VMC Vidyamandir Classes SINCE 1986
IIT JEE | MEDICAL | FOUNDATION

Solutions of JEE Advanced 2024 | Paper - 1

PHYSICS

1.(A) $[e]=[TA]$

$[\in_0]=\left[M^{-1}L^{-3}T^4A^2\right]$

$[h]=\left[ML^2T^{-1}\right]$

$[c]=\left[LT^{-1}\right]$

$$[TA]^{\alpha}\left[M^{-1}L^{-3}T^4A^2\right]^{\beta}\left[ML^2T^{-1}\right]^{\gamma}\left[LT^{-1}\right]^{\delta}=\left[M^0L^0T^0A^0\right]$$

For $[M]$: $-\beta+\gamma=0 \Rightarrow \beta=\gamma$

For $[L]$: $-3\beta+2\gamma+\delta=0$

$-\beta+\delta=0 \Rightarrow \delta=\beta$

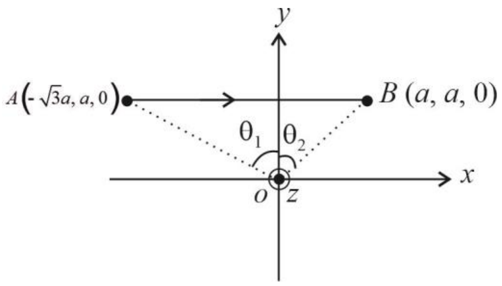
For $[A]$: $\alpha+2\beta=0 \Rightarrow \alpha=-2\beta$

For $[T]$: $\alpha+4\beta-\gamma-\delta=0$

This is also satisfied If $\beta=-n$ then $\alpha=2n, \gamma=-n, \delta=-n$

2.(A) Join A and B to O to form a closed loop

Using amperes' law



$$\oint \vec{B}\cdot\vec{dl}=\mu_0 I_{\text{enclosed}}$$

$$\tan\theta_1=\frac{\sqrt{3}a}{a}=\sqrt{3}\Rightarrow\theta_1=\frac{\pi}{3}$$

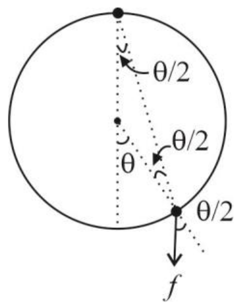
$$\tan\theta_2=\frac{a}{a}=1\Rightarrow\theta_2=\frac{\pi}{4}$$

$$I_{\text{enclosed}}=\frac{I}{2\pi}(\theta_1+\theta_2)=\frac{I}{2\pi}\times\frac{7\pi}{12}=\frac{7I}{24}$$

$$\oint\vec{B}\cdot\vec{dl}=\int_0^A\vec{B}\cdot\vec{dl}+\int_A^B\vec{B}\cdot\vec{dl}+\int_B^0\vec{B}\cdot\vec{dl}=0+\int_A^B\vec{B}\cdot\vec{dl}+0$$

$$\int_A^B\vec{B}\cdot\vec{dl}=\mu_0\left(\frac{-7I}{24}\right)$$

3.(B) $F = \frac{1}{4\pi \in_0} \frac{q^2}{\left(2R\cos\frac{\theta}{2}\right)^2}$



Restoring torque

$$\tau = -F\sin\frac{\theta}{2}R$$

$$mR^2\alpha = -\frac{q^2}{4\pi \in_0 4R^2\cos^2\frac{\theta}{2}}\sin\frac{\theta}{2}R$$

For small θ,

$$\sin\frac{\theta}{2} = \frac{\theta}{2}, \cos\frac{\theta}{2} = 1$$

$$mR^2\alpha = -\frac{q^2}{32\pi \in_0 R}\theta \qquad \therefore \qquad \omega^2 = \frac{q^2}{32\pi \in_0 R^3 m}$$

4.(C) $F = -20x + 10 = -20(x - 0.5)$

Motion is SHM with

$$\omega^2 = \frac{20}{5} = 4 \Rightarrow \omega = 2$$

Mean position is $x = 0.5$

$$\therefore \qquad x = 0.5 + A\sin(\omega t + \phi)$$

$$v = A\omega\cos(\omega t + \phi)$$

At $t = 0$, $\quad v = 0$, $\quad x = 1$

$$0 = A\omega\cos\phi \qquad \therefore\ \phi = \frac{\pi}{2}$$

$$1 = 0.5 + A \Rightarrow A = 0.5$$

$$\therefore \qquad x = 0.5 + 0.5\sin\left(2t + \frac{\pi}{2}\right) = 0.5 + 0.5\cos 2t$$

At $t = \frac{\pi}{4}, x = 0.5 + 0.5\cos\frac{\pi}{2} = 0.5\ m$

$$v = A\omega\cos\left(\omega t + \frac{\pi}{2}\right) = -\sin 2t$$

At $t = \frac{\pi}{4}, v = -\sin\frac{\pi}{2} = -1$

$$p = mv = -5\ kgm/s$$

5.(ABC) $kr = \dfrac{mv^2}{r}$ …(1)

$mvr = n\hbar$ …(2)

Using (1) and (2)

$$mv^2 = k\left(\frac{n\hbar}{mv}\right)^2 \Rightarrow \quad v^4 = \frac{k}{m}\left(\frac{n\hbar}{m}\right)^2 \qquad \therefore \qquad v^2 = n\hbar\sqrt{\frac{k}{m^3}}$$

$$r^2 = \frac{m}{k}v^2 = \frac{m}{k}n\hbar\sqrt{\frac{k}{m^3}} = n\hbar\sqrt{\frac{1}{mk}}$$

$$\frac{L}{mr^2} = \frac{n\hbar}{m\left(n\hbar\sqrt{\frac{1}{mk}}\right)} = \sqrt{\frac{k}{m}}$$

Potential energy $= \dfrac{kr^2}{2} = \dfrac{k}{2}\left(n\hbar\sqrt{\dfrac{1}{mk}}\right) = \dfrac{n\hbar}{2}\sqrt{\dfrac{k}{m}}$

Kinetic energy $= \dfrac{1}{2}mv^2 = \dfrac{1}{2}m\left(n\hbar\sqrt{\dfrac{k}{m^3}}\right) = \dfrac{n\hbar}{2}\sqrt{\dfrac{k}{m}}$

Total energy = Potential energy + kinetic energy $= n\hbar\sqrt{\dfrac{k}{m}}$

6.(ACD) With O as node,

Fundamental frequency of string *PO*

$$v_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = v_0$$

Fundamental frequency of string *OQ*

$$v_2 = \frac{1}{2(2L)}\sqrt{\frac{T}{4\mu}} = \frac{v_0}{4} \qquad \therefore \qquad \text{Minimum required frequency } = v_0$$

PO vibrates with fundamental frequency and *OQ* vibrates in 4th harmonic

$\therefore$ Total loops = 1 + 4 = 5

Total nodes = 6

With *O* as antinode

Fundamental frequency of string *PO*

$$v_1 = \frac{1}{4L}\sqrt{\frac{T}{\mu}} = \frac{v_0}{2}$$

Fundamental frequency of string *OQ*

$$v_2 = \frac{1}{4(2L)}\sqrt{\frac{\mathrm{T}}{4\mu}} = \frac{v_0}{8}$$

As only odd harmonics are allowed, so we can’t get a common vibrating frequency.

