SOLUTIONS

Joint Entrance Exam | IITJEE-2023

24th JAN 2023 | Evening Shift

PHYSICS

SECTION – 1

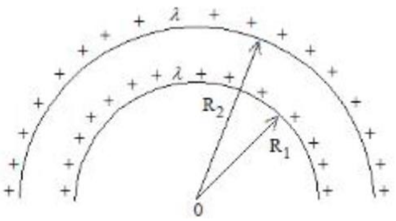
1.(1) Truth Table

A	B	Output
1	1	0
1	0	1
0	1	1
0	0	1

If either of the switch in opened, bulb will glow

If both are closed, bulb will be short and it will not glow.

2.(2) Charge at smaller half ring $= \lambda\pi R_1$



Charge at larger half ring $= \lambda\pi R_2$

Potential at centre $= \frac{kQ_1}{R_1} + \frac{kQ_2}{R_2}$

$= 2k\lambda\pi = \frac{2\lambda\pi}{4\pi\varepsilon_0} = \frac{\lambda}{2\varepsilon_0}$

3.(3) $v = E_0 / B_0$

$\frac{\omega}{k} = E_0 / B_0$

$B_0\omega = E_0 k$

4.(4) Fact Based

5.(1) $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mKE}}$

Same KE $\therefore$ $\lambda \propto \frac{1}{\sqrt{m}}$

$m_\alpha > m_p > m_e$ $\quad\therefore\quad$ $\lambda_\alpha < \lambda_p < \lambda_e$

6.(1) $B_{inside} = \mu_0 nI$

Solenoid $= 4\pi \times 10^{-7} \times 70 \times 100 \times 2 = 88 \times 2 \times 10^{-4} = 176 \times 10^{-4} T$

7.(4) Spring force will provide centripetal acceleration

$kx = m\omega^2(\ell + x)$

$12.5x = 0.2(25)(\ell + x)$

$x = 0.4(\ell + x)$

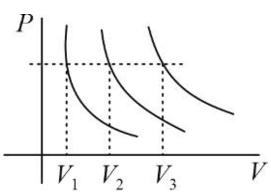
$0.6x = 0.4L;$ $\frac{x}{L} = \frac{0.4}{0.6} = \frac{2}{3}$

8.(1) For an isothermal process P v/s V graph is a rectangular hyperbola.

For same n

$P \times V = nRT$

if V increases, T also increases



$V_3 > V_2 > V_1$ $\quad\therefore\quad$ $T_3 > T_2 > T_1$

9.(3) Statement II is incorrect.
It is not same at same height and depth

10.(4) Motional EMF Question

$E = \frac{B\omega L^2}{2}$ {Basic Formula}

11.(2) Using Keplers laws

$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$

$\frac{1^2}{(2.83)^2} = \frac{(1.5\times10^6)^3}{R_2^3}$

$R_2^3 \sim 8\times(1.5\times10^6)^3$

$R_2 \sim 2\times1.5\times10^6$

$R_2 \sim 3\times10^6$

12.(2) $\bar{P}\cdot\bar{Q} = PQ\cos 90° = 0$

$(\hat{i} + 2m\hat{j} + m\hat{k})\cdot(4\hat{i} - 2\hat{j} + m\hat{k}) = 0$

$4 - 4m + m^2 = 0$

$(m-2)^2 = 0;\quad m = 2$

13.(4) $T = 2\pi\left(\frac{\ell}{g}\right)^{1/2}$

$g_{\text{at height}} < g_{\text{at surface}}$

$\therefore$ A is incorrect.

14.(3) Area above time axis $= 16 + 16 = 32$ m

Area below time axis $= 8 + 8 = 16$

Distance $= 32 + 16 = 48$

Displacement $= 32 - 16 = 16$ $\quad\therefore\quad$ Ratio $= \frac{16}{48} = \frac{1}{3}$

15.(2) Different lights have different foci due to different μ.
This is known as chromatic aberration.

16.(1) $E = 13.6\times1\times\left(1 - \frac{1}{16}\right) = 13.6\times\frac{15}{16} = 12.75eV$

$E = \frac{hc}{\lambda}$

$12.75 = \frac{4\times10^{-15}\times3\times10^8}{\lambda}$

$\lambda = 0.941\times10^{-7} = 94.1\times10^{-9}$ m

17.(4) $[v] = T^{-1}$

$[r] = L^{-1}$

$[\rho] = M^1L^{-3}$

$[s] = M^1T^{-2}$

$$T^{-1} = L^a (M^1 L^{-3})^b (M^1 T^{-2})^c$$

$$-1 = -2c; \qquad (c = 1/2)$$

$$c + b = 0 \quad \left(b = -\frac{1}{2}\right)$$

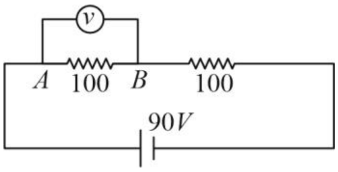
$$a - 3b = 0; \qquad \left(a = -\frac{3}{2}\right)$$

18.(4) $\gamma_1 = \frac{5}{3}; \qquad \gamma_2 = \frac{7}{5}$

$$\frac{\gamma_1}{\gamma_2} = \frac{25}{21}$$

19.(1) Voltmeter and Resistance in parallel

$$\therefore \; R_{AB} = \frac{400 \times 100}{500} = 80\Omega$$



$$\therefore \; (R_{eq})_{circuit} = 80 + 100 = 180\Omega$$

$$\therefore \; \text{I in circuit} = \frac{90}{180} A$$

$$V_{AB} = R_{AB} \times I = 80 \times \frac{90}{180} = 40V$$

20.(4) Both A and R are correct steel is more elastic.

