SUBJECTIVE MOCK TEST | MATHEMATICS | SOLUTION

CLASS – X | SET - 1

(SECTION – A)

1.(B) $2^8 \times 3^2$

2.(B) The number of zeroes is 1 as the graph given in the question intersects the *x*-axis at one point only.

3.(A) The number of solutions of two linear equations representing coincident lines are ∞ because two linear equations representing coincident lines has infinitely many solutions.

4.(C) If a quadratic equation $ax^2 + bx + c = 0,\ a \neq 0$, has two equal roots, then its discriminant value will be equal to zero i.e., $D = b^2 - 4ac = 0$

For equal roots, $D = b^2 - 4ac = 0$

$\Rightarrow \quad (-k)^2 - 4(2)(k) = 0$

$\Rightarrow \quad k^2 - 8k = 0$

$\Rightarrow \quad k(k-8) = 0$

$\therefore \quad k = 0, 8$

5.(C) $-5, x, 3$ in A.P.

$\therefore \quad x - (-5) = 3 - x$

$x + 5 = 3 - x$

$2x = -2$

$x = -1$

6.(A) Distance between $(a\cos\theta + b\sin\theta, 0)$ and $(0, a\sin\theta - b\sin\theta)$

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{\sqrt{(0 - (a\cos\theta + b\sin\theta))^2 + \{(a\sin\theta - b\cos\theta - 0)\}^2}}$

$= \sqrt{\{0 - (a\cos\theta + b\sin\theta)\}^2 + \{(a\sin\theta - b\cos\theta - 0)\}^2}$

$= \sqrt{\left(a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta\right)}$

$= \sqrt{a^2 \times 1 + b^2 \times 1} = \sqrt{a^2 + b^2}$

$\{\because \ \sin^2\theta + \cos^2\theta = 1\}$

7.(C) (–4, 2)

8.(C) In triangles APQ and ATS,

$\angle PAQ = \angle TAS$ [Vertically opposite angles] $\angle PQA = \angle ATS$ [Alternate angles]

$\therefore$ $\Delta APQ \sim \Delta AST$ [AA similarity]

$\therefore$ $\frac{AQ}{AT} = \frac{AP}{AS}$

$\Rightarrow$ $\frac{6}{6} = \frac{x}{3}$

$\Rightarrow$ $x = \frac{6 \times 3}{6} = 3$

And $\frac{AQ}{AT} = \frac{PQ}{ST}$

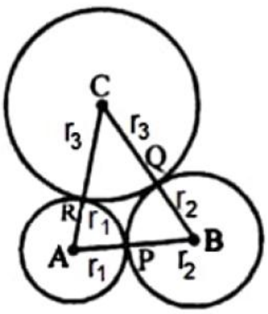
$\Rightarrow$ $\frac{6}{6} = \frac{y}{4}$

$\Rightarrow$ $y = \frac{4 \times 6}{6} = 4$

Therefore, $x = 3$, $y = 4$

9.(C) In the given figure, three circles with centre A, B and C are drawn touching each other externally $AB = 5\,cm. BC = 7\,cm$ and $CA = 6\,cm$

Let r_1, r_2, r_3 Be the radii of three circles respectively



$\therefore$ $AB = r_1 + r_2 = 5cm$ … (i)

$BC = r_2 + r_3 = 7cm$ …(ii)

$CA = r_3 + r_1 = 6cm$ …(iii)

Adding $2(r_1 + r_2 + r_3) = 18\,cm$ …(iv)

Now, subtracting (ii) from (iv) respectively we get $r_1 = 2cm$

Hence, radius of the circle with centre $A = 2cm$

10.(C) Given: PA and PB are two tangents a circle and $\angle APB = 80°$

Since $OA \perp PA$ and $OB \perp PB$, Then $\angle OAP = 90°$ and $\angle OBP = 90°$

In, ΔOAP and ΔOBP

$OA = OB$ (radius)

$OP = OP$ (common)

$PA = PB$ (lengths of tangents drawn from external points)

$\therefore \quad \Delta OAP \cong \Delta OBP$ (SSS congruence)

So, $\quad \angle OPA = \angle OPB \ (CPCT)$

So, $\quad \angle OPA = \frac{1}{2}\angle APB$

$= \frac{1}{2} \times 80^\circ = 40^\circ$

In ΔOPA,

$\angle POA + 40^\circ + 90^\circ = 180^\circ$

$\angle POA + 130^\circ = 180^\circ$

$\angle POA = 180^\circ - 130^\circ = 50^\circ$

Hence, the value of $\angle POA$ is 50°.

11.(A) Given: $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$

Squaring both sides, we get

$\Rightarrow \quad \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2\cos^2\theta$

$\Rightarrow \quad \cos^2\theta - 2\sin\theta\cos\theta = \sin^2\theta$

$\Rightarrow \quad \cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta = 2\sin^2\theta$

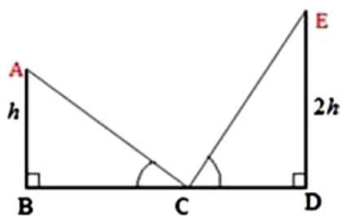
$\Rightarrow \quad (\cos\theta - \sin\theta)^2 = 2\sin^2\theta$

$\Rightarrow \quad \cos\theta - \sin\theta = \sqrt{2}\sin\theta$

12.(B) $(\cos 0^\circ + \sin 30^\circ + \sin 45^\circ)(\sin 90^\circ + \cos 60^\circ - \cos 45^\circ) = ?$

$$= \left(1 + \frac{1}{2} + \frac{1}{\sqrt{2}}\right)\left(1 + \frac{1}{2} - \frac{1}{\sqrt{2}}\right) = \left(\frac{3}{2} + \frac{1}{\sqrt{2}}\right)\left(\frac{3}{2} - \frac{1}{\sqrt{2}}\right) = \left(\frac{9}{4} - \frac{1}{2}\right) = \frac{7}{4}$$

13.(D)



Let the height of the building = AB = h meters, then

Height of the tower = $ED = 2h$ meters

According to question, $\angle ACB = \theta$ then $\angle EDC = 90^\circ - \theta$ and $BC = CD = 10\,m$

Now, in triangle ABC, $\tan\theta = \dfrac{AB}{BC} \Rightarrow \tan\theta = \dfrac{h}{10}$ …(i)

Now, in triangle EDC, $\tan(90° - \theta) = \dfrac{ED}{CD}$

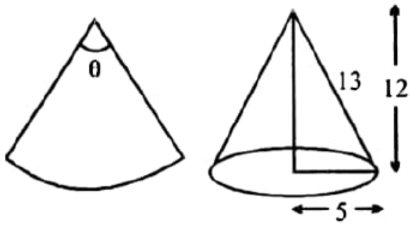
$\Rightarrow \quad \cot\theta = \dfrac{2h}{10} = \dfrac{h}{5}$ …(ii)

