IIT JEE | MEDICAL | FOUNDATION

MATHEMATICS
CLASS-X

SOLUTIONS

SECTION-A

1.(C) HCF (26, 169) = 13

$\therefore$ LCM (26, 169) = $\frac{26\times169}{13}=338$

2.(B) Let $p(x)=x^2-3x-m(m+3)$

$\Rightarrow \quad p(x)=x^2-(m+3)x+mx-m(m+3)$

$=x\{x-(m+3)\}+m\{x-(m+3)\}$

For zeros of $p(x)=(x+m)\{(x-(m+3)\}=0$

$\therefore$ Its zeros are $-m, m+3$

3.(B) Given quadratic equation, $2x^2+kx+2=0$

$\therefore$ Its discriminant, $D=(K)^2-4\times2\times2$

$\Rightarrow \quad D=K^2-16$

For roots to be real and equal

$D=0\Rightarrow K^2-16=0\Rightarrow K^2=16$

$\Rightarrow \quad K=\pm4$

4.(D) Given lines are $cx-y=2$ and $6x-2y=3$.

For infinitely many solutions, we have

$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}\Rightarrow\frac{c}{6}=\frac{-1}{-2}=\frac{-2}{-3}\Rightarrow\frac{c}{6}=\frac{2}{3}$

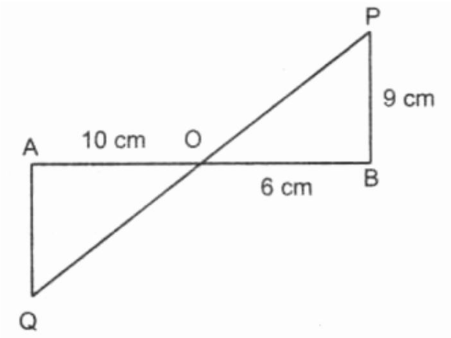
$\Rightarrow \quad c=\frac{6}{2}$ and $c=4 \Rightarrow c=3$ and $c=4$

Since, *c* has different values, hence, for no value of *c*, the pair of equations will have infinitely many solutions.

5.(A) We have,

$\frac{m}{3}=\frac{-6+(-2)}{2}\Rightarrow m=\frac{-8}{2}\times3\Rightarrow m=-12$

6.(A) In ΔAOQ and ΔBOP, we have



$\angle AOQ = \angle BOP$ (Vertically opposite angle)

$\angle OAQ = \angle OBP$ (Both of 90°)

$\therefore \quad \Delta AOQ \sim \Delta BOP$ (By AA similarity)

So $\frac{AO}{BO} = \frac{AQ}{PB} \Rightarrow \frac{10}{6} = \frac{AQ}{9} \Rightarrow AQ = \frac{10 \times 9}{6} = 15cm$

7.(C) Given $\frac{4 - \sin^2 45°}{\cot k \tan 60°} = 3.5$

$\Rightarrow \quad \frac{4 - \left(\frac{1}{\sqrt{2}}\right)^2}{\cot k \sqrt{3}} = 3.5 \Rightarrow \frac{4 - \frac{1}{2}}{\sqrt{3} \cot k} = 3.5$

$\Rightarrow \quad \frac{7}{2\sqrt{3} \times 3.5} = \cot k \Rightarrow \frac{7}{7\sqrt{3}} = \cot k$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \cot k \Rightarrow \cot 60° = \cot k \Rightarrow k = 60°$

8.(B) Given: $(1 + \tan^2 \theta)(1 - \sin \theta)(1 + \sin \theta)$

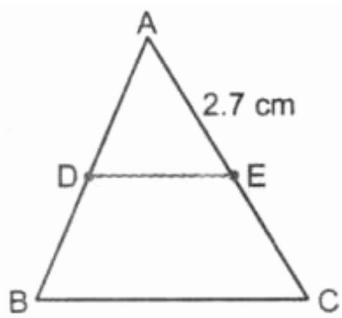
$= (\sec^2 \theta)(1 - \sin^2 \theta)$

$= (\sec^2 \theta)(\cos^2 \theta)$

$= \frac{1}{\cos^2 \theta} \times \cos^2 \theta = 1$

9.(D) We have,

In $\Delta ABC, DE \parallel BC$



$\Rightarrow \quad \frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{3}{2} = \frac{2.7}{EC}$