SECTION – 2

21.(54) Using lens maker formula

$$\frac{1}{18} = (1.5 - 1)(K) \qquad \{\text{In air}\}$$

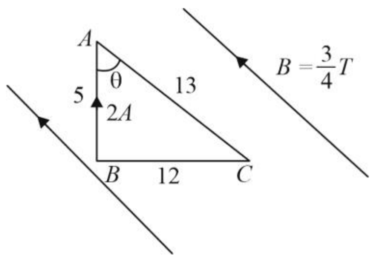
$$\frac{1}{f} = \left(\frac{1.5}{4/3} - 1\right)(K) \qquad \{\text{In water}\}$$

Dividing both we get:

$$\frac{f}{18} = \frac{0.5}{0.5} \times 4 \quad \Rightarrow \quad f = 72 \text{ cm}$$

$$\therefore \quad \text{change} = 72 - 18 - 54 \text{ cm}$$

22.(9) Force on wire $AB = I(\bar{L} \times \bar{B})$



$$= 2\left(\frac{5}{100}\right)\left(\frac{3}{4}\right)\sin\theta$$

$$= \frac{3}{40} \times \frac{12}{13} = \frac{9}{130} N$$

23.(1) $T = 2\pi\left(\frac{m}{k}\right)^{1/2}; \qquad 1 = 2\pi\left(\frac{m}{k}\right)^{1/2}$

$$2 = 2\pi\left(\frac{m+3}{k}\right)^{1/2}$$

Diving we get $\frac{m+3}{m} = 4 \quad \Rightarrow \quad m = 1\text{kg}$

24.(6) No. of atoms in 120 gm $=\frac{120}{240}\times 6\times 10^{23}=3\times 10^{23}$

$\therefore$ Total energy $=200\times 3\times 10^{23}$ $=6\times 10^{25}$ MeV

25.(32) Let d be density of material

$$I_1=\frac{MR^2}{2}=\frac{d(\pi R^2L)R^2}{2}$$

$$I_2=\frac{M'R'^2}{2}=\frac{d\left(\frac{\pi R^2}{4}\right)\left(\frac{L}{2}\right)\times\left(\frac{R}{2}\right)^2}{2}\ ;\ \frac{I_1}{I_2}=\frac{32}{1}$$

26.(100) $F=t\hat{i}+3t^2\hat{j}\,;\,a=t\hat{i}+3t^2\hat{j}$

$$\bar{v}=\int_0^2 tdt\hat{i}+\int_0^2 3t^2dt\hat{j}$$

$$\bar{v}(2)=2\hat{i}+8\hat{j}$$

$$\bar{F}(2)=2\hat{i}+12\hat{j}$$

$$P=\bar{F}\cdot\bar{v}=4+96=100W$$

27.(44) $L_i=L$; $L_f=1.2L$

$A_i=A$; $A_f=A/1.2$ {Volume Remains Constant}

$R_i=\rho L/A$; $R_f=1.44R_i$

$$\therefore\quad \frac{\Delta R}{R}\times 100=44\%$$

28.(7) At constant velocity, force = 0

$$mg=B_F+F_v$$

$$d\frac{4}{3}\pi x^3g=\rho\frac{4}{3}\pi x^3g+F_v$$

$$F_v=(d-p)\frac{4}{3}\pi x^3g=10^3\times 9\times\frac{4}{3}\times\frac{22}{7}\times 10^{-9}\times 9.8=3696\times 10^{-7}N$$

29.(3) Inductors will behave like wires

$$\therefore\quad \frac{1}{R_{eq}}=\frac{3}{12}\quad\Rightarrow\quad R_{eq}=4\Omega$$

$$I=\frac{12}{4}=3A$$

30.(105) $\frac{A\varepsilon_0}{d}=15pF$

Now $\frac{3.5A\varepsilon_0}{2d}=\frac{3.5}{2}\times 15=\frac{15\times 7}{4}pF=\frac{105}{4}pF$

CHEMISTRY

SECTION – 1

1.(3) $N_2 : \sigma_{1s^2} < \sigma^*_{1s^2} < \sigma_{2s^2} < \sigma^*_{2s^2} < \pi_{2px^2} < \sigma_{2pz^2} < \pi^*_{2px} < \sigma^*_{2pz}$

π_{2py^2} π^*_{2py}

Number of unpaired electron(s) in HOMO $(\sigma_{2pz}) = 0$

$N_2^+ : \sigma_{1s^2} < \sigma^*_{1s^2} < \sigma_{2s^2} < \sigma^*_{2s^2} < \pi_{2px^2} < \sigma_{2pz^1} < \pi^*_{2px} < \sigma^*_{2pz}$

π_{2py^2} π^*_{2py}

Number of unpaired electron(s) in HOMO $(\sigma_{2pz}) = 1$

$O_2 : \sigma_{1s^2} < \sigma^*_{1s^2} < \sigma_{2s^2} < \sigma^*_{2s^2} < \sigma_{2pz^2} < \pi_{2px^2} < \pi^*_{2px^1} < \sigma^*_{2pz}$