$h2h$ Multiplying equation (i) and (ii), we get

$\tan\theta.\cot\theta = \dfrac{h}{10} \times \dfrac{h}{5} \Rightarrow 1 = \dfrac{h^2}{50}$

$\Rightarrow \quad h^2 = 50 \Rightarrow h = 5\sqrt{2}\, m$

14.(C)



$\therefore$ Slant height = 13

As, $\quad \theta = \dfrac{S}{r}$

$\Rightarrow \quad S = r\theta$

$\Rightarrow \quad 2\pi(5) = 13\theta$

$\Rightarrow \quad \theta = \dfrac{10\pi}{13}$

15.(A) Area of a sector of a circle with radius r and making an angle of $x°$ at the centre $= \dfrac{x}{360} \times \pi r^2$

16.(C) Total number of digits from 1 to 9(n) = 9
Numbers which are odd (m) = 1, 3, 5, 7, 9 = 5

$\therefore \quad$ Probability $= \dfrac{m}{n} = \dfrac{5}{9}$

17.(C) Assuming a non-leap year
Ram can have the birthday on any day of the 365 days of the year
Shyam has a different birthday if his birthday is on any of the remaining 364 days of the year

Therefore P(Ram and Shyam have different birthdays) $= \dfrac{364}{365}$ and so, P (Ram and Shyam have birthdays on the same day)

= 1 – P(Ram and Shyam have different birthdays) $= 1 - \dfrac{364}{365} = \dfrac{1}{365}$

18.(D) Mode = 3 median –2 mean = 3(30) – 2(32) = 90 – 64 = 26

19.(C) A is true but *R* is false

20.(B) For $2k+1, 3k+3$ and $5k-1$ and to form an AP

$(3k+3)-(2k+1)=(5k-1)-(3k+3)$

$k+2=2k-4$

$2+4=2k-k=k$

$k=6$

So, both assertion and reason are correct but reason does not explain assertion.

(SECTION – B)

21. Let us assume that $2-3\sqrt{5}$ is rational. Then, there exist positive co-primes *a* and *b* such that $2-3\sqrt{5}=\frac{a}{b}$.

$3\sqrt{5}=2-\frac{a}{b}$

$3\sqrt{5}=\frac{2b-a}{b}$

$\sqrt{5}=\frac{2b-a}{3b}$

We observe that $\frac{2b-a}{3b}$ is a rational number.

It shows that $\sqrt{5}$ is a rational number.

This contradicts the fact that $\sqrt{5}$ is an irrational number

This contradiction has raised because we assumed that $2-3\sqrt{5}$ is a rational number

Hence, our assumption is wrong ,and $2-3\sqrt{5}$ is an irrational number.

22. It is given that *AB* = 10 cm, *AC* = 6 *cm* and *BC* = 12 *cm*

In ΔABC ,, *AD* is the bisector of *A*, meeting side *BC* at *D*

We have to find *BD* and *DC*

Since *AD* is $\angle A$ bisector

So $\frac{AC}{AB}=\frac{DC}{BD}$

Let *BD* = *x cm*

Then, $\frac{6}{10}=\frac{12-x}{x}$

$\Rightarrow \quad 6x=120-10x$

$\Rightarrow \quad 6x = 120$

$\Rightarrow \quad x = \frac{120}{16}$

$\Rightarrow \quad x = 7.5$

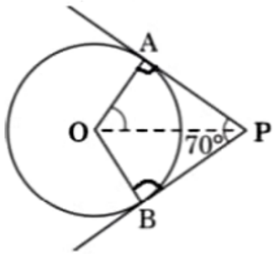
Now

$DC = 12 - BD$

$= 12 - 7.5 = 4.5$

Hence, $BD = 7.5\,cm$ and $DC = 4.5\,cm$

23.



PA and PB are tangents to the circle.

We know that the tangent to a circle is perpendicular to the radius through the point of contact.

Thus, $OA \perp PA$ and $OB \perp PB$

$\therefore \quad \angle OBP = 90°$ and $\angle OAP = 90°$

In quadrilateral $AOBP$, $\angle OAP + \angle APB + \angle PBO + \angle BOA = 360°$

(Sum of all interior angles of quadrilateral is 360°)

$90° + 70° + 90° + \angle BOA = 360°$

$\angle BOA = 110°$

In ΔOPA and ΔOPB,

$AP = BP$ (Tangents from a point outside the circle are equal in length)

$OA = OB$ (Radii of the same circle)

$OP = OP$ (Common side)

$\therefore \quad \Delta OPA \cong \Delta OPB$ (SSS congruence criterion)

$\Rightarrow \quad \angle POA = \angle POB$

$\Rightarrow \quad \angle POA = \frac{1}{2}\angle AOB = \frac{110°}{2} = 55°$

24. We have,

$\Rightarrow \quad \text{L.H.S} = \frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B}$

$\Rightarrow \quad \text{L.H.S} = \dfrac{(\sin A - \sin B)(\sin A + \sin B) + (\cos A + \cos B)(\cos A - \cos B)}{(\cos A + \cos B)(\sin A + \sin B)}$

$\Rightarrow \quad \text{L.H.S} = \dfrac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(\cos A + \cos B)(\sin A + \sin B)} \quad [\because\ a^2 - b^2 = (a+b)(a-b)]$

$\Rightarrow \quad \text{L.H.S} = \dfrac{(\sin^2 A + \cos^2 A) - (\sin^2 B + \cos^2 B)}{(\cos A + \cos B)(\sin A + \sin B)}$

$\Rightarrow \quad \text{L.H.S} = \dfrac{1-1}{(\cos A + \cos B)(\sin A + \sin B)} = 0 = \text{R.H.S}$

$\therefore \quad \dfrac{\sin A - \sin B}{\cos A + \cos B} + \dfrac{\cos A - \cos B}{\sin A + \sin B} = 0$

Hence proved

OR

We have $\sin\theta + \cos\theta = \sqrt{2}$

$(\sin\theta + \cos\theta)^2 = (\sqrt{2})^2$

$\sin^2 + \cos^2\theta + 2\sin\theta\cos\theta = 2$

$1 + 2\sin\theta\cos\theta = 2$

$2\sin\theta\cos\theta = 1$

$2\sin\theta\cos\theta = \sin^2\theta + \cos^2\theta \quad [\because\ 1 = \sin^2\theta + \cos^2\theta]$

$\Rightarrow \quad \dfrac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$\Rightarrow \quad 2 = \dfrac{\sin^2\theta}{\sin\theta\cos\theta} + \dfrac{\cos^2\theta}{\sin\theta\cos\theta}$

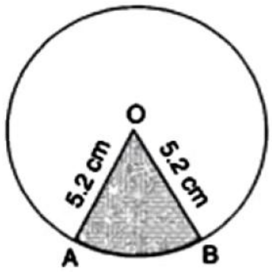
$\Rightarrow \quad 2 = \tan\theta + \cot\theta$

25. Here, $r = 45\,cm$ and $\theta = \dfrac{360°}{8} = 45°$

Area between two consecutive ribs of the umbrella $\dfrac{\theta}{360°} \times \pi r^2$

$= \dfrac{45°}{360°} \times \dfrac{22}{7} \times 45 \times 45 = \dfrac{22275}{28} cm^2$

OR



Let OAB be the given sector.

