop

$B_{centre} = \frac{\mu_0 I}{2r}$ and $B_{axis} = \frac{\mu_0 I r^2}{2(r^2+x^2)^{3/2}}$, where $x = r$ $\Rightarrow$ $B_{axis} = \frac{\mu_0 I r^2}{2(2r^2)^{3/2}}$

So, $\frac{B_{centre}}{B_{axis}} = \frac{2\sqrt{2}}{1}$

12.(3) **A** : Stoping potential depends on work function of metal and frequency of incident light.

B : Saturation current increases with increase in intensity of incident light (as number of incident photons per second also increases)

C : Maximum kinetic energy of a photoelectron depends on work function of metal and frequency of incident light.

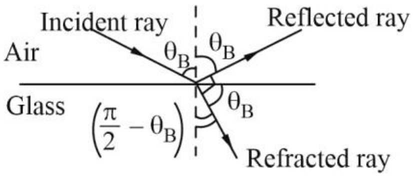
D : Photo electric effect cannot be explained using wave theory of light.

13.(4) $\frac{F_1}{0.1m}=\frac{\mu_0(10A)^2}{2\pi(5\times10^{-2}m)}$ . . . (i)

$\frac{F_2}{0.1m}=\frac{\mu_0(20A)^2}{2\pi(2.5\times10^{-2}m)}$ . . . (ii)

$\Rightarrow \quad \frac{F_2}{F_1}=\frac{8}{1}\Rightarrow F_2=8F_1$

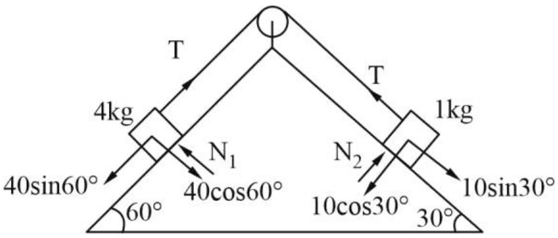
14.(2)



For light propagating from air to glass

$\theta_B=\tan^{-1}(\mu_g)$

15.(1)



$40\sin60°-T=4a$ . . . (i)

$T-10\sin30°=1a$ . . . .(ii)

Solving (i) and (ii) we get

$T=4(\sqrt{3}+1)N$

16.(3) $y(x,t)=[0.05\sin(8x-4t)]m$

Comparing with

$y(x,t)=A\sin(Kx-wt)$

And $V=\frac{\omega}{K}=\frac{4}{8}=0.5m/s$

17.(1) $\hat{B}=\hat{k}\times\hat{E}$

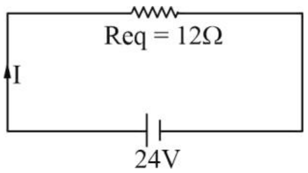
$\vec{B}=B(\hat{K}\times\hat{E})$

$\vec{B}=\frac{B}{EK}(\vec{K}\times\vec{E})$

And, $C=\frac{E}{B}=\frac{\omega}{K}$

$\Rightarrow \quad \vec{B}=\frac{1}{CK}(\vec{K}\times\vec{E})=\frac{1}{\omega}(\vec{K}\times\vec{E})$

18.(2) For the circuit given the following equivalent circuit can be used.



$I = \frac{24}{12} = 2A$

Since R_4 and R_5 are in parallel

$I_4 R_4 = I_5 R_5 \Rightarrow I_4(20) = I_5(5)$

and $I_4 + I_5 = 2A \Rightarrow I_4 = \frac{2}{5}A$ and $I_5 = \frac{8}{5}A$

19.(2) $\Delta\ell = \frac{F\ell}{AY} \Rightarrow \Delta\ell = \frac{250\times100}{6.25\times10^{-4}\times10^{10}} \Rightarrow \Delta\ell = 4\times10^{-3}m$

20.(3) $\frac{GMm}{R_e^2} = 18N \quad (R_e = 6400km)$ **. . . .(i)**

Let $\frac{GMm}{(R_e+h)^2} = W\ N$

Here $h = 3200\,Km = \frac{R_e}{2} \Rightarrow \frac{4GMm}{9R_e^2} = W\ N$ …(ii)

From (i) and (ii) we get $W = 8N$

SECTION – 2

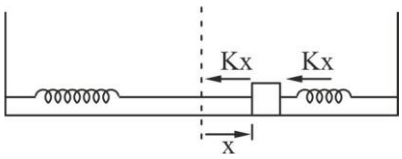
21.(40) $V = u + at \Rightarrow v = 0 + \left(\frac{F}{2kg}\right)(5s)$

And $KE = \frac{1}{2}mV^2 \Rightarrow 10000 = \frac{1}{2}(2)\left(\frac{5F}{2}\right)^2 \Rightarrow F = 40N$

22.(5) $2K(-\vec{x}) = m\vec{a}$

$\Rightarrow 2K(-\vec{x}) = m\omega^2(-\vec{x})$

$\Rightarrow \omega = \sqrt{\frac{2K}{m}} = T = 2\pi\sqrt{\frac{m}{2k}} \Rightarrow T = 2\pi\sqrt{\frac{2}{2(20)}} = \frac{\pi}{\sqrt{5}}s$



23.(11) Nuclear Density $(\rho_{Nucleus}) = \frac{M}{\frac{4}{3}\pi R^3}$

$\rho_{Nucleus} = \frac{A\times1.6\times10^{-27}}{\frac{4}{3}\pi(1.5\times10^{-15}\times A^{1/3})^3}$

$\rho_{Nucleus} \approx 11\times10^{16}\,kg/m^3$

$$\frac{\rho_{Nucleus}}{\rho_{water}}=\frac{11\times10^{16}\,kg/m^3}{1000\,Kg/m^3}$$

$$\frac{\rho_{Nucleus}}{\rho_{water}}=11\times10^{13}$$

24.(10) Q factor $=\frac{1}{R}\sqrt{\frac{L}{C}}$

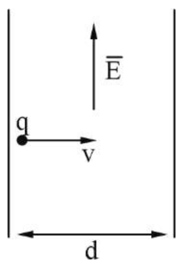
Bandwidth $=\Delta W=\frac{R}{L}$

$$\frac{Q_{factor}}{\Delta w}=\frac{1}{R^2}\sqrt{\frac{L^3}{C}}$$

$$\frac{Q_{factor}}{\Delta w}=\frac{1}{(10)^2}\sqrt{\frac{(3)^3}{27\times10^{-6}}}$$

$$\frac{Q_{factor}}{\Delta w}=10$$

25.(2)