$\Rightarrow \quad EC = \frac{2.7 \times 2}{3} = 1.8cm = 1.8cm$

10.(B) In ΔPQR we have

$PR = QR \Rightarrow \angle Q = \angle P = 40°$

$\angle R = 180° - (\angle P + \angle Q) = 180° - (40° + 40°) = 100°$

$\therefore \quad \angle R = 100°$

We have, $\frac{AB}{PR} = \frac{5}{10} = \frac{1}{2}, \frac{AC}{QR} = \frac{5}{10} = \frac{1}{2} \Rightarrow \frac{AB}{PR} = \frac{AC}{QR} = \frac{1}{2}$

Condition for both be triangles to be similar is

$\angle A = \angle R = 100°$

11.(B) We have,

$(1 + \cot\theta - \text{cosec}\theta)(1 + \tan\theta + \sec\theta)$

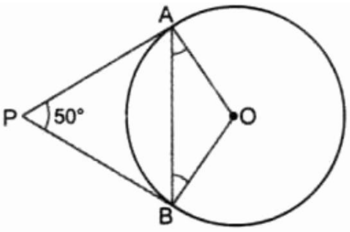
$= \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$

$= \left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)\left(\frac{\sin\theta + \cos\theta + 1}{\cos\theta}\right)$

$= \left(\frac{(\sin\theta + \cos\theta)^2 - (1)^2}{\sin\theta\cos\theta}\right) = \frac{\sin^2\theta + \cos^2\theta + 2\sin\cos\theta - 1}{\sin\theta\cos\theta}$

$= \frac{1 + 2\sin\theta\cos\theta 1}{\sin\theta\cos\theta} = \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2$

12.(A) $\angle AOB = 180° - 50° = 130°$



Let $\angle OAB = \angle OBA = x°$ (Angles opposite to equal sides)

Then $x° + x° + 130° = 180°$

$\Rightarrow \quad 2x° = 50° \Rightarrow x° = 25°$

$\Rightarrow \quad \angle OAB = 25°$

13.(B) Area of shaded region = area of circle – area of rectangle

Diagonal of rectangle = $\sqrt{12^2 + 5^2} = \sqrt{144 + 25} = 13\,cm$

Radius of circle = $\frac{13}{2}cm$

Area of shaded region = $\pi \times \left(\frac{13}{2}\right)^2 - 12 \times 5 = \frac{3.14}{100} \times \frac{169}{4} - 60$

$= \frac{53060}{400} - 60 = 132.66 - 60.00 = 72.66\ cm^2 = 73\,cm^2$(approx)

14.(D) First find the circumference of the wheel and then find number of revolution

$= \frac{\text{distance covered}}{\text{circumference of wheel}}$

Circumference of wheel = $2\pi r$

$= 2 \times \frac{22}{7} \times 42 = 12 \times 22$

Number of revolution = $\frac{79200}{12 \times 22} = 300$

15.(B) $\frac{n}{2} = \frac{60}{2} = 30$, median class = 160 – 165

Modal class = 150 – 155

Required sum = 150 + 165 = 315

16.(C) Given: Length of arc = 20 *cm*

$\Rightarrow \quad \frac{\theta}{360^\circ} \times 2\pi r = 20$

$\Rightarrow \quad \frac{60^\circ}{360^\circ} \times 2\pi r = 20$

$\Rightarrow \quad \frac{\pi r}{3} = 20$

$\Rightarrow \quad r\left(\frac{\pi}{3}\right) = 20$

$\Rightarrow \quad r = \frac{60}{\pi} cm$

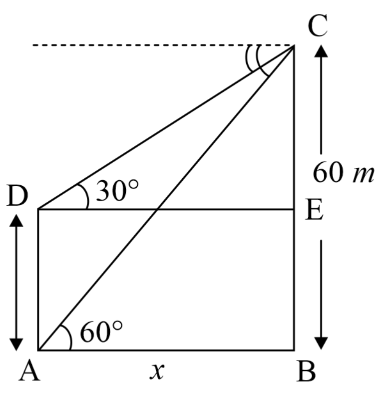
17.(C) Number of required tickets = 6000 × 0.08 = 480

18.(A) In triangle CDE,

$\tan 30^\circ = \frac{60 - h}{x}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{60 - h}{x}$

$\Rightarrow \quad x = \sqrt{3}(60 - h)m$ ….(i)



Again, in triangle CAB,

$\tan 60^\circ = \frac{60}{x}$

$\Rightarrow \quad \sqrt{3} = \frac{60}{x}$

$\Rightarrow \quad x = \frac{60}{\sqrt{3}}\ m$ ….(ii)

From equation (i) and (ii) we get,

$\sqrt{3}(60 - h) = \frac{60}{\sqrt{3}}$

$\Rightarrow \quad 60 - h = 20$

$\Rightarrow \quad h = 40\,m$

19.(C) We have

LCM $(a,b)\times HCF\,(a,b)=a\times b$

LCM × 5 = 150

$\therefore \quad LCM=\frac{150}{5}=30$

$\Rightarrow \quad LCM=30, i.e.,$ reason is incorrect and assertion is correct.