π_{2py^2} $\pi^*_{2py^1}$

Number of unpaired electron(s) in HOMO = 2

$O_2^+ : \sigma_{1s^2} < \sigma^*_{1s^2} < \sigma_{2s^2} < \sigma^*_{2s^2} < \sigma_{2pz^2} < \pi_{2px^2} < \pi^*_{2px^1} < \sigma^*_{2pz}$

π_{2py^2} π^*_{2py}

Number of electron(s) in HOMO = 1

2.(1) Electronic configuration of element having 55 protons in its unipositive state is

$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4p^6\ 5s^2\ 4d^{10}\ 5p^6$

Number of s electrons = 10

3.(1) $K_2Cr_2O_7 + H_2SO_4 + SO_2 \longrightarrow K_2SO_4 + \underset{\text{(green)}}{Cr_2(SO_4)_3} + H_2SO_4$

4.(3) Crystal field model successfully explains structure, stability, magnetic property and colour of metal complex but could not explain the order of spectrochemical series.

5.(4) For alkaline earth metals, value of standard potential.

Standard potential $E^\ominus / V$ for (M^{2+}/M)	Be	Mg	Ca	Sr	Ba	Ra
	–1.97	–2.36	–2.84	–2.89	–2.92	–2.92

Be has very large value of enthalpy of atomization compared to hydration energy of Be^{2+}.

6.(2) $\overset{+1-2+1}{HOCl} + H_2\overset{-1}{O_2} \longrightarrow H_3O^+ + \overset{-1}{Cl^-} + \overset{0}{O_2}$

(reduction: HOCl → Cl^-; oxidation: H_2O_2 → O_2)

In above reaction H_2O_2 is itself getting oxidized and reducing HOCl.

7.(1) $Ag(s) + CN^-(aq) + H_2O(aq) \xrightarrow{O_2} [Ag(CN)_2]^- + OH^-$

$[Ag(CN)_2]^- + Zn(s) \longrightarrow [Zn(CN)_4]^{2-} + Ag(s)$

Ag is first oxidized to $[Ag(CN)_2]^-$ and then reduced to Ag.

8.(4) **(A)**

Property	Lithium Li	Sodium Na	Potassium K	Rubidium Rb	Caesium Cs	Francium Fr
Standard potentials $E^{\ominus} / V$ for (M^+/M)	–3.04	–2.714	–2.925	–2.930	–2.927	–

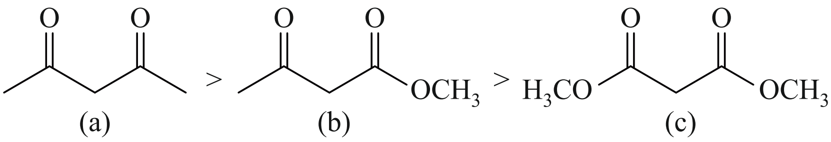
(B) CsI is less soluble in water

(C) $Li_2CO_3 \longrightarrow Li_2O + CO_2 \quad \because \quad Li_2CO_3$ is unstable to heat

(D) When alkali metal is dissolved in concentration liquid ammonia, solution of colour changes to bronze and become diamagnetic.

(E) All alkali metal hydrides are ionic

9.(3) Order of acidic strength is



'a' will undergo deprotonation fastest.

10.(4) Ce^{4+} and Tb^{4+} are good oxidizing agent.

$E^{\circ}_{Ce^{4+}/Ce^{3+}} = 1.74\,V$

11.(4)

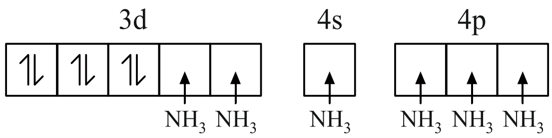
Norethindrone	–	Antifertility drug
Meprobomate	–	Transquilizer
Seldane	–	Antihistamine
Ampicillin	–	Antibiotic

12.(1) $\underset{\text{(Weak acid)}}{PhCOOH} + \underset{\text{(Strong base)}}{NaOH} \longrightarrow PhCOONa + H_2O$

Firstly, there will be increase in conductance due to formation of more conducting species PhCOONa than benzoic acid. After equivalence point, more conducting species OH^- will be responsible for Sharpe increase in conductance.

13.(3) Electronic configuration of $Co^{+3} = [Ar]3d^6$

NH_3 acts as strong field ligand when central atom is in +3 oxidation state.



Hybridization $= d^2sp^3$

Compound is diamagnetic.

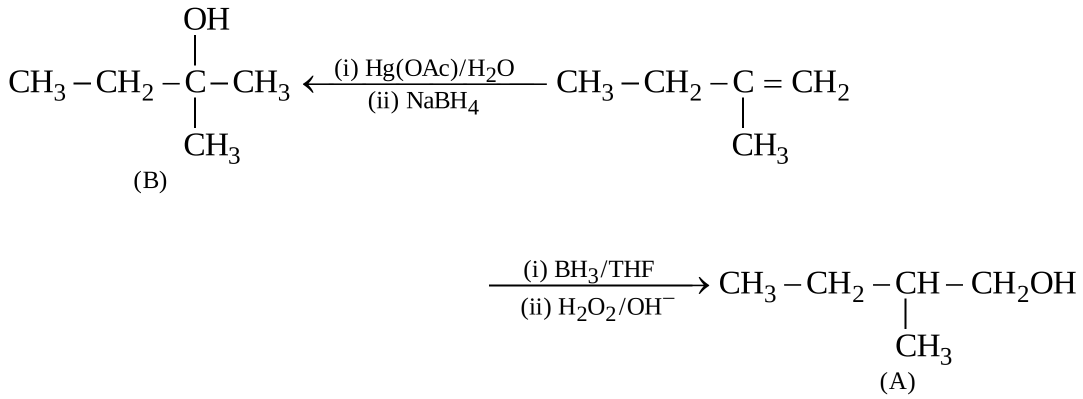
14.(4) $(t_{1/2}) \propto \dfrac{1}{(a)^{n-1}}$