$d=10cm=0.1m$

$v=3\times10^7\,m/s$

$E=1.8\times10^3 V/m$

$t=\frac{d}{v}$

$$y=\frac{1}{2}\left(\frac{Eq}{m}\right)t^2=\frac{1}{2}E\left(\frac{q}{m}\right)\left(\frac{d}{v}\right)^2$$

$$y=\frac{1}{2}(1.8\times10^3)(2\times10^{11})\left(\frac{0.1}{3\times10^7}\right)^2$$

$$y=2\times10^{-3}m=2mm$$

26.(110) $mK^2=\frac{2}{5}m(5cm)^2+m(10cm)^2$

$K=\sqrt{110}\,cm$

27.(12) $\Delta d=d\,\alpha\,\Delta T$

$\Delta d=(5)(1.6\times10^{-5})(177-27)=12\times10^{-3}\,cm$

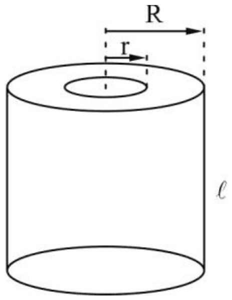
28.(1) $(a\hat{i}+b\hat{j}+\hat{k}).(2\hat{i}-3\hat{j}+4\hat{k})=0$

$\Rightarrow\quad 2a-3b+4=0 \Rightarrow\quad 2a-3b=-4$ . . . (i)

And $3a+2b=7$ . . . .(ii)

Solving (i) & (ii) we get a = 1, b = 2 $\Rightarrow\quad \frac{a}{b}=\frac{1}{2}$

29.(2) Resistance $= \dfrac{\rho\ell}{\pi(R^2 - r^2)}$



$= \dfrac{(2.4\times10^{-8})(3.14)}{\pi(4^2 - 2^2)\times10^{-6}}$

$= 2\times10^{-3}\,\Omega$

30.(120) $\dfrac{1}{f_1} = (1.75 - 1)\left(\dfrac{1}{(-\infty)} - \dfrac{1}{(+30)}\right)$

$\dfrac{1}{f_1} = \dfrac{-1}{40\,cm}$

$\dfrac{1}{f_2} = (1.75 - 1)\left(\dfrac{1}{(+30)} - \dfrac{1}{(+\infty)}\right)$

$\dfrac{1}{f_2} = +\dfrac{1}{40cm}$

Plano concave lens forms virtual image of the object at its focus i.e. 40 cm to the left of the plano concave lens.

Hence object distance from plano convex lens, $u_2 = -80\,cm$

$V_2 = \dfrac{u_2 f_2}{u_2 + f_2} = \dfrac{(-80)(+40)}{-80+40}$

$V_2 = 80cm$

(to the right of plano convex lens)

Hence image distance from plano concave lens = 80 + 40 = 120 cm

CHEMISTRY

SECTION - 1

1.(1) Based upon polarization of an ion by cation correct orders of covalent nature in bonding are

KF < KI (Larger anion can be easily polarize than smaller anion)

LiF > KF (smaller cation is more polarizing than larger cation)

$SnCl_4 > SnCl_2$ (Metal ion is more polarizing in higher valency state)

$CuCl > NaCl$ (Pseudo noble gas configuration cation is more polarizing than noble gas cation of comparable size)

2.(4) Nucleophilicity: $I^- > Cl^-$

Leaving group tendency: $I^- > Cl^-$

$R-Cl+H_2O \longrightarrow R-OH+HCl$ (Slower reaction)

$R-Cl+NaI \longrightarrow R-I+NACl$ (Faster reaction) (Based on nucleophicity of I^-)

$R-I+H_2O \longrightarrow R-OH+HI$ (Faster reaction) (Based on leaving group tendency of I^-)

3.(4) Correct matching are:

(A) Chlorophyll $\rightarrow$ (III) Mg^{2+}

(B) Soda ash $\rightarrow$ (I) Na_2CO_3

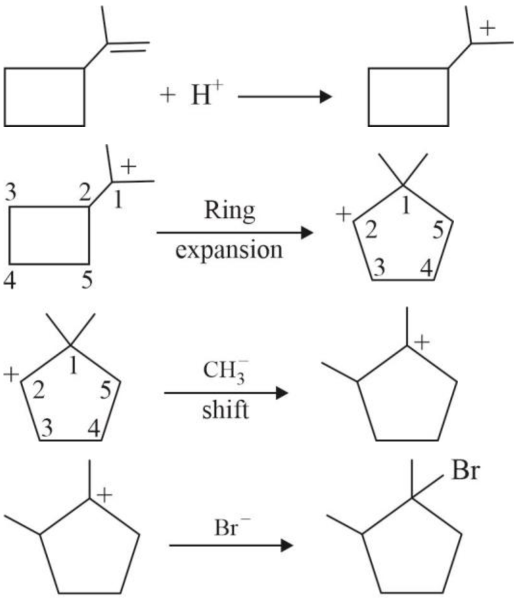
(C) Dentistry, ornamental work $\rightarrow$ (II) $CaSO_4$

(D) Used in white washing $\rightarrow$ (IV) $Ca(OH)_2$

4.(3) Hydrogen bonding is effective in solid state (ice) of H_2O as compared to its liquid state. By addition of some soluble impurity to water hydrogen bonding among H_2O molecules decrease. Hence correct decreasing order of hydrogen bonding is B > A > C.

