SUBJECTIVE MOCK TEST | MATHEMATICS | SOLUTION

CLASS – XII | SET – 1

SECTION-A

1.(C) As order of 3×3 matrix contains 9 elements. Each element can be selected in 2 ways (it can be either 0 or 1). Hence, all the nine netries can be choose in $2^9 = 512$ ways (by the multiplication principle).

2.(B)

$$f(z) = \begin{vmatrix} 5 & 3 & 8 \\ 2 & z & 1 \\ 1 & 2 & z \end{vmatrix} = 5(z^2-2) - 2(3z-16) + 1(3-8z)$$

$$= 5z^2 - 10 - 6z + 32 + 3 - 8z = 5z^2 - 14z + 25$$

$$f(5) = 5\times 5^2 - 14\times 5 + 25 = 125 - 70 + 25 = 150 - 70 = 80$$

3.(D)

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 2A$$

$$A = \frac{1}{2}\left[x_1(y_2-y_3) - x_2(y_1-y_3) + x_3(y_1-y_2)\right]$$

$$2A = \left[x_1(y_2-y_3) - x_2(y_1-y_3) + x_3(y_1-y_2)\right]$$

4.(C) $y = x\sqrt{1-x^2} + \sin^{-1}(x)$

$$\Rightarrow \frac{dy}{dx} = x\left\{\frac{1}{2\sqrt{1-x^2}} - (-2x)\right\} + \sqrt{1-x^2} + \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-x^2}{\sqrt{1-x^2}} + \sqrt{1-x^2} + \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-x^2 + 1 - x^2 + 1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2x^2 + 2}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx} = 2\sqrt{1-x^2}$$

5.(D)

6.(A)

7.(A)

Corner points	Value of $Z = 2x - y + 5$
$A(0,10)$	$Z = 2(0) - 10 + 5 = -5$ (Minimum)
$B(12, 6)$	$Z = 2(12) - 6 + 5 = 23$
$C(20, 0)$	$Z = 2(20) - 0 + 5 = 45$ (Maximum)
$O(0,0)$	$Z = 0(0) - 0 + 5 = 5$

So the minimum value of Z is –5.

8.(C) $(\vec{a} + 2\vec{b} - \vec{c}) \cdot \{(\vec{a} - \vec{b}) \times (\vec{a} - \vec{b} - \vec{c})\}$

$= (\vec{a} + 2\vec{b} - \vec{c}) \cdot (\vec{a} \times \vec{a} - \vec{a} \times \vec{b} - \vec{a} \times \vec{c} - \vec{b} \times \vec{a} + \vec{b} \times \vec{b} + \vec{b} \times \vec{c})$

$= (\vec{a} + 2\vec{b} - \vec{c}) \cdot (-\vec{a} \times \vec{b} - \vec{a} \times \vec{c} + \vec{a} \times \vec{b} + \vec{b} \times \vec{c})$

$= (\vec{a} + 2\vec{b} - \vec{c}) \cdot (-\vec{a} \times \vec{c} + \vec{b} \times \vec{c}) = [abc] + 2[abc] = 3[abc]$

9.(A) Given integral is $\int \frac{\sec^2(\log x)}{x} dx$

Let, $\log x = z$

$\Rightarrow \quad \frac{dx}{x} = dz$

$\Rightarrow$ So, $\int \frac{\sec^2(\log x)}{x} dx = \int f \sec^2 z dz = \tan z + c = \tan(\log x) + c.$

Which is the required solution.

10.(C) We have, $\begin{bmatrix} 1 & 2 \\ -2 & -b \end{bmatrix} + \begin{bmatrix} a & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} a+1 & 6 \\ 1 & 2-b \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow \quad a + 1 = 5,\ 2 - b = 0$

$\Rightarrow \quad a = 4,\ b = 2$

$\therefore \quad a^2 + b^2 = 4^2 + 2^2 = 16 + 4 = 20$

11.(a) Here, maximize $Z = 5x + 3y,$ subject to constraints $x + y \le 300, 2x + y \le 360, x \ge 0,\ y \ge 0.$

Coner points	$Z = 5x + 3y$

$P(0, 300)$	900
$Q(180, 0)$	900
$R(60, 240)$	1020….(Max.)
$S(0, 0)$	0

Hence, the maximum value is 1020.

12.(C) $\frac{2\pi}{3}$ is the correct answer. Apply the formula $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$.

13.(C) Given A is a square matrix of order 3 and also $|A| = 8$

$|3A| = (3)^3 \times |A| = 27 \times 8 = 216$

14.(D)

15.(C) Given, $xdy + ydx = 0$

$xdy = -ydx$

$-\frac{dy}{y} = \frac{dx}{x}$

On integration on both sides, we obtain

$-\log y = \log x + \log c$

$\log x + \log y = \log c$

$\log xy = \log c$

$xy = C$

16.(D) We have, $\vec{F} = 3\hat{i} + 4\hat{j} - 3\hat{k}$ and $\overrightarrow{OP} = r = 2\hat{i} - 2\hat{j} - 3\hat{k}$

Clearly, the magnitude of moment of the force about origin $= |\vec{r} \times \vec{F}|$ …(i)

Let us first find $\vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & -3 \\ 3 & 4 & -3 \end{vmatrix} = \hat{i}(6+12) - \hat{j}(-6+9) + \hat{k}(8+6) = 18\hat{i} - 3\hat{j} + 14\hat{k}$

$\therefore$ From equation (i)

$|r \times F| = \sqrt{(18)^2 + (-3)^2 + (14)^2} = 23 \text{ units}$

17.(C) $\frac{d}{dx}(\sin^3 x) = 3\sin^2 x \cos x$

$\frac{d^2}{dx^2} = (\sin^3 x) = \frac{d}{dx}(3\sin^2 x\cos x) = 6\sin x\cos^2 x - 3\sin^3 x$

$$\frac{d^3}{dx^3}(\sin^3 x) = \frac{d}{dx}(6\sin^2 x\cos^2 x - 3\sin^3 x) = 6\cos^3 x - 12\sin^2 x\cos x - 9\sin^2 x\cos x$$

$$= 6\cos^3 x - 21\sin^2 x\cos x$$

$$\frac{d^4}{dx^4}(\sin^3 x) = \frac{d}{dx}(6\cos^3 x - 21\sin^2 x\cos x) = -18\cos^2 x\sin x - 42\sin x\cos^2 x + 21\sin^3 x$$

$$= 60\sin x\cos^2 x + 21\sin^3 x = -60\sin x(1-\sin^2 x) + 21\sin^3 x$$

$$= -60\sin x + 60\sin^3 x + 21\sin^3 x = -60\sin x + 81\sin^3 x$$

$$= -60\sin x + 81\left[\frac{3\sin x - \sin 3x}{4}\right] = \frac{3\sin x - 3^4\sin 3x}{4}$$

18.(C) Let $\vec{a} = \hat{i} - \hat{j} - 2\hat{k}$ and $\vec{b} = 3\hat{i} - 5\hat{j} - 4\hat{k}$ and $|\vec{a}| = \sqrt{1+1+2^2} = \sqrt{6}$ $|\vec{b}| = \sqrt{3^2+5^2+4^2} = 5\sqrt{2}$

$$\cos\alpha = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\times|\vec{b}|}$$

$$\Rightarrow \quad \cos\alpha = \frac{(3\hat{i}-5\hat{j}-4\hat{k})(\hat{i}-\hat{j}-2\hat{k})}{5\sqrt{2}\times\sqrt{6}}$$

$$\Rightarrow \quad \cos\alpha = \frac{3+5+8}{5\sqrt{12}}$$

$$\Rightarrow \quad \cos\alpha = \frac{8\sqrt{3}}{5}$$

19.(A)

20.(D) $R = \{(1,3)(4,2)(2,7)(2,3)(3,1)\}$

As (2, 3) $\in R$ but (3, 2) $\notin R$

So, set ‘A’ is not symmetric.

SECTION-B

21. Given $\sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right)$

We know that $\cos^{-1}(-\theta) = \pi - \cos^{-1}\theta$

$$= \sin^{-1}\left(\frac{1}{3}\right) - \left[\pi - \cos^{-1}\left(\frac{1}{3}\right)\right]$$

$$= \sin^{-1}\left(\frac{1}{3}\right) - \pi + \cos^{-1}\left(\frac{1}{3}\right)$$

$$= \sin^{-1}\left(\frac{1}{3}\right) + \cos^{-1}\left(\frac{1}{3}\right) - \pi$$

$= \frac{\pi}{2} - \pi = -\frac{\pi}{2}$

Therefore we have,

$\sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right) = -\frac{\pi}{2}$

or

We have $\tan^{-1}\frac{n}{\pi} > \frac{\pi}{4}$

$\Rightarrow \quad \tan^{-1}\frac{n}{\pi} > \tan^{-1} 1 \qquad \left[\because \ \frac{\pi}{4} = \tan^{-1} 1\right]$

$\Rightarrow \quad \tan\left(\tan^{-1}\frac{n}{\pi}\right) > \tan(\tan^{-1} 1)$

$\because \quad$ (tan θ is an increasing function)

$\Rightarrow \quad \frac{n}{\pi} > 1$

$\Rightarrow \quad n > \pi \cong 3.14$

$\Rightarrow \quad$ *n* = 4,5,6, ….

Hence, the minimum value of *n* is 4.

22. Given: $f(x) = 2x^3 - 24x + 107$

$\Rightarrow \quad f'(x) = \frac{d}{dx}(2x^3 - 24x + 107)$

$\Rightarrow \quad f'(x) = 6x^2 - 24$

For $f(x)$ lets find critical point, for this we must have

$\Rightarrow \quad f'(x) = 0$

$\Rightarrow \quad 6x^2 - 24 = 0$

$\Rightarrow \quad 6(x^2 - 4) = 0$

$\Rightarrow \quad (x-2)(x+2) = 0$

$\Rightarrow \quad x = -2, 2$

Clearly, $f'(x) > 0$ if $x < -2$ and $x > 2$

and $f'(x) < 0$ if $-2 < x < 2$

Thus, the function $f(x)$ increases on $(-\infty, -2) \cup (2, \infty)$ and $f(x)$ is decreasing on interval $x \in (-2, 2)$.

