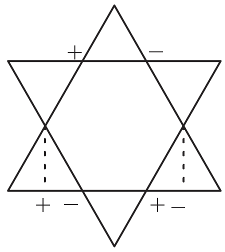
VMC Vidyamandir Classes SINCE 1986

IIT JEE | MEDICAL | FOUNDATION

Solutions of JEE Advanced 2024 | Paper - 2

PHYSICS

1.(A) The figure below shows a position where $x < \frac{3L}{2}$



$\Rightarrow$ Direction of induction changes at X = L

$\Rightarrow$ The changed direction continues

From X = L to X = 2L

This leaves us to check (A) & (B)

$\left|\frac{d\phi}{dt}\right|$ will be max at 2L

So (A) correct

2.(A) $F = -\frac{\partial Vc}{\partial r} = \frac{3m\alpha}{r^4}$

$$\frac{3m\alpha}{r^4} + \frac{GmM}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{3\alpha}{r^3} + \frac{GM}{r}}$$

$$\frac{2\pi r}{\sqrt{\frac{3\alpha}{r^3} + \frac{GM}{r}}} = T_1$$

$$\frac{2\pi r}{\sqrt{\frac{GM}{r}}} = T_0$$

$$\frac{(2\pi r)^2 \left\{ \frac{1}{\frac{3r}{\alpha^3} + \frac{GM}{r}} - \frac{1}{\frac{GM}{r}} \right\} r}{\frac{3\alpha}{r^3} + \frac{GM}{r}} = \frac{\frac{3\alpha}{r^3}}{\frac{GM}{r}} = \frac{3\alpha}{GMr^2}$$

3.(A) Energy of electron $= E_0$

$$\frac{\lambda_c}{\lambda_c} = E_0 \text{ given}$$

$$\Rightarrow \quad \frac{1}{\lambda_c} = \frac{E_0}{\lambda_c}$$

$$\frac{\lambda_c}{\lambda_a} = z^2 \times 13.6\left(1 - \frac{1}{4}\right)$$

$$\Rightarrow \quad \frac{1}{\lambda_a} = \frac{z^2 \times 13.6}{hc} \times \frac{3}{4}$$

$$2 = \frac{\lambda_\alpha}{\lambda_c} = \frac{hc \times 4}{z^2 \times 13.6 \times 3} \times \frac{E_0}{hc}$$

$$\Rightarrow \quad E_0 = \frac{2 \times (46)^2 \times 13.6 \times 3}{4}$$

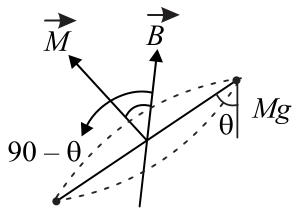
$$\frac{\lambda_a}{\lambda_c} = \frac{hc}{(41)^2 \times 13.6} \times \frac{4}{3} \times \frac{(46)^2 \times 13.6}{hc} \times \frac{3}{2} = \left(\frac{46}{41}\right)^2 \times \frac{4}{2} \times 2 \approx 2.53$$

4.(A) $M = (i)\pi r^2$

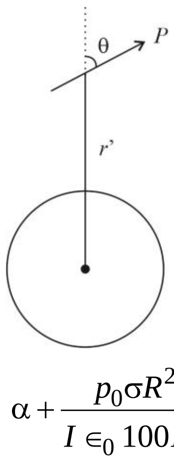
$$(B_0) i\pi r^2 \cos\theta = (mgr)\sin\theta$$

$$\tan\theta = \frac{(B_0)\ \pi r^2 l}{mgr}$$

$$\tan\theta = \pi r I B_0 / (mg)$$



5.(BD) $\dfrac{p(k)(\sigma)(4\pi R^2)}{r^2} + I\alpha = 0$



$$\alpha + \frac{p_0 \sigma R^2}{I \in_0 100 R^2}\theta = 0$$

$$\alpha + \frac{p_0 \sigma R^2}{I \in_0 r^2}\theta = 0$$

$\Rightarrow \quad \alpha + \frac{p_0\sigma}{100I \in_0}\theta = 0 \quad \Rightarrow \quad \omega = \sqrt{\frac{p_0\sigma}{100 \in_0 I}}$

$\omega^2 = \frac{po^- + R^2}{60r^2}$

At $r = 10R$

6.(ABD) Mass $= \pi \times 10^{-3} kg$

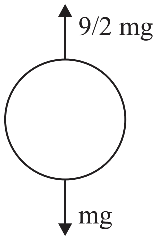
Vol $= \frac{4}{3}\pi \times \frac{27}{8} \times 10^{-6} = \frac{9\pi}{2} \times 10^{-6}$

$S = \frac{m}{v} = \frac{2}{9} \times 10^3$

$F_{net} = \frac{7mg}{2}$

$W = 7mgh = 0.077J$

$W = \frac{mv^2}{2} \qquad V = 7m/s$



$\frac{V^2}{2g} = \frac{49}{20} = 2.45$

$F_{viscous} = 6\pi nvr$

$= 6\pi \times 1 \times 10^{-3} \times \frac{7\times 3}{2} \times 10^{-2}$

$F_{other} = \frac{7}{2}mg$

Ratio $= \frac{500}{9}$

7.(AB) $r = \frac{mV}{qB} = \sqrt{\frac{2mqV}{qB}} = \sqrt{\frac{2mV}{q}}\left(\frac{1}{B}\right)$

$\sqrt{(2)\frac{\left(\frac{5}{3}\times 10^{-27}\right)(M)_{162}\times 10}{1.6\times 10^{-19}}} = \sqrt{\frac{(64)(10)^{-26}}{1.6\times 10^{-19}}M} \times 10 = \sqrt{40\times 10^{-7}M} \times 10$