5.(1) $[Co(NH_3)_5Cl]Cl_2 \rightleftharpoons \underset{\text{(secondary valency=6)}}{[Co(NH_3)_5Cl]^{2+}} + \underset{\text{(Primary valency=2)}}{2Cl^-}$

6.(4)



(Major product, A)

7.(1) $V^{2+} \to 3d^3$, 3 unpaired e^-s

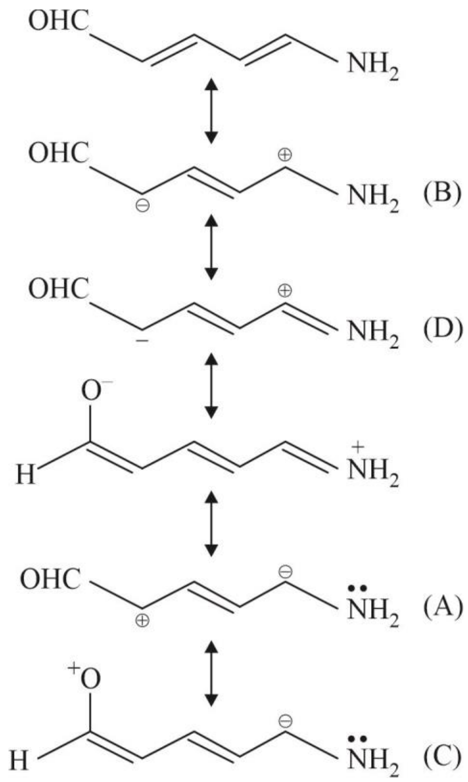
$Ti^{2+} \to 3d^2$, 2 unpaired e^-s

$Cr^{2+} \to 3d^4$, 4 unpaired e^-s

$Mn^{2+} \to 3d^5$, 5 unpaired e^-s

As the calculated magnetic moment is 3.87 BM, the metal ion has 3 unpaired electrons. Correct answer to the questions is V^{2+}.

8.(4)



Based upon rules governing stability of resonating structures the correct increasing order of stability is C < A < B < D. Out of the given options 4 is closest to the correct order.

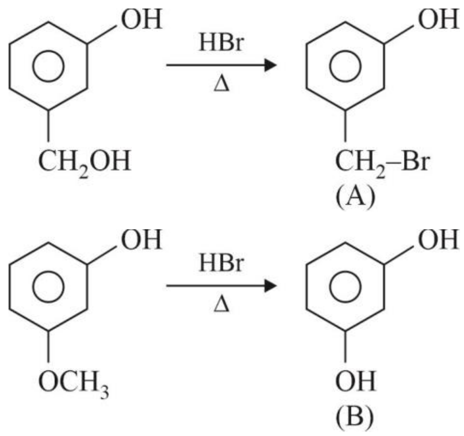
9.(1) Noradrenaline is a neurotransmitter. Low levels of noradrenaline causes depression in human. Correct option is 1.

10.(4) Due to lesser number of freely moving particles the compounds which can form colloids in a given solvent has lower values of their colligative properties as compared to compounds which do not form colloids and stays as true solution, under identical values of concentration. Zeta potential is the electrical potential difference between layers of opposite charges around a colloidal particle. Hence correct option is 4.

11.(4) As $\Delta E = h\nu = Rz^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \Rightarrow \quad \nu \propto z^2$

(or) $\nu^{1/2} \propto z$

12.(1)



13.(2) Correct matchings are:

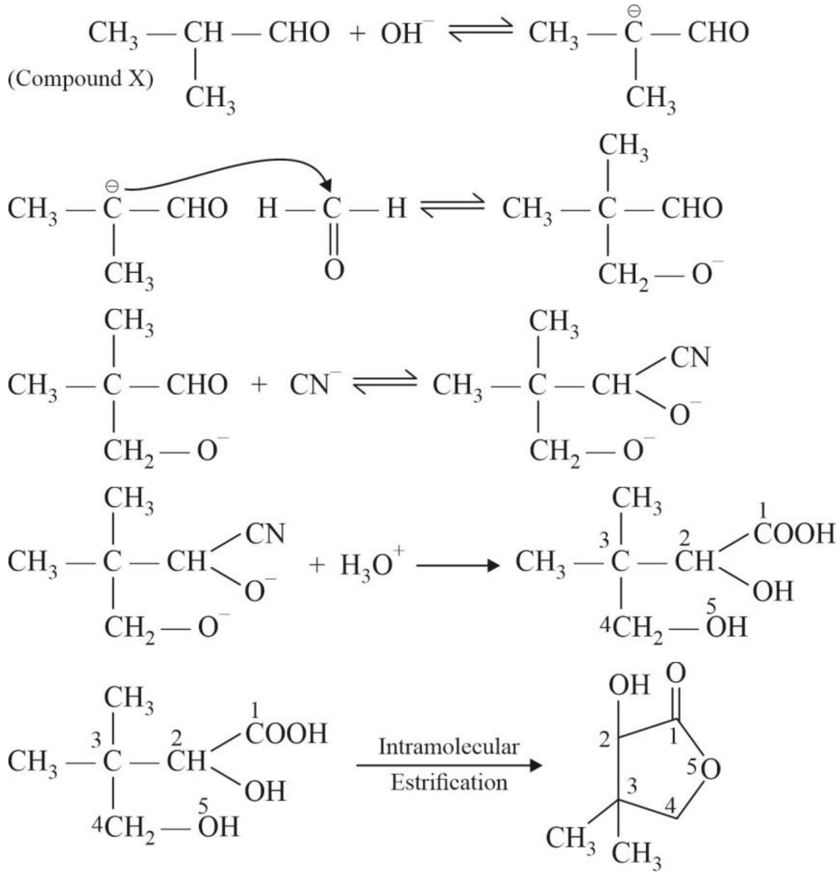
(A) Reverberatory furnace $\longrightarrow$ (IV) copper

(B) Electrolytic cell $\longrightarrow$ (II) Aluminium

(C) Blast furnace $\longrightarrow$ (I) pig iron

(D) Zone refining furnace $\longrightarrow$ (III) silicon

14.(1)



15.(1) For a binary solution of non-volatile solute in a volatile solvent, vapour pressure of the solution is less than that of pure solvent, and at freezing point only the solvent molecules solidify.

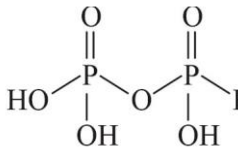
16.(3) $BeO + 2HF \rightleftharpoons BeF_2 + H_2O$

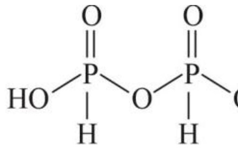
$BeF_2 + 2HF \rightleftharpoons H_2[BeF_4]$

$H_2[BeF_4] + 2NH_3 \rightleftharpoons (NH_4)_2[BeF_4] \xrightarrow{\Delta} 2NH_4F + BeF_2$

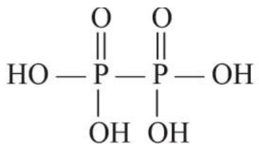
17.(2) Freons are chlorofluorocarbon compounds.