$$\frac{(t_{1/2})_1}{(t_{1/2})_2} = \left(\frac{a_2}{a_1}\right)^{n-1}$$

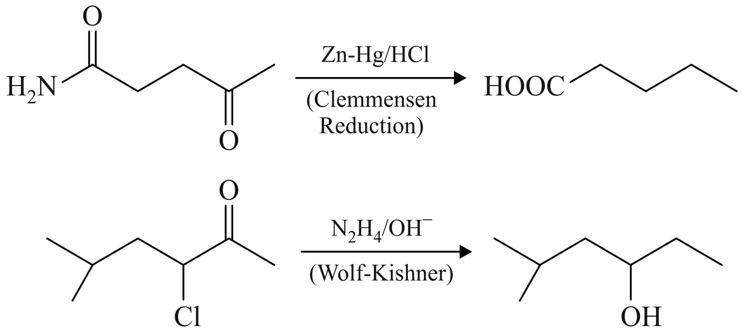
$$\left(\frac{4}{2}\right) = \left(\frac{100}{50}\right)^{n-1}$$

$2 = 2^{n-1} \quad \because \quad n-1 = 1 \quad \Rightarrow \quad n = 2$

15.(4)

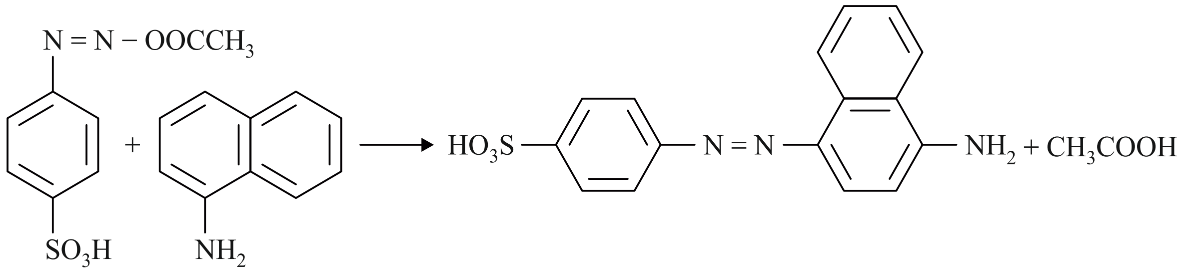


16.(3)



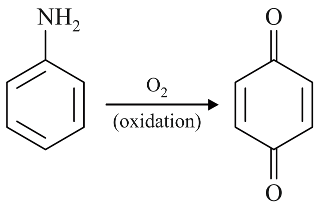
17.(2) Benzene is more stable due to aromatic nature in which delocalized π electron cloud is more strongly attracted by nuclei.

18.(3)



1-Naphthyl amine　　　　Red azo-dye

19.(4) Pure aniline is colourless.



20.(3) Fact based.

SECTION – 2

21.(85) Calcium lactate is salt of weak acid and strong base.

$$pH=\frac{1}{2}[pk_w+pka+\log c]$$

$$=\frac{1}{2}[14+5+\log 10^{-2}]=\frac{1}{2}[14+5-2]=\frac{1}{2}\times 17=8.5$$

$$\because \quad pH=8.5\times 10^{-1}$$

22.(314) $P_s = P_A^\circ x_A + P_B^\circ x_B$

$350 = P_A^\circ \times 0.7 + P_B^\circ \times 0.3$ …… (i)

$410 = P_A^\circ \times 0.2 + P_B^\circ \times 0.8$ …… (ii)

From (i) and (ii), we get

$$\frac{410}{350} = \frac{0.2P_A^\circ + 0.8P_B^\circ}{0.7P_A^\circ + 0.3P_B^\circ}$$

$P_A^\circ = 314 \text{ mm of Hg}$

23.(5) Mass percent
Mole fraction
Molarity
ppm
Molality

24.(3) **(A)** $T_1 > T_2 > T_3 > T_4$

(B) Planck's hypothesis implies that radiation of frequency can be generated only if an oscillator of that frequency has acquired the minimum energy required to start oscillation. Thus atoms in black body acts as SHM.

(C) As the temperature increases the maximum intensity of emission moves to shorter wavelength.

(D) The wavelength corresponding to maximum intensity is inversely proportional to absolute temperature.