It is given that Perimeter of sector $OAB = 16.4\,cm$

$\Rightarrow \quad OA + OB + \text{arc } AB = 16.4\,cm$

$\Rightarrow \quad 5.2 + 5.2 + \text{arc } AB = 16.4$

$\Rightarrow \quad \text{arc } AB = 6\,cm$

$\Rightarrow \quad \ell = 6\,cm$

$\therefore \quad$ Area of sector $OAB = \frac{1}{2}\ell r = \frac{1}{2} \times 6 \times 5.2\,cm^2 = 15.6\,cm^2$

26. Given numbers are 156, 208 and 260.

Here, 260 > 208 > 156

Thus, HCF of 156, 208 and 260 is 52.

Hence, the minimum number of buses

$= \frac{156}{52} + \frac{208}{52} + \frac{260}{52} = \frac{156 + 208 + 260}{52} = \frac{624}{52} = 12$

The number of buses is 12.

(SECTION – C)

27. Let $p(x) = x^2 - 2x - (7p + 3)$

Since –1 is a zero of $p(x)$. Therefore,

$p(-1) = 0$

$(-1)^2 - 2(-1) - (7p + 3) = 0$

$1 + 2 - 7p - 3 = 0$

$3 - 7p - 3 = 0$

$7p = 0$

$p = 0$

Thus, $p(x) = x^2 - 2x - 3$

For finding zeros of $p(x)$, we put,

$p(x) = 0$

$x^2 - 2x - 3 = 0$

$x^2 - 3x - x - 3 = 0$

$x(x-3)+1(x-3)=0$

$(x-3)(x+1)=0$

Put $x - 3 = 0$ and $x + 1 = 0$, we get,

Thus, $x = 3, -1$

Thus, the other zero is 3.

28. Let the present age of the father be x years and the sum of the present age of his two children be y years.

Then, according to the question,

$x = 3y$

$\Rightarrow \quad x - 3y = 0 \quad ...(1)$

and, $\quad x + 5 = 2(y + 5 + 5)$

$\Rightarrow \quad x + 5 = 2(y + 10)$

$\Rightarrow \quad x + 5 = 2y + 20$

$\Rightarrow \quad x - 2y - 15 = 0 \quad ...(2)$

$x = 45, y = 15$

By solving the equations (1) and (2)

OR

The given equations are

$\sqrt{2}x-\sqrt{3}y=0 \quad \ldots(i)$

$\sqrt{3}x-\sqrt{8}y=0 \quad \ldots(ii)$

From equation (i), we obtain:

$$x=\frac{\sqrt{3}y}{\sqrt{2}} \quad \ldots(iii)$$

Substituting this value in equation (ii), we obtain:

$$\sqrt{3}\left(\frac{\sqrt{3}y}{\sqrt{2}}\right)-\sqrt{8}y=0$$

$$\frac{3y}{\sqrt{2}}-2\sqrt{2}y=0$$

$$y\left(\frac{3}{\sqrt{2}}-2\sqrt{2}\right)=0$$

$$y=0$$

Substituting the value of y in equation (iii), we obtain

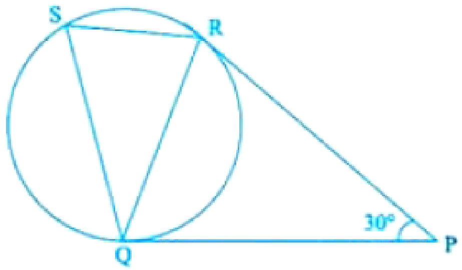
$x = 0$

$\therefore \quad x=0,\ y=0$

Hence the solution of given equation is (0,0).

29. In the given figure, we are given that, tangents PQ and PR are drawn toa circle such that $\angle RPQ=30^\circ$.

A chord *RS* is draw parallel to tangent *PQ*. We have to find the $\angle RQS$.



In APRQ, PQ and PR are tangents from an external point P to circle.

Therefore, *PR* = *PQ*

$\Rightarrow \angle PRQ = \angle PQR$ [opp. to equal sides in ΔPRQ are equal]

$\angle PRQ + \angle PQR + \angle RPQ = 180°$ [Int. $\angle s$ of Δ]

$\Rightarrow \quad \angle PRQ + \angle PRQ + 30° = 180°$

$\Rightarrow \quad 2\angle PRQ = 180° - 30°$

$\Rightarrow \quad \angle PRQ = \dfrac{150°}{2}$

Therefore, $\angle PRQ = \angle PQR = 75°$

Tangent $PQ \parallel SR$ [Given]

Therefore, $\angle PQR = \angle SRQ = 75°$ [Alternate segment of circle]

PQ is tangent at *Q* and *QR* is chord at *Q*.

Therefore, [$\angle SQ$ in alternate segment of circle]

In ΔSRQ ,

$\angle RSQ + \angle RSQ + \angle RSQ + = 180°$ [Angle sum property of a triangle]

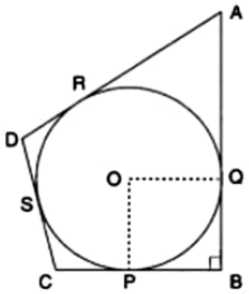
$\Rightarrow \quad 75° + 75° + \angle SQR = 180°$

$\Rightarrow \quad \angle SQR = 180° - 150°$

$\Rightarrow \quad \angle SQR = 30°$

OR

Given, a circle is inscribed in a quadrilateral *ABCD*



$OQ \perp AB$ [Radius is perpendicular to the tangent]

and $OP \perp BC$ [Radii of a circle]

$\therefore$ *OPBQ* is a square.