23. Given: $f(x) = \log(2+x) - \frac{2x}{2+x}, x \in R$

$\Rightarrow \quad f'(x) = \frac{1}{2+x} - \frac{(2+x)2 - 2x \times 1}{(2+x)^2}$ $\Rightarrow \quad f'(x) = \frac{1}{2+x} - \frac{4+2x-2x}{(2+x)^2}$

$\Rightarrow \quad f'(x) = \frac{1}{2+x} - \frac{4}{(2+x)^2}$

$\Rightarrow \quad f(x) = \frac{2+x-4}{(2+x)^2}$

$\Rightarrow \quad f'(x) = \frac{x-2}{(2+x)^2}$

For (x) to be increasing, we must have

$f(x) > 0 = (x) - 2 > 0 = 2 < x < 0$

For (x) to be decreasing, we must have,

$f(x) < 0 = x - 2 < 0$

$\Rightarrow \quad -\infty < x < 2$

$\Rightarrow \quad x \in (-\infty, 2)$

So, $f(x)$ is decreasing in $(-\infty, 2)$

OR

Given:

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ and $R_1 + R_2 = C$

$\Rightarrow \quad \frac{1}{R} = \frac{R_1 + R_2}{R_1 R_2} = \frac{C}{R_1 R_2} = \frac{c}{R_1(C - R_1)}$

$\Rightarrow \quad \frac{R_1 C - R_1^2}{C} = R_1 - \frac{R_1^2}{C}$

$\Rightarrow \quad \frac{dR}{dR_1} = 1 - \frac{2R_1}{C}$ and $\frac{d^2R}{dR_1^2} = -\frac{2}{C}$

The critical numbers of R are given by $\frac{dR}{dR_1} = 0$

$\because \frac{dR}{dR_1} = 0 \Rightarrow 1 - \frac{2R_1}{C} = 0 \Rightarrow R_1 = \frac{c}{2}$]

Now, $\frac{d^2R}{dR_1^2} = -\frac{c}{2} <$ for all value of R_1.

Thus, the value of R is maximum when $R_1 = \frac{c}{2} \quad \Rightarrow R_2 = \frac{C}{2}$

Putting, R is maximum when both $R_1 = R_2 = \frac{c}{2}$

24. $I = \int_{-1}^{1} |x^4 - x| dx = \int_{-1}^{1} (x^4 - x)dx - \int_{0}^{1} (x^4 - x)dx = \left[\left(\frac{x^5}{5} - \frac{x^2}{3}\right)\right]_{-1}^{0} - \left[\left(\frac{x^5}{5} - \frac{x^2}{2}\right)\right]_{0}^{1} = \frac{7}{10} + \frac{3}{10} = 1$

25. Here, for sphere volume is changing at the same rate as its radius

$\Rightarrow \frac{dV}{dt} = \frac{dr}{dt}$ d

To find: $\frac{dA}{dt}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$

$4\pi r^2 = 1$

Surface area of sphere $= 4\pi r^2 = 1$ square units.

SECTION-C

26. Using substitution Let $\cos x = t \quad \Rightarrow \quad \sin x dx = dt$

Now, $x = 0 \Rightarrow t = 1$

$x = \pi \Rightarrow t = -1$

$\therefore \int_{0}^{\pi} \sin^3 x(1 + 2\cos x)(1 + \cos x)^2 dx$

$= -\int_{1}^{-1} (1 - t^2)(1 + 2t)(1 + t)^2 dt \quad \left[\because \int_{a}^{b} f(x)dx = \int_{b}^{a} f(x)dx\right]$

$= \int_{-1}^{1} (1 - 2t - t^2 - 2t^5)(1 + t^2 + 2t)dt = \int_{-1}^{1} (1 + 4t + 4t^2 - 2t^3 - 5t^4 - 2t^5)dt$

$= \left[t + 2t^2 + \frac{4}{3}t^3 - \frac{t^4}{2} - t^5 - \frac{t^6}{3}\right]_{-1}^{1} = \left(1 + 2 + \frac{4}{3} - \frac{1}{2} - 1 - \frac{1}{3}\right) - \left(-1 + 2 - \frac{4}{3} - \frac{1}{2} + 1 - \frac{1}{3}\right) = \frac{8}{3}$

$\therefore \int_{0}^{5} \sin^3 x(1 + 2\cos x)(1 + \cos x)^2 dx = \frac{8}{3}$

27. Let E_1, E_2 and E_3 be the events of drawing a bolt produced by machine *A*, *B* and *C* respectively. Therefore, we have,

$P(E_1) = \frac{25}{100} = \frac{1}{4}, P(E_2) = \frac{35}{100} = \frac{7}{20},$ and $P(E_3) = \frac{40}{100} = \frac{2}{5}$

Let *E* be the event of drawing a defective bolt. Therefore,

$P\left(\frac{E}{E_1}\right) =$ probability of drawing a defective bolt, given that it is produced by the machine $A = \frac{5}{100} = \frac{1}{20}$

$P\left(\frac{E}{E_2}\right) =$ probability of drawing a defective bolt, given that it is produced by the machine $B = \frac{4}{100} = \frac{1}{25}$

$P\left(\frac{E}{E_3}\right) =$ probability of drawing a defective bolt, given that it is produced by the machine $C = \frac{2}{100} = \frac{1}{50}$

Therefore, we have,

Probability that the bolt drawn is manufactured by *C*, given that it is defective $= P\left(\frac{E_3}{E}\right)$

$$= \frac{P\left(\frac{E}{E_3}\right) \cdot P(E_3)}{P\left(\frac{E}{E_1}\right) \cdot P(E_1) + P\left(\frac{E}{E_2}\right) \cdot P(E_2) + P\left(\frac{E}{E_3}\right) \cdot P(E_3)}$$ [by Bayes's theorem]

$$\frac{\left(\frac{1}{50} \times \frac{2}{5}\right)}{\left(\frac{1}{20} \times \frac{1}{4}\right) + \left(\frac{1}{25} \times \frac{7}{20}\right) + \left(\frac{1}{50} \times \frac{2}{5}\right)} = \left(\frac{1}{125} \times \frac{2000}{69}\right) = \frac{16}{69}$$

Hence, the required probability is $\frac{16}{69}$.

28. using By part Method.

Here $\log(x+1)$ is first function and *x* is second function.

$$\int a \cdot b dx = a \int b dx - \int \left[\frac{da}{dx} \cdot \int f b dx\right] dx$$

$$\int x \log(x+1) = \log(x+1) \int x dx - \int \left(\frac{d \log(x+1)}{dx} \cdot \int x dx\right) dx$$

$$= \log(x+1)\frac{x^2}{2} - \int \frac{1}{x+1} \times \frac{x^2}{2} dx \;= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\int \frac{x^2-1+1}{x+1} dx$$

Adding and subtracting 1 in the numerator,

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\left(\int \frac{x^2-1}{x+1} + \frac{1}{x+1}\right)dx\right] = \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\left(\int \frac{(x+1)(x-1)}{x+1} + \frac{1}{x+1}\right)dx\right]$$

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\left(\int (x-1) + \frac{1}{x+1}\right)dx\right] = \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\frac{x^2}{2} - x + \log(x+1)\right] + c$$

$$= \log(x+1)\frac{x^2}{2} - \frac{x^2}{4} + \frac{x}{2} - \frac{\log(x+1)}{2} + c \;= \log(x+1)\frac{x^2-1}{2} - \frac{x^2}{4} + \frac{x}{2} + c$$

OR

Let the given integral be,

$$I = \int \frac{2x}{(2x+1)^2} dx$$

Now using partial fractions by putting, $\dfrac{2x}{(2x+1)^2} = \dfrac{A}{(2x+1)} + \dfrac{B}{(2x+1)^2}$ …(i)

$$A(2x+1) + B = 2x$$

Putting $2x+1=0,$

$$x = \frac{-1}{2}$$

$$A(0) + B = -1$$

$$B = -1$$

By equating the coefficient of x,

$$2A = 2$$

$$A = 1$$

From equation (i), we get,

$$\frac{2x}{(2x+1)^2} = \frac{1}{(2x+1)} - \frac{1}{(2x+1)^2}$$

$$\int \frac{2x}{(2x+1)^2}dx = \int \frac{1}{(2x+1)}dx - \int \frac{1}{(2x+1)^2}dx \;= \frac{\log|2x+1|}{2} + \frac{1}{2(2x+1)} + c$$

$$= \frac{1}{2}\left[\log|2x+1| + \frac{1}{2x+1}\right] + c$$

29. Given differential equation,

$$(x^2 - yx^2)dy + (y^2 + x^2y^2)dx = 0$$

$$\Rightarrow \quad x^2(1-y)dy + y^2(1+x^2)dx = 0$$

$\Rightarrow \quad -x^2(1-y)dy = y^2(1+x^2)dx$

$\Rightarrow \quad x^2(y-1)dy = y^2(1+x^2)dx$

$\Rightarrow \quad \frac{y-1}{y^2}dy = \frac{1+x^2}{x^2}dx$

On integration both sides, we get

$\int \frac{y-1}{y^2}dy = \int \frac{1+x^2}{x^2}dx$

$\Rightarrow \quad \int \frac{1}{y}dy - \int \frac{1}{y^2}dy = \int \frac{1}{x^2}dx + \int 1dx$

$\Rightarrow \quad \log|y| + \frac{1}{y} = \frac{-1}{x} + x + C$ …(ii)

Also, given that $y = 1,$ when $x = 1$

On putting $y = 1$ and $x = 1$ in equation (i), we get,

$\log|1| + 1 = -1 + 1 + C \Rightarrow C = 1$

On putting the value of *C* in equation (i), we get,

$\log|y| + \frac{1}{y} = \frac{-1}{x} + x + 1$

Which is the required solution.

OR

Let the population at any instant (*t*) be *y*.

Now it is given that the rate of increase of population is proportional to the number of inhabitants at any instant,

$\therefore \quad \frac{dy}{dt} \alpha y$

$\Rightarrow \quad \frac{dy}{dt} = ky$ (*k* is constant)

$\Rightarrow \quad \frac{dy}{y} = kdt$

Now, integrating both sides, we get,

$\log y = kt + C$ …(i)

According to given conditions,

In the year 1999, $t = 0$ and $y = 20000$

$\Rightarrow \quad \log 20000 = C$ …(ii)

Also, in the year 2004, $t = 5$ and $y = 25000$

$\Rightarrow \quad \log 25000 = k.5 + C$

$\Rightarrow \quad \log 25000 = 5k + \log 20000$

$\Rightarrow \quad 5k = \log\left(\frac{250000}{20000}\right) = \log\left(\frac{5}{4}\right)$

$\Rightarrow \quad k = \frac{1}{5}\log\left(\frac{5}{4}\right)$ ...(iii)

Also, in the year 2009, $t = 10$

Now, substituting the values of t, k and c in equation (io), we get

$$\log y = 10 \times \frac{1}{5}\log\left(\frac{5}{4}\right) + \log(20000)$$

$\Rightarrow \quad \log y = \log\left[20000 \times \left(\frac{5}{4}\right)^2\right] \quad \Rightarrow \quad y = 20000 \times \frac{5}{4} \times \frac{5}{4} \quad \Rightarrow \quad y = 31250$

Therefore, the population of the village in 2009 will be 31250.

30. Firstly, we will convert the given inequations into equations, now we will get the equations:
$x - y =, x + y = 3, x = 0$ and $y = 0$

Region represented by $x - y \le 1$: The line $x - y = 1$ meets the coordinate axes at $A(1,0)$ and $B(0, -1)$ respectively. By joining these points we obtain the line $x - y = 1$ Clearly (0,0) satisfies the inequation $x + y \le 8$. So, the region in $x\,y$ plane which contain the origin represents the solution set of the inequation $x - y \le 1$.

