

## Mathematics

## SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:  
*Full Marks* : +3 If **ONLY** the correct option is chosen;  
*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);  
*Negative Marks* : -1 In all other cases.

Q.1 Considering only the principal values of the inverse trigonometric functions, the value of

$$\tan \left( \sin^{-1} \left( \frac{3}{5} \right) - 2 \cos^{-1} \left( \frac{2}{\sqrt{5}} \right) \right)$$

is

(A)  $\frac{7}{24}$   
(C)  $\frac{-5}{24}$

(B)  $\frac{-7}{24}$   
(D)  $\frac{5}{24}$

Q.2 Let  $S = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x \geq 0, y \geq 0, y^2 \leq 4x, y^2 \leq 12 - 2x \text{ and } 3y + \sqrt{8}x \leq 5\sqrt{8}\}$ . If the area of the region  $S$  is  $\alpha\sqrt{2}$ , then  $\alpha$  is equal to

(A)  $\frac{17}{2}$

(B)  $\frac{17}{3}$

(C)  $\frac{17}{4}$

(D)  $\frac{17}{5}$

Q.3 Let  $k \in \mathbb{R}$ . If  $\lim_{x \rightarrow 0^+} (\sin(\sin kx) + \cos x + x)^{\frac{2}{x}} = e^6$ , then the value of  $k$  is

(A) 1

(B) 2

(C) 3

(D) 4

Q.4 Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a function defined by

$$f(x) = \begin{cases} x^2 \sin\left(\frac{\pi}{x^2}\right), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

Then which of the following statements is TRUE?

- (A)  $f(x) = 0$  has infinitely many solutions in the interval  $\left[\frac{1}{10^{10}}, \infty\right)$ .
- (B)  $f(x) = 0$  has no solutions in the interval  $\left[\frac{1}{\pi}, \infty\right)$ .
- (C) The set of solutions of  $f(x) = 0$  in the interval  $\left(0, \frac{1}{10^{10}}\right)$  is finite.
- (D)  $f(x) = 0$  has more than 25 solutions in the interval  $\left(\frac{1}{\pi^2}, \frac{1}{\pi}\right)$ .

**SECTION 2 (Maximum Marks: 12)**

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:  
*Full Marks* : +4 **ONLY** if (all) the correct option(s) is(are) chosen;  
*Partial Marks* : +3 If all the four options are correct but **ONLY** three options are chosen;  
*Partial Marks* : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;  
*Partial Marks* : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;  
*Zero Marks* : 0 If unanswered;  
*Negative Marks* : -2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then  
 choosing **ONLY** (A), (B) and (D) will get +4 marks;  
 choosing **ONLY** (A) and (B) will get +2 marks;  
 choosing **ONLY** (A) and (D) will get +2 marks;  
 choosing **ONLY** (B) and (D) will get +2 marks;  
 choosing **ONLY** (A) will get +1 mark;  
 choosing **ONLY** (B) will get +1 mark;  
 choosing **ONLY** (D) will get +1 mark;  
 choosing no option(s) (i.e. the question is unanswered) will get 0 marks and  
 choosing any other option(s) will get -2 marks.

Q.5 Let  $S$  be the set of all  $(\alpha, \beta) \in \mathbb{R} \times \mathbb{R}$  such that

$$\lim_{x \rightarrow \infty} \frac{\sin(x^2)(\log_e x)^\alpha \sin\left(\frac{1}{x^2}\right)}{x^{\alpha\beta}(\log_e(1+x))^\beta} = 0.$$

Then which of the following is (are) correct?

- (A)  $(-1, 3) \in S$
- (B)  $(-1, 1) \in S$
- (C)  $(1, -1) \in S$
- (D)  $(1, -2) \in S$

- Q.6 A straight line drawn from the point  $P(1, 3, 2)$ , parallel to the line  $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z-6}{1}$ , intersects the plane  $L_1 : x - y + 3z = 6$  at the point  $Q$ . Another straight line which passes through  $Q$  and is perpendicular to the plane  $L_1$  intersects the plane  $L_2 : 2x - y + z = -4$  at the point  $R$ . Then which of the following statements is (are) TRUE?

- (A) The length of the line segment  $PQ$  is  $\sqrt{6}$
- (B) The coordinates of  $R$  are  $(1, 6, 3)$
- (C) The centroid of the triangle  $PQR$  is  $\left(\frac{4}{3}, \frac{14}{3}, \frac{5}{3}\right)$
- (D) The perimeter of the triangle  $PQR$  is  $\sqrt{2} + \sqrt{6} + \sqrt{11}$

- Q.7 Let  $A_1, B_1, C_1$  be three points in the  $xy$ -plane. Suppose that the lines  $A_1C_1$  and  $B_1C_1$  are tangents to the curve  $y^2 = 8x$  at  $A_1$  and  $B_1$ , respectively. If  $O = (0, 0)$  and  $C_1 = (-4, 0)$ , then which of the following statements is (are) TRUE?

- (A) The length of the line segment  $OA_1$  is  $4\sqrt{3}$
- (B) The length of the line segment  $A_1B_1$  is 16
- (C) The orthocenter of the triangle  $A_1B_1C_1$  is  $(0, 0)$
- (D) The orthocenter of the triangle  $A_1B_1C_1$  is  $(1, 0)$

## SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:  
*Full Marks* : +4 If **ONLY** the correct integer is entered;  
*Zero Marks* : 0 In all other cases.

Q.8 Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a function such that  $f(x+y) = f(x) + f(y)$  for all  $x, y \in \mathbb{R}$ , and  $g: \mathbb{R} \rightarrow (0, \infty)$  be a function such that  $g(x+y) = g(x)g(y)$  for all  $x, y \in \mathbb{R}$ . If  $f\left(\frac{-3}{5}\right) = 12$  and  $g\left(\frac{-1}{3}\right) = 2$ , then the value of  $\left(f\left(\frac{1}{4}\right) + g(-2) - 8\right)g(0)$  is \_\_\_\_\_.