20.(D) $PQ=10\Rightarrow PQ^2=100\Rightarrow (10-2)^2+(y+3)^2=100$

$\Rightarrow \quad (y+3)^2 \quad \Rightarrow \quad 100-64=36\Rightarrow y+3=\pm 6$

$\Rightarrow \quad y=-3\pm 6 \quad \Rightarrow \quad y=3,-9$

So, A is incorrect but R is correct.

SECTION-B

21. Given, rectangle $ABCD$.

$\Rightarrow$ Opposite sides are equal.

Hence, $x+y=30$ ...(i)

$x-y=14$ ...(ii)

(i) + (ii) $2x=44\Rightarrow x=22$

Substituting in (i), $22+y=30\Rightarrow y=8$

$\Rightarrow \quad x=22, y=8$

22. We have,

In $\Delta ABC, DE \parallel AC$

$\therefore \quad \frac{BD}{DA}=\frac{BE}{EC}$

Also, in $\Delta ABP, DC \parallel AP$

$\therefore \quad \frac{BD}{DA}=\frac{BC}{CP}$ …(ii)

From (i) and (ii), we have

$\frac{BE}{EC}=\frac{BC}{CP}$

23. Given, $\angle QPR=120^\circ$

$\therefore$ Radius is perpendicular to the tangent at the point of contact.

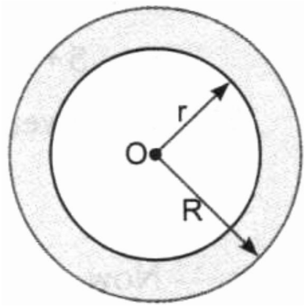
$\therefore \quad \angle OQP=90^\circ$ and

$\angle QPO=60^\circ$ (Tangents drawn to a circle from an external point are equally inclined to the segment, joining the centre to that point.)

In ΔQPO, $\cos 60^\circ=\frac{PQ}{PO}\Rightarrow \frac{1}{2}=\frac{PQ}{PQ}\Rightarrow 2PQ=PO$

24. Let the outer and inner radii of the ring be R m and r m respectively. Then,

$2\pi R=396$ and $2\pi r=352$



$\Rightarrow \quad 2\times\frac{22}{7}\times R=396$ and $2\times\frac{22}{7}\times r=352$

$\Rightarrow \quad R=396\times\frac{7}{22}\times\frac{1}{2}$ and $r=352\times\frac{7}{22}\times\frac{1}{2}$

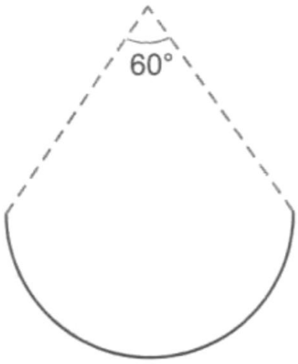
$\Rightarrow \quad R=63\,m$ and $r=56\,m$

Hence, width of the track = $(R-r)=(63-56)=7m$

OR

Let r be the radius and 0 be the angle subtended by the arc at the centre of the circle.

$\therefore$ Length of arc = length of piece of wire



$\frac{\theta}{360}\times 2\pi r=22$

$\Rightarrow \quad \frac{60}{360}\times 2\pi r=22 \Rightarrow \frac{\pi r}{3}=22$

$\Rightarrow \quad r=\frac{3\times 22}{\pi}=\frac{3\times 22}{\frac{22}{7}}=21cm$

$\therefore$ Radius = 21 cm

25. $\sin\theta=\cos\theta$ (Given)

It means value of $\theta=45°$

Now, $2\tan\theta+\cos^2\theta=2\tan 45°+\cos^2 45°$

$=2\times 1+\left(\frac{1}{\sqrt{2}}\right)^2=2+\frac{1}{2}=\frac{4+1}{2}=\frac{5}{2}\left(\because \tan 45°=1, \cos 45°=\frac{1}{\sqrt{2}}\right)$

OR

$\sin^2 A=2\sin A$

$\Rightarrow \quad \sin^2 A-2\sin A=0 \quad \Rightarrow \quad \sin A(\sin A-2)=0$

$\Rightarrow$ either $\sin A = 0$ or $\sin A - 2 = 0$

$\Rightarrow$ $A = 0°$ ($\sin A = 2$, which is not possible)

$\therefore$ Value of $\angle A = 0°$

SECTION-C

26. Given $\sqrt{2}$ is irrational.

To prove: $5+3\sqrt{2}$ is irrational.