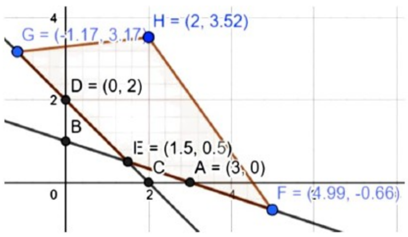
The region represented by $x + y \ge 3$:

The line $x + y = 3$ meets the coordinate axes at $C(3,0)$ and $D(0,3)$ respectively. By joining these points we obtain the line $x + y = 3$. Clearly (0,0) satisfies the in equation $x + y \ge 3$. So, the region in xy plane which does not contain the origin represents the solution set of the inequation $x + y \ge 3$

The region represented by $x \ge 0$ and $y \ge 0$ since every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations $x \ge 0$ and $y \ge 0$.

The feasible region determined by subject to the constraints are $x - y \le 1,\ x + y \ge 3,$ and the non-negative restrictions $x \ge 0$ and $y \ge 0$ are as follows.

31.



The feasible region is unbounded. We would obtain the maximum value at infinity. Therefore, maximum value will be infinity i.e. the solution is unbounded.

OR

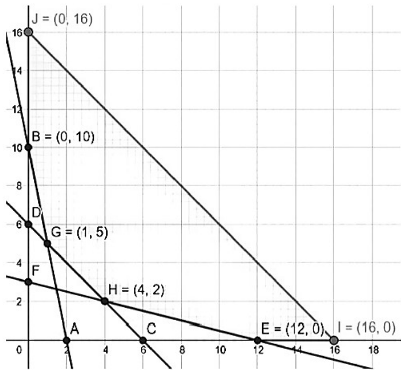
First, we will convert the given inequations into equations, we obtain the following equations:
$5x + y = 10, x + y = 6, x + 4y = 12, x = 0$ and $y = 0$

Region represented by $5x + y \geq 10$:

The line $5x + y = 10$ meets the coordinate axes at $A(2, 0)$ and $B(0, 10)$ respectively. By joining these points we obtain the line $5x + y = 10.$ Clearly (0, 0) does not satisfies the inequation $5x + y \geq 10.$ So, the region in *x y* plane which does not contain the origin represents the solution set of the inequation $5x + y \geq 10$ Region represented by $x + y \geq 6$:

The line $x + y = 6$ meets the coordinate axes at $C(6, 0)$ and $D(0, 6)$ respectively. By joining these points we obtain the line $2x + 3y = 30.$ Clearly (0, 0) does not satisfies the inequation $x + y \geq 6.$ So, the region which does not contain the origin represents the solution set of the inequation $2x + 3y \geq 30$

Region represented by $x + 4y \geq 12$ The line $x + 4y = 12x + 4y = 12$ meets the coordinate axes at $E(12, 0)$ and $F(0, 3)$ respectively. By joining these points we obtain the line $x + 4y = 12.$ Clearly (0, 0) does not satisfies the inequation . So, the first quadrant is the region represented by the inequations $x \geq 0,$ and $y \geq 0$. The feasible region determined by subject to the constraints are $5x + y \geq 10, x + y \geq 6, x + 4y \geq 12,$ and the non-negative restrictions $xx \geq 0,$ and $y \geq 0,$ are as follows.



The corner points of the feasible region are *B*(0, 10), *G*(1, 5), *H*(4, 2) and *E*(12, 0).

The values of objective function *Z* at these corner points are as follows.

Corner point $Z = 3x + 2y$

B(0, 10): $3 \times 0 + 3 \times 10 = 30$

G(1, 5): $3 + 1 + 2 \times 5 = 13$

H(4, 2) : $3 \times 4 + 2 \times 2 = 16$

B(12, 0): $3 \times 12 + 2 \times 0 = 36$

The corner points of the feasible region are *B*(0, 10), *G*(1, 5), *H*(4, 2) and *E*(12, 0)

The values of objective function *Z* at these corner points are as follows.

Corner point $Z = 3x + 2y$

B(0, 10) : $3 + 0 + 3 \times 10 = 30$

$G(1, 5)$: $3\times1+2\times5=13$

$H(4, 2)$: $3\times4+2\times2=16$

$B(12, 0)$: $3\times12+2\times0=36$

Therefore, the minimum value of Z is 13 at the point $G(1, 5)$. Hence, $x = 1$ and $y = 5$ is the optimal solution of the given LPP.

The optimal value of objective function Z is 13.

31. Given, $x=a(\cos t+t\sin t)$

On differentiating both sides w.r.t. t, we get

$$\frac{dx}{dt}=a\left[-\sin t+\frac{d}{dt}(t)\cdot\sin t+t\frac{d}{dt}(\sin t)\right]$$ [by using product rule of derivative]

$$\frac{dx}{dt}=a(-\sin t+1.\sin t+t\cos t)$$

$$\frac{dx}{dt}=at\cos t$$ …(i)

$$\frac{dy}{dt}=a(\cos t-\cos t.+t\sin t)=at\sin t$$ …(ii)

Now, $$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{at\sin t}{at\cos t}=\tan t$$ [From equation (i) and (ii)]

Again, differentiating both sides, w.r.t. x, we get

$$\frac{d^2t}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dt}(\tan t)\frac{dt}{dx}=\sec^2 t\,\frac{1}{dx/dt}=\frac{\sec^2 t}{at\cos t}=\frac{\sec^3 t}{dt}$$ [From equation (i)]

Also, $$\frac{d^2x}{dt^2}=\frac{d}{dt}(at\cos t)=a\frac{d}{dt}(t\cos t)$$

$$=a\left[\frac{d}{dt}(t)\cdot\cos t+t\frac{d}{dt}(\cos t)\right]$$ [by using product rule of derivative]

$=a[\cos t-\sin t]$ and $\frac{d^2y}{dt^2}=\frac{d}{dt}\left(\frac{dy}{dt}\right)=\frac{d}{dt}(at\sin t)=a(\sin t+t\cos t)$

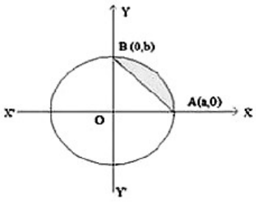
SECTION-D

32. The given curves are:

Ellipse: $$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Straight Line $\frac{x}{a}+\frac{y}{b}=1$

The required area bounded by given curves is shown in figure below by shaded portion;



Now area of bounded region is given as;

$A=\int y\,dx$

Here A = (Area of ellipse in 1st quadrant) – (Area of triangle OAB)

From given ellipse we have;

$y=\frac{b}{a}\sqrt{a^2-x^2}$ and from given line we have;

$y=\frac{b}{a}(a-x)$

Therefore the required area may be calculated as;

Area $=\int_0^a \frac{b}{a}\sqrt{(a^2-x^2)}dx-\int_0^a \frac{b}{a}(a-x)dx$

$$=\frac{b}{a}\left[\frac{x}{a}\sqrt{(a^2-x^2)}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a-\frac{b}{a}\left[ax-\frac{x^2}{2}\right]_0^a$$

$$=\frac{b}{a}\left[\frac{a^2}{2}\frac{\pi}{2}-0\right]-\frac{b}{a}\left[\frac{a^2}{2}\right]=\frac{ab}{4}[\pi-2]$$

Which is the required area.

33. (i) Let $(a_1 b_1)$ and $(a_2,b_2)\in A\times B$ such that

$f(a_1,b_1)=f(a_2,b_2)$

$\Rightarrow \quad (a_1,b_1)=(a_2,b_2)$

$\Rightarrow \quad a_1=a_2$ and $b_1=b_2$

$\Rightarrow \quad (a_1,b_1)=(a_2,b_2)$

Therefore, f is injective.

(ii) Let $(b,\,a)$ be an arbitrary

Element of $B\times A,$ then $b\in B$ and $a\in A$

$\Rightarrow \quad (a,b)\in(A\times B)$

Thus for all $(b,a)\in B\times A$ their exists $(a,b)\in(A\times B)$

Such that

$f(a,b)=(b,a)$

So, $f: A\times B \to B\times A$ is an onto function. Hence *f* is bijective.

OR

Let (a_1, b_1) and $(a_2, b_2)\in A\times B$

1. (i) $f(a_1,b_1)=f(a_2,b_2)$

$b_1=b_2$ and $a_1=a_2$

Then $f(a_1,b_1)=f(a_2,b_2)$

$(a_1,b_1)=(a_2,b_2)$ for all $(a_1,b_1)=(a_2,b_2)\in A\times B$

(ii) f is injective.

Let (b,a) be an arbitrary

Element of $B\times A.$ then $b\in B$ and $a\in A$

$\Rightarrow$ $(a,b)\in(A\times B)$

Thus for all $(b,a)\in B\times A$ their exists $(a,b)\in(A\times B)$

Hence that $f(a,\,b)=(b,\,a)$

So $f: A\times B\to B\times A$

f is an onto function.

34. Here,

$$D=\begin{vmatrix}1 & -1 & 3\\ 1 & 3 & -3\\ 5 & 3 & 3\end{vmatrix}=1(9+9)+1(3+15)+3(3-15)=18+18+3(-12)=0$$

$$D_1=\begin{vmatrix}6 & -1 & 3\\ -4 & 3 & -3\\ 10 & 3 & 3\end{vmatrix}=\begin{vmatrix}2 & 2 & 0\\ -4 & 3 & -3\\ 6 & 6 & 0\end{vmatrix}(R_1\to R_1+R_2, R_3\to R_3+R_2)\ =3\begin{vmatrix}2 & 2 & 0\\ -4 & 3 & -3\\ 2 & 2 & 0\end{vmatrix}=0$$

$$D_2=\begin{vmatrix}1 & 6 & 3\\ 1 & -4 & -3\\ 5 & 10 & 3\end{vmatrix}=\begin{vmatrix}2 & 2 & 0\\ 1 & -4 & -3\\ 6 & 6 & 0\end{vmatrix}(R_1\to R_1+R_2, R_3\to R_3+R_2)=3\begin{vmatrix}2 & 2 & 0\\ 1 & -4 & -3\\ 2 & 2 & 0\end{vmatrix}=0$$

$$D_3=\begin{vmatrix}1 & -1 & 6\\ 1 & 3 & -4\\ 5 & 3 & 10\end{vmatrix}=\begin{vmatrix}1 & -1 & 6\\ 0 & 4 & -10\\ 0 & 8 & -20\end{vmatrix}(R_2\to R_2+R_1, R_3+5R_1)=1(-80+80)+0+0=0$$

So, $D=D_1=D_2=D_3=0$

So, the given system is either inconsistent or has infinnite solution. Consider the first two equations, written as

$x-y=6-3z$

$x+3y=-4+3z$

Solving by Cramer's rule. Here,

$$D=\begin{vmatrix}1 & -1\\ 1 & 3\end{vmatrix}=3+1=4$$

$$D_1=\begin{vmatrix}6-3z & -1\\ -4+3z & 3\end{vmatrix}=3(6-3z)+(-4+3z)=14-6z$$

$$D_2=\begin{vmatrix}1 & 6-3z\\ 1 & -4+3z\end{vmatrix}=(-4+3z)-(6-3z)=-10+6z$$

$\therefore \quad x=\dfrac{D_1}{D}=\dfrac{14-6z}{4}=\dfrac{7-3z}{2}$

$y=\dfrac{D_2}{D}=\dfrac{6z-10}{4}=\dfrac{3z-5}{2}$

Let $z=k,$ then

$x=\dfrac{7-3k}{2}, y=\dfrac{3k-5}{2}, z=k$ are the infinite solutions of the given system of equations.