18.(2) Oxo-acid of phosphorus having P-H bonds can act as reducing agent for Ag^{+}

$H_4P_2O_7$:  , No P–H bond

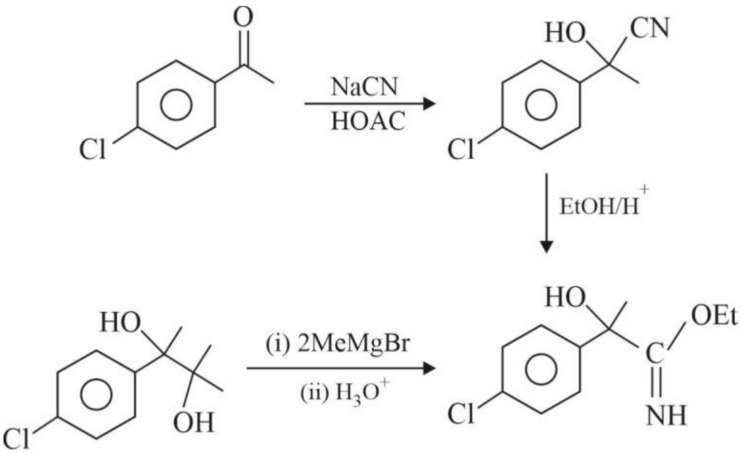
$H_4P_2O_5$:  , Two P–H bonds

$(HPO_3)_n$: Poly meta phosphoric acid , No P–H bond

$H_4P_2O_6$:  , No P–H bond

19.(3) $Ni^{2+} + 2dmg^{-} \rightleftharpoons \underset{\text{(Brilliant red ppt.)}}{Ni(dmg)_2}$

20.(4)



(Major product, R)

SECTION – 2

21.(492) Paschen series: $n_2 = 4, 5, 6, 7........ \text{to } n_1 = 3$

as $\frac{1}{\lambda} = R_H z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2}\right]$

first line : $n_2 = 4$ to $n_1 = 3$, λ_1

Second line : $n_2 = 5$ to $n_1 = 3$, λ_2 $\quad\Rightarrow\quad \frac{1/\lambda_1}{1/\lambda_2} = \frac{\left[\frac{1}{3^2} - \frac{1}{4^2}\right]}{\left[\frac{1}{3^2} - \frac{1}{5^2}\right]}$

$$\frac{\lambda_2}{\lambda_1} = \frac{\left(\frac{16-9}{9\times16}\right)}{\left(\frac{25-9}{9\times25}\right)} = \frac{7/16}{16/25} = \frac{25\times7}{16\times16} = \frac{175}{16\times16} = 0.683$$

$\Rightarrow \lambda_2 = \lambda_1(0.683) = 720(0.683) = 491.76 \approx 492\,\text{nm}.$

22.(917) $Cr_2O_7^{2-} + 14H^+ + 6e^- \rightleftharpoons 2Cr^{3+} + 7H_2O$

$$Q_C = \frac{[Cr^{3+}]^2}{[Cr_2O_7^{2-}][H^+]^{14}} = \frac{(0.1/1)}{(10^{-2})(10^{-3})^{14}} = 10^{42}$$

as $E = E^\circ - \frac{2.303RT}{nF}\log Q_C$

$$= 1.330 - \frac{0.059}{6}[42]$$

$$= 1.330 - 0.413 = 0.917 = 917 \times 10^{-3}\ \text{Volt}$$

$\therefore\ x = 917$

23.(2) Using $\Delta G = \Delta H - T(\Delta S)$; $\Delta G < 0$ for spontaneous process.

$$\Delta G_A = -25 - \frac{300(-80)}{1000} = -25 + 24 = -1\text{kJ},$$ spontaneous

$$\Delta G_B = -22 - \frac{300(-40)}{1000} = -22 - 12 = -34\text{kJ},$$ spontaneous

$$\Delta G_C = 25 - \frac{300(-50)}{1000} = 25 + 15 = 40\,\text{kJ},$$ Non spontaneous

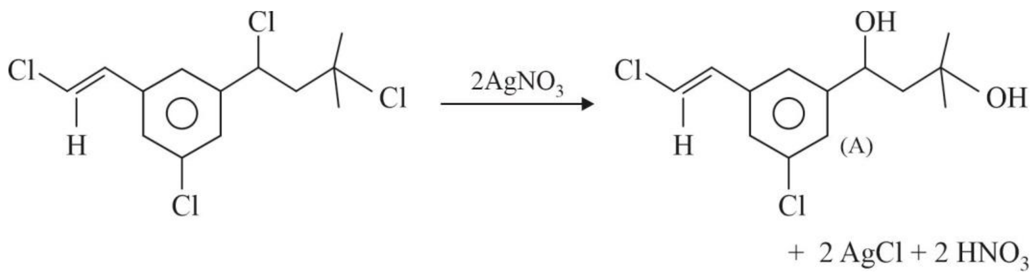
$$\Delta G_D = 22 - \frac{300(20)}{1000} = 22 - 6 = 16\,\text{kJ},$$ Non spontaneous

24.(180) Molarity of NaOH in the stock solution $= \frac{5/40}{0.45} = \frac{1}{8 \times 0.45} = \frac{1}{3.6}$ M

Using $M_1V_1 = M_2V_2$

$$V_1 = \frac{M_2V_2}{M_1} = \frac{0.1 \times 500}{1/3.6} = 3.6 \times 50 = 180\,\text{ml}$$

25.(2)