$\Rightarrow BQ = BP = OP = r$

Now, *RD* = *DS*

$\Rightarrow$ $RD = 5\ cm$

$\therefore$ $AR = AD - RD$

$= 23 - 5 = 18\ cm$

Also, $AR = AQ$

$\Rightarrow$ $AQ = 18\ cm$

Now, $AB = AQ + BQ$

$\Rightarrow$ $29 = 18 + r$

$\Rightarrow$ $r = 11\ cm.$

30. Here, $\sin\theta - \cos\theta = \frac{1}{2}$

Squaring both sides, we get

$$(\sin\theta - \cos\theta)^2 = \left(\frac{1}{2}\right)^2$$

$$\Rightarrow \quad \sin^2\theta + \cos^2\theta - 2\sin\theta.\cos\theta = \frac{1}{4}$$

$$1 - 2\sin\theta.\cos\theta = \frac{1}{4} \qquad (\because\ \sin^2\theta.\cos^2\theta = 1)$$

$$\Rightarrow \quad 1 - \frac{1}{4} = 2\sin\theta.\cos\theta$$

$$\Rightarrow \quad 2\sin\theta.\cos\theta = \frac{3}{4} \qquad \ldots\text{(i)}$$

Now $(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$= 1 + \frac{3}{4}$ (using(i)

$$\Rightarrow \quad (\sin\theta + \cos\theta) = \sqrt{\frac{7}{4}}$$

$$\Rightarrow \quad \frac{1}{\sin\theta + \cos\theta} = \frac{2}{\sqrt{7}} = \frac{2\sqrt{7}}{7}$$

31. Following table shows the given data & assumed mean deviation method to calculate the mean:

Class Interval	**Frequency** (f_i)	**Mid value** x_i	**Deviation** $d_i = x_i - 75.5$	$(f_i \times d_i)$
65 - 68	2	66.5	–9	–18
68 – 71	4	69.5	–6	–24
71 – 74	3	72.5	–3	–9
74 – 77	8	75.5 = A	0	0
77 – 80	7	78.5	3	21
80 – 83	4	81.5	6	24
83 – 86	2	84.5	9	18

	$\sum f_i = 30$			$\sum(f_i d_i) = 12$

Let, assumed mean (A) = 75.5 …(1)
Now, from table:
$\sum f_i = 30$ and $\sum f_i d_i = 12$ …(2)
Now,

$$\text{Mean} = A + \frac{\sum f_i d_i}{\sum f_i}$$

$$= 75.5 + \frac{12}{30} \quad \text{[from (1) \& (2)]}$$

$$= 75.5 + 0.4 = 75.9$$

Thus, the mean of heartbeats per minute for these patients is 75.9

32. If the present age of sister be x, then, by the first condition of the question, we have, present age of the girl = 2x.
By the second condition of the question, we have,

$(2x+4)(x+4) = 160$

$2x^2 + 8x + 4x + 16 = 160$

$2x^2 + 12x - 144 = 0$

$2x^2 + (24-12)x - 144 = 0$

$2x(x+12) - 12(x+12) = 0$

$(2x-12)(x+12) = 0$

$\therefore \; x = 6;\; x = -12$

Since age can't be negative, therefore x = 6
So, Age of sister = 6 and Age of girl = 2(6) = 12

OR

Here x = –2 is the root of the equation $3x^2 + 7x + p = 0$

then, $3(-2)^2 + 7(-2) + p = 0$

or, p = 2

Roots of the equation $x^2 + 4kx + k^2 - k + 2 = 0$ are equal then, $16k^2 - 4(k^2 - k + 2) = 0$

or $16k^2 - 4k^2 + 4k - 8 = 0$

or $12k^2 + 4k - 8 = 0$

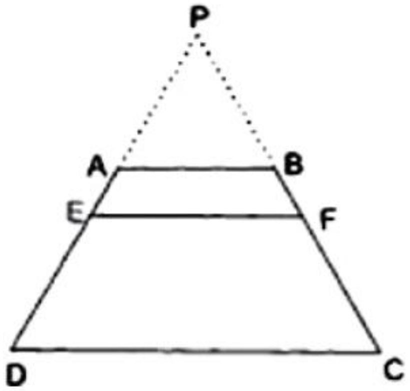
or, $3k^2 + k - 2 = 0$

or, $(3k-2)(k+1) = 0$

Hence, roots = $k = \frac{2}{3}, -1$

33. Given: According to the question, We have, $EF \parallel DC \parallel AB$ in the given figure.

To prove: $\frac{AE}{ED}=\frac{BF}{FC}$



Construction: Produce *DA* and *CB* to meet at *P*(say).

Proof: In ΔPEF , we have

$AB \parallel EF$

$\therefore \quad \frac{PA}{AE}=\frac{PB}{BF}$ [By Basic proportionality theorem]

$\Rightarrow \quad \frac{PA}{AE}+1=\frac{PB}{BF}+1$ [Adding 1 on both side]

$\Rightarrow \quad \frac{PA+AE}{AE}=\frac{PB+BF}{BF}$

$\Rightarrow \quad \frac{PE}{AE}=\frac{PF}{BF}$ …(1)

In ΔPDC , we have,

$EF \parallel DC$

$\therefore \quad \frac{PE}{ED}=\frac{PF}{FC}$ [By Basic Proportionality Theorem] ...(2)

Therefore, on dividing equation (i) by equation (ii), we get

$$\frac{\frac{PE}{AE}}{\frac{PE}{ED}}=\frac{\frac{PF}{BF}}{\frac{PF}{FC}}$$

$\Rightarrow \quad \frac{ED}{AE}=\frac{FC}{BF}$

$\Rightarrow \quad \frac{AE}{ED}=\frac{BF}{FC}$

(SECTION – D)

34. Radius of lower cylinder = 14 *cm*

Volume of pole $=\frac{22}{7}\times 14\times 14\times 200+\frac{22}{7}\times 7\times 7\times 50=130900\, cm^3$

Mass of the pole $=8\times 130900=1047200\, gm$ or $1047.2\, kg$

OR

Total volume = volume of cuboid + 1/2 × volume of cylinder.
length = 15 *m*, breadth =7 *m* and height = 8 *m*

$\therefore$ Volume of cuboidal part $= 1 \times b \times h = 15 \times 7 \times 8 m^2 = 840\, m^3$

Clearly,

r = Radius of half-cylinder = $\frac{1}{2}$ (Width of the cuboid) = $\frac{7}{2} m$

and, h = Height (length) of half-cylinder = Length of cuboid = 15 *m*

$\therefore$ Volume of half-cylinder $= \frac{1}{2}\pi r^2 h = \frac{1}{2} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 15 m^3 = \frac{1155}{4} m^3 = 288.75\, m^3$

Volume of air inside the shed when there is no people or machinery

$= (840 + 288.75)\, m^3 = 1128.75\, m^3$

Now, Total space occupied by 20 workers $= 20 \times 0.08\, m^3 = 1.6\, m^3$

Total space occupied by the machinery $= 300\, m^3$

$\therefore$ Volume of the air inside the shed when there are machine and workers inside it