7.(AB) The combination behaves as a mirror

$$\frac{1}{f}=\frac{1}{f_m}-\frac{2}{f_l}=-\frac{2}{f_l}=-2\left(\frac{1}{f_1}+\frac{1}{f_2}\right)$$

$$\frac{1}{f_1}=(n-1)\left(\frac{1}{\infty}-\frac{1}{9}\right)$$

$$\frac{1}{f_2}=(1.6-1)\left(\frac{1}{9}-\frac{1}{\infty}\right)$$

$$\frac{1}{f}=-2\left[(n-1)\left(-\frac{1}{9}\right)+(0.6)\left(\frac{1}{9}\right)\right]$$

For $n=1.42$

$$\frac{1}{f}=-2\left[(0.42)\left(-\frac{1}{9}\right)+(0.6)\left(\frac{1}{9}\right)\right]=-2\left[\frac{1}{9}\times 0.18\right] \quad \Rightarrow \quad f=-25\Rightarrow h=2|f|=50\,cm$$

For $n=1.35$

$$\frac{1}{f}=-2\left[0.35\left(-\frac{1}{9}\right)+0.6\left(\frac{1}{9}\right)\right]=-2\left[\frac{1}{9}\times 0.25\right]$$

$$f=-18\Rightarrow h=2|f|=36\,cm$$

For $n=1.45$

$$\frac{1}{f}=-2\left[0.45\left(-\frac{1}{9}\right)+0.6\left(\frac{1}{9}\right)\right]=-2\left[\frac{1}{9}\times 0.15\right]$$

$$f=-30\Rightarrow h=2|f|=60\,cm$$

For $n=1.48$

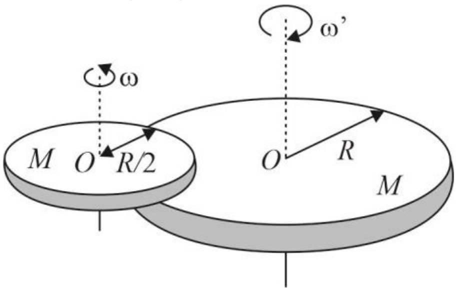
$$\frac{1}{f}=-2\left[0.48\left(-\frac{1}{9}\right)+0.6\left(\frac{1}{9}\right)\right]=-2\left[\frac{1}{9}\times 0.12\right]$$

$$f=-\frac{75}{2}\Rightarrow h=2|f|=75\,cm$$

8.(25000) $dQ=mCdT=1(kT)dT$

$$Q=\int dQ=\int_{200}^{300}kTdT=k\left[\frac{T^2}{2}\right]_{200}^{300}=\frac{k}{2}\left[90000-40000\right]=25000\,k$$

9.(12) Conserving angular momentum about the fixed axis,



$$0=\left(-\frac{MR^2}{2}\omega'\right)+\left(\frac{M\left(R/2\right)^2}{2}\omega-M\left(\omega' R\right)R\right)$$

$$0=-\frac{3}{2}MR^2\omega'+\frac{MR^2}{8}\omega$$

$$\omega'=\frac{\omega}{12}$$

10.(8) $r=\frac{I_A}{I_B}=2$

After placing two polaroids before B

$$I_B^{'}=\frac{I_B}{2}\cos^2 45°=\frac{I_B}{4}$$

$$\therefore \qquad r'=\frac{I_A}{I_B^{'}}=4\frac{I_A}{I_B}=4r=8$$

11.(200) Case I: $288=\left(\frac{c+v}{c-v}\right)240$ …(1)

Case 2: $n=\left(\frac{c-v}{c+v}\right)240$ …(2)

Multiplying (1) and (2)

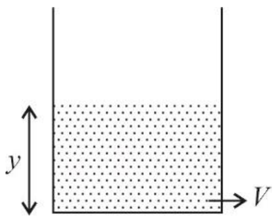
$$288n=240\times 240$$

$$n=\frac{240\times 240}{288}=200$$

12.(3) For Tank 1:

$$v=\sqrt{2gy}$$

$$A\left(-\frac{dy}{dt}\right)=a\sqrt{2gy}$$



$$\int_h^0\frac{dy}{\sqrt{y}}=-\frac{a}{A}\sqrt{2g}\int_0^{t_1}dt$$

$$2\sqrt{h}=\frac{a}{A}\sqrt{2g}\ t_1 \qquad \ldots(1)$$

For tank 2:

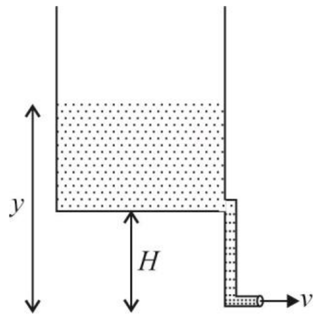
$$v=\sqrt{2gy}$$

$$A\left(-\frac{dy}{dt}\right)=a\sqrt{2gy}$$

$$\int_{H+h}^{H}\frac{dy}{\sqrt{y}}=-\frac{a}{A}\sqrt{2g}\int_0^{t_2}dt$$

$$2\left(\sqrt{H+h}-\sqrt{H}\right)=\frac{a}{A}\sqrt{2g}\,t_2$$

Using $H=\frac{16}{9}h$



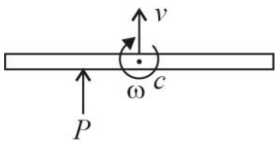
$$2\left(\sqrt{\frac{25}{9}h}-\sqrt{\frac{16}{9}h}\right)=\frac{a}{A}\sqrt{2g}\,t_2$$

$$2\left(\frac{1}{3}\sqrt{h}\right)=\frac{a}{A}\sqrt{2g}\,t_2 \qquad \ldots(2)$$

From (1) and (2)

$$3=\frac{t_1}{t_2}$$

13.(18) $p=mv \qquad \ldots(1)$



$$px=\frac{mL^2}{12}\omega \qquad \ldots(2)$$

If rod remains aligned with string, angular velocity of rod about its own axis is equal to angular velocity of string about O.

$\therefore \qquad \omega=\frac{V}{3L/2}\Rightarrow V=\frac{3}{2}L\omega$

Putting in (1), $p=m\frac{3}{2}L\omega$

Using in (2) $m\frac{3}{2}L\omega x=\frac{mL^2}{12}\omega\Rightarrow x=\frac{L}{18}$

14.(B) Let volume at $J=V_0$

Then volume at $K=3V_0$

Volume at $L=\frac{3V_0}{2}$

Volume at $M=\frac{V_0}{2}$

(P) $W_{JK}=p_0(3V_0-V_0)=2p_0V_0$

$$W_{KL}=nR3T_0\ln\left(\frac{1}{2}\right)=-3nRT_0\ln 2$$

$$W_{LM}=2p_0\left(\frac{V_0}{2}-\frac{3V_0}{2}\right)=-2p_0V_0$$

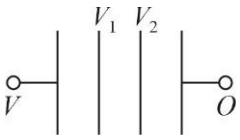
$$W_{MJ}=nRT_0\ln(2)$$

Total work done $=-2nRT_0\ln 2=-2RT_0\ln 2$

$(Q) \qquad \Delta U_{JK}=nC_v\Delta T=(1)\frac{3}{2}R(3T_0-T_2)=3RT_0$

(R) $\Delta Q_{KL} = W_{KL} = -3RT_0 \ln 2$ (S) $\Delta U_{MJ} = nC_v \Delta T = (1)\frac{3}{2}R(0) = 0$

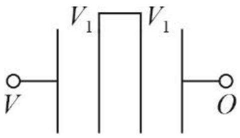
15.(C) (P)



Three capacitors in series.