Proof: Let us assume $5+3\sqrt{2}$ is rational. So it can be written in the form $\frac{a}{b}$ where $a, b \in Z, b \neq 0$, $HCF(a,b) = 1$.

$$5+3\sqrt{2} = \frac{a}{b} \Rightarrow 3\sqrt{2} = \frac{a}{b} - 5$$

$$3\sqrt{2} = \frac{a-5b}{b} \Rightarrow \sqrt{2} = \frac{a-5b}{3b}$$

This shows that $\sqrt{2}$ is rational ($a-5b$ and $3b$ are integers). But we know that $\sqrt{2}$ is irrational. This contradicts our assumption that $5+3\sqrt{2}$ is rational.

$\Rightarrow$ $5+3\sqrt{2}$ is irrational, hence proved.

27. It is given that α and β are zeros of the polynomial $2x^2 - 3x + 1$.

$\therefore$ $\alpha + \beta = \frac{-(-3)}{2} = \frac{3}{2}$ and $\alpha\beta = \frac{1}{2}$

Now, new quadratic polynomial whose zeros are 3α and 3β is given by $x^2 -$ (sum of zeros) $x +$ product of zeros

$$= x^2 - (3\alpha + 3\beta)x + 3\alpha \times 3\beta = x^2 - 3(\alpha + \beta)x + 9\alpha\beta$$

$$= x^2 - (3\alpha + 3\beta)x + 3\alpha \times 3\beta = x^2 - 3(\alpha + \beta)x + 9\alpha\beta$$

$$= x^2 - 3 \times \frac{3}{2}x + 9 \times \frac{1}{2}$$

$$= x^2 - \frac{9}{2}x + \frac{9}{2} = \frac{1}{2}(2x^2 - 9x + 9)$$

28. Let the speed of the boat be x km/h and speed of the stream be y km/h.

$\therefore$ Speed of boat in upstream = $(x - y)$ km/h

and speed of boat in downstream = $(x + y)$ km/h

$\therefore$ According to question,

$$\frac{30}{x-y} + \frac{44}{x+y} = 10 \qquad \ldots(i)$$

And $$\frac{40}{x-y} + \frac{55}{x+y} = 13 \qquad \ldots(ii)$$

$(i) \times 4 - (ii) \times 3$ gives

$$\frac{1}{x+y}(176 - 165) = 1$$

$\Rightarrow$ $x + y = 11$ $\ldots$(iii)

Putting the value of $x + y = 11$ in equation (i), we have

$$\frac{30}{x-y} + \frac{44}{11} = 10$$

$\Rightarrow \quad \frac{30}{x-y} + 4 = 10 \Rightarrow \frac{30}{x-y} = 104 = 6$

$\Rightarrow \quad x - y = 5$ …(iv)

By adding equations (iii) and (iv), we have

$x + y = 11$

$x - y = 5$

$2x = 16$

$\Rightarrow \quad x = 8$

Putting $x = 8$ in equation (iii), we have

$8 + y = 11$

$y = 11 - 8 = 3 \Rightarrow y = 3$

Speed of boat is 8 km/h and speed of stream is 3 km/h.

OR

Let the digits at units and tens places be x and y, respectively.

$\therefore \quad xy = 14$

$\Rightarrow \quad y = \frac{14}{x}$ ….(i)

According to the questions:

$10x + y + 45 = 10y + x$

$\Rightarrow \quad 9x - 9y + 45 = 0$

$\Rightarrow \quad x - y + 5 = 0$ ….(ii)

From (i) and (ii), we get:

$$x - \frac{14}{x} + 5 = 0$$

$\Rightarrow \quad x^2 + 5x - 14 = 0$

$\Rightarrow \quad x^2 + 7x - 2x - 14 = 0$

$\Rightarrow \quad x(x+7) - 2(x+7) = 0$

$\Rightarrow \quad x^2 - 7x + 2x - 14 = 0$

$\Rightarrow \quad (x+7)(x-2) = 0$

$\Rightarrow \quad x = -7$ or $x = 2$

$\Rightarrow \quad x = -7$ (the digit cannot be negative)