$= \sqrt{400\times 10^{-8}M} \times 10 = (20)\sqrt{M} \times 10^{-4} \times 10 = 2\sqrt{M}\ cm$

For, HTM = 1

$r = 2\ cm$

$x = 2r = 4\ cm$

For, Am = 144

$x = 2\sqrt{144} \times 2 = 98\ cm$

8.(3) $V = \frac{1}{3}\frac{\pi d^2}{4} \times h$

$$\frac{dV}{V} = \frac{2d(diameter)}{d} + \frac{dh}{h} = 3\%$$

9.(2)

$$v_0 \sin\theta_0 = v_1 \sin\theta_1$$

For $\theta_1 = 45 = \theta_0$ $v_0 = v_1$

$$\frac{L}{v_0\sqrt{2}}$$

For 60°, 30°

$$(v_0)\sin 60° = v_1 \sin 30°$$

$$v_1 = v_0\sqrt{3}$$

$$t = T_2 = \frac{L}{\frac{\left(V_0\sqrt{3}\right)\sqrt{3}}{2} + (v_0)\left(\frac{1}{2}\right)} = \frac{2}{2v_0}$$

$$\left(\frac{T_1}{T_2}\right)^2 = \left(T_2\right)^2 = 2$$

10.(3) For $\theta = 30°$

$$\phi_1 = \frac{q}{\varepsilon_0} - \frac{q}{2\varepsilon_0}(1 - \cos 30°) \times 2 = \frac{q\sqrt{3}}{2\varepsilon_0}$$

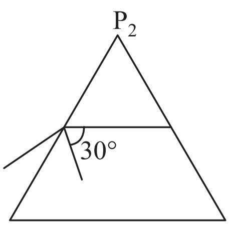
For $\theta = 60°$,

$$\phi_2 = \frac{q}{\varepsilon_0} - \frac{q}{2\varepsilon_0}(1 - \cos 60°) \times 2 = \frac{q}{2\varepsilon_0}$$

$$\phi_2 = \frac{\phi_1}{\sqrt{3}}$$

11.(12) For P_1 :

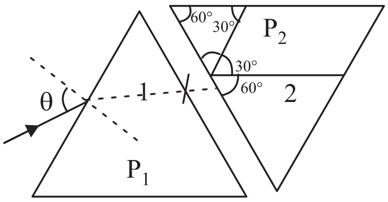
$$\sin 60 \times 1 = \sqrt{\frac{3}{2}} \times \theta_1$$



$\sin\theta_1 = \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{1}{\sqrt{2}}$

$\theta_1 = 45$

$r_1 + r_2 = A;$ $r_2 = 15°$



For P_2 : (for min & P_2 has $i = 60°$)

$\sin\theta = \sqrt{3}\sin 30°$

$\sin i = \frac{\sqrt{3}}{\sqrt{2}}\sin\left(\frac{\pi}{12}\right)$

12.(171) due to wire $\rightarrow$

$\int_{1/2}^{2}\frac{2k\lambda}{\gamma}dr$

$v_\rho - v_\gamma = 2k\lambda\ln 4 = 2(9\times10^9)(5\times10^{-9}) = (10)(9)(2)(0) = (7)(9)(2) = 126$

Due to shell

$V_p = \left[\frac{10\times10^{-9}}{1}\right]^{(9\times10^9)}$; $V_\Omega = \frac{(10\times10^{-9})}{2}(9\times10^9)$

$V_p - V_\Omega = \frac{(9\times10^9)(10^{-8})}{2} = 45V$; $N\in T = 126+45 = 171V$

13.(96) $\left(r_i 3\right) = \frac{8P_0}{27}(r_f 3)$

$r_f = \frac{3r_i}{2}$

$\frac{4T}{r_f} - \frac{(4T)(2)}{3r_i} = \left(\frac{2}{3}\right)\left[\frac{4T}{r_i}\right] = \frac{2}{3}(144) = 96$

14.(600) Paragraph (1):

$(d)\frac{(y)}{D} = 8\lambda$; $y = \frac{8\lambda D}{d} = \frac{(8)\lambda}{d}$

$\frac{(8)(6000)(10^{-10})}{0.76\times10^{-3}} - \frac{(8)(6000)(10^{-10})}{0.84\times10^{-3}} = \frac{(8)(600)}{0.70} - \frac{(8)(600)}{0.84}$

$= (8)(600)\left\{\frac{1}{0.70} - \frac{1}{0.84}\right\} = 600\mu m$

15.(24) $\frac{y}{d} = 8\lambda$

$y = \frac{8\lambda}{(0.8+0.04\sin wt)(10^{-3})} = \frac{(8)(6000)(10^{-10})}{(0.8+0.04\sin 0.08t)10^{-3}}$

$$y = \frac{4(600)(8)}{(0.8 + 0.04\sin 0.08t)}$$

$$\frac{du}{dt} = \frac{(600)(8)(0.04)(0.08t)\cos(0.08t)}{(0.8 + 0.04\sin 0.08t)^2}$$

$$\cos(0.08t) = 1$$

$$\sin(0.08t) = 0$$

$$\theta = 24\, ms^{-1}$$

16.(0.75) $V = a\omega\cos\omega t$

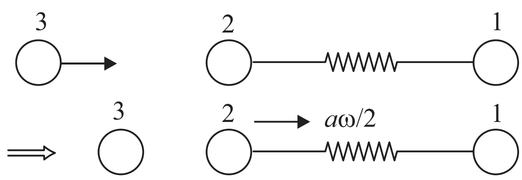
$$\frac{\frac{a\omega}{2} \times m + m \times a\omega}{2m}$$

$$V_{\omega m} = \frac{3a\omega}{4}$$

$$\frac{V\omega m}{a\omega} = 0.75$$

17.(4.25)