Q.9 A bag contains  $N$  balls out of which 3 balls are white, 6 balls are green, and the remaining balls are blue. Assume that the balls are identical otherwise. Three balls are drawn randomly one after the other without replacement. For  $i = 1, 2, 3$ , let  $W_i, G_i$ , and  $B_i$  denote the events that the ball drawn in the  $i^{\text{th}}$  draw is a white ball, green ball, and blue ball, respectively. If the probability  $P(W_1 \cap G_2 \cap B_3) = \frac{2}{5N}$  and the conditional probability  $P(B_3 | W_1 \cap G_2) = \frac{2}{9}$ , then  $N$  equals \_\_\_\_\_.

Q.10 Let the function  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined by

$$f(x) = \frac{\sin x (x^{2023} + 2024x + 2025)}{e^{\pi x} (x^2 - x + 3)} + \frac{2 (x^{2023} + 2024x + 2025)}{e^{\pi x} (x^2 - x + 3)}.$$

Then the number of solutions of  $f(x) = 0$  in  $\mathbb{R}$  is \_\_\_\_\_.

Q.11 Let  $\vec{p} = 2\hat{i} + \hat{j} + 3\hat{k}$  and  $\vec{q} = \hat{i} - \hat{j} + \hat{k}$ . If for some real numbers  $\alpha, \beta$ , and  $\gamma$ , we have  $15\hat{i} + 10\hat{j} + 6\hat{k} = \alpha(2\vec{p} + \vec{q}) + \beta(\vec{p} - 2\vec{q}) + \gamma(\vec{p} \times \vec{q})$ , then the value of  $\gamma$  is \_\_\_\_\_.

- Q.12 A normal with slope  $\frac{1}{\sqrt{6}}$  is drawn from the point  $(0, -\alpha)$  to the parabola  $x^2 = -4ay$ , where  $a > 0$ . Let  $L$  be the line passing through  $(0, -\alpha)$  and parallel to the directrix of the parabola. Suppose that  $L$  intersects the parabola at two points  $A$  and  $B$ . Let  $r$  denote the length of the latus rectum and  $s$  denote the square of the length of the line segment  $AB$ . If  $r : s = 1 : 16$ , then the value of  $24a$  is \_\_\_\_\_.

- Q.13 Let the function  $f : [1, \infty) \rightarrow \mathbb{R}$  be defined by

$$f(t) = \begin{cases} (-1)^{n+1} 2, & \text{if } t = 2n-1, n \in \mathbb{N}, \\ \frac{(2n+1-t)}{2} f(2n-1) + \frac{(t-(2n-1))}{2} f(2n+1), & \text{if } 2n-1 < t < 2n+1, n \in \mathbb{N}. \end{cases}$$

Define  $g(x) = \int_1^x f(t) dt$ ,  $x \in (1, \infty)$ . Let  $\alpha$  denote the number of solutions of the equation

$g(x) = 0$  in the interval  $(1, 8]$  and  $\beta = \lim_{x \rightarrow 1^+} \frac{g(x)}{x-1}$ . Then the value of  $\alpha + \beta$  is equal to \_\_\_\_\_.

**SECTION 4 (Maximum Marks: 12)**

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:  
*Full Marks* : +3 If ONLY the correct numerical value is entered in the designated place;  
*Zero Marks* : 0 In all other cases.

**PARAGRAPH “I”**

Let  $S = \{1, 2, 3, 4, 5, 6\}$  and  $X$  be the set of all relations  $R$  from  $S$  to  $S$  that satisfy both the following properties:

- $R$  has exactly 6 elements.
- For each  $(a, b) \in R$ , we have  $|a - b| \geq 2$ .

Let  $Y = \{R \in X : \text{The range of } R \text{ has exactly one element}\}$  and

$Z = \{R \in X : R \text{ is a function from } S \text{ to } S\}$ .

Let  $n(A)$  denote the number of elements in a set  $A$ .

**(There are two questions based on PARAGRAPH “I”, the question given below is one of them)**

Q.14 If  $n(X) = {}^m C_6$ , then the value of  $m$  is \_\_\_\_\_.

**PARAGRAPH “I”**

Let  $S = \{1, 2, 3, 4, 5, 6\}$  and  $X$  be the set of all relations  $R$  from  $S$  to  $S$  that satisfy both the following properties:

- $R$  has exactly 6 elements.
- For each  $(a, b) \in R$ , we have  $|a - b| \geq 2$ .

Let  $Y = \{R \in X : \text{The range of } R \text{ has exactly one element}\}$  and

$Z = \{R \in X : R \text{ is a function from } S \text{ to } S\}$ .

Let  $n(A)$  denote the number of elements in a set  $A$ .

**(There are two questions based on PARAGRAPH “I”, the question given below is one of them)**

Q.15 If the value of  $n(Y) + n(Z)$  is  $k^2$ , then  $|k|$  is \_\_\_\_\_.

**PARAGRAPH “II”**

Let  $f : \left[0, \frac{\pi}{2}\right] \rightarrow [0, 1]$  be the function defined by  $f(x) = \sin^2 x$  and let  $g : \left[0, \frac{\pi}{2}\right] \rightarrow [0, \infty)$  be the function defined by  $g(x) = \sqrt{\frac{\pi x}{2} - x^2}$ .

**(There are two questions based on PARAGRAPH “II”, the question given below is one of them)**

Q.16 The value of  $2 \int_0^{\frac{\pi}{2}} f(x)g(x)dx - \int_0^{\frac{\pi}{2}} g(x)dx$  is \_\_\_\_\_.

**PARAGRAPH “II”**

Let  $f : \left[0, \frac{\pi}{2}\right] \rightarrow [0, 1]$  be the function defined by  $f(x) = \sin^2 x$  and let  $g : \left[0, \frac{\pi}{2}\right] \rightarrow [0, \infty)$  be the function defined by  $g(x) = \sqrt{\frac{\pi x}{2} - x^2}$ .