$C = \frac{C_0}{3}$

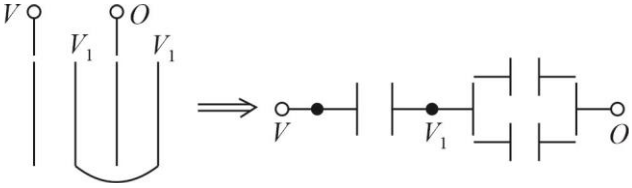
(Q)



Two capacitors in series

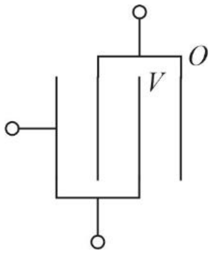
$C = \frac{Co}{2}$

(R)



$C = \frac{Co \times 2Co}{Co + 2Co} = \frac{2Co}{3}$

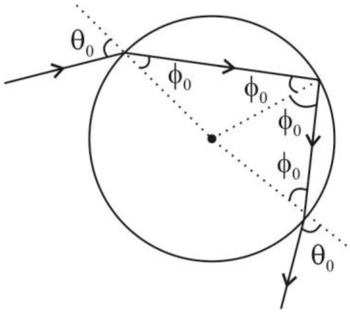
(S)



Three capacitors in parallel

$C = 3Co$

16.(A) The path of the ray is as shown



$\alpha = (\theta_0 - \phi_0) + (180° - 2\phi_0) + (\theta_0 - \phi_0)$

$\theta = 180° + 2\theta_0 - 4\phi_0$

(P) $\alpha = 180° \Rightarrow \theta_0 = 2\phi_0$

$\sin\theta_0 = 2\sin\phi_0$

$\sin 2\phi_0 = 2\sin\phi_0$

$2\sin\phi_0(\cos\phi_0 - 1) = 0 \qquad \therefore \qquad \phi_0 = 0 \Rightarrow \theta_0 = 0$

(*Q*, *R*) $\alpha = 180° \Rightarrow \theta_0 = 2\phi_0$

$\sin\theta_0 = \sqrt{3}\sin\phi_0$

$\sin 2\phi_0 = \sqrt{3}\sin\phi_0$

$\sin\phi_0(2\cos\phi_0 - \sqrt{3}) = 0 \qquad \therefore\ \phi_0 = 0 \text{ or } \phi_0 = 30°$

$\Rightarrow \qquad \theta_0 = 0 \text{ or } \theta_0 = 60°$

(*S*) $\theta_0 = 45°$

$\sin\theta_0 = \sqrt{2}\sin\phi_0$

$\frac{1}{\sqrt{2}} = \sqrt{2}\sin\phi_0 \Rightarrow \phi_0 = 30°$

$\alpha = 180° + 2\theta_0 - 4\phi_0 = 180° + 90° - 120° = 150°$

17.(A) (*P*) Current in inductor doesn't change suddenly, So $I_1 = 0$

(*Q*) $I_2 = \frac{20}{5} = 4A$

(*R*) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{25\times10^{-3}\times10\times10^{-6}}} = \frac{1}{5\times10^{-4}} = 2000\ \text{rad/s} = 2\ kilo-rad/s$

(*S*) Voltage across capacitor is maximum when current in circuit is zero. Applying energy conduction

$\frac{1}{2}L\,I_0^2 = \frac{1}{2}C\,V_0^2$

$\frac{1}{2}\times25\times10^{-3}\times16 = \frac{1}{2}\times10\times10^{-6}\times V_0^2$

$V_0 = 200$

CHEMISTRY

1.(D) $$\frac{u_{rms_X}}{u_{rms_Y}} = \sqrt{\frac{3R\times 300}{M_X}} \times \sqrt{\frac{M_Y}{3R\times 300}} = \sqrt{\frac{M_Y}{M_X}} = 2:1$$

$$2 = \frac{10}{M_X}\frac{RT}{V} \qquad \frac{2}{4} = \frac{10\cancel{RT}}{M_X\cancel{V}} \times \frac{M_Y\cancel{V}}{80\cancel{RT}}$$

$$4 = \frac{80}{M_Y}\times\frac{RT}{V} \qquad \frac{M_Y}{M_X} = \frac{2\times 8\cancel{0}}{4\times 1\cancel{0}} = 4$$

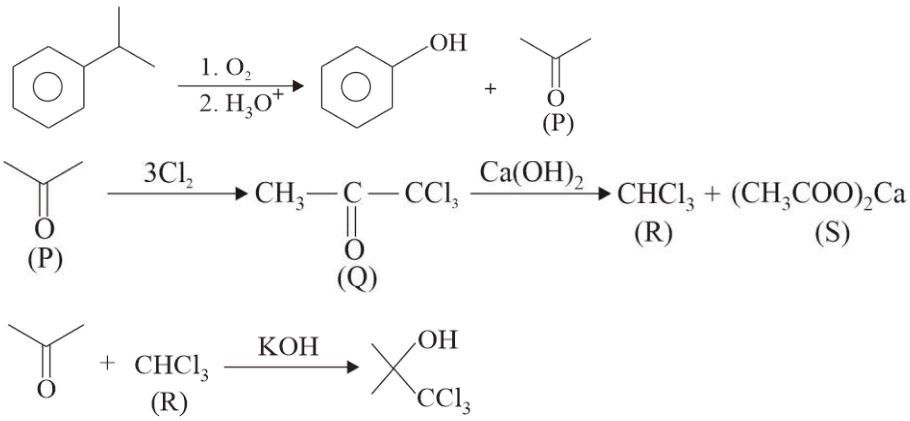
2.(A) $$\overset{+3}{HNO_2} \longrightarrow \overset{+5}{HNO_3} + \overset{+2}{NO}$$

3.(B)

4.(C)

5.(ABC) $V = \dfrac{2\pi KZe^2}{nh}$

6.(ABD)



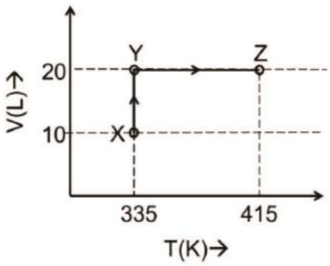
$$CHCl_3 + O_2 \xrightarrow{h\upsilon} \underset{\text{(phosgene)}}{COCl_2} + HCl$$

$$\underset{(Q)}{CH_3-\underset{\underset{O}{\parallel}}{C}-CCl_3} \xrightarrow{NaOH} CHCl_3 + CH_3COONa$$

$$\underset{(S)}{(CH_3COO)_2Ca} \xrightarrow{\Delta} \underset{(P)}{CH_3COCH_3} + CaO + CO_2$$

7.(AD) CO_2, C_2H_4, HCl and O_3 follow octet rule.