Putting $x = 2$ in equation (i), we get:

$y = 7$

$\therefore \quad$ Required number $= 10 \times 2 + 7 = 27$

29. $\sin\theta + \cos\theta = \sqrt{3}$

$\Rightarrow \quad (\sin\theta + \cos\theta)^2 = 3$

$\Rightarrow \quad \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\Rightarrow \quad 2\sin\theta\cos\theta = 2 \qquad (\because \sin^2\theta + \cos^2\theta = 1)$

$\Rightarrow \quad \sin\theta.\cos\theta = 1 = \sin^2\theta + \cos^2\theta$

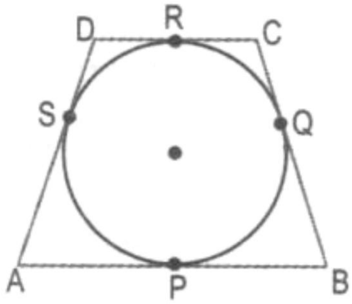
$\Rightarrow \quad 1 = \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} \Rightarrow 1 = \dfrac{\sin^2\theta}{\sin\theta\cos\theta} + \dfrac{\cos^2\theta}{\sin\theta\cos\theta}$

$\Rightarrow \quad 1 = \tan\theta + \cot\theta$

Therefore $\tan\theta + \cot\theta = 1$

30. **Given:** A quadrilateral circumscribing a circle, with centre *O*, such that it touches side *AB*, *BC*, *CD*, *AD* at *P*, *Q*, *R* and *S*.

To prove: *AB* + *CD* = *BC* + *DA*



Proof: Length of tangent drawn from external point are equal.

AP = *AS* — (at *A*) ...(i)

BP = *BQ* — (at *B*) ...(ii)

DR = *DS* — (at *C*) ...(iii)

CR = *CQ* — (at *D*) ...(iv)

Adding equation (i), (ii), (iii), (iv)

AP + *BP* + *DR* + *CR* = *AS* + *DS* + *BQ* + *CQ*

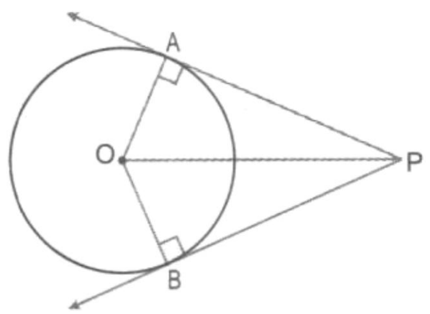
$\Rightarrow$ *AB* + *CD* = *AD* + *BC*

Hence Proved.

OR

Given: Circle (*O*, *r*) *AP* and *PB* are tangents drawn to the circle.

To prove : *PA* = *PB*.



Construction: Join *OA*, *OB* and *OP*.

Proof: *OA* = *OB* [Radii]

$\angle OAP = \angle OBP = 90^\circ$ (Right angle)

[$\because$ Radius is perpendicular to tangent at point of contact]

$OP = OP$ (Hypotenuse)

So in $\Delta OAP \cong \Delta OBP$

$\Rightarrow$ $\Delta OAP \cong \Delta OBP$ [By RHS congruency]

$\Rightarrow$ $AP = BP$ [By *CPCT*]

Hence proved

31. Total number of outcomes = 36

(i) Favourable outcomes are (1, 1,) (1, 2) (1, 3) (1, 4) (1, 5) (2, 1) (2, 2) (2, 3) (2, 4) (3, 1) (3, 2) (3, 3) (4, 1) (4, 2) (5, 1) i.e., 15.

$\therefore$ P (sum less than 7) = $\frac{15}{36}$ or $\frac{5}{12}$

(ii) Favourable outcomes are (1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6) (2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6) (3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (4, 1) (4, 2) (4, 3) (5, 1) (5, 2) (5, 3) (6, 1) (6, 2) i.e., 25.

$\therefore$ P (product less than 16) = $\frac{25}{36}$

(iii) Favourable outcomes are (1, 1) (3, 3) (5, 5) i.e., 3.

$\therefore$ P (doublet of odd number) = $\frac{3}{36}\, or \,\frac{1}{12}$

SECTION-D

32. Let, time taken by faster tap to fill the tank be x hours.

Therefore, time taken by slower tap to fill the tank = (x + 3) hours

Since the faster tap takes x hours to fill the tank.