**(There are two questions based on PARAGRAPH “II”, the question given below is one of them)**

Q.17 The value of  $\frac{16}{\pi^3} \int_0^{\frac{\pi}{2}} f(x)g(x)dx$  is \_\_\_\_\_.

**END OF THE QUESTION PAPER**

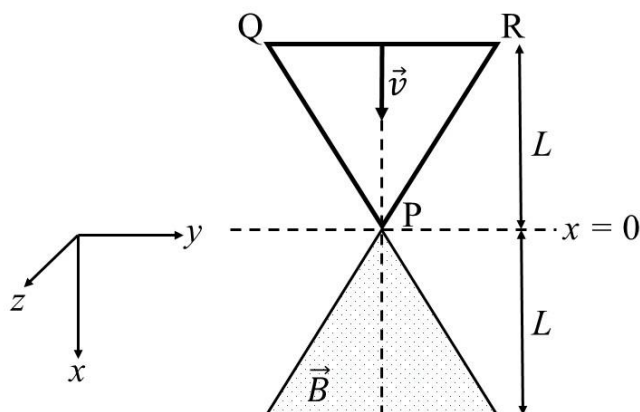


## Physics

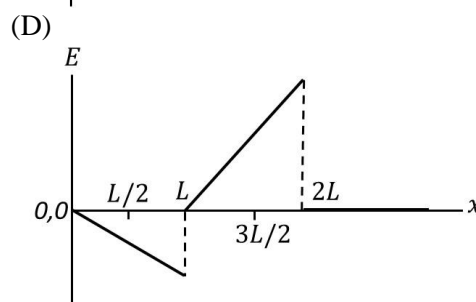
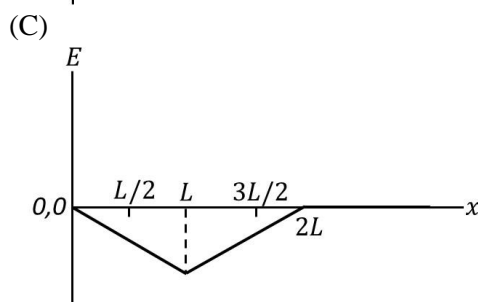
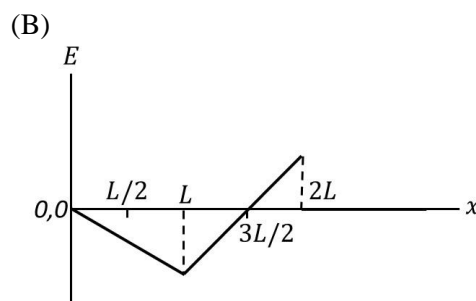
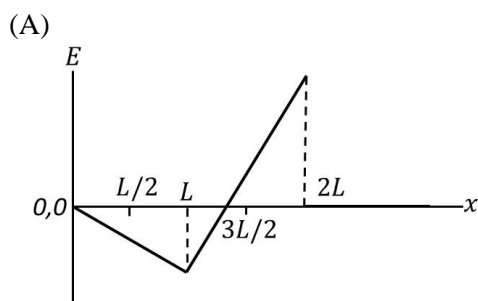
## SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:  
*Full Marks* : +3 If **ONLY** the correct option is chosen;  
*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);  
*Negative Marks* : -1 In all other cases.

- Q.1 A region in the form of an equilateral triangle (in  $x$ - $y$  plane) of height  $L$  has a uniform magnetic field  $\vec{B}$  pointing in the  $+z$ -direction. A conducting loop PQR, in the form of an equilateral triangle of the same height  $L$ , is placed in the  $x$ - $y$  plane with its vertex P at  $x = 0$  in the orientation shown in the figure. At  $t = 0$ , the loop starts entering the region of the magnetic field with a uniform velocity  $\vec{v}$  along the  $+x$ -direction. The plane of the loop and its orientation remain unchanged throughout its motion.



Which of the following graph best depicts the variation of the induced emf ( $E$ ) in the loop as a function of the distance ( $x$ ) starting from  $x = 0$ ?



- Q.2 A particle of mass  $m$  is under the influence of the gravitational field of a body of mass  $M$  ( $\gg m$ ). The particle is moving in a circular orbit of radius  $r_0$  with time period  $T_0$  around the mass  $M$ . Then, the particle is subjected to an additional central force, corresponding to the potential energy  $V_c(r) = m\alpha/r^3$ , where  $\alpha$  is a positive constant of suitable dimensions and  $r$  is the distance from the center of the orbit. If the particle moves in the same circular orbit of radius  $r_0$  in the combined gravitational potential due to  $M$  and  $V_c(r)$ , but with a new time period  $T_1$ , then  $(T_1^2 - T_0^2)/T_1^2$  is given by

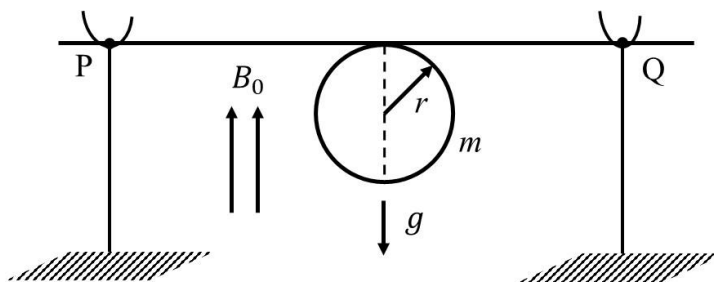
[ $G$  is the gravitational constant.]