35. Here, it is given equations of lines:

$L_2=\dfrac{x-6}{3}=\dfrac{y-7}{1}=\dfrac{x-4}{1}$

$L_2=\dfrac{x}{-3}=\dfrac{y+9}{2}=\dfrac{x-2}{4}$

Direction ratios of L_1 and L_2 are (3, –1, 1) and $(-3,2,4)$ respectively.

Suppose general point on line L_1 is $P=(x_1,y_1,z_1)$

$x_1=3x+6, y_1=-x+7, z_1=s+4$ and suppose general point on line L_2 is $Q=(x_2,y_2,z_1)$

$x_2=-3t, y_2=2t-9, z_2=4t+2$

$\therefore \quad \overrightarrow{PQ}=(x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}+(z_2-z_1)\hat{k}$

$=(-3t-3s-6)\hat{i}+(2t-9+s-7)\hat{j}+(4t+2-s-4)\hat{k}$

$\therefore \quad \overrightarrow{PQ}=(-3t-3s-6)\hat{i}+(2t+s-16)\}+(4t-s-2)\hat{k}$

Direction ratios of $\overrightarrow{PQ}$ are $((-3t-3s-6),(2t+s-16),(4t-s-2))$ PQ will be the shortest distance if it is perpendicular to both the given lines

Thus, by the condition of perpendicularity,

$\Rightarrow \quad 3(-3t-3s-60)-1(2t+s-16)+1(4t-s-2)=0$ and

$\Rightarrow \quad -3(-3t-3s-6)+2(2t+s-16)+4(4t-s-2)=0$

$\Rightarrow \quad -9t-9s-18-2t-s+16+4t-s-2=0$ and $9t+9s+18+4t+2s+-32+16t-4s-8=0$

$\Rightarrow \quad -7t-11s=4$ and $29t+7s=-22$

Solving above two equations, the we obtain

$t=1$ and $s=-1$ Therefore, $P=(3,8,3)$ and $Q=(-3,-7,6)$

Now, distance between points *P* and *Q* is

$$d=\sqrt{(3+3)^2+(8+7)^2+(3-6)^2}=\sqrt{(6)^2+(15)^2+(-3)^2}$$

$$=\sqrt{36+225+9}=\sqrt{270}=3\sqrt{30}$$

Thus, the shortest distance between two given lines is

$d=3\sqrt{30}$ units

Now, the equation of the line passing through points *P* and *Q* is

$$\frac{x-x_1}{x_1-x_2}=\frac{y-y_1}{y_1-y_2}=\frac{z-z_1}{z_1-z_2}$$

$\therefore \quad \frac{x-3}{3+3}=\frac{y-8}{8+7}=\frac{z-3}{3-6} \qquad \therefore \quad \frac{x-3}{6}=\frac{y-8}{15}=\frac{x-3}{-3}$

$\therefore \quad \frac{x-3}{2}=\frac{y-8}{5}=\frac{z-3}{-1}$ thus, the equation of the line of the shortest distance between two given lines is $\frac{x-3}{2}=\frac{y-8}{5}=\frac{z-3}{-1}$

OR

Suppose,

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\lambda$$

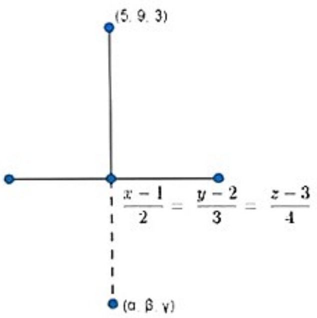
So the foot of the perpendicular is $(2\lambda+1, 3\lambda+2, 4\lambda+3)$

The direction ratios of the perpendicular is

$(2\lambda+1-5):(3\lambda+2-9):(4\lambda+3-3)$

$\Rightarrow \quad (2\lambda-4):(3\lambda-7):(4\lambda)$

Direction ratio of the line is $2:3:4$



From the direction ratio of the line and the direct ratio of its perpendicular, we have

$2(2\lambda-4)+3(3\lambda-7)+4(4\lambda)=0$

$\Rightarrow \quad 4\lambda-8+9\lambda-21+16\lambda=0$

$\Rightarrow \quad 29\lambda=29$

$\Rightarrow \quad \lambda=1$

Therefore, the foot of the perpendicular is (3, 5, 7)

The foot of the perpendicular is the mid-point of the line joining (5, 9, 3) and (α, β, γ)

Therefore, we have

$\frac{\alpha+5}{2}=3 \Rightarrow \alpha=1$

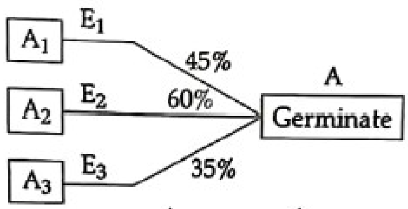
$\frac{\beta+9}{2}=5 \Rightarrow \beta=1$

$\frac{y+3}{2}=7 \Rightarrow \gamma=11$

Therefore, the image is (1, 1, 11)

SECTION-E

36. (i)



Here, $P(E_1)=\frac{4}{10}, P(E_2)=\frac{4}{10}, P(E_3)=\frac{2}{10}$

$$P\left(\frac{A}{E_1}\right)=\frac{45}{100}, P\left(\frac{A}{E_2}\right)+P(E_2)\cdot P\left(\frac{A}{E_2}\right)+P(E_3)\cdot P\left(\frac{A}{E_3}\right)$$

$$=\frac{4}{10}\times\frac{45}{100}+\frac{4}{10}\times\frac{60}{100}+\frac{2}{10}+\frac{35}{100}=\frac{180}{1000}+\frac{240}{1000}+\frac{70}{100}=\frac{490}{1000}=4.9$$

(ii) Required probability $=P\left(\frac{E_2}{A}\right)=\frac{P(E_2)\cdot P\left(\frac{A}{E_2}\right)}{P(A)}=\frac{\frac{4}{10}+\frac{60}{100}}{\frac{490}{1000}}=\frac{240}{490}=\frac{24}{49}$

(iii) Let,

$E_1 =$ Event for getting an even number on die and

$E_2 =$ Event that a spade card is selected

$\therefore$ $P(E_1)=\frac{3}{6}=\frac{1}{2}$ and $P(E_2)=\frac{13}{52}=\frac{1}{4}$

Then, $P(E_1\cap E_2)=P(E_1)\cdot P(E_2)=\frac{1}{2},\frac{1}{4}=\frac{1}{8}$

OR

$P(A)+P(VB)-P(A\text{ and }B)=P(A)$

$\Rightarrow$ $P(A)+P(B)-(A\cap B)=P(A)$

$\Rightarrow$ $P(B)-P(A\cap B)=0$

$\Rightarrow$ $P(A\cap B)=P(B)$

$\therefore$ $P(A\mid B)=\frac{P(A\cap B)}{P(B)}=\frac{P(B)}{P(B)}=1$

37. (i) Let F be the combined force,

$\therefore$ $\vec{F}=\vec{F_1}+\vec{F_2}+\vec{F_3}$

$=(4\hat{i}+0\hat{j})+(-2\hat{i}+4\hat{j})+(-3\hat{i}-3\hat{j})=(4-2-3)\hat{i}+(0+4-3)=-\hat{i}+\hat{j}$

$\therefore$ $\vec{F}\mid=\left|\sqrt{(-1)^2+1^2}\right|=\left|\sqrt{2}\right| KN$

(ii) Magnitude of force of Team B =

$\left|\vec{F_2}\right|=\left|\sqrt{(-2)^2+4^2}\right|=\sqrt{20}\,KN=2\sqrt{5}\ KN$

(iii) We have,

$\left|\vec{F_1}\right|=\left|\sqrt{(4)^2+0^2}\right|=4\ KN$

$\left|\vec{F_2}\right| = \left|\sqrt{(-2)^2 + 4^2}\right| = \sqrt{20}\ KN$

and $\left|\vec{F_3}\right| = \left|\sqrt{(-3)^2 + (-3)^2}\right| = \sqrt{18}\ KN$

Here, the magnitude of force F_2 is greater, therefore team Q will win the game.

OR

We have,

Combined force, $\vec{F} = -\hat{i} + \hat{j}$

$\therefore$ $\theta \tan^{-1}\left(\frac{F_g}{F_x}\right) = \tan^{-1}\left(\frac{1}{-1}\right) = \tan^{-1}(1) = \tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \frac{3\pi}{4}$ radians.

38. (i) $C = 40000h^2 + 5000x^2$

as $x^2h = 250$

$\Rightarrow$ $C = \frac{40000(250)^2}{4} + 5000x^2$

(ii) $\frac{dC}{dx} = \frac{-160000(250)^2}{x^5} + 10000x$

(iii) For minimum cost $\frac{dc}{dx} = 0$

$\Rightarrow$ $10000x^6 = 250 \times 250 \times 160000$

$\Rightarrow$ $x = 10$

Showing $\frac{d^2C}{dx^2} > 0$ at $x = 10$

$\therefore$ Cost is minimum when $x = 10$

OR

$\frac{dC}{dx} = \frac{-160000(250)^2}{x^4} + 10000x$

$\frac{dC}{dx} = 0$ gives $x = 10$

$\frac{dC}{dx} > 0$ in $(10, \infty)$ and $\frac{dC}{dx} < 0$ in (0, 10).

Hence, cost function is neither increasing nor decreasing for $x > 0$

SUBJECTIVE MOCK TEST | MATHEMATICS | SOLUTION

CLASS – XII | SET – 2

SECTION-A

1.(A) The diagonal elements of a skew- symmetric matrix is always zero and the elements. $a_{ij} = -aji$.