$= (1128.75 - 1.6 - 300)\, m^3 = (1128.75 - 301.6)\, m^3 = 827.15\, m^3$

Hence, volume of air when there are machinery and workers is $827.15\, m^3$

35. Let the missing frequencies are *a* and *b*.

Class Interval	**Frequency** f_i	**Cumulative frequency**
0 – 5	12	12
5 – 10	*a*	12 + *a*
10 – 15	12	24 + *a*
15 – 20	15	39 + *a*
20 – 25	*b*	39 + *a* + *b*
25 – 30	6	45 + *a* + *b*
30 – 35	6	51 + *a* + *b*
35 – 40	4	55 + *a* + *b* = 70

Then, 55 + *a* + *b* = 70
a + *b* = 15 ...(1)
Median is 16, which lies in 15 – 20
So, The median class is 15 – 20
Therefore, ℓ = 15, *h* = 5, *N* = 70, *f* = 15 and *cf* = 24 + *a*
Median is 16, which lies in the class 15 - 20. Hence, median class is 15 – 20.

$\therefore \quad \ell = 15,\ h = 5,\ f = 15,\ c.f. = 24 + a$

$55 + a + b = 70$

$a + b = 15$

Median = 16

Median $= \ell + \dfrac{\frac{n}{2} - cf}{f} \times h$

$16 = 15 + \left[\dfrac{35 - 24 - a}{15} \times 5\right]$

$1 = \dfrac{11 - a}{3}$

$3 = 11 - a$

$a = 8$, $b = 7$

36. (i) A.P. for the number of squares in each row is 1, 3, 5, 7, 9 …

(ii) A.P. for the number of triangles in each row is 2, 6, 10, 14 …

(iii) Area of each square $= 2 \times 2 = 4\,cm^2$

Number of squares in 15 rows $= \dfrac{15}{2}(2 + 14 \times 2) = 225$

Shaded area $= 225 \times 4 = 900\,cm^2$

OR

$S_n = \dfrac{n}{2}[4 + (n-1)4] = 2n^2$

$\therefore\ S_{10} = 2 \times 10^2 = 200$

37. (i) Distance of charu from y-axis = 8

(ii)

Distance between Anishka and Bhawna $= \sqrt{(6-3)^2 + (4-1)^2}$

$= \sqrt{3^2 + 3^2} = 3\sqrt{2}$

(iii) $AB = 3\sqrt{2}$

$BC = \sqrt{(8-6)^2 + (6-4)^2} = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

$AC = \sqrt{(8-3)^2 + (6-1)^2} = \sqrt{25 + 25} = 5\sqrt{2}$

$AC = 5\sqrt{2}$

$AB + BC = AC$

OR

Yes, because $AB + BC = AC$

(SECTION – E)

38. (i) Given height of tree = 80 m, P is the initial position of bird and Q is position of bird after 2 sec the distance between observer and the bottom of the tree.

In ΔABP

$$\tan 45° = \frac{BP}{AB}$$

$$\Rightarrow \quad 1 = \frac{80}{AB}$$

$$\Rightarrow \quad AB = 80\,m$$

(ii) The speed of the bird

In ΔAQC

$$\tan 30° = \frac{QC}{AC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{80}{AC}$$

$$\Rightarrow \quad AC = 80\sqrt{3}\,m$$

$$\Rightarrow \quad BC = 80\sqrt{3} - 80 = 80(\sqrt{3}-1)\,m$$

Speed of bird $= \dfrac{\text{Distance}}{\text{Time}}$

$$\Rightarrow \quad \frac{BC}{2} = \frac{80(\sqrt{3}-1)}{2} = 40(\sqrt{3}-1)$$

$\Rightarrow$ Speed of the bird $= 29.28\,\text{m/ sec}$

(iii) The distance between second position of bird and observer

In ΔAQC

$$\sin 30° = \frac{QC}{AQ}$$

$$\Rightarrow \frac{1}{2} = \frac{80}{AQ}$$

$$\Rightarrow AQ = 160\,m$$

OR

The distance between initial position of bird and observer.

In ΔABP

$$\sin 45° = \frac{BP}{AP}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{80}{AP}$$

$$\Rightarrow \quad AP = 80\sqrt{2}\,m$$

SUBJECTIVE MOCK TEST | MATHEMATICS | SOLUTION

CLASS – X | SET - 2

(SECTION – A)

1.(D) $a = pq^2$

$b = p^3q$

$\text{HCF}(a,b) = pq$

2.(C) $a > b,\ b > 0,\ c > 0$

3.(C) No solution

4.(C) $b^2 - 4ac < 0$

5.(A)

$a_n = 181$

$181 = 5(n-1)8$

$\frac{176}{8} = n-1$

$22+1 = n$

$n = 23$

$S_n = \frac{23}{2}[5+181]$

$= \frac{23}{2}\times 186$

$= 23\times 93$

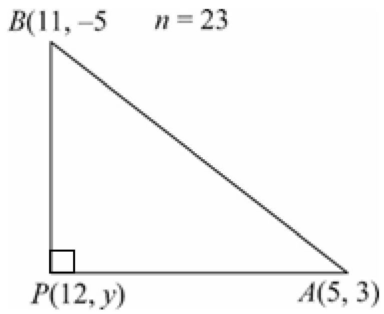
$= 2139$

6.(D) $AB^2 = AP^2 + PB^2$

$(11-5)^2 + (3+5)^2 = (12-5)^2 + (y-3)^2 + (12-11)^2 + (y+5)^2$

$6^2 + 8^2 = 7^2 + y^2 + 9 - 6y + 1 + y^2 + 25 + 10y$

$36 + 64 = 2y^2 + 4y + 84$



$2y^2 + 4y + 84 - 100 = 0$

$2y^2 + 4y - 16 = 0$

$y^2 + 2y - 8 = 0$

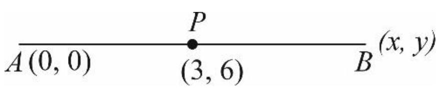
$y^2 + 4y - 2y - 8 = 0$

$y(y+4) - 2(y+4) = 0$

$(y+4)(y-2) = 0$

$y = -4, y = 2$

7.(B)



$\frac{0+x}{2} = 3$; $\frac{0+y}{2} = 6$

$x = 6$ $y = -12$

8.(D) Isosceles and similar

9.(C) 80°

10.(C) $AC = PC$ and $PC = PB$

$\Rightarrow \angle CAP = \angle CPA = x(\text{say})$ $\angle CPB = \angle CBP = y(\text{say})$