- (A)  $\frac{3\alpha}{GM r_0^2}$  (B)  $\frac{\alpha}{2GM r_0^2}$  (C)  $\frac{\alpha}{GM r_0^2}$  (D)  $\frac{2\alpha}{GM r_0^2}$

- Q.3 A metal target with atomic number  $Z = 46$  is bombarded with a high energy electron beam. The emission of X-rays from the target is analyzed. The ratio  $r$  of the wavelengths of the  $K_\alpha$ -line and the cut-off is found to be  $r = 2$ . If the same electron beam bombards another metal target with  $Z = 41$ , the value of  $r$  will be

- (A) 2.53 (B) 1.27 (C) 2.24 (D) 1.58

- Q.4 A thin stiff insulated metal wire is bent into a circular loop with its two ends extending tangentially from the same point of the loop. The wire loop has mass  $m$  and radius  $r$  and it is in a uniform vertical magnetic field  $B_0$ , as shown in the figure. Initially, it hangs vertically downwards, because of acceleration due to gravity  $g$ , on two conducting supports at P and Q. When a current  $I$  is passed through the loop, the loop turns about the line PQ by an angle  $\theta$  given by

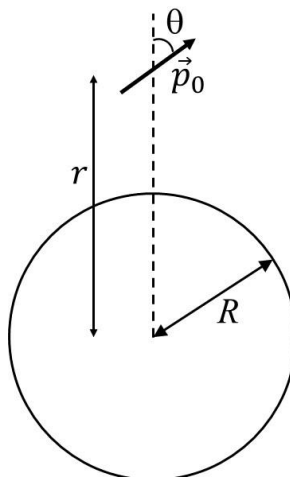


- (A)  $\tan \theta = \pi r I B_0 / (mg)$  (B)  $\tan \theta = 2\pi r I B_0 / (mg)$   
(C)  $\tan \theta = \pi r I B_0 / (2mg)$  (D)  $\tan \theta = mg / (\pi r I B_0)$

**SECTION 2 (Maximum Marks: 12)**

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:  
*Full Marks* : +4 **ONLY** if (all) the correct option(s) is(are) chosen;  
*Partial Marks* : +3 If all the four options are correct but **ONLY** three options are chosen;  
*Partial Marks* : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;  
*Partial Marks* : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;  
*Zero Marks* : 0 If unanswered;  
*Negative Marks* : -2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then  
 choosing **ONLY** (A), (B) and (D) will get +4 marks;  
 choosing **ONLY** (A) and (B) will get +2 marks;  
 choosing **ONLY** (A) and (D) will get +2 marks;  
 choosing **ONLY** (B) and (D) will get +2 marks;  
 choosing **ONLY** (A) will get +1 mark;  
 choosing **ONLY** (B) will get +1 mark;  
 choosing **ONLY** (D) will get +1 mark;  
 choosing no option(s) (i.e. the question is unanswered) will get 0 marks and  
 choosing any other option(s) will get -2 marks.

- Q.5 A small electric dipole  $\vec{p}_0$ , having a moment of inertia  $I$  about its center, is kept at a distance  $r$  from the center of a spherical shell of radius  $R$ . The surface charge density  $\sigma$  is uniformly distributed on the spherical shell. The dipole is initially oriented at a small angle  $\theta$  as shown in the figure. While staying at a distance  $r$ , the dipole is free to rotate about its center.



If released from rest, then which of the following statement(s) is(are) correct?

[ $\epsilon_0$  is the permittivity of free space.]

- (A) The dipole will undergo small oscillations at any finite value of  $r$ .  
 (B) The dipole will undergo small oscillations at any finite value of  $r > R$ .  
 (C) The dipole will undergo small oscillations with an angular frequency of  $\sqrt{\frac{2\sigma p_0}{\epsilon_0 I}}$  at  $r = 2R$ .  
 (D) The dipole will undergo small oscillations with an angular frequency of  $\sqrt{\frac{\sigma p_0}{100\epsilon_0 I}}$  at  $r = 10R$ .

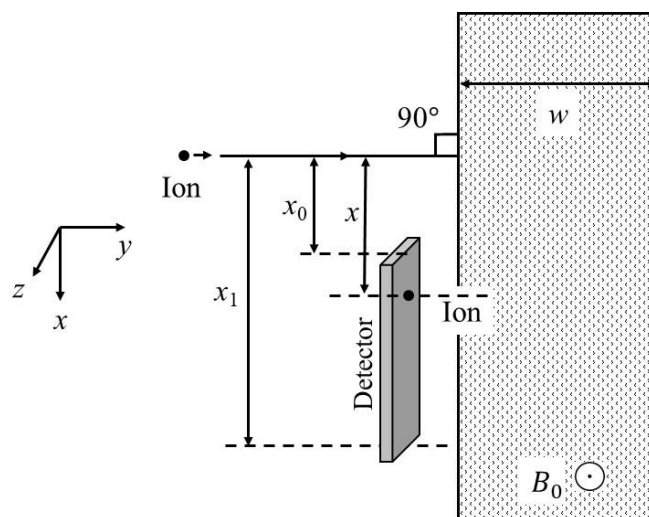
- Q.6 A table tennis ball has radius  $(3/2) \times 10^{-2}$  m and mass  $(22/7) \times 10^{-3}$  kg. It is slowly pushed down into a swimming pool to a depth of  $d = 0.7$  m below the water surface and then released from rest. It emerges from the water surface at speed  $v$ , without getting wet, and rises up to a height  $H$ . Which of the following option(s) is(are) correct?

[Given:  $\pi = 22/7$ ,  $g = 10 \text{ m s}^{-2}$ , density of water =  $1 \times 10^3 \text{ kg m}^{-3}$ , viscosity of water =  $1 \times 10^{-3} \text{ Pa-s}$ .]

- (A) The work done in pushing the ball to the depth  $d$  is 0.077 J.  
 (B) If we neglect the viscous force in water, then the speed  $v = 7 \text{ m/s}$ .  
 (C) If we neglect the viscous force in water, then the height  $H = 1.4 \text{ m}$ .  
 (D) The ratio of the magnitudes of the net force excluding the viscous force to the maximum viscous force in water is 500/9.

- Q.7 A positive, singly ionized atom of mass number  $A_M$  is accelerated from rest by the voltage 192 V. Thereafter, it enters a rectangular region of width  $w$  with magnetic field  $\vec{B}_0 = 0.1\hat{k}$  Tesla, as shown in the figure. The ion finally hits a detector at the distance  $x$  below its starting trajectory.

[Given: Mass of neutron/proton =  $(5/3) \times 10^{-27}$  kg, charge of the electron =  $1.6 \times 10^{-19}$  C.]