$\therefore \quad \lambda \alpha \frac{1}{T}$

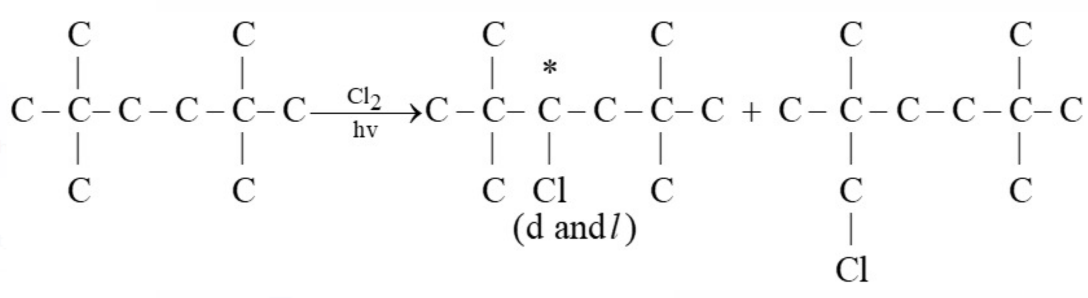
$\lambda T = \text{constant}$

But $\nu \times \lambda = c$

$\therefore \quad \lambda = \frac{c}{\nu} \quad \therefore \quad \frac{T}{\nu} = \text{constant}$

(E) If the oscillating atom releases an energy E into the surroundings, then radiation of frequency $\nu = E/h$ will be detected.

25.(3)



26.(2) **(A)** At point 'b', liquification starts.
(B) At point 'c', all gas is present in liquid state.
(C) Between b and c, both liquid and gaseous state coexist.
(D) $CO_2(g) \rightleftharpoons CO_2(\ell)$

As volume is decreased, pressure of CO_2 increases and according to Le chat principle, reaction will go forward hence amount liquid increases.

27.(620) $w_{1-2} = -P_{ext}(v_2 - v_1)$ [isobaric process]

$= -1[40-20] = -20\,L\text{-}bar$

$w_{2-3} = 0$ [isochoric process]

$w_{3-1} = -P_1V_1 \ln\frac{v_2}{v_1}$ [isothermal process]

$= -0.5\times 40\times 2.303\log\frac{20}{40} = -20\times 2.303\times\log\frac{1}{2}$

$= 20\times 2.303\times 0.3 = 13.818 \approx 13.8\,L\text{-}bar$

$w_{net} = w_1 + w_2 + w_3$

$= -20+0+13.8 = 6.2\,L\text{-}bar$

$= 6.2\,L\text{-}bar = 6.2\times 100\,J \cong 620\,J$

28.(8) V – V – V

V – V – P

V – P – V $V\rightarrow$ Valine

P – V – V $P\rightarrow$ Proline

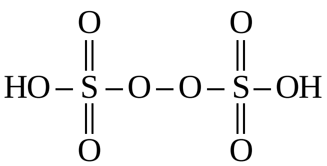
P – P – P

P – P – V

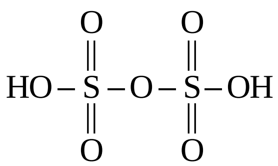
P – V – P

V – P – P

29.(8)



(peroxodisulphuric acid)



(pyrosulphuric acid)

Number of π bonds = 4 Number of π bonds = 4

Total number of π bonds $= 4+4 = 8$

30.(2) **(a)** It is not specific in nature

(b) Enthalpy of adsorption is low

(c) It decreases with increase in temperature

(d) It results into multimolecular layer

(e) No appreciable activation energy required

MATHEMATICS

SECTION – 1

1.(4) $f(x)+f(1-x)=\dfrac{2^{2x}}{2^{2x}+2}+\dfrac{2^{2-2x}}{2^{2-2x}+2}=1$

$\therefore \quad f\left(\dfrac{1}{2023}\right)+f\left(\dfrac{2}{2023}\right)+f\left(\dfrac{3}{2023}\right)+...+f\left(\dfrac{2022}{2023}\right)=1011$

2.(2) $\displaystyle\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}}\frac{48}{\sqrt{9-4x^2}}dx=24\left[\sin^{-1}\frac{2x}{3}\right]_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}}=24\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=2\pi$

3.(2) Since vertex A lies on y-axis

$\therefore \quad \dfrac{4}{\lambda}=\dfrac{\lambda}{\lambda-1}\Rightarrow\lambda^2-4\lambda+4=0\Rightarrow\lambda=2$

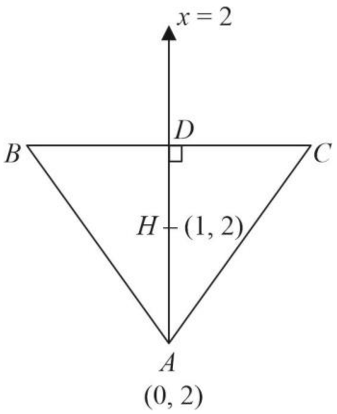
$\therefore \quad A\equiv(0,2)$

$AB:3x+2y=4$

$AC:2x-y+2=0$

Equation of CH $y\simeq\dfrac{2}{3}x+\dfrac{4}{3}$

$\therefore \quad C\equiv\left(\dfrac{-1}{2},1\right)$



Equation of tangent at $P\left(6t^2,6t\right)$ $3x-6ty+18t^2=0$ Passes through $C\left(\dfrac{-1}{2},1\right)$

Then $\dfrac{-3}{2}-6t+18t^2=0\Rightarrow12t^2-4t-1=0$

$\Rightarrow \quad t=\dfrac{1}{2},\dfrac{-1}{6}$

$\therefore$ Point on passes $\equiv P\left(\dfrac{3}{2},3\right)$

$\therefore$ Length of tangent, CP $=\sqrt{4+4}=2\sqrt{2}$

4.(1) No. of square matrix = $5C_1 . 4C_1 . 3C_1 . 2C_1 . 1C_1 .$

$= \lfloor 5$

5.(1) Plane P $[x+(\lambda+4)y+z-1]+k[2x+y+z-2]$

$\therefore \quad (\lambda+4)-1+K(1-2)=0 \Rightarrow k=\lambda+3$

and $\quad 1+K=0 \Rightarrow K=-1$

$\therefore \quad \lambda=-4$

$\therefore$ Distance of point (–8, –4, 4)