26.(10) For an acidic buffer mixture of CH_3COOH and $CH_3COO^-Na^+$

$$pH = pKa + \log_{10}\frac{[CH_3COO^-]}{[CH_3COOH]}\text{salt}$$

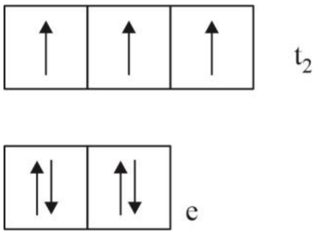
$$5 = pKa + \log_{10}\frac{(25 \times 0.2)}{(25 \times 0.02)}$$

$$5 = pKa + \log_{10} 10$$

$$pKa = 5 - 1 = 4$$

$$Ka = 10^{-4} = 10 \times 10^{-5} \quad \therefore\ x = 10$$

27.(7) $[CoCl_4]^{2-}$ has $Co^{2+}:3d^7$ configuration



$\Rightarrow e^4t_2^3$ $\quad\therefore\ m=4,\ n=3$ unpaired electron = 3

28.(25) M.F of uracil $\Rightarrow C_4N_2H_4O_2$

$$\%\text{ of }N=\frac{2\times14}{4\times12+2\times14+4+2\times16}\times100=\frac{28}{112}\times100=25$$

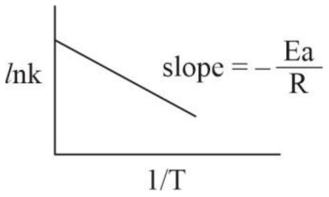
29.(3) As $k=Ae^{-E_a/RT}$: larger the value of E_a, smaller is the value of rate constant K

(Statement A is correct)

As $\ell n\frac{k_2}{k_1}=\frac{Ea}{R}\left[\frac{1}{T_1}-\frac{1}{T_2}\right]$: Higher is the value of E_a, higher will be the value of temperature coefficient $\left(\ell n\frac{k_2}{k_1}\right)$ (statement B is correct)

As $\ell nk=\ell nA-\frac{Ea}{RT}$

$\frac{d}{dT}(\ell nk)=\frac{+Ea}{RT^2},\ \frac{d}{dT}(\ell nk)$ is higher at lower value of T. (Statement C is correct)



(Statement D is correct)

30.(4) $Fe_{0.93}O : 93Fe^{2+} + 100\ O^{2-}$

+ ve charge = + 186

–ve charge = – 200

$\therefore$ Number of Fe^{2+} got converted into $Fe^{3+}=14$

Remaining $Fe^{2+}=79$

Hence 1 mole of $Fe_{0.93}O$ has only 0.79 moles of Fe^{2+}

$$Fe_{0.93}O\xrightarrow[\Delta]{O_2}\underset{(\text{all }Fe^{3+})}{Fe_2O_3}$$

n-factor of $Fe_{0.93}O$ is 0.79 and $E^\circ=\frac{M^\circ}{0.79}$

% of Fe^{3+} in original lattice $=\frac{14}{93}\times100=15.053\%\approx15\%$

Hence all the four given statements are correct.

MATHEMATICS

SECTION - 1

1.(1) $\frac{(c-a)^3}{(a-b)(b-c)(c-a)}+\frac{(a-b)^3}{(a-b)(b-c)(c-a)}+\frac{(b-c)^3}{(a-b)(b-c)(c-a)}$

$A=a-b$

$B=b-c$

$C=c-a$

$A+B+C=0$

$A^3+B^3+C^2=3ABC$

$S=\frac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)}=3$

2.(3) dot with $\bar{w}$ $\quad 0=\bar{u}\cdot\bar{w}+\lambda\bar{v}\cdot\bar{w}$

$\Rightarrow \bar{u}\cdot\bar{w}=-\lambda\times 2$

Dot with $\bar{v}$ $\quad 0=\bar{u}\cdot\bar{v}+\lambda|\bar{v}|^2$

$\Rightarrow\ 0=3+\lambda\times 6\ \Rightarrow \lambda=-\frac{1}{2}$

$\therefore\ \bar{u}\cdot\bar{w}=1$

3.(2) $\lim_{t\to 0}\left(1^{\operatorname{cosec}^2 t}+2^{\operatorname{cosec}^2 t}+.......n^{\operatorname{cosec}^2 t}\right)$

Take $n^{\operatorname{cosec}^2 t}$ common

$\lim_{t\to 0} n\left[\left(\frac{1}{n}\right)^{\operatorname{cosec}^2 t}+\left(\frac{2}{n}\right)^{\operatorname{cosec}^2 t}+....\left(\frac{n}{n}\right)^{\operatorname{cosec}^2 t}\right]=\lim_{t\to 0} n[(\to 0)+(\to 0)+....1]^{\sin^2 t}=n$

4.(3) $(a,\ a)\notin R$

$\because$ GCD of a and $a=a$

$(2,\ 1)\in R$ but $(1,\ 2)\notin R$

$(4,\ 9)\in R$ and $(9,\ 16)\in R$

But $(4,\ 16)\notin R$

So not transitive

5.(2) $\frac{dy}{dx}+\frac{y}{x^2}=\frac{1}{x^3}$

I.F. $=e^{-\frac{1}{x}}$

$y\times e^{-\frac{1}{x}}=\int e^{-\frac{1}{x}}\times\frac{1}{x^3}\,dx$

RHS $=\int e^{-\frac{1}{x}}\times\frac{1}{x^2}\times\frac{1}{x}\,dx$

$$-\frac{1}{x}=t$$

RHS $=-\int te^t dt=-te^t+e^t=\frac{1}{x}e^{-\frac{1}{x}}+e^{-\frac{1}{x}}+C$

$$y\times e^{-\frac{1}{x}}=\frac{1}{x}e^{-\frac{1}{x}}+e^{-\frac{1}{x}}+C$$

$$x=\frac{1}{2},\ y=3-e$$

$$\Rightarrow 3-e=3+Ce^2 \Rightarrow c=-\frac{1}{e} \Rightarrow y=\frac{1}{x}+1+ce^{\frac{1}{x}}$$

$$x=1, y=1+1+\left(-\frac{1}{e}\right)\times=1$$

6.(4) $(x-1)(x-3)=[x](x-1)$

$x=1$ are solutions.