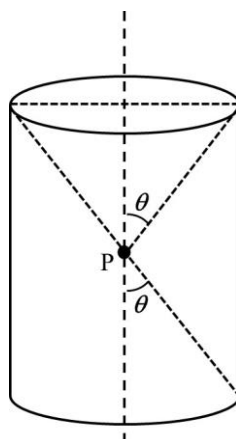
Which of the following option(s) is(are) correct?

- (A) The value of  $x$  for  $H^+$  ion is 4 cm.
- (B) The value of  $x$  for an ion with  $A_M = 144$  is 48 cm.
- (C) For detecting ions with  $1 \leq A_M \leq 196$ , the minimum height ( $x_1 - x_0$ ) of the detector is 55 cm.
- (D) The minimum width  $w$  of the region of the magnetic field for detecting ions with  $A_M = 196$  is 56 cm.

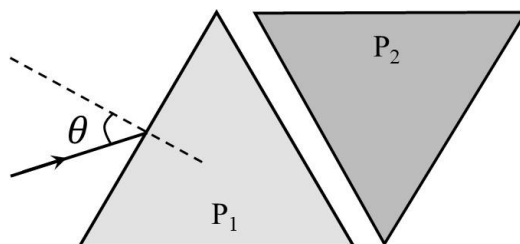
**SECTION 3 (Maximum Marks: 24)**

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:  
*Full Marks* : +4 If **ONLY** the correct integer is entered;  
*Zero Marks* : 0 In all other cases.

- Q.8 The dimensions of a cone are measured using a scale with a least count of 2 mm. The diameter of the base and the height are both measured to be 20.0 cm. The maximum percentage error in the determination of the volume is \_\_\_\_\_.
- Q.9 A ball is thrown from the location  $(x_0, y_0) = (0, 0)$  of a horizontal playground with an initial speed  $v_0$  at an angle  $\theta_0$  from the  $+x$ -direction. The ball is to be hit by a stone, which is thrown at the same time from the location  $(x_1, y_1) = (L, 0)$ . The stone is thrown at an angle  $(180 - \theta_1)$  from the  $+x$ -direction with a suitable initial speed. For a fixed  $v_0$ , when  $(\theta_0, \theta_1) = (45^\circ, 45^\circ)$ , the stone hits the ball after time  $T_1$ , and when  $(\theta_0, \theta_1) = (60^\circ, 30^\circ)$ , it hits the ball after time  $T_2$ . In such a case,  $(T_1/T_2)^2$  is \_\_\_\_\_.
- Q.10 A charge is kept at the central point P of a cylindrical region. The two edges subtend a half-angle  $\theta$  at P, as shown in the figure. When  $\theta = 30^\circ$ , then the electric flux through the curved surface of the cylinder is  $\Phi$ . If  $\theta = 60^\circ$ , then the electric flux through the curved surface becomes  $\Phi/\sqrt{n}$ , where the value of  $n$  is \_\_\_\_\_.

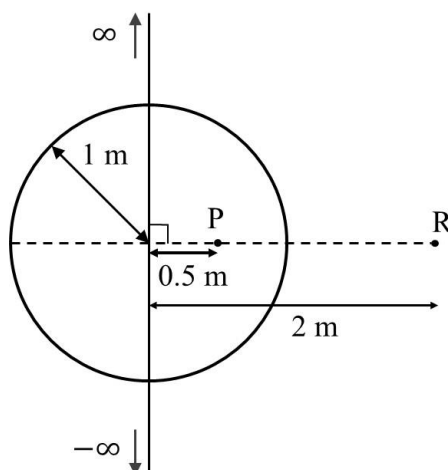


- Q.11 Two equilateral-triangular prisms  $P_1$  and  $P_2$  are kept with their sides parallel to each other, in vacuum, as shown in the figure. A light ray enters prism  $P_1$  at an angle of incidence  $\theta$  such that the outgoing ray undergoes minimum deviation in prism  $P_2$ . If the respective refractive indices of  $P_1$  and  $P_2$  are  $\sqrt{\frac{3}{2}}$  and  $\sqrt{3}$ , then  $\theta = \sin^{-1} \left[ \sqrt{\frac{3}{2}} \sin \left( \frac{\pi}{\beta} \right) \right]$ , where the value of  $\beta$  is \_\_\_\_\_.



- Q.12 An infinitely long thin wire, having a uniform charge density per unit length of  $5 \text{ nC/m}$ , is passing through a spherical shell of radius  $1 \text{ m}$ , as shown in the figure. A  $10 \text{ nC}$  charge is distributed uniformly over the spherical shell. If the configuration of the charges remains static, the magnitude of the potential difference between points P and R, in Volt, is \_\_\_\_\_.

[Given: In SI units  $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ ,  $\ln 2 = 0.7$ . Ignore the area pierced by the wire.]



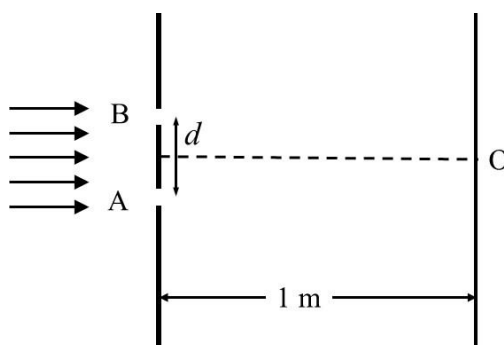
- Q.13 A spherical soap bubble inside an air chamber at pressure  $P_0 = 10^5 \text{ Pa}$  has a certain radius so that the excess pressure inside the bubble is  $\Delta P = 144 \text{ Pa}$ . Now, the chamber pressure is reduced to  $8P_0/27$  so that the bubble radius and its excess pressure change. In this process, all the temperatures remain unchanged. Assume air to be an ideal gas and the excess pressure  $\Delta P$  in both the cases to be much smaller than the chamber pressure. The new excess pressure  $\Delta P$  in Pa is \_\_\_\_\_.