From plane $2x+y+z=2$

$$= \frac{|2(-8)+(-4)+(4)-2|}{\sqrt{(2)^2+(1)^2+(1)^2}} = \frac{18}{\sqrt{6}} = 3\sqrt{6}$$

6.(2) $\left(x^2-3y^2\right)dx+3xy\ dy=0$

$\Rightarrow \quad x^2dx+3y(xdy-ydy)=0 \quad \Rightarrow \quad \frac{dx}{x}+3\left(\frac{y}{x}\right)d\left(\frac{y}{x}\right)=0$

$\Rightarrow \quad \ln x+\frac{3\left(\frac{y}{x}\right)^2}{2}=c$ Passes through (1, 1)

$\therefore \quad c=\frac{3}{2} \qquad \therefore \quad \ln x=\frac{3}{2}\left[1-\frac{y^2}{x^2}\right]$

Put $x = e$

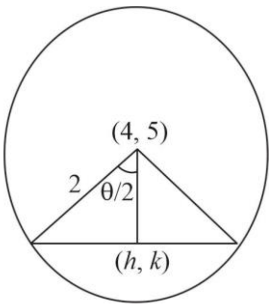
$$1=\frac{3}{2}\left[1-\frac{(y(e))^2}{e^2}\right]$$

$\Rightarrow \quad (y(e))^2=\frac{e^2}{3} \qquad \Rightarrow \quad 6y^2(e)=2e^2$

7.(3) $|adj(adj(adjA))|=|A|^{2^3}=12^4 \Rightarrow |A|=\sqrt{12}$

$\therefore \quad \left|A^{-1}adjA\right|=\left|A^{-2}\cdot|A|\cdot I\right|=|A|^3\cdot|A|^{-2}=|A|=\sqrt{12}$

8.(3) $(h-4)^2+(k-5)^2=\left(2\cos\frac{\theta}{2}\right)^2$



$\Rightarrow$ locus of mid-point of chord

$$(x-4)^2+(y-5)^2=4\cos^2\frac{\theta}{2}$$

$\Rightarrow \quad (x-4)^2+(y-5)^2=2+2\cos\theta$

$$r_1 = 2\cos\frac{\theta_1}{2} = 2\cos\frac{\pi}{6} = \sqrt{3}$$

$$r_3 = 2\cos\frac{\theta_3}{2} = 2\cos\frac{\pi}{3} = 1$$

$$\therefore \quad r_2 = \sqrt{r_1^2 - r_3^2} = \sqrt{2} = 2\cos\frac{\theta_2}{2} \Rightarrow \cos\frac{\theta_2}{2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \theta_2 = \frac{\pi}{2}$$

9.(1) $\vec{\beta}_1 = \lambda\left(4\hat{i} + 3\hat{j} + 5\hat{k}\right)$

$\vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1 = (1-4\lambda)\hat{i} + (2-3\lambda)\hat{j} + (-4-5\lambda)\hat{k}$

$\vec{\beta}_2 \perp \vec{\alpha} \Rightarrow (1-4\lambda)4 + (2-3\lambda)3 + (-4-5\lambda)5 = 0$

$\Rightarrow \quad \lambda = \frac{-1}{5}$

$5\vec{\beta}_2 \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = 5[-1-12\lambda] = 5\left(-1 + \frac{12}{5}\right) = 7$

10.(2) $f(x+y) = f(x) \cdot f(y)$ and $f(1) = 3 \Rightarrow f(x) = 3^x$

$\therefore \quad \sum_{k=1}^{n} f(k) = 3279 \Rightarrow \sum_{k=1}^{n}\left(3^k\right) = 3279$

$\Rightarrow \quad \frac{3\left(3^n - 1\right)}{2} = 3279 \Rightarrow 3^n = 2187 \Rightarrow n = 7$

11.(3) All 5! 5digit numbers.

and ${}^2C_1 \times {}^4C_1 \times 3!$ 4 digits numbers are greater than 7000.

12.(3) $x - 5 = 2x + 2$ at $x = -7$.

Also, for any $a, b \in R$.

$[a] = [b] \Rightarrow a - f_1 = b - f_2$ where

$f_1 = \{a\}$ and $f_2 = \{b\}$

$\Rightarrow \quad a - b = f_1 - f_2$

$\Rightarrow \quad a - b \in (-1, 1)$

as $\quad f_1, f_2 \in [0, 1)$

$2x + 2 - (x - 5) > -1 \Rightarrow x > -8$

$2x + 2 - (x - 5) < -1 \Rightarrow x < -6$

Hence, $x \in (-8, -6)$

Analyzing graph of $[x - 5]$ and $[2x + 2]$ in (–8, –6), or using properties of GIF,

$$[x-5]=\begin{cases}-13 & x\in(-8,-7)\\ -12 & x\in[-7,-6)\end{cases}$$

$$[2x+2]=\begin{cases}-14 & x\in(-8,-7.5)\\ -13 & x\in[-7.5,-7)\\ -12 & x\in[-7,-6\cdot5)\\ -11 & x\in[-6\cdot5,-6)\end{cases}$$

$\Rightarrow \quad [x-5]=[2x+2]\ \forall x\in[-7.5,-6.5)$

$\Rightarrow \quad a\in(-7.5-6.5)$ as $\lim\limits_{x\to-7.5}[x-5]-[2x+2]$ doesn't exist

13.(2) Put $f(x)=x^3-Ax^2+Bx-C$

To get $C=6, A=3 \text{ and } B=6$

14.(2) DRs of the line: $(-5,1,3)$

Any point on it: $(3-5k, 2+k, 1+3k)$

Since, line joining this point to (1,9,7) is $\perp r$ to given line, we get: $k=1$

15.(3) $\Delta=0\Rightarrow\lambda=\frac{72}{5}$

and solve for μ by using the fact that given system of equation has infinitely many solutions.