8.(8120)



$\Delta H = \Delta H_{X-Y} + \Delta H_{Y-Z}$

$\Delta H_{X-Y} = 0$

$\Delta H_{Y-Z} = 5\times(12+8.3)\times(415-335)$

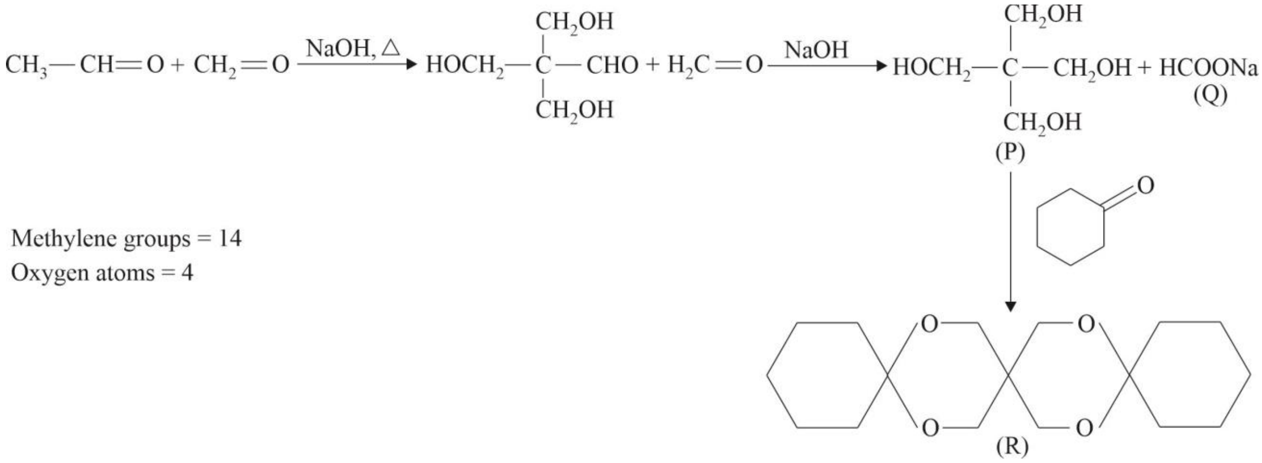
$= 5\times 20.3\times 80$

= 8120 J

9.(3) $r = K[N_2O_2][H_2]$ $\quad r = k\,K_{eq}[NO]^2[H_2]$

$K_{eq} = \dfrac{[N_2O_2]}{[NO]^2}$ $\quad r = K'[NO]^2[H_2]$

10.(18)

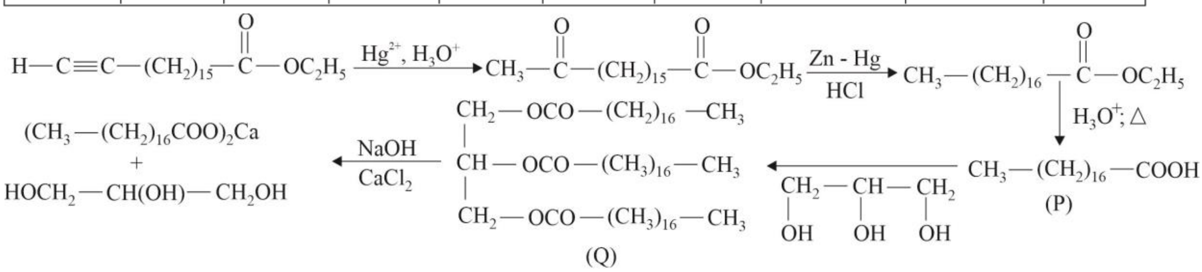


Methylene groups = 14
Oxygen atoms = 4

11.(1)

Species	$Ni(CO)_4$	$V(CO)_6$	$Cr(CO)_5$	$Cu(CO)_3$	$Mn(CO)_5$	$Fe(CO)_5$	$[Co(CO)_3]^{3-}$	$[Cr(CO)_4]^{4-}$	$Ir(CO)_3$
No. of electrons	28 + 56 = 84	23 + 84 = 107	24 + 70 = 94	29 + 42 = 71	25 + 70 = 95	26 + 70 = 96	27 + 3 + 42 = 72	24 + 4 + 56 = 84	77 + 42 = 119

12.(909)

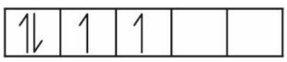


One mole of Q gives 3 moles of $CH_3(CH_2)_{16}COO^-$

Two mole of $CH_3(CH_2)_{16}COO^-$ gives one mole of $(CH_3(CH_2)_{16}COO)_2Ca$

Mole of R formed = $\dfrac{1}{2}\times 3$

Mass of R formed = $\dfrac{3}{2}\times 606 = 909\,\text{gm}$

13.(1) $[Mn(NH_3)_6]^{3+}$ $\quad Mn^{3+} \Rightarrow 3d^4$  paramagnetic

Complex	Ion	d-orbitals					Magnetic nature
$[MnCl_6]^{3-}$	$Mn^{3+} \Rightarrow 3d^4$	↑	↑	↑	↑		paramagnetic
$[FeF_6]^{3-}$	$Fe^{3+} \Rightarrow 3d^5$	↑	↑	↑	↑	↑	paramagnetic
$[CoF_6]^{3-}$	$Co^{3+} \Rightarrow 3d^6$	↑↓	↑	↑	↑	↑	paramagnetic
$[Fe(NH_3)_6]^{3+}$	$Fe^{3+} \Rightarrow 3d^5$	↑↓	↑↓	↑			paramagnetic
$[Co(en)_3]^{3+}$	$Co^{3+} \Rightarrow 3d^6$	↑↓	↑↓	↑↓			diamagnetic

14.(C) **P-3, Q-4, R-2, S-5**

(P) $K^+ + \underset{\text{More conductivity}}{Cl^-} + AgNO_3 \rightarrow K^+ + \underset{\text{Less conductivity}}{NO_3^-} + AgCl(s)$

Conductance decreases slightly and then increase

(Q) $\underset{\text{Less conductivity}}{Ag^+} + NO_3^- + KCl \rightarrow \underset{\text{More conductivity}}{K^+} + NO_3^- + AgCl(s)$

Conductance increases slightly and then increases

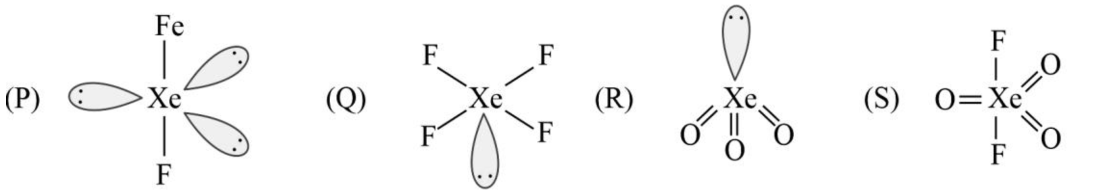
(R) $Na^+ + \underset{\text{More conductivity}}{OH^-} + HCl \rightarrow Na^+ + \underset{\text{less conductivity}}{Cl^-} + H_2O$