2.(D) $\Rightarrow \quad A(A+I)-2I=0 \quad \Rightarrow \quad A^2+A-2A-2I=0$

$\Rightarrow \quad A^2-A=2I \quad \Rightarrow \quad 2I=A^2-A \quad \Rightarrow \quad I=\frac{1}{2}(A^2-A)$

$\Rightarrow \quad A^{-1}=\frac{1}{2}(A^{-1}A^2-A^{-1}A)=\frac{1}{2}(A^{-1}A)(A-I)=\frac{1}{2}(IA-I)=\frac{1}{2}(A-I)$

3.(B) We know that Adj. (Adj. A) $=|A|^{n-2}A$ if $|A|\neq 0$, where n is the order of matrix A. Therefore, Adj. (Adj. A) $=|A|.A$, Det. (Adj. (Adj. A)) $=|A|^3.|A|=|A|^4=14^4$

4.(B) $x = A\cos 4t + B\sin 4t$

$$\frac{d}{dt}x=A\frac{d}{dt}\cos 4t+B\frac{d}{dt}\sin 4t$$

$$\frac{dx}{dt}=A(-\text{sins}\,4t)\frac{d}{dt}+B(\cos 4t)\frac{d}{dt}At = -4A\sin 4t+4B\cos 4t$$

$$\frac{d}{dt}\left(\frac{dx}{dt}\right)=-4A\frac{d}{dt}\sin 4t+4B\frac{d}{dt}\cos 4t$$

$$\frac{d^2x}{dt^2}=-16A\cos 4t-16B\sin 4t$$

$$\frac{d^2x}{dt^2}=-16(A\cos 4t+B\sin 4t) = -16x$$

5.(B) $13;\frac{12}{13},\frac{4}{13},\frac{3}{13}$

If a line makes angles α,β and γ with the axis, then $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$ ...(i)

Let r be the length of the line segment. Then,

$r\cos\alpha=12, r\cos\beta=4, r\cos\gamma=3$ …(ii)

$\Rightarrow \quad (r\cos\alpha)^2+(r\cos\beta)^2+(r\cos\gamma)^2=12^2+4^2+3^2$

$\Rightarrow \quad r^2(\cos^2\alpha+\cos^2\beta+\cos^2\gamma)=169 \quad \Rightarrow \quad r^2(1)=169$ [From (i)]

$\Rightarrow \quad r=\sqrt{169} \quad \Rightarrow \quad r=\pm 13 \quad \Rightarrow \quad r=13$ (since length cannot be negative)

Substituting r = 13 in (ii)

We get,

$$\cos\alpha=\frac{12}{13}, \cos\beta=\frac{4}{13}, \cos\gamma=\frac{1}{13}$$

Thus, the direction cosines of the line are $\frac{12}{13},\frac{4}{13},\frac{3}{13}$

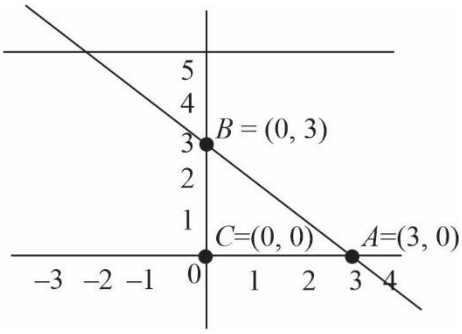
6.(B)

7.(B) Converting the given inequations into equations, we obtain $y = 6$, $x + y = 3$, $x = 0$ and $y = 0$, $y = 6$ is the line passing through (0, 6) and parallel to the x- axis. The region below the line $y = 6$ will satisfy the given inequation.

The line $x + y = 3$ meets the coordinate axis at $A(3, 0)$ and $B(0, 3)$. Join these points to obtain the line $x + y = 3$. Clearly, (0, 0) satisfies the inequation $x + y \leq 3$. So, the region in x, y -plane that contains the origin represents the solution set of the given equation.

The region represented by $x \geq 0$ and $y \geq 0$:

Since every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations.



8.(C) Hint: $\vec{a} = (\hat{i} + \hat{j} + \sqrt{2}\hat{k}) \quad \Rightarrow \quad |\vec{a}| = |\vec{a}| = \sqrt{1^2 + 1^2 + (\sqrt{2})^2} = \sqrt{4} = 2$

Direction ratios of $\vec{a}$ are $(1, 1, \sqrt{2})$

Direction of cosines of $\vec{a}$ are $\left(\frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2}\right)$, i.e., $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}\right)$

Direction cosines along to z- axis (0, 0, 1)

$\therefore \quad \cos\gamma = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad \gamma = \frac{\pi}{4}$

9.(B) $= xe^x - \int e^z dx = e^x(x-1) + C$

10.(B) $A = \begin{vmatrix} 3 & -5 \\ -4 & -2 \end{vmatrix}, A^2 = \begin{vmatrix} 3 & -5 \\ -4 & 2 \end{vmatrix} \begin{vmatrix} 3 & -5 \\ -4 & 2 \end{vmatrix} = \begin{vmatrix} 9+20 & -15-10 \\ -12-8 & 20+4 \end{vmatrix} = \begin{vmatrix} 29 & -25 \\ -20 & 24 \end{vmatrix}$

$A^2 - 5A = \begin{vmatrix} 29 & -25 \\ -20 & 24 \end{vmatrix} + \begin{vmatrix} -15 & 25 \\ 20 & -10 \end{vmatrix} = \begin{vmatrix} 14 & 0 \\ 0 & 14 \end{vmatrix} = 14I$

11.(C)

Corner Point	$Z = 0.7x + y$
(0, 0)	$0.7 \times 0 + 0 = 0$
(40, 0)	$0.7 \times 40 + 0 = 28$
(30, 20)	$0.7 \times 30 + 20 = 41$ ⟵Maximum
(0, 40)	$0.7 \times 0 + 40 = 40$

12.(C) $(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})$

$= 3(\vec{a} + \vec{a}) - (\vec{a} \times \vec{b}) + 9(\vec{b} \times \vec{a}) - 3(\vec{b} \times \vec{b}) = -10(\vec{a} + \vec{b}) \,|\, (\vec{a} + \overrightarrow{3b}) \times (\overrightarrow{3a} - \vec{b})^2 \,|$

$= (-10(\vec{a} \times \vec{b}))^2 = 100\,(\vec{a} \times \vec{b})^2 = 100(\vec{a} \times \vec{b})^2 = 100\,|\vec{a}|^2\,|\vec{b}|^2 \sin^2 120^\circ$

$= 100 \times 4 \times \left(\frac{\sqrt{3}}{2}\right)^2 = 300$

13.(D) If det. $A = 0$, (adj A) $B = 0 \Rightarrow$ The system $AX = B$ of n equations in n unknowns may be consistent with infinitely many solutions or it may be inconsistent.

14.(D)

15.(D) We have, $\dfrac{dy}{dx} + y\cot x = \operatorname{cosec} x$

Comparing with $\dfrac{dy}{dx} + Py = Q$ of the above equation then, we get

$\Rightarrow \quad P = \cot x, Q = \operatorname{cosec} x$

I.F. = $e^{\int P dx} = e^{\int \cot x dx} = e^{\int \log \sin x} = \sin x$

Multiplying on both sides by $\sin x$

$\sin x\dfrac{dy}{dx} + y\cos x = 1$

$\Rightarrow \quad \dfrac{d}{dx}(y\sin x) = 1 \qquad \Rightarrow \qquad y\sin x = \int 1 dx$

$\Rightarrow \quad y\sin x = x + C$

16.(B) O is the circumcenter of the triangle ABC

$\Rightarrow \quad |\overrightarrow{AO}| = |\overrightarrow{BO}| = |\overrightarrow{CO}| = 2\sqrt{2} = R$

Position vector of O be $x\hat{i} + y\hat{j}$

$\Rightarrow \quad |\overrightarrow{AO}| = \sqrt{x^2 + y^2} = 2\sqrt{2} = R$

$\Rightarrow \quad x^2 + y^2 = 8 \qquad \ldots$(i)

And $|\overrightarrow{BO}| = |\overrightarrow{CO}|$

$\sqrt{(x-3)^2 + (y-3\sqrt{3})^2} = \sqrt{(x+3\sqrt{3})^2 + (y-3)^2}$

$\Rightarrow \quad y = \dfrac{x(1+\sqrt{3})}{1-\sqrt{3}}$

Put it in (i)

$x^2 + y^2 = 8$

$x^2 + \left[\dfrac{x(1+\sqrt{3})}{1-\sqrt{3}}\right]^2 = 8$

$\Rightarrow \quad x = 1-\sqrt{3}$

$\Rightarrow \quad y = 1+\sqrt{3}$

$\overrightarrow{AO} = (1-\sqrt{3})\hat{i} + (1+\sqrt{3})\hat{j}$

17.(C) Differentiating both sides we get $\quad \Rightarrow \quad y_1\left(y^{\frac{1}{n}-1} + y^{\frac{-1}{n}-1}\right) = 2ny$

Again differentiating both sides we get

$y_2\left(y^{\frac{1}{n}} - y^{\frac{-1}{n}}\right) + \dfrac{y_1}{n}\left(y^{\frac{1}{n}-1} + y^{\frac{-1}{n}-1}\right) = 2ny_1 \Rightarrow \qquad ny_2\left(y^{\frac{1}{n}} - y^{\frac{-1}{n}}\right) + \dfrac{y_1}{n}\left(y^{\frac{1}{n}} + y^{\frac{-1}{n}}\right) = 2n^2 y_1$

$\Rightarrow \quad nyy_2\left(y^{\frac{1}{n}} - y^{\frac{-1}{n}}\right) + 2xy_1^2 = 2n^2 yy_1$

18.(D) Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = (\sqrt{3}-1)\hat{i} + (-\sqrt{3}-1)\hat{j} + 4\hat{k}$

$$|\vec{a}| = \sqrt{6}, |\vec{b}| = \sqrt{(4-2\sqrt{3}) + (4+2\sqrt{3}) + 16} = 2\sqrt{6}$$

$$\cos\alpha = \frac{(\hat{i} + \hat{j} + 2\hat{k}) . ((\sqrt{3}-1)\hat{i} + (-\sqrt{3}-1)\hat{j} + 4\hat{k})}{\sqrt{6} \times 2\sqrt{6}}$$

$$\cos\alpha = \frac{1}{2}$$

$$\alpha = 60^{o}$$

19.(A) Circumference of circle with radius *r* is given by $C = 2\pi r$

Differentiating w.r.t. '*t*', we get

$$\frac{dC}{dt} = 2\pi\frac{dr}{dt}$$

Given, $\frac{dC}{dt} = 10\, cm/s$

$$\therefore \quad 10 = 2\pi\frac{dr}{dt}$$

$$\Rightarrow \quad \frac{dr}{dt} = \frac{5}{\pi} cm/s$$

Now, area of circle, $A = \pi r^2$

$$\therefore \quad \frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$

Substituting *r* = 3 cm and $\frac{dr}{dt} = \frac{5}{\pi} cm/s$, we get , $\frac{dA}{dt} = 2\pi \times 3 \times \frac{5}{\pi}$

$$\therefore \quad \frac{dA}{dt} = 30\, cm^2/s$$

20.(D) **Assertion:** Given function is $f(x) = x^2 + bx + c$

It is a quadratic equation in *x*.

So, we will get a parabola either downward or upward.

Hence, it is a many-one mapping and not onto mapping.

Hence, it is neither one-one nor onto mapping.

Reason: Total number of functions = $(n(B))^{n(A)} = 2^n$

Clearly, a function will not be onto if all elements of A map to either *a* or *b*.