16.(2) Using $\frac{1+e^{i\theta}}{1+e^{-i\theta}}=e^{i\theta}$, we get final answer as $e^{i3\theta}$, where $\theta=\frac{5\pi}{18}$.

17.(3) $a_1+a_3=10\Rightarrow a_2=5$

Sum of 6 numbers = 57

$\Rightarrow \quad a_4+a_5+a_6=42 \quad \Rightarrow \quad a_5=14$

$\Rightarrow \quad d=3 \quad \Rightarrow \quad A.P.=2,5,8,11,14,17$

$$\sigma^2=\frac{1}{6}\times 2\left(\left(\frac{3}{2}\right)^2+\left(\frac{9}{2}\right)^2+\left(\frac{15}{2}\right)^2\right)=\frac{105}{4}$$

18.(4) Let $t=\frac{x+1}{x}\Rightarrow t\in(-\infty,-2]\cup[2,\infty)$

$\Rightarrow \quad 3(t^2-2)-2t+5=0 \quad \Rightarrow \quad 3t^2-2t-1=0 \quad \Rightarrow \quad t=1,\frac{-1}{3}$

$\Rightarrow$ No real solution.

19.(2) $\sum\limits_{r=1}^{n} r\left({}^nC_r\right)=\sum\limits_{r=1}^{n} n\left({}^{n-1}C_{r-1}\right)\left({}^nC_{n-r}\right)$

$$=\sum_{r=1}^{n} n\left({}^{n-1}C_{r-1}\right)\left({}^nC_{n-r}\right)$$

= coefficient. of x^{n-1} in $n(1+x)^{n-1}\times(1+x)^n$

$=n\times {}^{2n-1}C_{n-1}\Rightarrow\alpha=2$

20.(4) $\sim\left(p\wedge\left(p\to\sim q\right)\right)$

$=\sim\left(p\wedge\left(\sim p\vee\sim q\right)\right)=\sim\left(p\wedge\sim q\right)=\left(\sim p\right)\vee q$

SECTION – 2

21.(27) $f'(x)+f(x)\sqrt{1-\ln^2 f(x)}=0$

$\Rightarrow \quad \frac{dy}{dx}+y\sqrt{1-\ln^2 y}=0 \quad \Rightarrow \quad \int\frac{dy}{y\sqrt{1-\ln^2 y}}=\int -dx$

$\Rightarrow \quad \sin^{-1}(\ln y)=-x+c$

At $x=0\Rightarrow y=e$

$\Rightarrow \quad c=\frac{\pi}{2} \quad \Rightarrow \quad f(x)=e^{\cos x} \quad \Rightarrow \quad \left\{6\ln f\left(\frac{\pi}{6}\right)\right\}^2=27$

22.(2) $\frac{\sin(\pi\cos\theta)}{\cos(\pi\cos\theta)}+\frac{\sin(\pi\sin\theta)}{\cos(\pi\sin\theta)}=0$

$\Rightarrow \quad \sin[\pi(\cos\theta+\sin\theta)]=0 \quad \Rightarrow \quad \cos\theta+\sin\theta=-1,0,1$

$\Rightarrow \quad \theta=0,\frac{\pi}{2},\frac{3\pi}{4},\pi,\frac{3\pi}{2},\frac{7\pi}{4}$

$\Rightarrow \quad \sin^2\left(\frac{\pi}{4}\right)+\sin^2\left(\frac{3\pi}{4}\right)+\sin^2(\pi)+\sin^2\left(\frac{5\pi}{4}\right)+\sin^2\left(\frac{7\pi}{4}\right)+sin^2 2\pi \Rightarrow 2$

23.(5) $\frac{\sum_{r=1}^{h} r^3}{\sum_{r=1}^{h} r.(2r+1)}$

$\Rightarrow \quad \frac{n^2(n+1)^2}{4\left[\frac{2\,.\,n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}\right]}=\frac{9}{5}$

$\Rightarrow \quad 5n^2-19n-30=0 \quad \therefore \quad n=5$

24.(405)

$\Rightarrow \quad {}^nC_0\,.\,(-3)^0+{}^nC_1\,.\,(-3)^1+{}^nC_2\,.\,(-3)^2=376$

$\Rightarrow \quad 1-3n+9\frac{n(n-1)}{2}=376 \quad \Rightarrow \quad 3n^2-5n-250=0$

$\therefore \quad n=10$

Now, In $\left(x-\frac{3}{x^2}\right)^{10}$

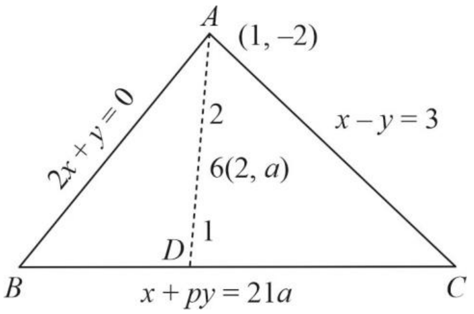
$$T_{r+1} = {}^{10}C_r \ x^{10-r}\left(\frac{-3}{x^2}\right)^r$$