In ΔAPB

$\angle BAP + \angle APB + \angle PBA = 180°$

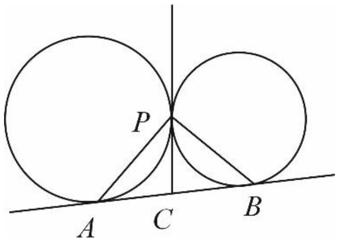
$x + \angle CPA + \angle CPB + y = 180°$

$x + x + y + y = 180°$

$2(x + y) = 180°$

$x + y = 90°$

$\angle APB = 90°$



11.(C) $\frac{\sqrt{\sec\theta - 1}}{\sqrt{\sec\theta + 1}} + \frac{\sqrt{\sec\theta + 1}}{\sqrt{\sec\theta - 1}}$

$= \frac{\sec\theta - 1 + \sec\theta + 1}{\sqrt{(\sec\theta - 1)(\sec\theta + 1)}}$

$= \frac{2\sec\theta}{\sqrt{\sec^2\theta - 1}}$ $(\because \quad 1 + \tan^2\theta = \sec^2\theta)$

$= \frac{2\sec\theta}{\tan\theta}$

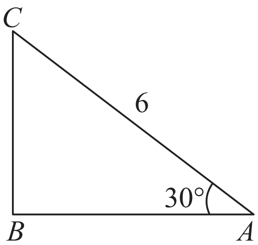
$= \frac{2}{\sin\theta}$

$= 2\cos ec\theta$

12.(A) $\sin 30° = \frac{BC}{AC}$

$\frac{1}{2} = \frac{BC}{6}$

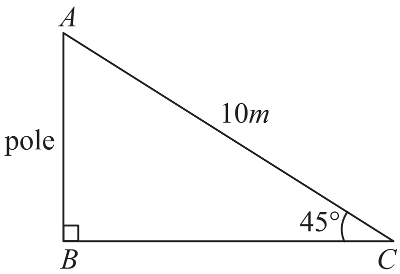
$BC = 3cm$



13.(D) $\sin 45° = \frac{AB}{AC}$

$\frac{1}{\sqrt{2}} = \frac{AB}{10}$

$AB = \frac{10}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}m$



14.(C) Area of sector $= \frac{1}{2}lr$ sq. units

15.(D) Slant height of cone $(\ell) = \sqrt{12^2 + 5^2}$

$= \sqrt{144 + 25} = \sqrt{169} = 13cm.$

Radius of sector = slant height of cone = 13*cm*

16.(D) Nos. divisible by 2 and 3 both

6, 12, 18, 24

p(number divisible by 2 and 3 both) $= \frac{4}{25}$

17.(C) 0

18.(A) Mean $= \frac{59+46+31+23+27+44+52+40+29}{9}$

$= \frac{351}{9} = 39$

19.(D) A is false but R is true.

20.(D) AP is 10, 20, 30,,……… 100 times

$a_n = a + (n-1)d$

$a_n = 10 + 99(10) = 1000$

$S_n = \frac{100}{2}[10 + 1000]$

$= 50 \times 1010 = 50500$

A is false but R is true

(SECTION – B)

21. LCM $(2, 4, 6, 8, 10, 12) = 2^3 \times 3 \times 5$

= 120 seconds

No. of times it will ring in 30 minutes $= \frac{30 \times 60}{120} = 15$

22. In ΔPQR

$\frac{PS}{QS} = \frac{PT}{TR}$ given

By converse of $BPT \quad ST \parallel QR$

$\angle PST = \angle PQR$ and $\angle PTS = \angle PRQ$

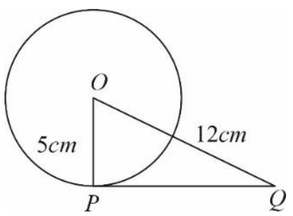
(corresponding angles)

$\angle PST = \angle PRQ$ given

$\Rightarrow \quad \angle PRT = \angle PRQ$ and $\angle PST = \angle PTS$

$\Rightarrow \quad \angle PQR$ is an isosceles Δ.

23. $OP \perp PQ$



$\therefore$ (radius is perpendicular to the tangent).

$PQ = \sqrt{12^2 - 5^2} = \sqrt{144-25} = \sqrt{119}\, cm$

24. $(m^2 + n^2)\cos^2\beta = \left(\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}\right)\cos^2\beta$

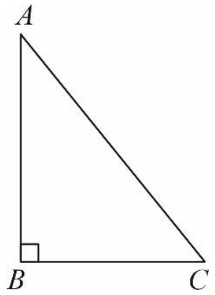
$$= \cos^2\alpha\left[\frac{1}{\cos^2\beta}+\frac{1}{\sin^2\beta}\right]cos^2\beta = \cos^2\alpha\left[\frac{\sin^2\beta+\cos^2\beta}{\sin^2\beta\cos^2\beta}\right]\cos^2\beta$$

$$= \cos^2\alpha \times \frac{1}{\sin^2\beta.\cos^2\beta}\times\cos^2\beta = \frac{\cos^2\alpha}{\sin^2\beta} = n^2 \qquad \because \qquad \sin^2\theta+\cos^2\theta = 1$$

OR

Let in set $\Delta ABC, \angle B = 90°,$

$AB = 3k$ units and $BC = 4k$ units



$AC = \sqrt{(4k)^2+(3k)^2} = \sqrt{25k^2} = 5k$ units

$$\cos\theta = \frac{AR}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

$$\frac{1-\cos\theta}{1+\cos\theta} = \frac{1-3/5}{1+3/5}$$

$$= \frac{5-3}{5+3} = \frac{2}{3} = \frac{1}{4}$$

25. θ in 1 minute $= 360° \div 60 = 6°$

θ in 56 minutes $= 6° \times 56 = 336°$

Area of sector $= \dfrac{Q}{360°}\times \pi r^2$

$$= \frac{336}{360}\times\frac{22}{7}\times 7.5\times 7.5 = \frac{10}{10}\times 22\times 7.5 = 165cm^2$$

OR

$$\ell = \frac{\theta}{360°}\times 2\pi r$$

$$= \frac{72}{360}\times 2\times\frac{22}{7} = 44cm$$

Area of sector $= \dfrac{\theta}{360}\times \pi r^2$

$$= \frac{72}{360}\times\frac{22}{7}\times 35 = 770cm^2$$

(SECTION – C)

26. $105 = 3\times 5\times 7$

$140 = 2\times 5\times 7$

$175 = 5^2\times 7$

$HCF(105,140,175) = 5\times 7 = 35$

Hence the no. of animals went in each trip is 35.