**SECTION 4 (Maximum Marks: 12)**

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:  
*Full Marks* : +3 If ONLY the correct numerical value is entered in the designated place;  
*Zero Marks* : 0 In all other cases.

**PARAGRAPH I**

In a Young's double slit experiment, each of the two slits A and B, as shown in the figure, are oscillating about their fixed center and with a mean separation of 0.8 mm. The distance between the slits at time  $t$  is given by  $d = (0.8 + 0.04 \sin \omega t)$  mm, where  $\omega = 0.08 \text{ rad s}^{-1}$ . The distance of the screen from the slits is 1 m and the wavelength of the light used to illuminate the slits is  $6000 \text{ \AA}$ . The interference pattern on the screen changes with time, while the central bright fringe (zeroth fringe) remains fixed at point O.

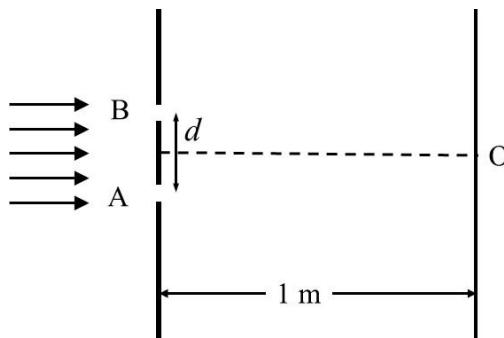


- Q.14 The 8<sup>th</sup> bright fringe above the point O oscillates with time between two extreme positions. The separation between these two extreme positions, in micrometer ( $\mu\text{m}$ ), is \_\_\_\_\_.



### PARAGRAPH I

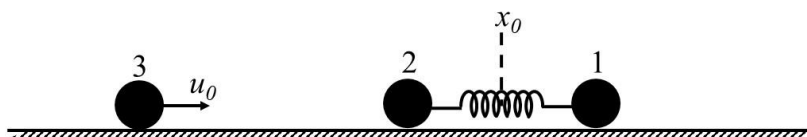
In a Young's double slit experiment, each of the two slits A and B, as shown in the figure, are oscillating about their fixed center and with a mean separation of 0.8 mm. The distance between the slits at time  $t$  is given by  $d = (0.8 + 0.04 \sin \omega t)$  mm, where  $\omega = 0.08 \text{ rad s}^{-1}$ . The distance of the screen from the slits is 1 m and the wavelength of the light used to illuminate the slits is  $6000 \text{ \AA}$ . The interference pattern on the screen changes with time, while the central bright fringe (zeroth fringe) remains fixed at point O.



Q.15 The maximum speed in  $\mu\text{m/s}$  at which the 8<sup>th</sup> bright fringe will move is \_\_\_\_\_.

### PARAGRAPH II

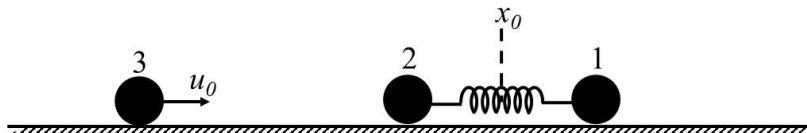
Two particles, 1 and 2, each of mass  $m$ , are connected by a massless spring, and are on a horizontal frictionless plane, as shown in the figure. Initially, the two particles, with their center of mass at  $x_0$ , are oscillating with amplitude  $a$  and angular frequency  $\omega$ . Thus, their positions at time  $t$  are given by  $x_1(t) = (x_0 + d) + a \sin \omega t$  and  $x_2(t) = (x_0 - d) - a \sin \omega t$ , respectively, where  $d > 2a$ . Particle 3 of mass  $m$  moves towards this system with speed  $u_0 = a\omega/2$ , and undergoes instantaneous elastic collision with particle 2, at time  $t_0$ . Finally, particles 1 and 2 acquire a center of mass speed  $v_{\text{cm}}$  and oscillate with amplitude  $b$  and the same angular frequency  $\omega$ .



Q.16 If the collision occurs at time  $t_0 = 0$ , the value of  $v_{\text{cm}}/(a\omega)$  will be \_\_\_\_\_.

### PARAGRAPH II

Two particles, 1 and 2, each of mass  $m$ , are connected by a massless spring, and are on a horizontal frictionless plane, as shown in the figure. Initially, the two particles, with their center of mass at  $x_0$ , are oscillating with amplitude  $a$  and angular frequency  $\omega$ . Thus, their positions at time  $t$  are given by  $x_1(t) = (x_0 + d) + a \sin \omega t$  and  $x_2(t) = (x_0 - d) - a \sin \omega t$ , respectively, where  $d > 2a$ . Particle 3 of mass  $m$  moves towards this system with speed  $u_0 = a\omega/2$ , and undergoes instantaneous elastic collision with particle 2, at time  $t_0$ . Finally, particles 1 and 2 acquire a center of mass speed  $v_{\text{cm}}$  and oscillate with amplitude  $b$  and the same angular frequency  $\omega$ .



Q.17 If the collision occurs at time  $t_0 = \pi/(2\omega)$ , then the value of  $4b^2/a^2$  will be \_\_\_\_\_.

**END OF THE QUESTION PAPER**

## Chemistry

## SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:  
*Full Marks* : +3 If **ONLY** the correct option is chosen;  
*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);  
*Negative Marks* : -1 In all other cases.

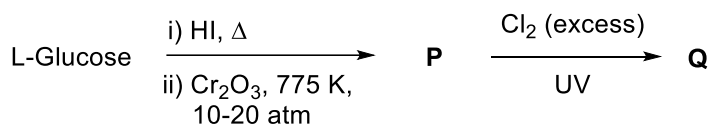
Q.1 According to Bohr's model, the highest kinetic energy is associated with the electron in the

- (A) first orbit of H atom  
 (B) first orbit of  $\text{He}^+$   
 (C) second orbit of  $\text{He}^+$   
 (D) second orbit of  $\text{Li}^{2+}$

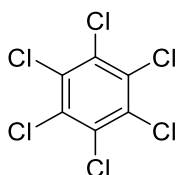
Q.2 In a metal deficient oxide sample,  $\text{M}_x\text{Y}_2\text{O}_4$  (**M** and **Y** are metals), **M** is present in both +2 and +3 oxidation states and **Y** is in +3 oxidation state. If the fraction of  $\text{M}^{2+}$  ions present in **M** is  $\frac{1}{3}$ , the value of **X** is \_\_\_\_\_.