$\therefore \quad 10 - r - 2r = 4$

$\therefore \quad r = 2$

$\therefore \quad$ Coefficient of $x^4 = {}^{10}C_2\,(-3)^2 = 405$

25.(122)



$\therefore \quad D\left(\frac{5}{2}, \frac{3a+2}{2}\right)$

Let B $(b, -2b)$

$C(c, c-3)$

$\therefore \quad b + c = 5 \qquad \ldots$(i)

and

$-2b + c = 3a + 5 \qquad \ldots$(ii)

From (i) and (ii)

$b = -a$ and $c = 5 + a$

Now satisfy $B(-a, 2a)$

and $C(5+a, 2+a)$ in BC to get $p = 11$ and $a = 3$

$\therefore \quad B(-3, 6)$ and $C(8, 5)$

$\therefore \quad BC^2 = 122$

26.(13)

$$R = \begin{Bmatrix} (a,b),(b,a),(b,c),(c,b),(b,d),(d,b),(a,a),(b,b), \\ (c,c),(d,d),(a,c),(c,a),(a,d),(d,a),(c,d),(d,c) \end{Bmatrix}$$

$\therefore$ 13 elements must be added to R such that it is an equivalence relation.

27.(384)

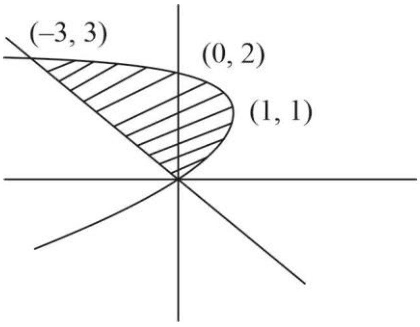
$$\frac{(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} = \pm 6 \quad \Rightarrow \quad \frac{\left\{\left(\lambda + \sqrt{6}\right)\hat{i} + \sqrt{6}\hat{j} - 3\sqrt{6}\hat{k}\right\}.\left(-\hat{i} + 2\hat{j} - \hat{k}\right)}{\sqrt{6}} = \pm 6$$

$\therefore \quad \lambda = -2\sqrt{6}$ and $10\sqrt{6}$

$\therefore \quad sum = 8\sqrt{6}$

$\therefore \quad$ Square = 384

28.(36)



$$A=\int_0^3 \left(x_{\text{Parabola}}-x_{\text{line}}\right)dy$$

$$A=\int_0^3 2y-y^2+y\ \ dy$$

$\therefore \quad A=\frac{9}{2}$

$\therefore \quad 8A=36$

29.(8) $\vec{a}\times\vec{c}=\vec{b}\times\vec{c}$

$\Rightarrow \quad \left(\vec{a}-\vec{b}\right)\times\vec{c}=\vec{0}$

$\therefore \quad \vec{c}=t\left(\vec{a}-\vec{b}\right)$

$\therefore \quad \vec{c}=-2t\hat{i}+7t\hat{j}+2\lambda t\hat{k}$

Now $\quad \vec{a}\cdot\vec{c}=12t+2\lambda^2 t=7 \qquad \ldots$(i)

$\quad \vec{b}\cdot\vec{c}=-41t-2\lambda^2 t=\frac{-43}{2} \qquad \ldots$(ii)

From (i) and (ii)

$t=\frac{1}{2}$ and $\lambda^2=1$

$\therefore \quad \left|\vec{a}.\,\vec{b}\right|=\left|3-10-\lambda^2\right|$

$= 8$

30.(432)

A	B	C
$4R$	$5R$	λR
$6B$	$5B$	$4B$

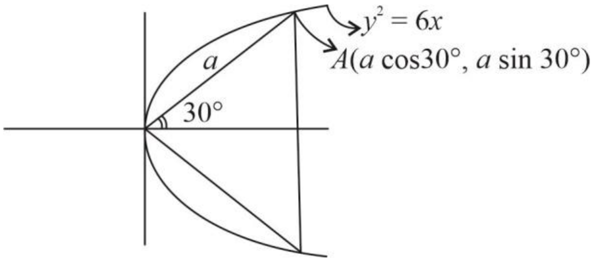
Let A be the event of getting a red ball.

Let E_1 be the event of selecting urn A.

Let E_2 be the event of selecting urn B.

Let E_3 be the event of selecting urn C.

$$P\left(E_3/A\right)=\frac{\frac{1}{3}\cdot\frac{\lambda}{\lambda+4}}{\frac{1}{3}\cdot\frac{4}{10}+\frac{1}{3}\cdot\frac{5}{10}+\frac{1}{3}\cdot\frac{\lambda}{\lambda+4}}=\frac{4}{10}$$



$\therefore \quad \lambda = 6$

$y^2 = 6x$

$A(a\cos 30°, a\sin 30°)$

$A\left(\frac{a\sqrt{3}}{2}, \frac{a}{2}\right)$

$\therefore \quad \frac{a^2}{4} = \frac{6a.\sqrt{3}}{2}$

$\therefore \quad a = 12\sqrt{3}$

$\therefore \quad a^2 = 432$