Or $x-3=[x]$

But $[x]\in(x-1,\ x]$

So $[x]\neq x-3$

So only 1 solution

7.(1) $1-\sqrt{3}i=2e^{-i\pi/3}$

$$(1-\sqrt{3}i)^{2w}\times e^{-i200\pi/3}=2^{200}\times e^{i4\pi/3}$$

$$=2^{200}w^2=2^{2w}\left[\frac{-1-i\sqrt{3}}{2}\right]=2^{199}\left[-1-i\sqrt{3}\right]$$

$$p=-1,\ q=-\sqrt{3}$$

$$p+q+q^2=2-\sqrt{3}$$

$$p-g+g^2=2+\sqrt{3}$$

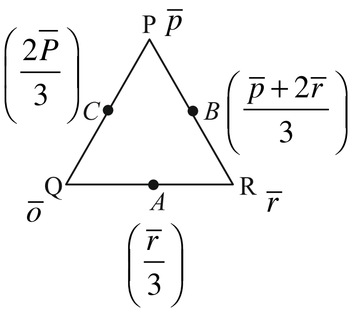
$$S_1=4,\ S_2=1$$

$$x^2-S_1x+S_2=0$$

8.(1) For answer we can take $Q(0,\ 0),\ R(3,\ 0)$

$$P\left(\frac{3}{2},\ \frac{3\sqrt{3}}{2}\right)$$

For proper solution



$\overline{AB}=\frac{\overline{p}+\overline{r}}{3}$ $\quad\overline{AC}=\frac{2\overline{p}-\overline{r}}{3}$

$\frac{1}{2}\overline{AB}\times\overline{AC}=\frac{1}{2}\left[\frac{-\overline{p}\times\overline{r}+2\overline{r}\times\overline{p}}{9}\right]=\frac{1}{2}\left[\frac{3\overline{r}\times\overline{p}}{9}\right]=\frac{1}{2}\times\frac{\overline{r}\times\overline{p}}{3}$

Area of $\Delta ABC=\frac{1}{3}\times$area of ΔQRP

9.(1) $f(x)$ is cont. and diff at $x = 0$

$f'(x)=2x\cos\frac{1}{x}-\cos\frac{1}{x}$

$\lim_{x\to 0} f'(x)$ does not exist

So $f'(x)$ is NOT continuous at $x=0$

10.(1) Equation of line

$\frac{x+1}{3}=\frac{y-9}{-4}=\frac{z+16}{12}=\lambda$

Point on line $(3\lambda-1,-4\lambda+9,12\lambda-16)$

It lies on $2x+3y-z=5$

$\Rightarrow\quad 6\lambda-2-12\lambda+27-12\lambda+16=5\quad\Rightarrow\quad 18\lambda=36\quad\Rightarrow\quad\lambda=2$

$\therefore$ Point of intersection is $(5,1,8)$

Dist. $=\sqrt{6^2+8^2+24^2}=\sqrt{26^2}=26$

11.(4) $\sum_{r=0}^{22}{}^{22}C_r\,{}^{23}C_r={}^{20}C_0\,{}^{23}C_0+{}^{22}C_1\,{}^{23}C_1+{}^{22}C_2\,{}^{23}C_2$

Consider $\quad(1+x)^{22}={}^{22}C_0+{}^{22}C_1x+{}^{22}C_2x^2+...+{}^{22}C_{22}x^{22}$

$(x+1)^{23}={}^{23}C_0x^{23}+{}^{23}C_1x^{22}+{}^{23}C_2x^{21}+...+{}^{23}C_{22}x+{}^{23}C_{23}$

$(1+x)^{22}(x+1)^{23}=\left({}^{22}C_0+{}^{22}C_1x+{}^{22}C_2x^2+....+{}^{22}C_{22}x^{22}\right)$

$\left({}^{23}C_0x^{23}+{}^{23}C_1x^{22}+....+{}^{23}C_{22}x+{}^{23}C_{23}\right)$

$(1+x)^{45}=\left({}^{22}C_0\,{}^{23}C_0x^{23}+{}^{22}C_1\,{}^{23}C_1x^{23}+{}^{22}C_2\,{}^{23}C_2x^{23}+...+{}^{22}C_{22}\,{}^{23}C_{22}x^{23}\right)+...$

$\therefore$ Required value = Coefficient of x^{23} in $(1+x)^{45}={}^{45}C_{23}$

12.(1) $X=A^2$

$X+B=XB$

Pre multiplying both side by X^{-1}

$I+X^{-1}B=B$

Post multiplying both side by B^{-1}

$B^{-1}+X^{-1}=I$

Taking inverse on both side

$B+X=I^{-1}=I\quad\therefore\quad XB=I\quad\therefore\quad X$ & B are inverse of each other

$\therefore\quad XB=BX\quad\therefore\quad A^2B=BA^2$

13.(1) If $P(A)=0$ does not imply $A=\phi$

Let sample space have infinite cardinality and A is an event with finite (but non-zero) outcomes

e.g. Sample space = Set of Natural numbers

A = A Natural number is chosen then probability of getting 3

$\therefore \quad P(A)=0$

Similarly, $P(A)=1$ does not imply A = Sample space

Here, we can take 'A' as complement of Set A taken in Statement 1.

Both statement false.

14.(1) $\Delta=\begin{vmatrix}1 & 1 & 1\\ 2 & N & 2\\ 3 & 3 & N\end{vmatrix}$

For unique solution

$\Delta \neq 0$

$\begin{vmatrix}1 & 1 & 1\\ 2 & N & 2\\ 3 & 3 & N\end{vmatrix} \neq 0 \quad \Rightarrow \quad N \neq 2 \text{ or } 3$

($\because$ for $N=2$, C_1 & C_2 are same, Q for $N=3$ C_1 & C_3 are same)

$$\Delta_1=\frac{\begin{vmatrix}1 & 1 & 1\\ 2 & N & 2\\ 3 & 3 & N\end{vmatrix}}{\begin{vmatrix}1 & 1 & 1\\ 2 & N & 2\\ 3 & 3 & N\end{vmatrix}}=1 \quad (\text{for } N \neq 2 \text{ or } 3)$$

$$\Delta_2=\frac{\begin{vmatrix}1 & 1 & 1\\ 2 & 2 & 2\\ 3 & 3 & N\end{vmatrix}}{\Delta}=0 \qquad (C_1 \,\&\, C_2 \text{ same})$$

$$\Delta_3=\frac{\begin{vmatrix}1 & 1 & 1\\ 2 & N & 2\\ 3 & 3 & 3\end{vmatrix}}{\Delta}=0 \qquad (C_1 \,\&\, C_3 \text{ same})$$

$\because \quad N=\{1, 2, 3, 4, 5, 6\}$

Favorable values of N for which given system of equations has unique

Solution = {1, 4, 5, 6}

$\therefore$ Required probability $=\frac{4}{6} \qquad \therefore \; k=4$

Sum of possible values of N = 1 + 4 + 5 + 6 = 16 $\quad \therefore$ required answer = 4 + 16 = 20

15.(2) $x^{pq^2}=y^{qr}=z^{p^2r}$ & $r=pq+1 \qquad$ (p, q, r (+ve) tangents)

$3, 3\log_y x, 3\log_z y, 7\log_x z$ are in A.P.