SECTION-B

21. We have, $\cos^{-1}\left(\frac{1}{2}\right) = \cos^{-1}\left(\cos\frac{\pi}{3}\right) = \frac{\pi}{3} \quad \left[\because \quad \frac{\pi}{3} \in [0, \pi]\right]$

Also, $\sin^{-1}\left(-\frac{1}{2}\right) = \sin^{-1}\left(-\sin\frac{\pi}{6}\right) = -\frac{\pi}{6} \quad \left[\because \quad -\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

$$\therefore \quad \cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{3} - 2\left(-\frac{\pi}{6}\right) = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

OR

Let $x = \cos\theta$

$\therefore \quad \theta = \cos^{-1} x$

$$\sec^{-1}\left(\frac{1}{2x^2-1}\right) = \sec^{-1}\left(\frac{1}{2\cos^2\theta - 1}\right) = \sec^{-1}\left(\frac{1}{\cos 2\theta}\right) = \sec^{-1}(\sec 2\theta) = 2\theta = 2\cos^{-1} x$$

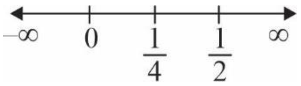
22. Given function: $f(x) = x^3(2x-1)^3$

$\Rightarrow \quad f'(x) = 3x^2(2x-1)^3 + 6x^2(2x-1)^2$

To find the local maxima or minima, we must have $f'(x) = 0$

$\Rightarrow \quad 3x^2(2x-1)^3 + 6x^3(2x-1)^2 = 0 \Rightarrow \quad 3x^2(2x-1)^2(2x-1)^2(2x-1+2x) = 0$

$\Rightarrow \quad x^2(2x-1)^2(4x-1) = 0 \Rightarrow \quad x = 0, \frac{1}{2}$ and $\frac{1}{4}$



Since $f'(x)$ changes from negative to positive when x increases through $\frac{1}{4}$. Hence, $x = \frac{1}{4}$ is a point of local minima. Thus the local minimum value of $f(x)$ at $x = \frac{1}{4}$ is given by $\left(\frac{1}{4}\right)^3\left(\frac{1}{2}-1\right)^3 = \frac{-1}{512}$.

23. $f(x) = \cos x + a^2x + b$

differentiating function $f(x)$ w.r.t 'x'

$f'(x) = a^2 - \sin x$

Given $f(x)$ is strictly increasing on R

$\Rightarrow \quad f'(x) > 0, \forall x \in R \Rightarrow \quad a^2 - \sin x > 0, \forall x \in R \Rightarrow \quad a^2 > \sin x, \forall x \in R$

We know that $\sin(x) \in [-1, 1]$

$a^2 > \sin x, a^2$ is always greater than 1.

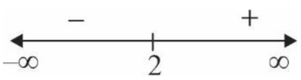
$\Rightarrow \quad a^2 > 1 \Rightarrow \quad a^2 > 1 > 0 \Rightarrow \quad (a+1)(a-1) > 0 \Rightarrow \quad a \in (-\infty, -1) \cup (1, \infty)$

OR

Given function : $f(x) = \frac{x}{2} + \frac{2}{x}, x > 0 \Rightarrow \quad f'(x) = \frac{1}{2} - \frac{2}{x^2}$

To find the local maxima or minima, we must have $f'(x) = 0$

$\Rightarrow \quad \frac{1}{2} - \frac{2}{x^2} = 0 \Rightarrow \quad \frac{1}{2} - \frac{2}{x^2} \Rightarrow \quad x^2 = \pm 2$



Since $x > 0$, $f(x)$ changes from negative to positive when x increases through 2, So, $x = 2$ is a point of local minima. Thus the local minimum value of $f(x)$ at $x = 2$ is given by $\frac{2}{2} + \frac{2}{2} = 2$.

24. Let $I=\int_0^{\frac{\pi}{2}}|\cos 2x|dx$, then

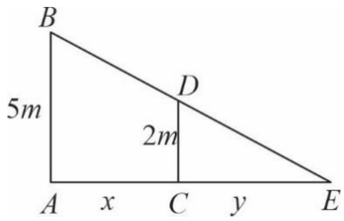
$$I=\int_0^{\frac{\pi}{4}}-\cos 2x\,dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}+\cos 2x\,dx=\left[\frac{+\sin 2x}{2}\right]_0^{\frac{\pi}{4}}+\left[\frac{-\sin 2x}{2}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$=\frac{1}{2}\left[\sin\frac{\pi}{2}-\sin 0\right]+\frac{1}{2}\left[\sin\pi+\sin\frac{\pi}{2}\right]=\frac{1}{2}[1]+\frac{1}{2}[1]=\frac{1}{2}+\frac{1}{2}=1$$

25. Let *AB* be the lamp-post. Let at any time *t*, the man *CD* be at a distance *x* metres from the lamp-post and let the length shadow be *y* metres. Then,

$$\frac{dx}{dt}=6\text{ metres / minute}$$

Clearly, triangle *ABE* and *CDE* are similar.



$\therefore\quad \frac{AB}{CD}=\frac{AE}{CE}$

$\Rightarrow\quad \frac{5}{2}=\frac{x+y}{y}$

$\Rightarrow\quad 5y=2x+2y$

$\Rightarrow\quad 3y=2x$

$\Rightarrow\quad 3\frac{dy}{dt}=2\frac{dx}{dt}$

$\Rightarrow\quad 3\frac{dy}{dt}=2(6)$

$\Rightarrow\quad \frac{dy}{dt}=4$ Thus, the shadow increases at a rate of 4 metres/minute.

SECTION-C

26. Let $I=\int\frac{1}{e^2+1}dx=\int\frac{1}{e^x\left[1+\frac{1}{e^x}\right]}dx$

$\Rightarrow\quad I=\int\frac{1}{e^x[1+e^{-x}]}dx$

Let $1+e^{-x}=t$, then

$d(1+e^{-x})=dt$

$\Rightarrow\quad -e^{-x}dx=dt\quad\Rightarrow\quad dx=\frac{-dt}{e^{-x}}\quad\Rightarrow\quad dx=e^{-x}dt$

Putting $1+e^{-x}=t$ and $dx=-e^x dt$ in equation (i), we get,

$$I=\int\frac{1}{e^z\times t}\times -e^x dt\text{ b}=-\int\frac{dt}{t}=-\log|t|+C=-\log|1+e^{-x}|+C$$

27. Let E_1 = event that A is selected, and E_2, = event that B is selected.

Therefore, we have,

$\Rightarrow \quad P(\bar{E}_1) = \left(1 - \frac{1}{3}\right) = \frac{2}{3}$ and $P(\bar{E}_2) = \left(1 - \frac{2}{5}\right) = \frac{3}{5}$

$\therefore \quad P$ (event that only one of them is selected)

$= P[E_1 \text{ and not } E_2) \text{ or } (E_2 \text{ and not } E_1) \; = P[(E_1 \cap \bar{E}_2) \text{ or } (E_2 \cap \bar{E}_1)]$

$= P(E_1 \cap \bar{E}_2) + P(E_2 \cap \bar{E}_1) \quad [\because \quad (E_1 \cap \bar{E}_2) \cap (E_2 \cap \bar{E}_1) = \phi]$

$= P(E_1) \,.\, P(\bar{E}_2) + P(E_2) \,.\, P(\bar{E}_1)$

[$\because$ E_1 and E_2 are independent, and E_2 and $\bar{E}_1$ are independent]

$= \left(\frac{1}{3} \times \frac{3}{5}\right) + \left(\frac{2}{5} \times \frac{2}{3}\right) = \left(\frac{1}{5} + \frac{4}{15}\right) = \frac{7}{15}$

This is the required probability.

28. By using the property of definite integrals we have

$$\int_0^{2\pi} f(x)dx = \int_0^{2\pi} f(2\pi - x)dx$$

Hence,

$$\int_0^{2\pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}} dx = \int_0^{2\pi} \frac{e^{\sin(2\pi - x)}}{e^{\sin(2\pi - x)} + e^{-\sin(2\pi - x)}} dx$$

We know,

$$\sin(2\pi - x) = -\sin x$$

$$I = \int_0^{2\pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}} dx = \int_0^{2\pi} \frac{e^{-\sin x}}{e^{-\sin x} + e^{\sin x}} dx$$

If $I = \int_0^{2\pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}} dx$

Then also,

$$\int_0^{2\pi} \frac{e^{-\sin x}}{e^{-\sin x} + e^{\sin x}} dx$$

Hence, $2I = \int_0^{2\pi} \frac{e^{-\sin x}}{e^{-\sin x} + e^{\sin x}} dx + \int_0^{2\pi} \frac{e^{\sin x}}{e^{-\sin x} + e^{\sin x}} dx$

$$2I = \int_0^{2\pi} \frac{e^{-\sin x}}{e^{-\sin x} + e^{\sin x}} + \frac{e^{\sin x}}{e^{-\sin x} + e^{\sin x}} dx$$

$$2I = \int_0^{2\pi} dx$$

$$2I = 2\pi$$

$$I = \pi$$

OR

Let the given integral be,

$$I=\int \frac{1}{4\cos x-1}dx$$

Putting $\cos x=\dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$\Rightarrow \qquad I=\int \frac{1}{4\left(\dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)-1}dx=\int \frac{1}{\dfrac{4\left(1-\tan^2\frac{x}{2}\right)-\left(1+\tan^2\frac{x}{2}\right)}{\left(1+\tan^2\frac{x}{2}\right)}}$$

$$=\int \frac{\left(1+\tan^2\frac{x}{2}\right)dx}{4-4\tan^2\left(\frac{x}{2}\right)-1-\tan^2\left(\frac{x}{2}\right)}=\int \frac{\sec^2\left(\frac{x}{2}\right)dx}{3-5\tan^2\left(\frac{x}{2}\right)}$$

Let $\tan\left(\frac{x}{2}\right)=t \Rightarrow \qquad \frac{1}{2}\sec^2\left(\frac{x}{2}\right)dx=dt \Rightarrow \qquad \sec^2\left(\frac{x}{2}\right)dx=2dt$

$$\therefore\ I=2\int \frac{dt}{3-5t^2}=\frac{2}{5}\int \frac{dt}{\frac{3}{5}-t^2}=\frac{2}{5}\int \frac{dt}{\left(\frac{\sqrt{3}}{\sqrt{5}}\right)^2-t^2}=\frac{2}{5}\times\frac{\sqrt{5}}{2\sqrt{3}}\text{In}\left|\frac{\frac{\sqrt{3}}{\sqrt{5}}+t}{\frac{\sqrt{3}}{\sqrt{5}}-t}\right|+C$$

$$=\frac{1}{\sqrt{15}}\text{In}\left|\frac{\sqrt{3}+\sqrt{5}t}{\sqrt{3}-\sqrt{5}t}\right|+C=\frac{1}{\sqrt{15}}\text{In}\left|\frac{\sqrt{3}+\sqrt{5}\tan\left(\frac{x}{2}\right)}{\sqrt{3}-\sqrt{5}\tan\left(\frac{x}{2}\right)}\right|+C$$

29. We can rewrite the given differential equation as $\dfrac{dy}{dx}=\dfrac{x^2+y^2}{x^2+xy}$

On dividing the Nr and Dr of RHS of (i) by x^2, we get $\dfrac{dy}{dx}=\dfrac{\left\{1+\left(\frac{y}{x}\right)^2\right\}}{\left\{1+\frac{y}{x}\right\}}=f\left(\dfrac{y}{x}\right)$

Therefore, the given differential equation is homogeneous.