At $\frac{\pi}{2\omega}$ $\quad v = a\cos\frac{\pi}{2} = 0$



$$m\frac{a\omega}{2} = 2mv_0$$

$$v_0 = \frac{a\omega}{4}$$

$$\frac{1}{2}m\left(\frac{a\omega}{2}\right)^2 + \frac{1}{2}k(2a)^2 = \frac{1}{2}k \times m^2 + \frac{1}{2} \times 2m\frac{a^2\omega^2}{16}$$

$$m\frac{a^2\omega^2}{8} + m\omega^2 a^2 - \frac{m\omega^2 a^2}{16} = \frac{1}{2} \times \frac{m\omega^2}{2} \times X_m^2$$

$$\frac{m\omega^2 a^2}{16} + m\omega^2 a = \frac{1}{4}m\omega^2 \times m^2$$

$$\frac{17m\omega^2 a^2}{16} = \frac{m\omega^2 \times m^2}{4}$$

$$\frac{17a^2}{4} = 4b^2 \Rightarrow \qquad X_m = 2b; X_{m^2} = 4b^2 \qquad \Rightarrow \frac{4b^2}{a^2} = \frac{17}{4}$$

CHEMISTRY

1.(B) $K.E. = -E = 13.6\,\dfrac{Z^2}{n^2}\text{ eV/atom}$

For maximum kinetic energy, Z should be high and n should be low.

2.(D) $M_X Y_2 O_4$

Given $Y = +3$ oxidation state, so total charge on X atom of M is +2

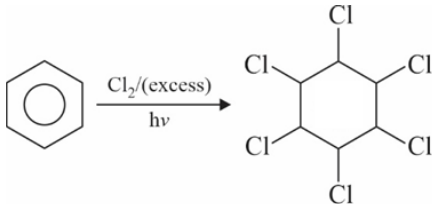
$$\frac{x}{3}(+2) + \frac{2x}{3}(+3) = +2$$

$$\frac{2x}{3} + \frac{6x}{3} + 2$$

$$\Rightarrow \quad 8x = 6 \Rightarrow x = \frac{6}{8}$$

$$\Rightarrow \quad x = \frac{3}{4}$$

$$\Rightarrow \quad x = 0.75$$

3.(D) $\text{L-glucose} \xrightarrow[\Delta]{HI} \text{n-hexene} \xrightarrow[775\,K,\ 10-20\,atm]{Cr_2O_3}$ 

4.(B) $[PCl_4]^+[PCl_4F_2]^-$ colourless crystal

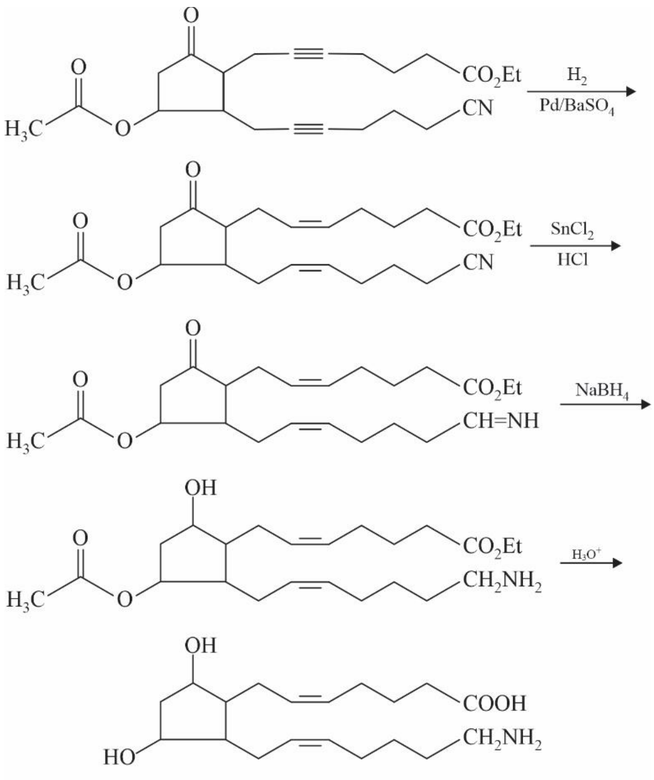
$[PCl_4]^+[PF_6]^-$ white crystal

5.(AC)

Anode	:	$N_2H_4(aq) + 4OH^-(aq) \rightarrow N_2(g) + 4H_2O(\ell) + 4e^-$
Cathode	:	$O_2(g) + 2H_2O(\ell) + 4e^- \rightarrow 4OH^-(aq)$
Overall reaction	:	$N_2H_4(aq) + O_2(g) \rightarrow N_2(g) + 2H_2O(\ell)$

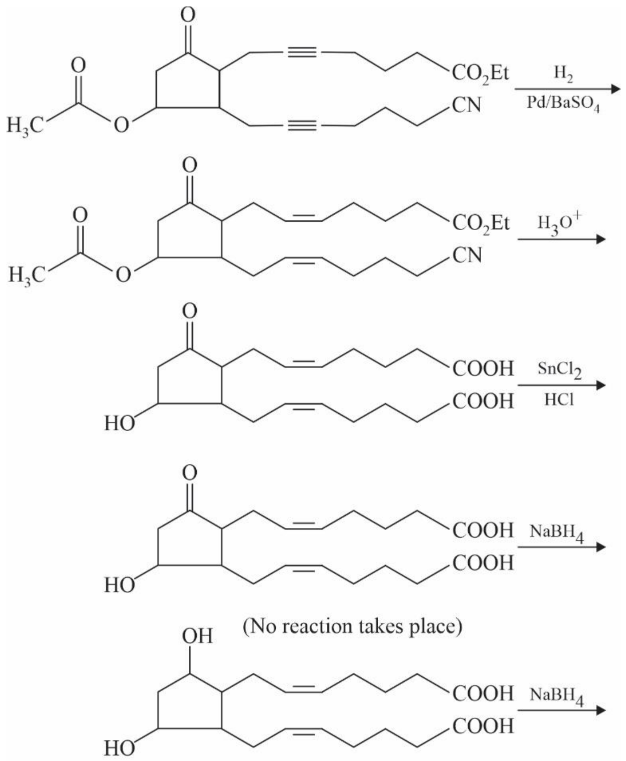
6.(CD)