Common difference $d=\frac{1}{2}$

$x^{pq^2} = y^{qr}$

Taking log on both side

$$pq^2 \log x = qr \log y \quad \Rightarrow \quad \log_y x = \frac{qr}{pq^2} = \frac{r}{pq}(g \neq 0) = \frac{pq+1}{pq} = 1 + \frac{1}{pq}$$

$$\therefore \qquad 3\log_y x = 3 + \frac{3}{pq}$$

Similarly, $\log_z y = \frac{p^2 r}{qr} = \frac{p^2}{q}$

$$3\log_x y = \frac{3p^2}{q} \ \& \ \log_x z = \frac{pq^2}{pr} = \frac{q^2}{pr} \qquad \therefore \ 7\log_x z = \frac{7q^2}{pr}$$

$$3\log_y x = 3 = d \qquad \Rightarrow \qquad 3 + \frac{3}{pq} - 3 = \frac{1}{2} \quad \Rightarrow \quad pq = 6 \qquad \therefore \qquad r = pq + 1 = 7$$

Also, $\quad 7\log_x z - 3 = 3d$

$$\Rightarrow \qquad \frac{7q^2}{pr} - 3 = \frac{3}{2} \ \Rightarrow \ \frac{q^2}{p} = \frac{9}{2} \ \Rightarrow \ \frac{q^3}{6} = \frac{9}{2} \ \Rightarrow \ q = 3 \ \& \ p = 2$$

$$\therefore \qquad r - p - q = 7 - 2 - 3 = 2$$

16.(2) Let $\quad A(2, -3, 1), B(-1, 1, -2) \ \& \ C(3, -4, 2)$ and $P(7, -3, -4)$

Distance of point P from plane containing A, B and C

$$= \left| \frac{\left[\overrightarrow{AP}\ \overrightarrow{AB}\ \overrightarrow{BC}\right]}{\left|\overrightarrow{AB} \times \overrightarrow{BC}\right|} \right| = \left| \frac{\begin{vmatrix} 7-2 & -3-(-3) & -4-1 \\ -1-2 & 1-(-3) & -2-1 \\ 3-(-1) & -4-1 & 2-(-2) \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 4 & -3 \\ 4 & -5 & 4 \end{vmatrix}} \right|$$

$$= \left| \frac{\begin{vmatrix} 5 & 0 & -5 \\ -3 & 4 & -3 \\ 4 & -5 & 4 \end{vmatrix}}{\left|\hat{i} - \hat{k}\right|} \right| = \frac{5(1) - 5(-1)}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 5\sqrt{2} \text{ units}$$

17.(1) Equation of tangent to the curve $y^2 = 24x$ of slope m.

$$y = mx + \frac{6}{m}\left[y^2 = 4ax, \ a = 6\right]$$

This line cuts the curve $xy = 2$ at $A(x_1, y_1)$ and $B(x_2, y_2)$

$$\Rightarrow \qquad y_1 = \frac{2}{x_1} \text{ and } y_2 = \frac{2}{x_2}$$

$$x\left(mx + \frac{6}{m}\right) = 2 \qquad \Rightarrow \qquad m^2x^2 + 6x - 2m = 0$$

$\therefore \quad x_1 + x_2 = \frac{-6}{m^2}$

$x_1 x_2 = \frac{-2m}{m^2} = -\frac{2}{m}$

Mid point M of A & B is $M\left(\frac{x_1+x_2}{2}, \frac{2}{2}\left(\frac{1}{x_1}+\frac{1}{x_2}\right)\right) = M\left(-\frac{3}{m^2}, \left(\frac{-\frac{6}{m^2}}{-\frac{2}{m^2}}\right)\right) = M\left(-\frac{3}{m^2}, \frac{3}{m}\right)$

$\therefore$ Locus of $M(h, k)$

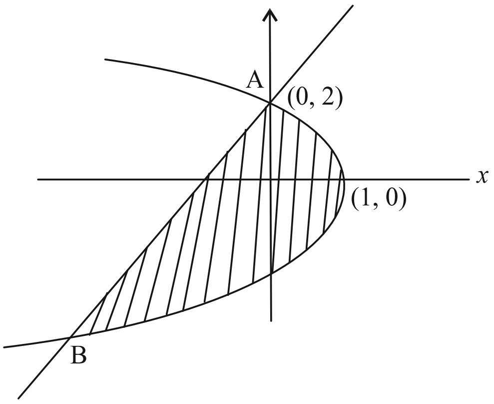
$h = -\frac{3}{m^2}, \; k = \frac{3}{m}$

$\Rightarrow \quad h = -\frac{3}{\left(\frac{3}{k}\right)^2} \quad \Rightarrow \quad k^2 = -3h \quad \therefore \quad$ Locus is $y^2 = -3x$

It is a parabola whose directrix is $x = \frac{3}{4}$

18.(1) $y^2 = 4 - 4x \quad \Rightarrow \quad y^2 = -4(x-1) \Rightarrow x = -\frac{y^2}{4} + 1$

$y = 2 + 2x \quad \Rightarrow \quad y = 2(x+1) \Rightarrow x = \frac{y}{2} - 1$



A & B are points of intersection of parabola and line

$4(x+1)^2 = -4(x-1)$

$\Rightarrow \quad x^2 + 3x = 0 \quad \Rightarrow \quad x = 0, -3$

$\therefore \quad A(0, 2) \; \& \; B(-3, -4)$

Area enclosed $= \int_{-4}^{2} \left(x_{parabola} - x_{line}\right) dy$

$= \int_{-4}^{2} \left(-\frac{y^2}{4} + 1 - \left(\frac{y}{2} - 1\right)\right) dy = \int_{-4}^{2} \left(-\frac{y^2}{4} - \frac{y}{2} + 2\right) dy = \left[-\frac{y^3}{12}\right]_{-4}^{2} - \left[\frac{y^2}{4}\right]_{4}^{2} + [2y]_{-4}^{2}$