Conductance decrease rapidly and then increases rapidly

(S) $CH_3COOH + NaOH \rightarrow CH_3COO^- + Na^+ + H_2O$

Conductance decreases rapidly and then no much change in conductance

15.(B) **P-5, Q-3, R-2, S-4**



(5) Trigonal bipyramidal two lp

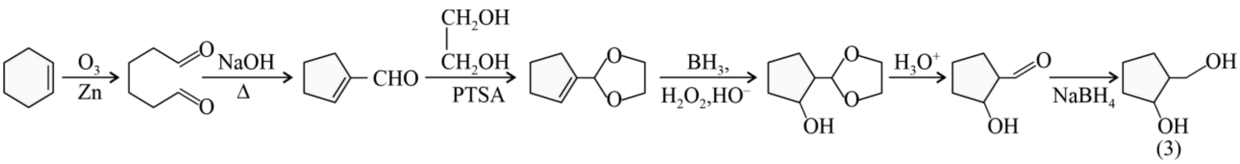
(3) Octahedral two lp

(2) Tetrahedral one lp

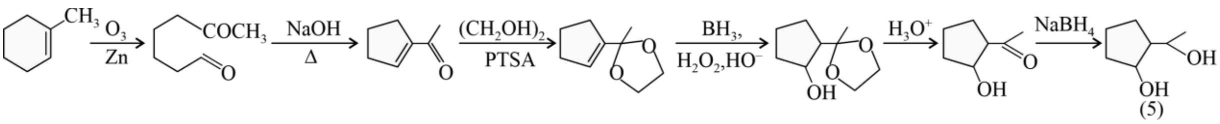
(4) Trigonal bipyramidal no lp

16.(A) **P-3, Q-5, R-4, S-1**

(P)

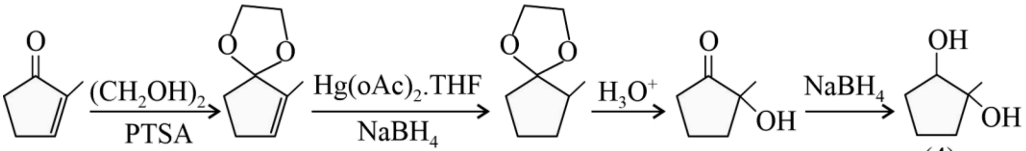


(Q)



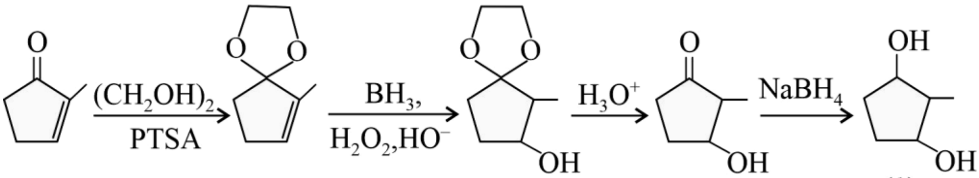
(5)

(R)



(4)

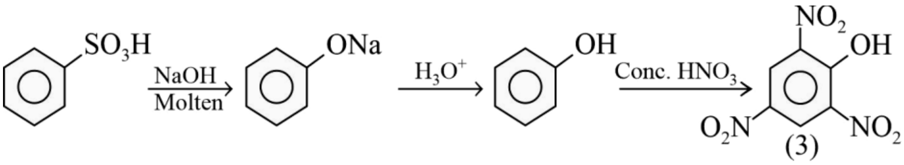
(S)



(1)

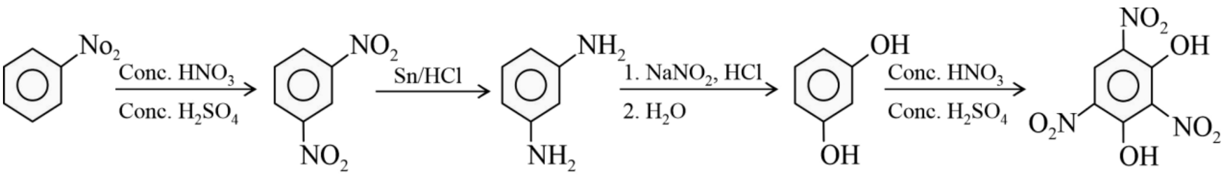
17.(C) P-3; Q-5; R-4; S-1

(P)



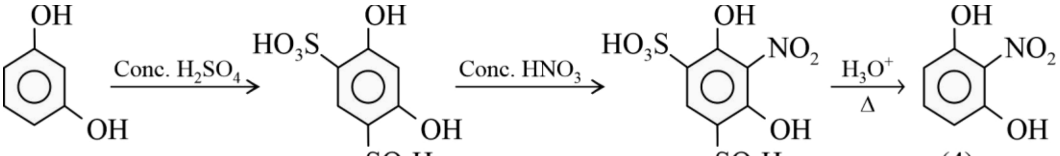
(3)

(Q)



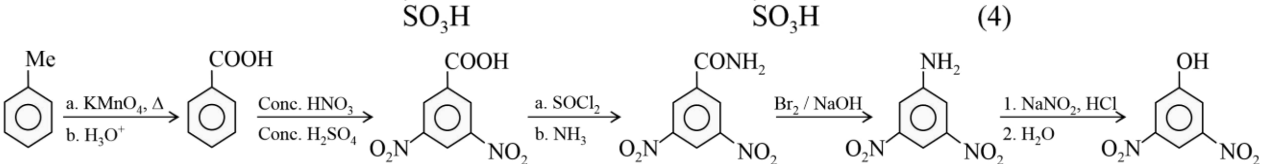
(5)

(R)



(4)

(S)



(1)