$\therefore$ Portion of the tank filled by the faster tap in one hour = $\frac{1}{x}$

Portion of the tank filled by the slower tap in one hour = $\frac{1}{x+3}$

Portion of the tank filled by the two tap together in one hour = $\frac{1}{\frac{40}{13}} = \frac{13}{40}$

According to question,

$\Rightarrow \frac{1}{x}+\frac{1}{x+3}=\frac{13}{40}$ $\Rightarrow \frac{x+3+x}{x(x+3)}=\frac{13}{40}$

$\Rightarrow 40(2x+3)=13x(x+3)$ $\Rightarrow 80x+120=13x^2+39x$

$\Rightarrow 13x^2-41x-120=0$ $\Rightarrow 13x^2-65x+24x-120=0$

$\Rightarrow 13x(x-5)+24(x-5)=0$ $\Rightarrow (x-5)(13x+24)=0$

Eithre $x-5=0$ or $13x+24=0$

$\Rightarrow x=5$ or $x=\frac{-24}{13}$

$\Rightarrow x=5$ [$\because$ x cannot be negative]

Hence, time taken by faster tap to fill the tank = x = 5 hours and time taken by slower tap = x + 3 = 5 + 3 = 8 hours.

OR

Let the speed of stream be x km/h.

$\therefore$ The speed of the boat upstream = $(18 - x)$ km/h

and Speed of the boat downstream = $(18 + x)$ km/h

According to question, $\dfrac{24}{18-x} - \dfrac{24}{18+x} = 1$

$\Rightarrow \quad 24(18+x) - 24(18-x) = 324 - x^2$

$\Rightarrow \quad 48x = 324 - x^2 \qquad \Rightarrow \quad x^2 + 48x - 324 = 0$

$\Rightarrow \quad x^2 + 54x - 6x - 324 = 0 \qquad \Rightarrow \quad x(x+54) - 6(x+54)$

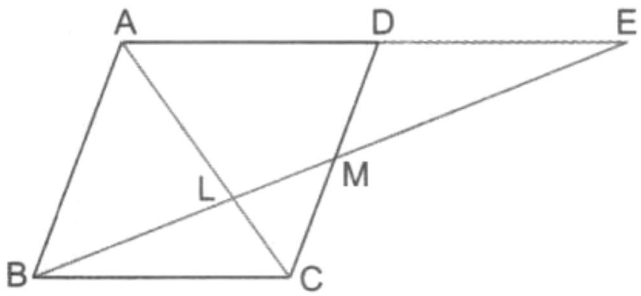
$\Rightarrow \quad (x+54)(x-6) = 0$

But $x \neq -54$, $\therefore$ $x = 6$ [Speed can't be negative]

$\therefore$ Speed of the stream = 6 km/h.

33. In ΔBMC and ΔEMD, we have

$MC = MD$ ($\because$ M is the mid-point of CD)



$\angle CMB = \angle DME$ (Vertically opposite angles)

and $\angle MBC = \angle MED$ (Alternate angles)

$\Delta BMC \cong \Delta EMD$ (By AAS criterion of congruence)

$\Rightarrow \quad BC = DE$ (CPCT)

Also, $BC = AD$ ($\because$ $ABCD$ is a parallelogram)

Now, in ΔAEL and ΔCBL, we have

$\angle ALE = \angle CLB$ (Vertically opposite angles)

$\angle EAL = \angle BCL$ (Alternate angles)

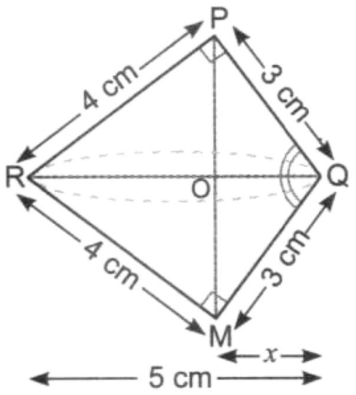
$\therefore$ $\Delta AEL \sim \Delta CBL$ (By AA similarity)

$\Rightarrow \quad \dfrac{EL}{BL} = \dfrac{AE}{CB} \Rightarrow \dfrac{EL}{BL} = \dfrac{2BC}{BC}$ ($\because$ $AE = AD + DE = BC + BC = 2BC$)