$= -\frac{72}{12} - \frac{(-12)}{4} + 2(6) = -6 + 3 + 12 = 9 \; sq. \; units$

19.(2)

P	~P	Q	~Q	$\sim(P\wedge Q)$	$(\sim P)\wedge Q$	$(\sim(P\wedge Q)\vee (\sim P\wedge Q))$	$(\sim P)\wedge(\sim Q)$	$(\sim(P\wedge Q)\vee ((\sim P)\wedge Q)) \Rightarrow ((\sim P)\wedge(\sim Q))$
T	F	T	F	F	F	F	F	T
T	F	*F*	T	T	F	T	F	F
F	T	*T*	F	T	T	T	F	F
F	T	*F*	T	T	F	T	T	T

Option 1 $((\sim P)\vee Q)\wedge(\sim Q)$	**Option 2** $((\sim P)\vee Q)\wedge(\sim Q)\vee P)$	**Option 3** $(\sim Q)\wedge P$	**Option 4** $(\sim P)\vee Q$
F	T	T	T
F	F	T	F
F	F	F	T
T	T	T	T

20.(4) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)+\sec^{-1}\left(\sqrt{\frac{4}{3}}\right)=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)+\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)=\frac{\pi}{6}+\frac{\pi}{6}=\frac{\pi}{3}$

SECTION - 2

21.(1012)

$$\sum_{r=0}^{n} r^2\ {}^nC_r=\sum_{r=0}^{n}(r^2-r)+\sum_{r=0}^{n} r\ {}^nC_r=n(n-1)\sum_{r=0}^{n-2}{}^{n-2}C_r+n\sum_{r=0}^{n-1}{}^{n-1}C_r$$

$$=n(n-1)2^{n-2}+n2^{n-1}=n2^{n-2}(n+1)=n(n+1)2^{n-2}$$

Put $n=2023$ to get $x=1012$

22.(546) ${}^7C_5\times{}^5C_0+{}^7C_4\times{}^5C_1+{}^7C_3\times{}^5C_2=546$

23.(22) $I=12\int_0^3\left|\left(x-\frac{3}{2}\right)^2-\frac{1}{4}\right|dx=12\int_{-3/2}^{3/2}\left|y^2-\frac{1}{4}\right|dy=24\int_0^{3/2}\left|y^2-\frac{1}{4}\right|dy$

$$=24\left[\int_0^{1/2}\left(\frac{1}{4}-y^2\right)dy+\int_{1/2}^{3/2}\left(y^2-\frac{1}{4}\right)dy\right]=22$$

24.(7) Equation of tangent at $(4\cos\theta,\ 3\sin\theta)$

$\frac{x}{4}\cos\theta+\frac{y}{3}\sin\theta=1 \quad\Rightarrow\quad A(4\sec\theta,\ 0),\ B(0,\ 3\operatorname{cosec}\theta)$

$$L^2=\frac{16}{\cos^2\theta}+\frac{9}{\sin^2\theta}$$

Now using $AM\ge H.M.$, we get $\frac{1}{7}\left(\left(\frac{\cos^2\theta}{4}\right)\times4+\left(\frac{\sin^2\theta}{3}\right)\times3\right)\ge\frac{7}{\frac{16}{\cos^2\theta}+\frac{9}{\sin^2\theta}}$

$$\Rightarrow\quad L^2=\frac{16}{\cos^2\theta}+\frac{9}{\sin^2\theta}\ge7^2\quad\Rightarrow\quad L\ge7$$

25.(12) $S_6 - S_5 = T_6 > 1$

$S_7 - S_6 = T_7 < \frac{1}{2}$

$T_6 = \frac{500}{m^2}$ and $T_7 = \frac{500}{m^3}$

$\Rightarrow \quad m^2 < 500$ and $m^3 > 1000 \quad \Rightarrow \quad m \in \{11, 12, ..., 22\}$

26.(60) ${}^5C_2 \times {}^4C_2 = 60$

27.(5) Let $y = |x| \Rightarrow y \geq 0$

E $\quad y^2 - 2y + |\lambda - 3| = 0$

Product of roots ≥ 0

$\Rightarrow$ Roots can't have opposite signs

Also for set S, we're only considering integral solutions of E and $y \geq 0$

Hence, only 2 cases arise

Roots are $y = 0$ and $y = 2$ i.e. $|\lambda - 3| = 0$

$\Rightarrow \quad x = 0,\ x = \pm 2$ and $\lambda = 3$ OR $y = 1$ *i.e.* $|\lambda - 3| = 1$

$\Rightarrow \quad x \pm 1$ and $\lambda = 2, 4$

Highest value of $x + \lambda = 5$ when $x = 1$ and $\lambda = 4$ OR $x = 2$ and $\lambda = 3$

28.(2) Apply king rule to solve

29.(14) Shortest distance is projection of line segment

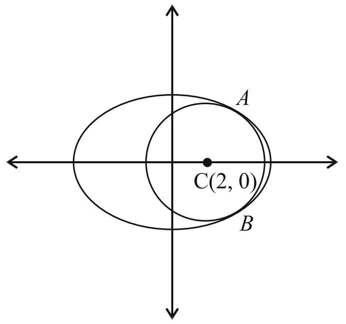
P_1P_2 *on* $\vec{L}_1 \times \vec{L}_2$

Where $P_1 : (2, -1, 6)$

$P_2 : (6, 1, -8)$

$\vec{L}_1 : 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{L}_2 : 3\hat{i} - 2\hat{j}$

30.(118)



Equation of circle $\quad (x-2)^2 + y^2 = R^2$

Equation of ellipse $\quad \frac{x^2}{36} + \frac{y^2}{16} = 1$

Finding abscissa of points of intersection

$$R^2 - (x-2)^2 = 16\left(1 - \frac{x^2}{36}\right)$$

Since, the largest circle touches ellipse, the above quadratic equation has repeated root.

$\Rightarrow R^2 = \frac{64}{5}$