MATHEMATICS

1.(B) $\displaystyle \lim_{t\to x}\frac{t^{10}f(x)-x^{10}f(x)+x^{10}f(x)-x^{10}f(t)}{t^9-x^9}=1$

$$\Rightarrow \quad \lim_{t\to x}\left(\frac{t^{10}-x^{10}}{t^9-x^9}\right).f(x)+\lim_{t\to x}x^{10}.\frac{[f(x)-f(t)]}{t^9-x^9}=1$$

$$\Rightarrow \quad \lim_{t\to x}\frac{10t^9}{9t^8}f(x)+\lim_{t\to x}x^{10}\frac{[0-f'(t)]}{9t^8}=1$$

$$\Rightarrow \quad \frac{10}{9}.x.f(x)+\frac{x^{10}.[-f'(x)]}{9x^8}=1 \Rightarrow \quad 10.x.y-x^2.\frac{dy}{dx}=9$$

$$\Rightarrow \quad \frac{dy}{dx}-\frac{10y}{x}=\frac{-9}{x^2}$$

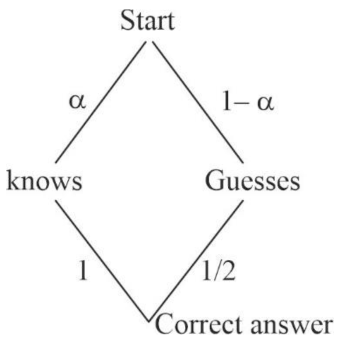
$$\text{I.f.}=e^{-\int\frac{10}{x}dx}=e^{-10\log x}=x^{-10}$$

$$y\times x^{-10}=\int\frac{-9}{x^2}\times x^{-10}dx$$

$$\Rightarrow \quad \frac{y}{x^{10}}=-9\int\frac{1}{x^{12}}dx \quad \Rightarrow \quad \frac{y}{x^{10}}=\frac{-9}{-11}\times\frac{1}{x^{11}}+c \Rightarrow y=\frac{9}{11x}+cx^{10}$$

$$x=1,\ y=2 \Rightarrow 2=\frac{9}{11}+c \Rightarrow c=\frac{13}{11}$$

2.(C)



$$\text{ATP} \quad \frac{(1-\alpha)\frac{1}{2}}{\alpha+\frac{1}{2}-\frac{\alpha}{2}}=\frac{1}{6} \Rightarrow \frac{1-\alpha}{2\alpha+1-\alpha}=\frac{1}{6}$$

$$\Rightarrow \quad 6-6\alpha=\alpha+1$$

$$\Rightarrow \quad 7\alpha=5$$

$$\Rightarrow \quad \alpha=\frac{5}{7}$$

3.(B) $\left(\sin\dfrac{11x}{2}\right)(\sin 6x-\cos 6x)+\left(\cos\dfrac{11x}{2}\right)(\sin 6x+\cos 6x)$

$$=\cos\left(6x-\frac{11}{2}\right)+\sin\left(6x-\frac{11x}{2}\right)$$

$$=\cos\frac{x}{2}+\sin\frac{x}{2}$$

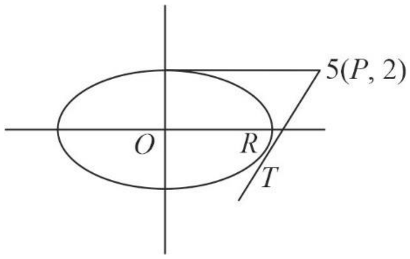
$$\cot x=\frac{-5}{\sqrt{11}} \Rightarrow \cos 2x=\frac{-5}{6}$$

$$\Rightarrow \quad \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}=\frac{-5}{6} \qquad \Rightarrow \qquad \tan^2\frac{x}{2}=11$$

$$\Rightarrow \quad \tan\frac{x}{2}=\sqrt{11} \Rightarrow \sin\frac{x}{2}=\frac{\sqrt{11}}{2\sqrt{3}}$$

$$\cos\frac{x}{2}=\frac{1}{2\sqrt{3}}$$

4.(A)



$q = 2$

$$y=mx\pm\sqrt{a^2m^2+b^2}$$

$$\Rightarrow \quad 2=p.m\pm\sqrt{9m^2+4} \qquad \Rightarrow \qquad (2-pm)^2=9m^2+4$$

$$\Rightarrow \quad 4+p^2m^2-4pm=9m^2+4 \qquad \Rightarrow \qquad (p^2-9)m^2-4pm=0$$

$m\neq 0$ For ST $\Rightarrow$ $m=\dfrac{4p}{p^2-9}$ …(1)

Equation tangent is, $mx-y-\sqrt{9m^2+4}=0$

$$\Rightarrow \quad mx-y=\sqrt{9m^2+4}$$

Let if (x_1, y_1) touches at $\Rightarrow$ $\dfrac{xx_1}{9}+\dfrac{yy_1}{4}=1$

Comparing $\dfrac{x_1}{9m}+\dfrac{y_1}{-4}=\dfrac{1}{\sqrt{9m^2+4}}$

$\Rightarrow \quad x_1 = \dfrac{9m}{\sqrt{9m^2+4}} \Rightarrow y_1 = \dfrac{-4}{9m^2+4}$

Area of $\Delta ORT = \dfrac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ \dfrac{9m}{\sqrt{9m^2+4}} & \dfrac{-4}{\sqrt{9m^2+4}} & 1 \\ 3 & 0 & 1 \end{vmatrix} = \dfrac{6}{\sqrt{9m^2+4}}$

A.T.P $\dfrac{6}{\sqrt{9m^2+4}} = \dfrac{3}{2} \Rightarrow 4^2 = 9m^2+4 \Rightarrow m^2 = \dfrac{12}{9} = \dfrac{4}{3}$

From (1) $\dfrac{16p^2}{(p^2-q)^2} = \dfrac{4}{3} \Rightarrow p = 3\sqrt{3}$

5.(ACD) For $T_2 = (\sqrt{2}+1)^n,\ n \in N$

(a) $\quad {}^nc_0 + {}^nc_2 \cdot 2 + ... + \sqrt{2}\ ({}^nc_2 + 2 \cdot {}^nc_3 + ...)$

$T_1 = \{(-1+\sqrt{2})^n :\ n \in N\}$

${}^nc_0(-1)^n + {}^nc_2(-1)^{n-2}2 + ... + \sqrt{2}(\ {}^nc_1(-1)^{n-1} + ...)$

and $S = \left\{a + b\sqrt{2}, a, b \in Z\right\}$

Notice $T_1, T_2 \in S$ and $ab = 0$ for $Z\,C\,S$

(b) $\quad$ Same $\lim\limits_{n\to\infty}\left(-1+\sqrt{2}\right)^n = 0$

So, $\exists\ N : \forall n \geq N \Rightarrow (-1+\sqrt{2})^n < \dfrac{1}{2024}$

(c) $\quad$ Notice $\left(\sqrt{2}+1\right)^n$ is unbounded and $2^{11} > 2024$

$\Rightarrow (\sqrt{2}+1)^n > 2024$ for all $n \geq 11$

(d) $\quad$ For $\cos\left(\pi(a+b\sqrt{2})\right) + i\sin\left(\pi(a+b\sqrt{2})\right)$ to be an integer.