(A)



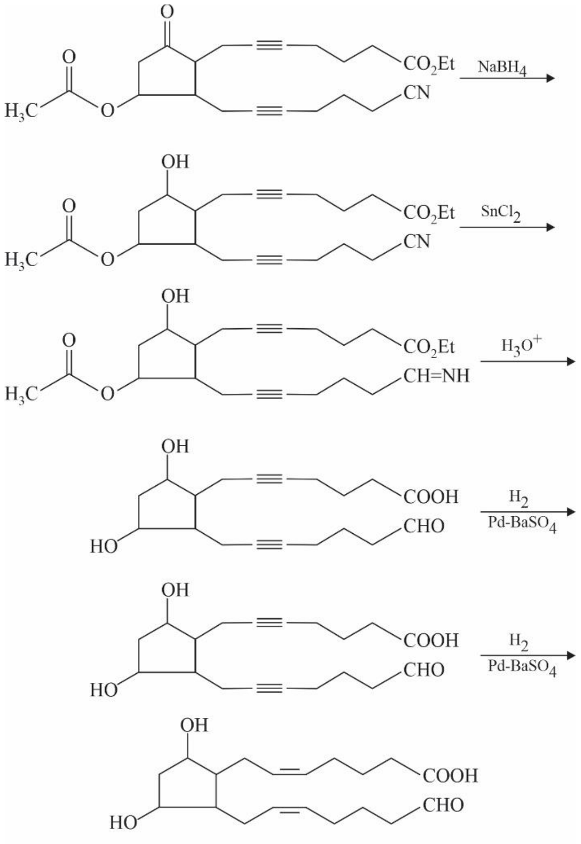
Not Given Product

(B)



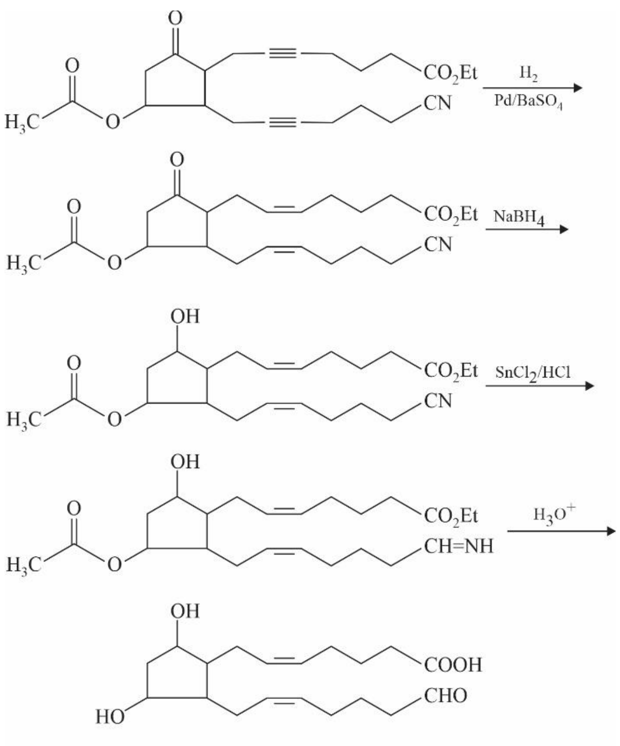
(Not given product)

(C)



Given Product

(D)



Given Product

7.(BD) H_2SO_5

$$\begin{array}{c} \quad\;\; O \\ \quad\;\; \| \\ HO-S-O-O-H \\ \quad\;\; \| \\ \quad\;\; O \end{array}$$

$H_2S_2O_8$

$$\begin{array}{ccccccccc} & & O & & & & O & & \\ & & \| & & & & \| & & \\ HO & - & S & - & O-O & - & S & - & OH \\ & & \| & & & & \| & & \\ & & O & & & & O & & \end{array}$$

$H_2S_2O_7$

$$\begin{array}{ccccccc} & & O & & O & & \\ & & \| & & \| & & \\ HO & - & S & -O- & S & - & OH \\ & & \| & & \| & & \\ & & O & & O & & \end{array}$$

$H_2S_2O_5$

$$\begin{array}{ccccccc} & & O & & O & & \\ & & \| & & \| & & \\ HO & - & S & -O- & S & - & OH \\ & & \| & & & & \\ & & O & & & & \end{array}$$

8.(2500)

100 mL of 0.5 M $\Rightarrow$50 ml CH_3COOH initially added

40 mL of 1 M $\Rightarrow$40 mmol NaOH required for unadsorbed CH_3COOH

So, amount of adsorbed $CH_3COOH = 10\,\text{mmol}$ = $10\times10^{-3}\times6\times10^{23} = 1.5\times10^{2}$ molecules of CH_3COOH

Surface area of charcoal = $1.5\times10^{2}\ m^2/g$

Surface area occupied by $10\times10^{-3}\times6\times10^{23}\text{molecules} = 1.5\times10^{2}\ m^2$

Surface area occupied by 1 molecule = $\dfrac{1.5\times10^{2}}{10\times10^{-3}\times6\times10^{23}} = 2500\times10^{-23}$