- (A) 0.25                      (B) 0.33                      (C) 0.67                      (D) 0.75

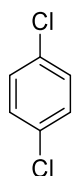
Q.3 In the following reaction sequence, the major product **Q** is



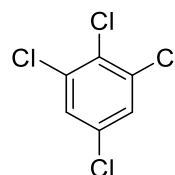
(A)



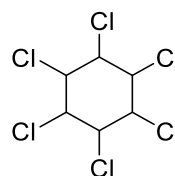
(C)



(B)



(D)



Q.4 The species formed on fluorination of phosphorus pentachloride in a polar organic solvent are

- (A)  $[\text{PF}_4]^+[\text{PF}_6]^-$  and  $[\text{PCl}_4]^+[\text{PF}_6]^-$       (B)  $[\text{PCl}_4]^+[\text{PCl}_4\text{F}_2]^-$  and  $[\text{PCl}_4]^+[\text{PF}_6]^-$   
(C)  $\text{PF}_3$  and  $\text{PCl}_3$       (D)  $\text{PF}_5$  and  $\text{PCl}_3$

**SECTION 2 (Maximum Marks: 12)**

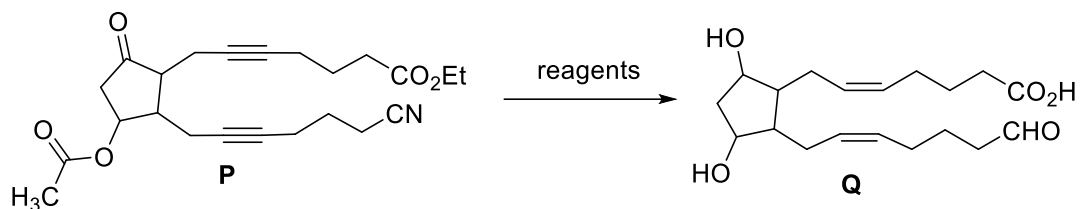
- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  - Full Marks* : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  - Partial Marks* : +3 If all the four options are correct but **ONLY** three options are chosen;
  - Partial Marks* : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  - Partial Marks* : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  - Zero Marks* : 0 If unanswered;
  - Negative Marks* : -2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then
  - choosing **ONLY** (A), (B) and (D) will get +4 marks;
  - choosing **ONLY** (A) and (B) will get +2 marks;
  - choosing **ONLY** (A) and (D) will get +2 marks;
  - choosing **ONLY** (B) and (D) will get +2 marks;
  - choosing **ONLY** (A) will get +1 mark;
  - choosing **ONLY** (B) will get +1 mark;
  - choosing **ONLY** (D) will get +1 mark;
  - choosing no option(s) (i.e. the question is unanswered) will get 0 marks and
  - choosing any other option(s) will get -2 marks.

Q.5 An aqueous solution of hydrazine ( $\text{N}_2\text{H}_4$ ) is electrochemically oxidized by  $\text{O}_2$ , thereby releasing chemical energy in the form of electrical energy. One of the products generated from the electrochemical reaction is  $\text{N}_2(\text{g})$ .

Choose the correct statement(s) about the above process

- (A)  $\text{OH}^-$  ions react with  $\text{N}_2\text{H}_4$  at the anode to form  $\text{N}_2(\text{g})$  and water, releasing 4 electrons to the anode.
- (B) At the cathode,  $\text{N}_2\text{H}_4$  breaks to  $\text{N}_2(\text{g})$  and nascent hydrogen released at the electrode reacts with oxygen to form water.
- (C) At the cathode, molecular oxygen gets converted to  $\text{OH}^-$ .
- (D) Oxides of nitrogen are major by-products of the electrochemical process.

Q.6 The option(s) with correct sequence of reagents for the conversion of **P** to **Q** is(are)



- (A) i) Lindlar's catalyst,  $\text{H}_2$ ; ii)  $\text{SnCl}_2/\text{HCl}$ ; iii)  $\text{NaBH}_4$ ; iv)  $\text{H}_3\text{O}^+$   
 (B) i) Lindlar's catalyst,  $\text{H}_2$ ; ii)  $\text{H}_3\text{O}^+$ ; iii)  $\text{SnCl}_2/\text{HCl}$ ; iv)  $\text{NaBH}_4$   
 (C) i)  $\text{NaBH}_4$ ; ii)  $\text{SnCl}_2/\text{HCl}$ ; iii)  $\text{H}_3\text{O}^+$ ; iv) Lindlar's catalyst,  $\text{H}_2$   
 (D) i) Lindlar's catalyst,  $\text{H}_2$ ; ii)  $\text{NaBH}_4$ ; iii)  $\text{SnCl}_2/\text{HCl}$ ; iv)  $\text{H}_3\text{O}^+$

Q.7 The compound(s) having peroxide linkage is(are)

- (A)  $\text{H}_2\text{S}_2\text{O}_7$   
 (B)  $\text{H}_2\text{S}_2\text{O}_8$   
 (C)  $\text{H}_2\text{S}_2\text{O}_5$   
 (D)  $\text{H}_2\text{SO}_5$

**SECTION 3 (Maximum Marks: 24)**

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:  
*Full Marks* : +4 If **ONLY** the correct integer is entered;  
*Zero Marks* : 0 In all other cases.