$\Rightarrow\ a + b\sqrt{2} = I$ (some integer) $\Rightarrow\ b = 0$

6.(BCD) $ax^2 + 2bxy + cy^2 > 0 \quad \forall (x, y) \in R^2 - \{(0,\ 0)\}$

If $y = 0$, we get $ax^2 > 0 \ \ \forall x \in R - \{0\}$

$\Rightarrow\ a > 0$

when $y \neq 0$, we get

$$a\left(\frac{x}{y}\right)^2 + 2b\left(\frac{x}{y}\right) + c > 0 \ \ \forall (x, y) \in R \times R$$

$\Rightarrow$ $a > 0$ and discriminant < 0

$\Rightarrow$ $b^2 < ac \ \Rightarrow c > 0$

(A) incorrect

(B) correct

(C) correct because $\begin{vmatrix} a & b \\ b & c \end{vmatrix} = ac - b^2 > 0$

(D) correct

Infinite solution iff $(a+1)(c+1) = b^2$

$\Rightarrow$ $(a+1)(c+1) < ac$

Not possible because $a, c > 0$

7.(ABCD)

$$S = (h-1)^2 + (k-2)^2 + (l-3)^2 - (h-4)^2 - (k-2)^2 - (l-7)^2 = 50$$

$\Rightarrow$ $(2h-5)(3) + (2l-10)(4) = 50$

$\Rightarrow$ $6x + 8y = 105$

For T: $6x + 8y = 5$

Distance between planes $= \dfrac{100}{10} = 10$ units

(A) Correct $\because$ S is a plane on which we can find such triangle

(B) Correct $\because$ T is a plane in which we can find L & M

(C) and (D)

Correct $\because$ we can use 10 unit distinct as 2 sides to correct 2 parallel planes without any loss of generality assume we have a *xy* plane 4 another plane at 10 unit distance from it we can take (0, 0, 10), (12, 0, 10), $(0, 2\sqrt{6}, 0), (12, 2\sqrt{6}, 0)$ for rectangle we can think about many such points, we can take the $\perp$ distance of 10 units to be 2 sides as well and one side each on the places.

8.(8) $\log_a (18)^{5/4} = \dfrac{5}{4}\log_{3\sqrt{2}} 18 = \dfrac{5}{4} \times 2 = \dfrac{5}{2}$

$\Rightarrow$ $3x + 2y = \dfrac{5}{2}$

$$b^6 = \frac{1}{5 \times 6^3} = \frac{1}{1080} = (1080)^{-1}$$

$\Rightarrow$ $b = (1080)^{-1/6}$

$$2x - y = \log_b (1080)^{\frac{1}{2}} = \frac{1}{2} \times \frac{1}{\left(\frac{-1}{6}\right)} \times 1$$

$\Rightarrow \quad 2x - y = -3$

$\Rightarrow \quad 4x - 2y = -6 \qquad \ldots(2)$

$3x + 2y = \frac{5}{2} \qquad \ldots(1)$

Add $\quad 9x = \frac{-7}{2} \Rightarrow x = \frac{-1}{2}$

$\Rightarrow \quad 4x = -2$

$\Rightarrow \quad y = 2 \Rightarrow 5y = 10$

$4x + 5y = -2 + 10 = 8$

9.(20) **Part-1**

$f'(x) = x[4x^2 + 3ax + 2b]$ all coefficient $\in R$ & $i\sqrt{3}$ is are root of $f'(x)$ other root is $-i\sqrt{3}$ real root of $f(x)$ is clearly '0'

sum of roots of $f'(x) = 0 \Rightarrow a = 0$

Product of root of $4x^2 + 3ax + 2b = 0$ is $\frac{2b}{4} = i\sqrt{3} \times (-i\sqrt{3})$

$\Rightarrow b = 6$

Part-2

$f(x) = x^4 + 6x^2 + c$

$f(1) = -9$

$\Rightarrow f(x) = x^4 + 6x^2 - 16$

$= \left(x^2 + 8\right)(x^2 - 2)$

$\therefore$ Roots are $\alpha_1 = 2\sqrt{2}i$

$\alpha_2 = -2\sqrt{2}i$

$\alpha_3 = \sqrt{2}$

$\alpha_4 = -\sqrt{2}$

$\sum |\alpha_1|^2 = 8 + 8 + 2 + 2 = 20$

10.(16) Part-1

$$|A| = \begin{vmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 1 & c \\ 0 & a-b & d-e \\ 1 & b & e \end{vmatrix} \quad \{R_2 \to R_2 - R_3\}$$

$$|A| = (d-e) - c(a-b)$$

Since : $a,\ b,\ c,\ d,\ e \in \{0,\ 1\}$

Total possibilities = 32

Part-2

Only odd integral values taken by $|A|$ can be ± 1

$\Rightarrow$ We want $|A|$ to be odd

So, bad cases are those with $|A|$ even

Let's count bad cases.

Case : 1 $(d - e)$ and $c(a - b)$ are even

For $d - e$ to be even : $d = e$

$\Rightarrow$ 2 cases

Part-3

For $c(a-b)$ to be even

$c = 0 \Rightarrow$ 4 cases

$c = 1 \Rightarrow a = b \Rightarrow$ 2 case

Total 12 cases

Case-2: $d - e$ and $c(a - b)$ are odd

d-e is odd $\Rightarrow d \neq e \Rightarrow$ 2 cases

$c(a - b)$ is odd $\Rightarrow c = 1$ and $a \neq b \Rightarrow$ 2 cases

Total 4 cases

$\Rightarrow$ Bad cases = 16

And Good cases = 16 too

ALTERNATIVELY : There is a bijective between good and bad cases. We can switch the value of d to map a good case to bad case.

But we want $|A| \in \{-1,\ 1\}$

$d - e \in \{-1,\ 0,\ 1\}$

$c(a-b) \in \{-1,\ 0,\ 1\}$

11.(665) A_1 : selection in any manner

A_2 : S_1 in team X

A_3 : S_2 in team Y

A_4 : S_1 & S_2 in team X & Y respectively

By principle inclusion exclusion

Answer = $n(A_1)-n(A_2)-n(A_3)+n(A_4)$

$=\frac{9!}{4!3!2!}-\frac{8!}{1!3!4!}-\frac{8!}{2!2!4!}+\frac{7!}{1!2!4!}$ = 1260 – 280 – 420 + 105 = 665

12.(5)

$$=\begin{vmatrix}\frac{\alpha-1}{\alpha} & 1 & 1\\ 1 & \frac{\beta-1}{\beta} & 1\\ 1 & 1 & \frac{1}{2}\end{vmatrix}=0=\begin{vmatrix}\alpha-1 & \alpha & \alpha\\ \beta & \beta-1 & \beta\\ 2 & 2 & 1\end{vmatrix}$$

$$=\begin{vmatrix}-1 & 0 & \alpha\\ 1 & -1 & \beta\\ 0 & 1 & 1\end{vmatrix}=-1(-1-\beta)+\alpha(1)=0 \Rightarrow \quad 1+\beta+\alpha=0$$

$\Rightarrow \quad 3\alpha+3\beta-2+l=0 \quad \Rightarrow \quad 1+\beta+\alpha=0$

$\Rightarrow \quad -3-2+l=0$

$l=5$

13.(42) Let $P(X=0)=a$

$P(X=1)=a+d \qquad \Rightarrow$ Slope of straight line = d

and y-intercept = a

$\Rightarrow P(X=2)=a+2d$

$P(X=3)=a+3d$

$P(X=4)=a+4d$

Since, they sum up to 1

$\Rightarrow P(X=2)=\frac{1}{5}$

Mean $=0\left(\frac{1}{5}-2d\right)+1\left(\frac{1}{5}-d\right)+2\left(\frac{1}{5}\right)+3\left(\frac{1}{5}+d\right)+4\left(\frac{1}{5}+2d\right)=2+10d=2.5$

$\Rightarrow \; d=0.05$

$E(X^2)=0.1(0^2)+0.15(1^2)+0.2(2^2)+0.25(3^2)+0.3(4^2)=8$

Variance $=8-(2.5)^2=1.75$

14.(C) Given $\alpha+\beta+1=0$

(P) Only possibility is every row and column must be a permutation of $(1,\alpha,\beta)$ for row 1, we can arrange $1,\alpha,\beta$ in 3! ways.