27. $f(x) = 2x^2 - 5x + 7$

$\alpha+\beta=\frac{-5}{2}$

$\alpha\beta=\frac{7}{2}$

Sum of zeroes of new polynomial $=3\alpha+3\beta+3\alpha+2\beta$

$=5\alpha+5\beta=5(\alpha+\beta)=5\times\frac{(-5)}{2}=\frac{-25}{2}$

Product of zeroes of new polynomial $=(2\alpha+3\beta)(3\alpha+2\beta)$

$=6\alpha^2+4\alpha\beta+9\alpha\beta+6\beta^2=6\alpha^2+6\beta^2+13\alpha\beta$

$=6\alpha^2+6\beta^2+12\alpha\beta+\alpha\beta=6(\alpha^2+\beta^2+2\alpha\beta)+\alpha\beta$

$=6(\alpha+\beta)^2+\alpha\beta=6\left(\frac{-5}{2}\right)^2+\frac{7}{2}=6\times\frac{25}{4}+\frac{7}{2}$

$=3\times\frac{25}{2}+\frac{7}{2}=\frac{75}{2}+\frac{7}{2}=\frac{82}{2}=41$

Required polynomial = $K\left[x^2-(\text{sum of zeroes})x+\text{product of zeroes}\right]$

$=K\left[x^2-\left(\frac{-25}{2}\right)x+\frac{82}{2}\right]=\frac{K}{2}\left[2x^2+25x+82\right]$

One of the required polynomial is $2x^2+25x+82$

28. $\angle A+\angle B+\angle C=180°$

$x°+3x°-2°+y°=180°$

$4x+y=180°\quad\ldots(1)$

$\angle C-B=9°$

$y-(3x-2)°=9°$

$-3x+y+2°=9°$

$-3x+y=7°$

$3x-y=-7°\quad\ldots(2)$

$4x+\not{y}=182$

$3x-\not{y}=-7$

$7x=175$

$x=\frac{175}{7}$

$x=25$

$y(25)+y=182°$

$y=182°-100°$

$y=82°$

OR

Considering $\frac{x+y-8}{2}=\frac{x+2y-14}{3}$

$3(x+y-8)=2(x+2y-14)$

$3x+3y-24=2x+4y-28$

$3x=2x+3y-4y=-28+24$

$x-y=-4$ ….(1)

Now considering

$\frac{x+2y-14}{3}=\frac{3x+y-12}{11}$

$11(x+2y-14)=3(3x+y-12)$

$11x+22y-154=9x+3y-36$

$11x-9x+22y-3y=-36+154$

$2x+19y=118$ …(2)

Solving equations (1) & (2)

$2x-2y=-8$	$x-y=-4$
$+2x+19y=118$	$x-6=-4$
$-21y=-126$	$x=-4+6$
$y=\frac{126}{21}$	$x=-4+6$
$y=6$	$x=2$

29.

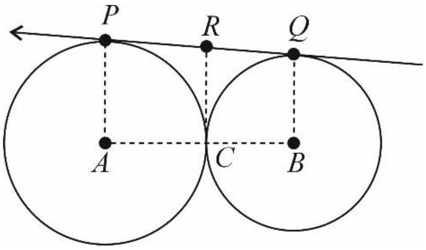
Height (in m)	**No. of village**	x_i	**di**	u_i	f_iu_i
0 – 200	142	100	–700	–7	–994
200 – 600	265	400	–400	–4	–1060
600 – 1000	560	800	0	0	0
1000 – 1400	271	1200	400	4	1084
1400 – 1800	89	1600	800	8	712
1800 – 2200	16	2000	1200	12	192
	$\sum f_i=1343$				$\sum f_iu_i=66$

$$\bar{X}=A+\frac{\sum f_iu_i}{\sum fi}\times h$$

$$\bar{X}=800+\frac{-66}{1343}\times 100$$

$$=800-\frac{6600}{1343}=800-4.91=795.09m$$

30.



Construction

Draw common tangents RC

Proof: $RP = RC$ (length of tangents)

Similarly,

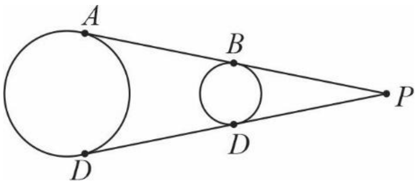
$RQ = RC$

$\Rightarrow RP = RQ$

$\Rightarrow R$ is the midpoint of PQ

$\Rightarrow$ the common tangent RC bisects the tangent PQ.

Or



Construction: Produce AB and CD such that they intersect at point P.

For larger circle, $AP = CP$ (i) (length of tangents)

Similarly for smaller circle, $BP = DP$ …(ii)

By subtracting (ii) from (i) we get

$AP - BP = CP - DP$

$AB = CD$

31. $LHS = \dfrac{\cot A + \text{cosec}\, A - 1}{\cot A - \text{cosec}\, A + 1} = \dfrac{\cot A + \text{cosec}\, A - (\text{cosec}^2 A - \cot^2 A)}{\cot A + 1 - \text{cosec}\, A}$

$$= \frac{(\cot A + \text{cosec}\, A) - \left[(\text{cosec}\, A + \cot A)(\text{cosec}\, A - \cot A)\right]}{(\cot A + 1 - \text{cosec}\, A)}$$

$$(\cot A + \text{cosec}\, A)\left[\frac{1 - (\text{cosec}\, A - \cot A)}{\cot A + 1 - \text{cosec}\, A}\right] = (\cot A + \cos ecA)\left[\frac{1 - \cos ecA + \cot A}{\cot A + 1 - \cos ecA}\right]$$

$$= \cot A + \cos ecA = \frac{\cos A}{\sin A} + \frac{1}{\sin A} = \frac{1 + \cos A}{\sin A} = RHS.$$

(SECTION – D)

32. $2x^2 + px + 15 = 0$

$2(-5)^2 + p(-5) + 15 = 0$

$50 + 15 - 5p = 0$

$-5p = -65$

$p = 13$

$p(x^3 + x) + k = 0$

$15(x^3 + x) + k = 0$

$13x^2 + 13x + k = 0$

$D = b^2 - 4ac$

$13^2 - 4(15)(k) = 0$

$169 - 52k = 0$

$169 = 52k$

$k = \frac{169}{52}$

$k = \frac{13}{4}$

33. $\Delta EEC \cong \Delta GBD$ (given)

$EC = BD$ (By CPCT) …(i)

$\angle 1 = \angle 2$ (given)

$\Rightarrow \quad AE = AD$ …(ii)

Using (i) and (ii) we get

$\frac{AE}{EC} = \frac{AD}{BD}$

By converse of BPT

$DE \parallel BC$

$\angle 1 = \angle 3$ and $\angle 2 = \angle 4$ corresponding angles

In ΔADE and ΔABC

$\angle A = \angle A$ (common)

$\angle 1 = \angle 3$ (proved above)

By AA similarity rule

$\Delta ADE \sim \Delta ABC$

34.