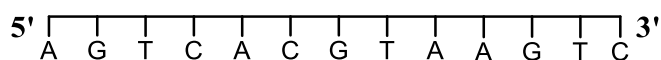
Q.8 To form a complete monolayer of acetic acid on 1g of charcoal, 100 mL of 0.5 M acetic acid was used. Some of the acetic acid remained unadsorbed. To neutralize the unadsorbed acetic acid, 40 mL of 1 M NaOH solution was required. If each molecule of acetic acid occupies  $P \times 10^{-23} \text{ m}^2$  surface area on charcoal, the value of **P** is \_\_\_\_\_.

[Use given data: Surface area of charcoal =  $1.5 \times 10^2 \text{ m}^2\text{g}^{-1}$ ; Avogadro's number ( $N_A$ ) =  $6.0 \times 10^{23} \text{ mol}^{-1}$ ]

Q.9 Vessel-1 contains  $w_2$  g of a non-volatile solute **X** dissolved in  $w_1$  g of water. Vessel-2 contains  $w_2$  g of another non-volatile solute **Y** dissolved in  $w_1$  g of water. Both the vessels are at the same temperature and pressure. The molar mass of **X** is 80% of that of **Y**. The van't Hoff factor for **X** is 1.2 times of that of **Y** for their respective concentrations.

The elevation of boiling point for solution in Vessel-1 is \_\_\_\_\_ % of the solution in Vessel-2.

Q.10 For a double strand DNA, one strand is given below:



The amount of energy required to split the double strand DNA into two single strands is \_\_\_\_\_ kcal  $\text{mol}^{-1}$ .

[Given: Average energy per H-bond for A-T base pair =  $1.0 \text{ kcal mol}^{-1}$ , G-C base pair =  $1.5 \text{ kcal mol}^{-1}$ , and A-U base pair =  $1.25 \text{ kcal mol}^{-1}$ . Ignore electrostatic repulsion between the phosphate groups.]

Q.11 A sample initially contains only U-238 isotope of uranium. With time, some of the U-238 radioactively decays into Pb-206 while the rest of it remains undisintegrated.

When the age of the sample is  $P \times 10^8$  years, the ratio of mass of Pb-206 to that of U-238 in the sample is found to be 7. The value of  $P$  is \_\_\_\_\_.

[Given: Half-life of U-238 is  $4.5 \times 10^9$  years;  $\log_e 2 = 0.693$ ]

Q.12 Among  $[\text{Co}(\text{CN})_4]^{4-}$ ,  $[\text{Co}(\text{CO})_3(\text{NO})]$ ,  $\text{XeF}_4$ ,  $[\text{PCl}_4]^+$ ,  $[\text{PdCl}_4]^{2-}$ ,  $[\text{ICl}_4]^-$ ,  $[\text{Cu}(\text{CN})_4]^{3-}$  and  $\text{P}_4$  the total number of species with tetrahedral geometry is \_\_\_\_\_.

Q.13 An organic compound **P** having molecular formula  $\text{C}_6\text{H}_6\text{O}_3$  gives ferric chloride test and does not have intramolecular hydrogen bond. The compound **P** reacts with 3 equivalents of  $\text{NH}_2\text{OH}$  to produce oxime **Q**. Treatment of **P** with excess methyl iodide in the presence of  $\text{KOH}$  produces compound **R** as the major product. Reaction of **R** with excess *iso*-butylmagnesium bromide followed by treatment with  $\text{H}_3\text{O}^+$  gives compound **S** as the major product.

The total number of methyl ( $-\text{CH}_3$ ) group(s) in compound **S** is \_\_\_\_\_.



**SECTION 4 (Maximum Marks: 12)**

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:  
*Full Marks* : +3 If **ONLY** the correct numerical value is entered in the designated place;  
*Zero Marks* : 0 In all other cases.

*“PARAGRAPH I”*

An organic compound **P** with molecular formula  $C_9H_{18}O_2$  decolorizes bromine water and also shows positive iodoform test. **P** on ozonolysis followed by treatment with  $H_2O_2$  gives **Q** and **R**. While compound **Q** shows positive iodoform test, compound **R** does not give positive iodoform test. **Q** and **R** on oxidation with pyridinium chlorochromate (PCC) followed by heating give **S** and **T**, respectively. Both **S** and **T** show positive iodoform test.

Complete copolymerization of 500 moles of **Q** and 500 moles of **R** gives one mole of a single acyclic copolymer **U**.

[Given, atomic mass: H = 1, C = 12, O = 16]

Q.14 Sum of number of oxygen atoms in **S** and **T** is \_\_\_\_\_.

*“PARAGRAPH I”*

An organic compound **P** with molecular formula  $C_9H_{18}O_2$  decolorizes bromine water and also shows positive iodoform test. **P** on ozonolysis followed by treatment with  $H_2O_2$  gives **Q** and **R**. While compound **Q** shows positive iodoform test, compound **R** does not give positive iodoform test. **Q** and **R** on oxidation with pyridinium chlorochromate (PCC) followed by heating give **S** and **T**, respectively. Both **S** and **T** show positive iodoform test.

Complete copolymerization of 500 moles of **Q** and 500 moles of **R** gives one mole of a single acyclic copolymer **U**.

[Given, atomic mass: H = 1, C = 12, O = 16]

Q.15 The molecular weight of **U** is \_\_\_\_\_.

*"PARAGRAPH II"*

When potassium iodide is added to an aqueous solution of potassium ferricyanide, a reversible reaction is observed in which a complex **P** is formed. In a strong acidic medium, the equilibrium shifts completely towards **P**. Addition of zinc chloride to **P** in a slightly acidic medium results in a sparingly soluble complex **Q**.

Q.16 The number of moles of potassium iodide required to produce two moles of **P** is \_\_\_\_\_.

*"PARAGRAPH II"*

When potassium iodide is added to an aqueous solution of potassium ferricyanide, a reversible reaction is observed in which a complex **P** is formed. In a strong acidic medium, the equilibrium shifts completely towards **P**. Addition of zinc chloride to **P** in a slightly acidic medium results in a sparingly soluble complex **Q**.

Q.17 The number of zinc ions present in the molecular formula of **Q** is \_\_\_\_\_.

**END OF THE QUESTION PAPER**