9.(150) $\dfrac{\Delta T_x}{\Delta T_y} = \dfrac{i_1\times K_{b_1}\times m_1}{i_2\times K_{b_2}\times m_2}$

$$\frac{\Delta T_x}{\Delta T_y} = \frac{1.2\times\dfrac{\dfrac{W_2}{M_X}}{\dfrac{W_1}{1000}}}{1\times\dfrac{\dfrac{W_2}{M_Y}}{\dfrac{W_1}{1000}}}$$

$$\frac{\Delta T_x}{\Delta T_y} = 1.2\times\frac{M_X}{M_Y} = 1.2\times\frac{100}{80} = 1.5$$

$\% = 1.5\times100 = 150$

10.(41) Each AT pair has 2 H-Bonds

Each GC pair has 3 H-Bonds

So total energy required $= (7\times 2\times 1) + (6\times 3\times 1.5) = 14 + 27 = 41\,\text{kcal mol}^{-1}$

11.(143)

$$\ln\left(1+\frac{Pb}{U}\right) = k\times t$$

$$k = \frac{\ln 2}{4.5\times 10^9}$$

$$\ell n\left(1+\frac{\frac{7}{206}}{\frac{1}{238}}\right) = \frac{\ln 2}{4.5\times 10^9}\times t$$

$$\ell n\left(1+\frac{7\times 238}{206}\right) = \frac{\ell n 2}{4.5\times 10^9}\times t$$

$$\ell n(1+8.09) = \frac{\ell n 2}{4.5\times 10^9}\times t$$

$$\ell n(9.09) = \frac{0.693}{4.5\times 10^9}\times t$$

$$14.33\times 10^9 = t$$

$$t = 143.3\times 10^8$$

$$t \sim 143\times 10^8 \text{ years}$$

12.(5) $[Co(CN)_4]^{4-}$ = Tetrahedral

$[Co(CO)_3NO]$ = Tetrahedral

XeF_4 = Square planar

$[PCl_4]^+$ = Tetrahedral

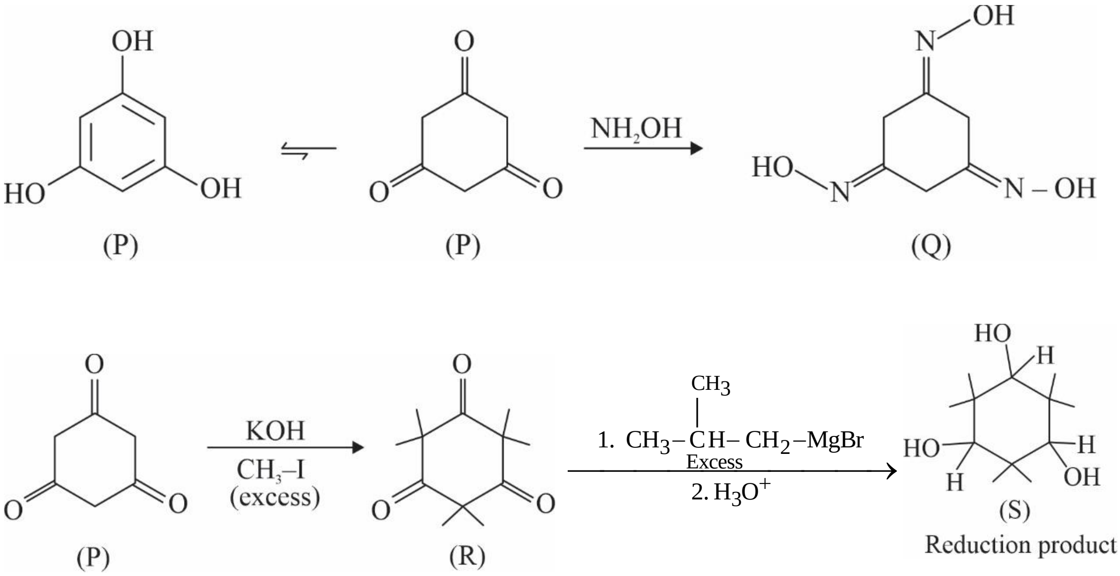
$[PdCl_4]^-$ = Square planar

$[ICl_4]^-$ = Square planar

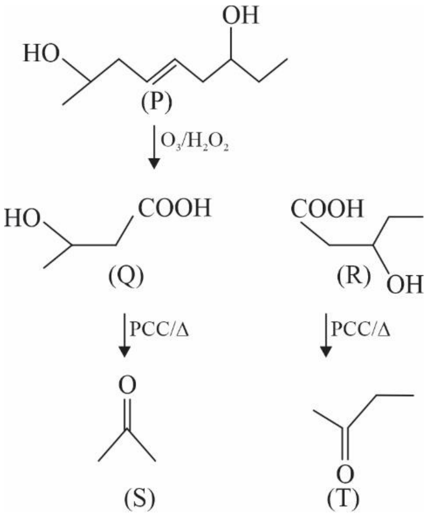
$[Cu(CN)_4]^{3-}$ = Tetrahderal

P_4 = Tetrahdedral

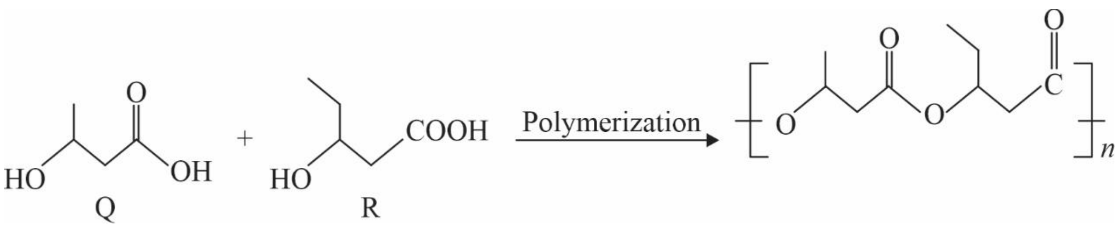
13.(6)



14.(2)



15.(93018)



Molecular mass of polymer formed is $(104+118)\,500-18\,(999)\quad \Rightarrow\ 93018\,\text{u}$.