$\Rightarrow \quad \dfrac{EL}{BL} = 2 \Rightarrow EL = 2BL$

34. In the given Fig., ΔPQR is a right triangle, where $PQ = 3cm$, $PR = 4cm$ and $QR = 5cm$.

Let $OQ = x \Rightarrow OR = 5 - x$ and $OP = y$



Now in right angled-triangle *POQ*, we have

$PQ^2 = OQ^2 + OP^2$ (By Pythagoras Theorem)

$\Rightarrow \quad (3)^2 = x^2 + y^2 + y^2 \Rightarrow y^2 = 9 - x^2$ …(i)

Also from right angled triangle *POR*, we have

$OP^2 + OR^2 = PR^2$

$\Rightarrow \quad y^2 + (5-x)^2 = (4)^2$ …(ii)

$\Rightarrow$ From (i) and (ii), we get

$9 - x^2 = 16 - (5-x)^2$

$\Rightarrow \quad 9 - x^2 = 16 - \left(25 + x^2 - 10x\right)$

Or $\quad 9 - x^2 = -9 - x^2 + 10x \quad \Rightarrow \quad 10x = 18 \Rightarrow x = \frac{9}{5}$

$\therefore \quad OR = 5 - x = 5 - \frac{9}{5} = \frac{16}{5}$

Now putting $x = \frac{9}{5}$ in (i), we get

$y^2 = 9 - \left(\frac{9}{5}\right)^2 = 9 - \frac{81}{25} = \frac{144}{25} \Rightarrow y = \frac{12}{5}$

$\therefore \quad OP = y = \frac{12}{5}$

Now for the cone *PQM*, radius $OP = \frac{12}{5} cm$, height $OQ = \frac{9}{5} cm$

Volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{12}{5}\right)^2 \times \frac{9}{5} = \frac{432\pi}{125} cm^3$

Also for the cone *PRM*, radius OP = $\frac{12}{5} cm$, height $OR = \frac{16}{5} cm$

$\therefore \quad$ Volume = $\frac{1}{3}\pi\left(\frac{12}{5}\right)^2 \times \frac{16}{5} = \frac{768\pi}{125} cm^3$

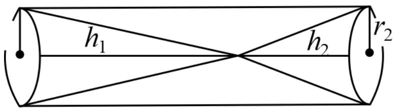
Hence total volume, i.e., volume of the double cone

$$=\left(\frac{432\pi}{125}+\frac{768\pi}{125}\right)=\frac{1200\pi}{125}=9.6\times 3.14cm^3=30.144cm^3$$

OR

Let height of the cone 1 be 'h' cm and the height of the cone 2 be (21 cm – h).

As the ratio of volumes of cone c_1 and c_2 is 2 : 1, their radii are same equal to $r=\frac{6}{2}cm=3cm$.



$$\therefore \quad \frac{V_1}{V_2}=\frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$$

$$\Rightarrow \quad \frac{2}{1}=\frac{h}{21cm-h}$$

Or 42 cm – 2 h = h

Or, $3h$ = 42 cm

$$\Rightarrow \quad h=\frac{42}{3}=14\ cm$$

Hence, height of cone 1 = 14 cm and height of cone 2 = 7 cm

Cone I	Cone II	Cylinder
$r_1=\frac{6}{3}=3\ cm$	$r_2=3cm$	$r=3cm$
$h_1=14cm$	$h_2=7cm$	$h=21cm$

Volume of cone $1=\frac{1}{3}\pi r_1^2 h_1=\frac{1}{3}\times\frac{22}{7}\times 3\times 3\times 14=132\,cm^3$

Volume of cone 2 = $\frac{1}{3}\pi r_2^2 h_2=\frac{1}{3}\times\frac{22}{7}\times 3\times 3\times 7=22\times 3=66cm^3$

Volume of remaining portion of tube = Volume of cylinder – Volume of cone 1 – Volume of cone 2

$=\pi r^2 h-132-66$

$=\frac{22}{7}\times 3\times 3\times 21-198$

$=22\times 27-198=594-198=396\ cm^3$.

Hence the required volume is 396 cm^3.

35.

Class	Frequency	Cumulative Frequency
0 – 10	f_1	f_1
10 – 20	5	$5+f_1$
20 – 30	9	$14+f_1$
30 – 40	12	$26+f_1$
40 – 50	f_2	$26+f_1+f_2$
50 – 60	3	$29+f_1+f_2$
60-70	2	$31+f_1+f_2$
	$\sum f = 40$	

Median = 32.5 $\Rightarrow$ Median class is 30 – 40.