Classes	*Frequency*	*xi*	*di*	*fi*	*fiui*
10 – 20	4	15	–30	–3	–12
20 – 30	8	25	–20	–2	–16
30 – 40	10	35	–10	–1	–10
40 – 50	12	45	0	0	0
50 – 60	10	55	10	1	10
60 – 70	4	65	20	2	8
70 – 80	2	75	30	3	6
	$\sum fi = 50$				$\sum fiui = 14$

$$\bar{x} = A + \frac{\sum fiui}{\sum fi} \times h = 45 + \frac{-14}{50} \times 10 = 45 - \frac{14}{5} = 45 - 2.8 = 42.2$$

Model class is 40 – 50

$f_1 = 12$

$f_0 = 10$

$f_2 = 10$

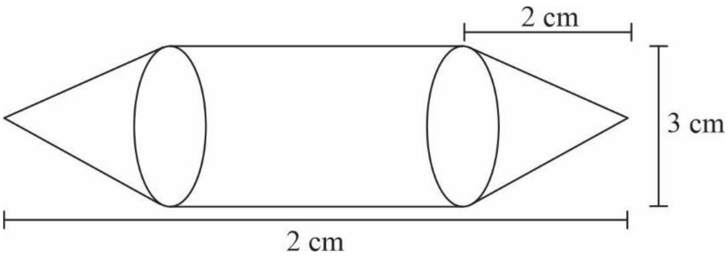
$h = 10$

Mode $= \ell + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

$= 40 + \frac{12-10}{24-10-10} \times 10$

$= 40 + \frac{2}{4} \times 10 = 40 + 5 = 45$

35.



For conical part

$r = \frac{3}{2} cm$

$h = 2cm$

$l = \sqrt{(1.5)^2 + 4} = \sqrt{2.25 + 4} = \sqrt{6.25} = 2.5\, cm$

Volume of 2 cones $= 2 \times \frac{1}{3}\pi r^2 h - \frac{2}{3} \times \pi \left(\frac{3}{2}\right)^2 \times 2 = \frac{4}{3} \pi \times \frac{9}{4} = 3\pi\, cm^3$

Curved surface area of 2 cones $= 20 \times \pi rl = 2 \times \pi \times \frac{3}{2} \times 2.5\, cm^2 = 7.5\pi\, cm^2$

For cylindrical part,

$r = \frac{3}{2}\, cm$

$h = 12 - (2+2) = 8\, cm$

Volume of cylinder $= \pi r^2 h = \pi \times \frac{3}{2} \times \frac{3}{2} \times 8 = 18\pi\, cm^3$

Curved surface area of cylinder $= 2\pi rh = 2\pi \times \frac{3}{2} = 24\pi cm^2$

Volume of air $= 3\pi + 18\pi = 21\pi = 21 \times \frac{22}{7} = 66\, cm^3$

Area to be painted $= 7.5\pi + 24\pi = 31.5\pi = \frac{315}{10} \times \frac{22}{7} = 99\,cm^2$

Cost of painting = 12.50 × 99 = Rs. 1237.50

OR

Diameter of wire = 6 mm = 0.6 cm.

r = radius of wire $= \frac{0.6}{2} = 0.3\,cm$

Height of cylinder = 18 cm

R = radius of cylinder $= \frac{49}{2}$ *cm.*

Number of rotations $= \frac{\text{length of cylinder}}{\text{diameter of wire}} = \frac{18}{0.6} = 30$

Length of wire = Circumference of base of cylinder × Number of rotations

$= 2\pi R \times 30 = 2 \times \frac{22}{7} \times \frac{49}{2} \times 30\,cm = 4620\,cm$ = 46.20 m

Volume of wire $= \pi r^2 h = \frac{22}{7} \times 0.3 \times 0.3 \times 4620 = 22 \times 0.3 \times 0.3 \times 660 = 1306.8\,cm^3$

Weight of wire = Volume of wire × density of wire = 1306.8 × 8.8 g = 11499.84 g = 11.49984 kg
= 11.5 kg (Approx)

(SECTION – E)

36. AP is 3, 5, 7,……….

$S_n = 360$

$a = 3,\ d = 5 - 3 = 2$

(i) $S_n = \frac{n}{2}[2a + (n-1)d]$

$360 = \frac{n}{2}[2 \times 3 + (n-1)2]$

$2 \times 360 = 2n[3 + (n-1)]$

$360 = n[3 + n - 1]$

$360 = n(2 + n)$

$360 = 2n + n^2$

$n^2 + 2n - 360 = 0$

$n^2 + 20n - 18n - 360 = 0$

$n(n+20) - 18(n+20) = 0$

$(n+20)(n-18) = 0$

$n = -20$ (rejected) or $n = 18$

$\Rightarrow$ No. of rows = 18

(ii) $a_n = a + (n-1)d$

$a_n = 3 + (18-1)(2) = 3 + 34 = 37$

OR

$a_{12} = a + 11d = 3 + 11(2) = 3 + 22 = 23$

(iii) $n = 15$

$a_{15} = a + 14d = 3 + 14(2) = 3 + 28 = 31$

37. Coordinates of Aaksh (2, 3)

Coordinates of Neena (3, 6)

Coordinates of Pinu (5, 2)

Coordinates of Karan (6, 5)

(i) distance between Neena and Karan $= \sqrt{(6-3)^2 + (5-6)^2} = \sqrt{3^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}\ units$

(ii) Coordinates of seat of Aakash (2, 3)

OR

Distance between Prinu and Karan $= \sqrt{(6-5)^2 + (5-7)^2} = \sqrt{1+3^2} = \sqrt{10}\ units$

(iii) Coordinates of midpoint between

Akash and Binu

$x = \frac{2+5}{2};\ y = \frac{3+2}{2}$

$x = \frac{7}{2};\ y = \frac{5}{2}$

$\left(\frac{7}{2}, \frac{5}{2}\right)$

38. (i) $\cos 60° = \frac{CX}{DX}$

$\frac{1}{2} = \frac{8}{DX}$

$DX = 8 \times 2$

$DX = 16\ m$

(ii) In ΔBAX

$BX = 20 + 16 = 36\ m$

$\cos 60° = \frac{BX}{AX}$

$\frac{1}{2} = \frac{36}{AX}$

$AX = 72\ m$

$AC = AX - CX = 72 - 8 = 64\ m$

OR

(ii) $\frac{DC}{CX} = \tan 60^\circ$

$\sqrt{3} = \frac{DC}{8}$

$DC = 8\sqrt{3}\ m$

(iii) $\tan 60^\circ = \frac{AB}{XA}$

$\sqrt{3} = \frac{AB}{72}$

$AB = 72\sqrt{3}\ m$