Step 1: $\begin{bmatrix}1 & \alpha & \beta\end{bmatrix}$

Now in column 1, the remaining 2 entries can be arranged in 2! Ways.

Step 2: $\begin{bmatrix}1 & \alpha & \beta \\ \beta & & \\ \alpha & & \end{bmatrix}$

Also after this, for the remaining 4 entries, only 1 arrangement is possible.

$$\begin{bmatrix}1 & \alpha & \beta \\ \beta & 1 & \alpha \\ \alpha & \beta & 1\end{bmatrix}$$

Total matrices = 3! × 2! = 12

$\Rightarrow \ (p)\to(2)$

(Q) Note that M being symmetric

$\Rightarrow \quad C_j=0\ \forall_j$ iff $R_j=0\ \forall j$

Hence, we count similar to the previous case except that in step 2, we don't have 2! Options as symmetric nature of the matrics determines 1st column from 1st row.

$\Rightarrow \quad 3!\text{matrices} \Rightarrow (Q)\to(4)$

(S) (5) because $R_i=0\forall_i$

$\Rightarrow$ the operation $C_1\to C_1+C_2+C_3$ will make the 1st column = 0

$\Rightarrow$ Determinant is always 0.

(R) M is skew symmetric of odd order

$\Rightarrow \quad |M|=0$

Hence, either no solution or infinitely many solution of $M\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}a_{12}\\0\\-a_{23}\end{bmatrix}$

$\Rightarrow (R)\to(3)$

Note: Clearly: $x=0$, $y=1$, $z=0$ is a possible solution.

Also we can confirm infinite solution using Cramer's rule with $\Delta_1=\Delta_2=\Delta_3=0=\Delta$

15.(C) P$\to$ 4, Q$\to$ 2, R$\to$ 5, S$\to$ 3

$CP=r \Rightarrow \dfrac{|2\times 0-\alpha|}{\sqrt{5}}=r$

$\Rightarrow \quad \alpha=\sqrt{5}r$

$\alpha+r=5+\sqrt{5}$

$\Rightarrow \quad \sqrt{5}r + r = 5 + \sqrt{5} \quad \Rightarrow \quad r = \sqrt{5}$

$\therefore$ circle is $x^2 + (y-5)^2 = 5$

Solving with $y = 2x$

$\Rightarrow \quad x^2 + (2x-5)^2 = 5 \quad \Rightarrow \quad 5x^2 - 20x + 25 = 5$

$\Rightarrow \quad 5x^2 - 20x + 20 = 0 \quad \Rightarrow \quad x^2 - 4x + 4 = 0 \quad \Rightarrow \quad x = 2$

$\therefore \quad A_1 = (2,\ 4)$

$B_1 = (h,\ k)$

$\frac{h+2}{2} = 0 \Rightarrow h = -2$

$\frac{k+4}{2} = 5 \Rightarrow k = 6$

$B = (-2, 6)$

16.(C) P $\rightarrow$ 3, Q $\rightarrow$ 4, R $\rightarrow$ 1, S $\rightarrow$ 5

Point on $L_1 = (\lambda - 11,\ 2\lambda - 21,\ 3\lambda - 29)$

Point on $L_2 = (3\mu - 16,\ 2\mu - 11,\ \mu\gamma - 4)$

A.T.P. $3\mu - 16 = \lambda - 11$

$\Rightarrow \lambda = 3\mu - 5$

& $\quad 2\lambda - 21 = 2\mu - 11 \Rightarrow 2\mu = 2\lambda - 10$

$\Rightarrow \quad 2\mu = 2(3\mu - 5) - 10$

$\Rightarrow \quad 2\mu = 6\mu - 20$

$\Rightarrow \quad \mu = 5,\ \lambda = 10$

$R_1 = (-1, -1, 1)$

$5\lambda - 4 = 1 \Rightarrow \lambda = 1$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 4\hat{i} - 8\hat{j} + 4\hat{k}$$

$$\hat{n} = \frac{\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{6}}$$

$\overline{OR_1} = -\hat{i} - \hat{j} + \hat{k}$

$$\overline{OR_1} \cdot \hat{n} = \frac{-1+2+1}{\sqrt{6}} = \frac{2}{\sqrt{6}} = \frac{2}{\sqrt{2} \times \sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}$$

17.(C) x takes all values except between $\left[0, \frac{1}{2}\right]$ but for $0 \le x \le \frac{1}{2}$, $3x - 1$ takes all the values in the interval $\left[0, \frac{1}{2}\right]$

Range $= \mathbb{R}$

$$f(x) = \begin{cases} x|x|\sin\left(\frac{1}{x}\right), & x \ne 0 \\ 0 & x = 0 \end{cases} \quad \textit{and} \quad g(x) = \begin{cases} 1-2x, & 0 \le 2x \le 1 \\ 0, & \text{other wise} \end{cases}$$

$$g\left(\frac{1}{2} - x\right) = \begin{cases} 2x, & 0 \le x \le \frac{1}{2} \\ 0, & \text{other wise} \end{cases}$$

$$\Rightarrow \quad g(x) + g\left(\frac{1}{2} - x\right) = \begin{cases} 1, & 0 \le x \le \frac{1}{2} \\ 0 & \text{other wise} \end{cases}$$

(P) when $a = 0$, $b = 1$, $c = 0$, $d = 0$

$$h(x) = \begin{cases} 1, & 0 \le x \le \frac{1}{2} \\ 0 & \text{other wise} \end{cases}$$

Range of h is {0, 1}

(Q) When $a = 1$, $b = c = d = 0$

$h(x) = f(x)$ which is differentiate on $\mathbb{R}$

$$\text{As } \lim_{h \to 0^+} \frac{h^2 \sin\left(\frac{1}{h}\right)}{h} = \lim_{h \to 0} -\frac{h^2 \sin\left(\frac{1}{h}\right)}{h} = 0$$

(R) when $a = b = d = 0$ and $c = 1$

$$h(x) = \begin{cases} 3x-1, & 0 \le x \le \frac{1}{2} \\ x & \text{other wise} \end{cases}$$

x takes all values except between $\left[0, \frac{1}{2}\right]$ but for takes all the values in the interval $\left[0, \frac{1}{2}\right]$

Range $= \mathbb{R}$

(S) $a = b = c = 0$ and $d = 1$

$$h(x) = g(x) = \begin{cases} 1-2x, & 0 \le x \le \frac{1}{2} \\ 0, & \text{other wise} \end{cases}$$

Clearly, g is continuous and decreasing in $\left[0, \frac{1}{2}\right] \Rightarrow g\left(\frac{1}{2}\right) \le g(x) \le g(0)$

$\Rightarrow \ 0 \le g(x) \le 1$

Range [0, 1]