Put $y = vx$ and $\dfrac{dy}{dx}=\dfrac{x^2+v^2x^2}{x^2+vx^2}=\dfrac{1+v^2}{1+v}$

$$v+x\frac{dv}{dx}=\frac{x^2+v^2x^2}{x^2+vx^2}=\frac{1+v^2}{1+v}$$

$$\Rightarrow \qquad x\frac{dv}{dx}=\frac{(1+v^2)}{(1+v)}-v=\frac{1+v^2-v-v^2}{(1+v)}=\frac{(1-v)}{(1+v)} \Rightarrow \qquad \frac{(1+v)}{(1-v)}dv=\frac{1}{x}dx$$

$\Rightarrow \quad \int \frac{(1+v)}{(1-v)} dv = \int \frac{1}{x} dx$ [on integrating both sides]

$\Rightarrow \quad \int \frac{|2-(1-v)\}}{(1-v)} dv = \int \frac{1}{x} dx \Rightarrow \quad \int \left\{ \frac{2}{(1-v)} - 1 \right\} dv = \int \frac{1}{x} dx$

$\Rightarrow \quad -2\log|1-v| - v = \log|x| + \log|C|$ where C is an arbitrary constant.

$\Rightarrow \quad \log|x| + \log|C| + 2\log|1-v| = -v \Rightarrow \quad \log|Cx(1-v)^2| = -v$

$\Rightarrow \quad |Cx(1-v)^2| = e^{-v} \Rightarrow \quad |Cx(1-\frac{y}{x})^2| = e^{-y/x}$ $[\because \; v = \frac{y}{x}]$

$\Rightarrow \quad C|(x-y)^2| = |x| e^{-y/x}$, which is the required solution.

OR

Given differential equation is,

$(1+e^{2x})dy + (1+y^2)e^x dx = 0$

Above equation may be written as

$$\frac{dy}{1+y^2} = \frac{-e^x}{1+e^{2x}} dx$$

On integrating both sides, we get

$$\int \frac{dy}{1+y^2} = -\int \frac{e^x}{1+e^{2x}} dx$$

On putting $e^x = 1 \Rightarrow e^x dx = dt$ in RHS, we get

$$\tan^{-1} y = -\int \frac{1}{1+t^2} dt$$

$\Rightarrow \quad \tan^{-1} y = -\tan^{-1} t + C \Rightarrow \quad \tan^{-1} y = -\tan^{-1}(e^x) + C \quad \ldots$(i) [put $t = e^x$]

Also, given that $y = 1$, when $x = 0$.

On putting above values in Eq. (i), we get

$$\tan^{-1} t = -\tan^{-1}(e^0) + C$$

$\Rightarrow \quad 2\tan^{-1} 1 = C \Rightarrow \quad 2\tan^{-1}\left(\tan\frac{\pi}{4}\right) = C \Rightarrow \quad C = 2 \times \frac{\pi}{4} = \frac{\pi}{2}$

On putting $C = \frac{\pi}{2}$ in equation (i), we get

$$\tan^{-1} y = -\tan^{-1} e^x + \frac{\pi}{2}$$

$\Rightarrow \quad y = \tan\left[\frac{\pi}{2} - \tan^{-1}(e^x)\right] = \cot[\tan^{-1}(e^x)] \; = \cot\left[\cot^{-1}\left(\frac{1}{e^x}\right)\right]$ $[\because \; \tan^{-1} x = \cot^{-1}\frac{1}{x}]$

$\therefore \quad y = \frac{1}{e^x}$

Which is the required solution.

30. Subject to the constraints are

$x+2y\le 28$

$3x+y\le 24$

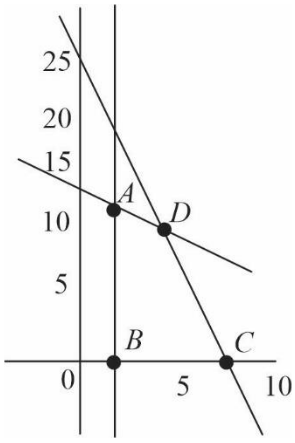
$x\ge 2$

and the non-negative restrictions $x, y \ge 0$

Converting the given inequations into equations, we get

$x + 2y = 28$, $3x + y = 24$, $x = 2$, $x = 0$ and $y = 0$

These lines are drawn on the graph and the shaded region ABCD represents the feasible region of the given LPP.



It can be observed that the feasible region is bounded. The coordinates of the corner points of the feasible region are $A(2, 13)$, $B(2, 0)$, $C(4, 12)$ and $D(8, 0)$.

The values of the objective function, Z at these corner points are given in the following table:

Corner Point Value of the Objective Function

$Z = 20x + 10y$

$A(2, 13)$: $Z=20 \times 2 + 10 \times 13 =170$

$B(2, 0)$: $Z = 20 \times 2 + 10 \times 0 = 40$

$C(8, 0)$: $Z = 20 \times 8 + 10 \times 0 = 160$

$D(4, 12)$:$Z = 20 \times 4 + 10 \times 12 = 200$

From the table, Z is maximum at $x = 4$ and $y = 12$ and the maximum value of objective function Z is 200.

OR

First, we will convert the given inequations into equations, we obtain the following equations:

$x_1-x_2=-1, -x_1+x_2=0, x_1=0$ and $x_2=0$

Region represented by $x_1-x_2\le -1$;

The line $x_1-x_2=-1$ meets the coordinate axes at $A(-1,0)$ and $B(0,1)$ respectively. By joining these points we obtain the line $x_1-x_2=-1$.

Clearly (0, 0) does not satisfies the inequation $x_1-x_2\le -1$. So, the region in the plane which does not contain the origin represents the solution set of the inequation $x_1-x_2\le -1$

Region represented by $-x_1+x_2\le 0$ or $x_1\ge x_2$

The line $-x_1+x_2=0$ or $x_1=x_2$, is the line passing through (0, 0). The region to the right of the line $x_1=x_2$ will satisfy the given inequation $-x_1+x_2\le 0$.

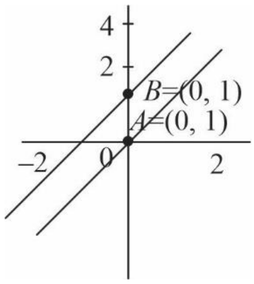
If we take a point (1, 3) to the left of the line $x_1 = x_2$. Here, $1 \le 3$ which is not satisfying the inequation $x_1 \ge x_2$.

Therefore, region to the right of the line $x_1 = x_2$ will satisfy the given inequation $-x_1 + x_2 \le 0$

Region represented by $x_1 \ge 0$ and $x_2 \ge 0$.

Since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations $x_1 \ge 0$ and $x_2 \ge 0$.

The feasible region determined by subject to the constraints are $x_1 - x_2 \le -1, -x_1 + x_2 \le 0$, and the non-negative restrictions; $x_1 \ge 0$ and $x_2 \ge 0$, are as follows:



We observe that the feasible region of the given LPP does not exist because the following equations have no common region.

31. Given function is

$$f(x) = \begin{cases} |x| + 3, & x \le -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \ge 3 \end{cases} = \begin{cases} -x + 3, & x \le -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \ge 3 \end{cases}$$

First, we verify continuity at x = – 3 and then at x = 3 Continuity at x = –3

$$LHL = \lim_{x \to (-3)^-} f(x) = \lim_{x \to (-3)^-} (-x + 3)$$

$\Rightarrow$ $LHL = \lim_{h \to 0^-} [-(-3 - h) + 3] = \lim_{h \to 0^-} (3 + h + 3)$ = 3 + 3 = 6

And $RHL = \lim_{x \to (-3)^+} f(x) = \lim_{x \to (-3)^+} (-2x)$

$\Rightarrow$ $RHL = \lim_{h \to 0^-} [-2(-3 + h)] = \lim_{h \to 0} (6 - 2h) \Rightarrow \quad RHL = 6$

Also, f (–3) = value of f (x) at x = –3 = (–3) + 3 = 3 + 3 = 6

$\because$ LHL = RHL = f (–3)

$\therefore$ f (x) is continuous at x = –3. So, x = – 3 is the point of continuity.

Continuity at x = 3

$LHL = \lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} [(-2x)] \Rightarrow \quad LHL = \lim_{h \to 0^-} [-2(3 - h)] = \lim_{h \to 0} (-6 + 2h) \Rightarrow$ LHL = – 6

And $RHL = \lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (6x + 2) \Rightarrow \quad RHL = \lim_{h \to 0} [6(3 + h) + 2] \Rightarrow$ RHL = 20

$\because$ $LHL \ne RHL$

$\therefore$ f is discontinuous at x = 3 Now, as f (x) is a polynomial function for x < –3, –3 < x < 3 so it is continuous in these intervals.

Hence, only x = 3 is the point of discontinuity of f (x).

SECTION-D

32. According to the question

Given curves are

$x - y + 2 = 0$ ...(i)

$x = \sqrt{y}$ ..(ii)

Consider $x = \sqrt{y} \Rightarrow x^2 = y$, which represents the parabola vertex of parabola is (0, 0) axis of parabola is Y-axis.

Now, the point of intersection of Eqs.(i) and (ii) is given by

$x = \sqrt{x+2}$

Squaring on both sides,

$\Rightarrow x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0 \Rightarrow x = -1, 2$

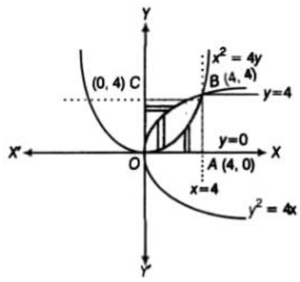
When $x = -1$, does not satisfy the Eq. (ii).

When $x = 2$, then $2 = \sqrt{y} \Rightarrow y = 4$

Hence, the point of intersection is (2, 4).

But actual equation of given parabola is $x = \sqrt{y}$, it means a semi-parabola which is on right side of Y-axis.