16.(3) $3KI + 2K_3[Fe(CN)_6] \longrightarrow 2K_4[Fe(CN)_6] + KI_3$

17.(3) $2K_4[Fe(CN)_6] + 3ZnCl_2 \longrightarrow K_2Zn_3[Fe(CN)_6]_2$

MATHEMATICS

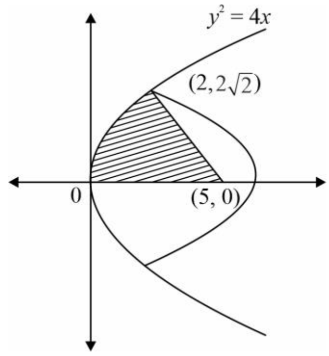
1.(B) Let $A=\sin^{-1}\frac{3}{5} \Rightarrow \tan A=\frac{3}{4}$

$$B=2\cos^{-1}\frac{2}{\sqrt{5}} \Rightarrow \cos\frac{B}{2}=\frac{2}{\sqrt{5}}$$

$$\Rightarrow \quad \tan\frac{B}{2}=\frac{1}{2} \quad \Rightarrow \quad \tan B=\frac{2\times\frac{1}{2}}{1-\frac{1}{4}}=\frac{4}{3}$$

$$\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\, to\, B}=\frac{\frac{3}{4}-\frac{4}{3}}{1+1}=\frac{-7}{24}$$

2.(B) $\int_0^2 \sqrt{4x}+\frac{1}{2}\times 3\times 2\sqrt{2}$



$$=\frac{2}{3}\times 2\times 2\sqrt{2}+3\sqrt{2}$$

$$=\left(\frac{8}{3}+3\right)\sqrt{2}=\frac{17}{3}\sqrt{2}$$

3.(B) $L=\lim_{x\to 0^+}\left(\sin(\sin kx)+\cos x+x\right)^{2/x}$

$$=e^{\lim_{x\to 0^+}\frac{2}{x}(\sin kx+x+\cos x-1)}=e^{\lim_{x\to 0^+}\frac{2}{x}\left(\sin(\sin kx)+x-2\sin^2\frac{x}{2}\right)}=e^6$$

$$\Rightarrow \lim_{x\to 0^+}\left(\frac{\sin(\sin kx)+x-2\sin^2\frac{x}{2}}{x}\right)=3 \qquad \Rightarrow \qquad k=2$$

4.(D) In this question, roots of $f(x)=0$ are being considered.

For $x\neq 0$, $f(x)=0$ for $\frac{1}{x^2}=n,\ n\in N \qquad \Rightarrow \qquad x^2=\frac{1}{n}, n\in N$

(A) False because: $x\in\left[10^{-10},\infty\right)$

$$\Rightarrow \qquad \frac{1}{x^2}\in\left(0,10^{20}\right]$$

So, 10^{20} solutions of $\frac{1}{x^2}=n, n\in N$

(B) False because: $x \in \left[\frac{1}{\pi}, \infty\right)$

$\Rightarrow \quad \frac{1}{x^2} \in \left(0, \pi^2\right]$ and

Can be equal to $\{1, 2 \ldots 9\}$

(C) False because: $x \in \left(0, 10^{-10}\right)$

$\Rightarrow \quad \frac{1}{x} \in \left(10^{10}, \infty\right) \quad \Rightarrow \quad \frac{1}{x^2} \in \left(10^{20}, \infty\right)$

Infinitely many natural numbers in $\left(10^{20}, \infty\right)$

(D) True: $x \in \left(1/\pi^2, 1/\pi\right) \Rightarrow \frac{1}{x^2} \in \left(\pi^2, \pi^4\right)$

More than 25 natural numbers in $\left(\pi^2, \pi^4\right)$.

5.(BC) Since $L = \lim_{x\to\infty} \frac{\sin(x^2)(\ln x)^{\alpha} \sin\left(1/x^2\right)}{x^{\alpha\beta}\left(\ln(1+x)\right)^{\beta}}$

Note as $x \to \infty$, $\frac{\sin(1/x^2)}{1/x^2} \to 1$

$\Rightarrow$ We can replace $\sin\left(1/x^2\right)$ with $1/x^2 \quad \Rightarrow \quad L = \lim_{x\to\infty} \frac{(\ln x)^{\alpha}\sin\left(x^2\right)}{x^{(\alpha\beta+2)}\left(\ln(1+x)\right)^{\beta}}$

Note $\alpha\beta + 2 > 0$ ensures that denominator dominates AND $L \to 0$

$\Rightarrow$ (B) and (C) are correct.

When $\alpha\beta + 2 < 0$, L doesn't exist as x has a positive power in numerator.

$\Rightarrow$ (A) is incorrect.

When $\alpha\beta + 2 = 0$,

$$L = \lim_{x\to\infty} \frac{\sin\left(x^2\right)(\ln x)^{\alpha}}{\left(\ln\left(1+x\right)\right)^{\beta}}$$

$L = 0$ iff $\alpha < \beta$ but (D) is incorrect because: $\alpha = 1 > \beta = -2$.