Now $32.5 = 30 + \frac{10}{12}(20 - 14 - f_1)$ $\Rightarrow$ $f_1 = 3$

Also $31 + f_1 + f_2 = 40$ $\Rightarrow$ $f_2 = 6$

SECTION-E

36. (i) Let n be the minimum number of days for AP. 51, 49, 47 ....

$\therefore$ $a_n = 31$

$\Rightarrow$ $a + (n-1)d = 31$ $\Rightarrow$ $51 + (n-1)\times(-2) = 31$

$\Rightarrow$ $(n-1)\times(-2) = -20$ $\Rightarrow$ $n - 1 = 10$ $\Rightarrow$ $n = 11$

(ii) Since Veer currently runs the distance in 51 seconds and with each day of practice it takes him 2 seconds less.

Thus, AP formed is 51, 49, 47, ...

(iii) Given n^{th} term of an AP is

$a_n = 2n + 3$

$\therefore$ $d = a_n - a_{n-1} = (2n+3) - (2(n-1)+3)$

$\Rightarrow$ $d = 2$

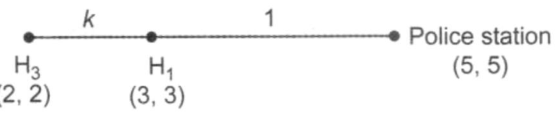
37. (i) Coordinates of school are (1,6).

Coordinates of House 1 are (3, 3).

Distance between school and House 1, is

$\sqrt{(3-1)^2 + (3-6)^2} = \sqrt{4+9} = \sqrt{13}\ km$

(ii) Coordinates of House $1(H_1)$ are (3, 3).



House $3(H_3)$ coordinates are (2,2).

Coordinates of police station are (5, 5).

Let H_1, divides path joining H_3 and police station in the ratio k : 1.

By section formula we have

$$3 = \frac{k(5)+1(2)}{k+1}$$

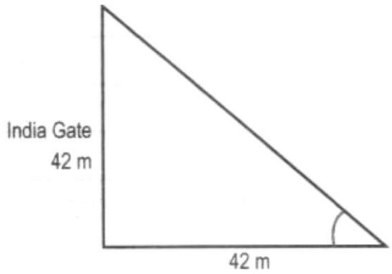
$\Rightarrow \quad 3k+3 = 5k+2 \quad \Rightarrow \quad 3-2 = 5k-3k$

$\Rightarrow \quad 2k = 1 \quad \Rightarrow \quad k = \frac{1}{2}$

$\therefore \quad k:1 = \frac{1}{2}:1 = 1:2$

(iii) Shortest distance = $\sqrt{(2-1)^2+(2-6)^2} = \sqrt{1+16} = \sqrt{17}\ km$

38. (i) Let θ be the angle of elevation.

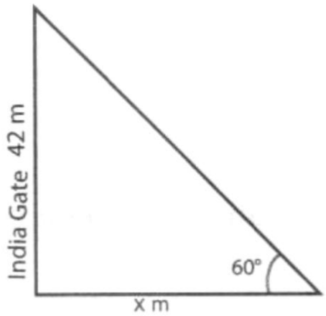


$\therefore \quad \tan\theta = \frac{42}{42} = 1$

$\Rightarrow \quad \tan\theta = 1 = \tan 45° \Rightarrow \theta = 45°$

$\therefore$ Angle of elevation is 45°

(ii) Let x m be the required distance.



$\therefore \quad \tan 60° = \frac{42}{x}$

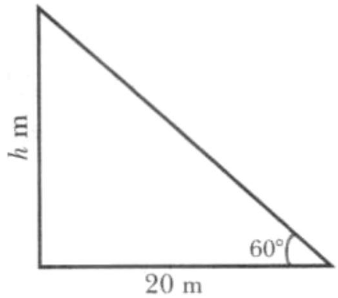
$\Rightarrow \quad \sqrt{3} = \frac{42}{x} \Rightarrow x = \frac{42}{\sqrt{3}}$

$\Rightarrow \quad x = \frac{42}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{42\sqrt{3}}{3} = 14\sqrt{3} = 14 \times 1.732 = 24.248m$

(iii) Let h m be the height of the vertical tower.

We have,

$\therefore \quad \tan 60° = \frac{h}{20}$



$\Rightarrow \qquad \sqrt{3} = \frac{h}{20} \Rightarrow h = 20\sqrt{3}m$