The graph of given curves are shown below:



Clearly, area of bounded region = Area of region $OABO$

$$= \int_0^2 [y_{(line)} - y_{(parabola)}]\,dx = \int_0^2 (x+2)\,dx - \int_0^2 x^2\,dx = \left[\frac{x^2}{2} + 2x\right]_0^2 - \left[\frac{x^3}{3}\right]_0^2$$

$$= \left[\frac{4}{2} + 4 - 0\right] - \left[\frac{8}{3} - 0\right] = 6 - \frac{8}{3} = \frac{18-8}{3} = \frac{10}{3} \text{sq.units}$$

33. For $x_1, x_2 \in R$, consider

$f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1} \Rightarrow x_1x_2^2 + x_1 = x_2x_1^2 + x_2 \Rightarrow x_1x_2(x_2 - x_1) = x_2 - x_1$$

$$\Rightarrow x_1 = x_2 \quad \text{or} \quad x_1x_2 = 1$$

We note that there are point, x_1 and x_2 with $x_1 \neq x_2$ and $f(x_1) - f(x_2)$ for instance, if we take $x_1 = 2$ and $x_2 = \frac{1}{2}$, then we have $f(x_1)\frac{2}{5}$ and $f(x_2) = \frac{2}{5}$ but $2 \neq \frac{1}{3}$. Hence f is not one-one. Also, f is not onto for if so then for $1 \in R \, \exists \, x \in R$ such that $f(x) = 1$ which gives $\frac{x}{x^2+1} = 1$. But there is no such x in the domain R, since the equation $x^2 - x + 1 = 0$ does not give any real value of x.

OR

$L_1 \| L_1$ i.e., $(L_1, L_1) \in R$ Hence reflexive

Let $(L_1, L_2) \in R$, then

$L_1 \| L_2$ which implies $L_2 \| L_1$

$\Rightarrow \quad (L_2, L_1) \in R$ Hence symmetric

We know the

$L_1 \| L_2$ and $L_2 \| L_3$

Then $L_1 \| L_3$

Therefore, $(L_1 \| L_2) \in R$ and $(L_2 \| L_3) \in R$ implies $(L_1 \| L_3) \in R$

Hence Transitive

Hence, R is an equivalence relation.

Any line parallel to $y = 2x + 4$ is of the form $y = 2x + K$, where k is a real number.

Therefore, set of all lines parallel to $y = 2x + 4$ is $(y : y = 2x + k$, k is a real number}

34. For the given system of equations, we have

$$D = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} \Rightarrow \quad D = \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & b^2-a^2 & c^2-a^2 \end{vmatrix} \text{[Applying } C_2 \to C_2 - C_1 \text{ and } C_3 \to C_3 - C_1]$$

$$\Rightarrow \quad D = (b-a)\,(c-a) \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & 1 \\ a^2 & b+a & c+a \end{vmatrix} \Rightarrow \quad D = (b-a)\,(c-a) \begin{vmatrix} 1 & 1 \\ b+a & c+a \end{vmatrix}$$

$$\Rightarrow \quad D = (b-a)\,(c-a)(c+a-b-a) = (b-c)(c-a)(a-b)$$

$$D_1 = \begin{vmatrix} 1 & 1 & 1 \\ k & b & c \\ k^2 & b^2 & c^2 \end{vmatrix} = (b-c)\,(c-k)\,(k-b);\ D_2 = \begin{vmatrix} 1 & 1 & 1 \\ a & k & c \\ a^2 & k^2 & c^2 \end{vmatrix} = (k-c)\,(c-a)\,(a-k)$$

And $D_3 = \begin{vmatrix} 1 & 1 & 1 \\ a & b & k \\ a^2 & b^2 & k^2 \end{vmatrix} = (a-b)\,(b-k)\,(k-a)$

$$\therefore \quad x = \frac{D_1}{D},\ y = \frac{D_2}{D} \text{ and } z = \frac{D_3}{D}$$

$$\Rightarrow \quad x = \frac{(b-c)\,(c-k)\,(k-b)}{(b-c)\,(c-a)\,(a-b)},\ y = \frac{(k-c)\,(c-a)\,(a-k)}{(b-c)(c-a)(a-b)} \text{ and } z = \frac{(a-b)\,(b-k)\,(k-a)}{(a-b)\,(b-c)\,(c-a)}$$

Hence, $x = \dfrac{(c-k)\,(k-b)}{(c-a)\,(a-b)},\ y = \dfrac{(k-c)(a-k)}{(b-c)(a-b)}$ and $z = \dfrac{(b-k)\,(k-a)}{(b-c)\,(c-a)}$

is the solution of given system of equations.

35. Here, it is given that

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\vec{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$$

Here, $\quad \vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{a}_2 = 4\hat{i} + \hat{j}$; $\vec{b}_2 = 5\hat{i} + 2\hat{j} + \hat{k}$

Thus, $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix} = \hat{i}(3-8) - \hat{j}(2-20) + \hat{k}(4-15)$

$\therefore \quad \vec{b}_1 \times \vec{b}_2 = -5\hat{i} + 18\hat{j} - 11\hat{k}$

$\therefore \quad |\vec{b}_1 \times \vec{b}_2| = \sqrt{(-5)^2 + 18^2 + (-11)^2} = \sqrt{25+324+121} = \sqrt{470}$

$\vec{a}_2 - \vec{a}_1 = (4-1)\hat{i} + (1-2)\hat{j} + (0-3)\hat{k}$

$\therefore \quad \vec{a}_2 - \vec{a}_1 = 3\hat{i} - \hat{j} - 3\hat{k}$

Now, we have

$(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1) = (-5\hat{i} + 18\hat{j} - 11\hat{k}).(3\hat{i} - \hat{j} - 3\hat{k})$ = ((–5) × 3) + (18 × (–1) + (–11) × (–3))

= –15 – 18 + 33 = 0

Thus, the distance between the given lines is

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$\therefore \quad d = \left| \frac{0}{\sqrt{470}} \right| \qquad \therefore \quad d = 0$ units

As $d = 0$

Thus, the given lines intersect each other.

Now, to find a point of intersection, let us convert given vector equations into Cartesian equations.

For that putting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in given equations,

$\Rightarrow \quad \vec{L}_1 : x\hat{i} + y\hat{j} + z\hat{k} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$

$\Rightarrow \quad \vec{L}_2 : x\hat{i} + y\hat{j} + z\hat{k} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$

$\Rightarrow \quad \vec{L}_1 : (x-1)\hat{i} + (y-2)\hat{j} + (z-3)\hat{k} = 2\lambda\hat{i} + 3\lambda\hat{j} + 4\lambda\hat{k}$

$\Rightarrow \quad \vec{L}_2 : (x-4)\hat{i} + (y-1)\hat{j} + (z-0)\hat{k} = 5\mu\hat{i} + 2\mu\hat{j} + \mu\hat{k}$

$\Rightarrow \quad \vec{L}_1 : \frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{2} = \lambda \qquad \therefore \quad \vec{L}_2 : \frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{2} = \mu$

General point on L_1 is

$x_1 = 2\lambda + 1, y_1 = 3\lambda + 2, z_1 = 4\lambda + 3$

Suppose, $P(x_1, y_1, z_1)$ be point of intersection of two given lines.

Thus, point P satisfies the equation of line $\vec{L}_2$

$\Rightarrow \quad \frac{2\lambda + 1 - 4}{5} = \frac{3\lambda + 2 - 1}{2} = \frac{4\lambda + 3 - 0}{1} \qquad \therefore \quad \frac{2\lambda - 3}{5} = \frac{3\lambda + 1}{2}$

$\Rightarrow \quad 4\lambda - 6 = 15\lambda + 5 \quad \Rightarrow \quad 11\lambda = -11 \quad \Rightarrow \quad \lambda = -1$

Thus, $x_1 = 2(-1) + 1, y_1 = 3(-1) + 2, z_1 = 4(-1) + 3 \Rightarrow \quad x_1 = -1, y_1 = -1, z_1 = -1$

Therefore, point of intersection of given lines is (–1, –1, –1).

OR

Line passing through (1, 2, 3)

i.e., $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and parallel to the given planes is perpendicular to the vectors

$\vec{b}_1 = \hat{i} - \hat{j} + 2\hat{k}$

$\vec{b}_2 = 3\hat{i} + \hat{j} + \hat{k}$

Required line is parallel to $\vec{b}_1 \times \vec{b}_2$

$$\vec{b} = \vec{b}_1 \times \vec{b}_2 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix} = \vec{i}(-1-2) - \vec{j}(1-6) + \vec{k}(1+3) = -3\vec{i} + 5\vec{j} + 4\vec{k}$$

Required education of line is:

$\vec{r} = \vec{a} + \lambda\vec{b}$

$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$

Section E

36. **i.** It is given that if India loose any match, then the probability that it wins the next match is 0.3.

$\therefore$ Required probability = 0.3

ii. It is given that, if India loose any match, then the probability that it wins the next match is 0.3.

$\therefore$ Required probability = 1 – 0.3 = 0.7

iii. Required probability = *P*(India losing first match) . P(India losing second match when India has already lost first match) = 0.4 × 0.7 = 0.28

OR

Required probability = *P*(India winning first match) . *P*(India winning second match if India has already won first match)

P(India winning third match if India has already won first two matches) = 0.6 × 0.4 × 0.4 = 0.096

37. **i.** Displacement between Ram's house and school = $= \sqrt{4^2 + 3^2} = \sqrt{25}$ = 5 km

ii. Distance travelled to reach school by Ram = 4 + 3 = 7 km

iii. Position vector of school $4\hat{i} + 3\hat{j}$

Position vector of Suresh's $= (4 + 6\cos 30)\hat{i} + (3 + 6\sin 30)\hat{j}$

$$= \left(4 + \frac{6\sqrt{3}}{2}\right)\hat{i} + \left(3 + \frac{6\times 1}{2}\right)\hat{j} \quad = \left(4 + 3\sqrt{3}\right)\hat{i} + 6\hat{j}$$

Vector distance from school to Suresh's home $= [(4 + 3\sqrt{3})\hat{i} + 6\hat{j}] - [(4\hat{i} + 3\hat{j})]$

$3\sqrt{3}\hat{i} + 3\hat{j}$

OR

Position vector of Ram's house = $0\hat{i} + 0\hat{j}$

Position vector of Suresh's house $= (4 + 3\sqrt{3})\hat{i} + 6\hat{j}$

$\therefore$ Displacement from Ram's house to Suresh's house

$= (4 + 3\sqrt{3})\hat{i} + 6\hat{j} - (0\hat{i} + 0\hat{j}) \quad (4 + 3\sqrt{3})\hat{i} + 6\hat{j}$

38. **i.** If P is the rent price per apartment and N is the number of rented apartments, the profit is given by

$NP - 500N = N(P - 500)$ [$\because$ Rs. 500/month is the maintenance charge for each occupied unit)

ii. Let R be the rent price per apartment and N is the number of rented apartments.

Now, if x be the number of non-rented apartments, then $N(x) = 50 - x$ and $R(x) = 10000 + 250x$

Thus, profit $P(x) = NR = (50 - x)(10000 + 250x - 500)$

$= (50 - x)(9500 + 250x) = 250(50 - x)(38 + x)$

iii. We have, $P(x) = 250(50 - x)(38 + x)$

Now, $P'(x) = 250[50 - x - (38 + x)] = 250[12 - 2x]$

For maxima/minima, put $P'(x) = 0$

$\Rightarrow \quad 12 - 2x = 0 \quad \Rightarrow \quad x = 6$

Number of apartments are 6.

OR

$P'(x) = 250(12 - 2x)$

$\Rightarrow \quad P''(x) = -500 < 0$

$\Rightarrow \quad P(x)$ is maximum at $x = 6$