6.(AC) Given $P:(1, 3, 2)$

Let Q: $(1+a, 3+2a, 2+a)$

Q lies on $L_1 \Rightarrow a = 1$

$Q:\ (2, 5, 3)$

Let $R(2+b, 5-b, 3+3b)$

R lies on $L_2 \Rightarrow b=-1$

$\Rightarrow \quad R:(1, 6, 0)$

$PQ=\sqrt{6}, QR=\sqrt{11}, PR=\sqrt{3}$

Centroid of $\Delta PQR = \left(\frac{4}{3}, \frac{14}{3} \cdot \frac{5}{3}\right)$

7.(AC) Let $A_1:\left(2t_1^2, 4t_1\right)$,

$B_1:\left(2t_2^2, 4t_2\right)$

$\Rightarrow \quad C_1:\left(2t_1t_2, 2(t_1+t_2)\right)=(-4,0) \quad \Rightarrow \quad t_1+t_2=0$

$\Rightarrow \quad t_1, t_2 \in \left\{-\sqrt{2}, \sqrt{2}\right\} \quad \Rightarrow \quad A_1$ and B_1 are $\left(4, \pm 4\sqrt{2}\right)$

$OA_1 = 4\sqrt{3}$

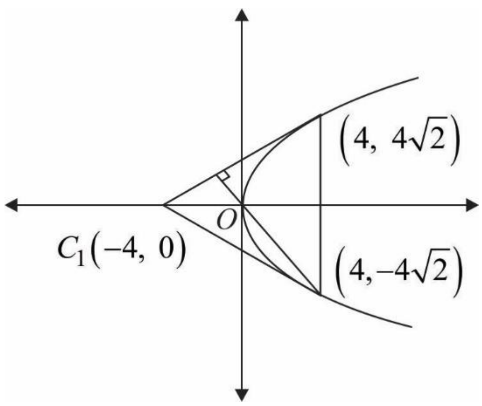
$A_1B_1 = 8\sqrt{2}$

For orthocentre, clearly X-axis is the line containing altitude from C_1 to A_1B_1.

Slope of line joining $(-4, 0)$ to $\left(4, 4\sqrt{2}\right)$ is $\frac{1}{\sqrt{2}}$.

Slope of line joining $(0, 0)$ to $\left(4, -4\sqrt{2}\right)$ is $-\sqrt{2}$.

Clearly the altitude from $\left(4, -4\sqrt{2}\right)$ intersects the altitude from C_1 at $O:(0, 0)$



8. (51) Clearly: $f(x)=kx$ with $k=-20$

and $g(x)=a^x$ with $a=\frac{1}{8}$

9.(11) $P(W_1 \cap G_2 \cap B_3) = \frac{3}{N} \times \frac{6}{N-1} \times \frac{N-9}{N-2} = \frac{2}{5N}$

Also, $\frac{2}{9} = \frac{P(W_1 \cap G_2 \cap B_3)}{P(W_1 \cap G_2)} = \frac{\frac{3}{N} \times \frac{6}{N-1} \times \frac{N-9}{N-2}}{\frac{3}{N} \times \frac{6}{N-1}}$

$\Rightarrow N = 11$

10.(1) $\sin x\left[x^{2023} + 2024x + 2025\right] + 2\left[x^{2023} + 2024x + 2025\right]$

$(\sin x + 2)\left[x^{2023} + 2024x + 2025\right]$

$f(x) = 2023x^{2022} + 2024 > 0$

Inc. $\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = -\infty$

1 root: $x = -1$.

11.(2) $\vec{p} \times \vec{q} = 4\hat{i} + \hat{j} - 3\hat{k}$

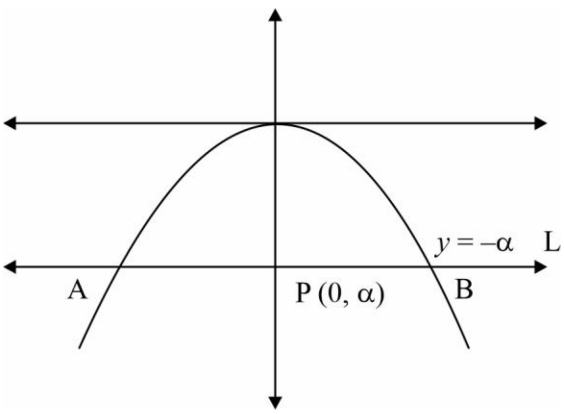
Taking dot product with $\vec{p} \times \vec{q}$ on both sides of the given equation.

$(15\hat{i} + 10\hat{j} + 6\hat{k}).(4\hat{i} + \hat{j} - 3\hat{k}) = \gamma\,|\vec{p} \times \vec{q}|^2$

$52 = \gamma \times 26$

$\Rightarrow \quad \gamma = 2.$

12.(12) Let $M : (2at, -at^2)$ be a point on the parabola.



Slope of Normal $= \frac{1}{t}$

$\Rightarrow \quad t = \sqrt{6} \quad \Rightarrow \quad M : (2a\sqrt{6}, -6a)$

Equation of Normal: $y + 6a = \frac{1}{\sqrt{6}}(x - 2a\sqrt{6})$

$\Rightarrow \quad \alpha = 8a$ (Putting $x = 0$)

Put $y = -8a$ in $x^2 = -4ay$

$\Rightarrow \quad x = \pm 4\sqrt{2}a \quad \Rightarrow \quad AB = 8\sqrt{2}a$

$\Rightarrow \quad S = 128a^2$ AND $r = 4a$

$\frac{r}{s} = \frac{1}{32a} = \frac{1}{16} \Rightarrow a = \frac{1}{2}$

13.(5) Note: $f(2n+1) = -f(2n-1)\ \forall n \in N$

$\Rightarrow$ When $t \in (2n-1, 2n+1)$,

$$f(t) = f(2n-1)\left[\frac{2n-(t-1)}{2} + \frac{(2n-1)-t}{2}\right]$$

$= f(2n-1)[2n-t]$ {Straight line with slope $= -f(2n-1)$ }

$At\ t \to (2n-1)^{+}$

$f(t) \to f(2n-1)$

At $t \to (2n+1)^{-}$

$f(t) \to -f(2n-1)$

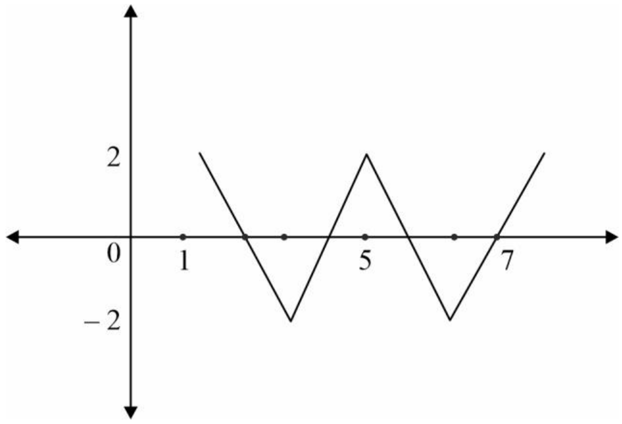
$\to f(2n+1)$

$\Rightarrow$ In $[2n-1, 2n+1]$, the graph of $f(t)$ is a straight line drawn from $f(2n-1)$ to $f(2n+1)$

Also, $f(1) = 2 = f(5)$

$f(3) = -2$

Hence, graph of *f* is:



Interestingly: This is the graph of $\frac{4}{\pi}\sin^{-1}\left(\sin\frac{\pi x}{2}\right)$ in $[1, \infty)$

From graph of $f(x)$, it's clear that $\int_1^x f(t)dt = \forall x = 2n-1, n \in N$

So, $\alpha = 3$ as there are 3 odd natural numbers in $(1, 8]$.

To Evaluate β :

Let $x = 1 + h$

$$\int_1^{1+h} f(t)\,dt = \int_1^{1+h} 2(2-t)\,dt$$

$$= 4h - (1+h)^2 + 1 = 2h - h^2$$

$$\Rightarrow \quad \beta = \lim_{h\to 0} \frac{h(2-h)}{h} = 2.$$

Solution of 14.(20) and 15.(36):

Elements in S×S satisfy $|a-b| \geq 2$.

$(1,3),(1,4),(1,5),(1,6) = 4$ Possible images of 1

$(2,4),(2,5),(2,6) = 3$ Possible images of 2

$(3,1),(3,5),(3,6) = 3$ Possible images of 3

$(4,1),(4,2),(4,6) = 3$ Possible images of 4

$(5,1),(5,2),(5,6) = 3$ Possible images of 5

$(6,1),(6,2),(6,3),(6,4) = 4$ Possible images of 6

Total 20 elements in S × S satisfying the 2nd property.

Since X has exactly 6 elements, $|X| = {}^{20}C_6$.

Y will be an empty set because if range of a relation R in X has exactly 1 element, all 6 element of S must map to it but that's clearly not possible.

$n(Z) = 4^2 \times 3^4$ because for a relation *R* in *X* to be a function, *X* must contain a unique image of each element of $S \Rightarrow k = 36$

16.(0) $$I = \int_0^{\frac{\pi}{2}} f(x)g(x)dx = \int_0^{\frac{\pi}{2}} \sin^2 x \sqrt{x\left(\frac{\pi}{2}-x\right)}dx \qquad \textbf{… (i)}$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2\left(\frac{\pi}{2}-x\right)\sqrt{x\left(\frac{\pi}{2}-x\right)}dx \qquad \textbf{… (ii)}$$

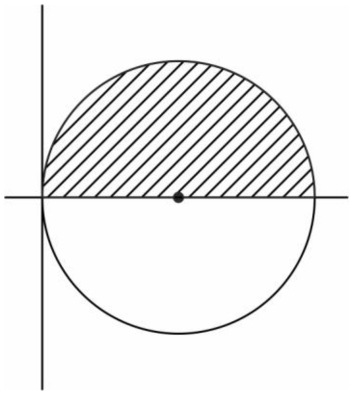
Add (i) and (ii)

$$2I = \int_0^{\frac{\pi}{2}} \sqrt{x\left(\frac{\pi}{2}-x\right)}dx = \int_0^{\frac{\pi}{2}} g(x)dx \qquad \Rightarrow 2I - \int_0^{\frac{\pi}{2}} g(x)dx = 0$$

17.(0.25) $I=\frac{16}{\pi^3}\int_0^{\frac{\pi}{2}}\sin^2 x\sqrt{\frac{\pi}{2}x-x^2}$

Let $I=\frac{16}{\pi^3}\int_0^{\frac{\pi}{2}}\sin^2 x\sqrt{x\left(\frac{\pi}{2}-x\right)}$

$I=\frac{16}{\pi^3}\int_0^{\frac{\pi}{2}}\sin^2\left(\frac{\pi}{2}-x\right)\sqrt{\left(\frac{\pi}{2}-x\right)(x)}$



$I=\frac{8}{\pi^3}\int_0^{\frac{\pi}{2}}\sqrt{\frac{\pi}{2}x-x^2}=\frac{8}{\pi^3}\int_0^{\frac{\pi}{2}}\sqrt{\frac{\pi}{16}-\left(\frac{\pi}{4}-x\right)^2}\,dx$

Consider the integral $I^1=\int_0^{\frac{\pi}{2}}\sqrt{\frac{\pi^2}{16}-\left(x-\frac{\pi}{4}\right)^2}\,dx$

Consider the curve $\left(x-\frac{\pi}{4}\right)^2+y^2=\left(\frac{\pi}{4}\right)^2$

Clearly I^1 is the area of the shaded semi-circle = $\frac{\pi\left(\frac{\pi}{4}\right)^2}{2}$

So, $I=\frac{8}{\pi^3}.\frac{\pi^3}{32}=\frac{1}{4}$