



JEE Main – 2023

29th JAN 2023 (Morning Shift)

General Instructions

1. The test is of **3 hours** duration and the maximum marks is **300**.
2. The question paper consists of **3 Parts** (Part I: **Physics**, Part II: **Chemistry**, Part III: **Mathematics**). Each Part has **two** sections (Section 1 & Section 2).
3. **Section 1** contains **20 Multiple Choice Questions**. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE CHOICE** is correct.
4. **Section 2** contains **10 Numerical Value Type Questions** Out of which **ONLY 5 (any)** questions have to be attempted. You will **NOT** be allowed to attempt the sixth question. If you wish to attempt any other question apart from the five already attempted, then you will have to delete any one response from the five previously answered and then proceed to answer the new one.
The answer to each question should be **rounded off to the nearest integer**.
5. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. inside the examination room/hall.

Marking Scheme

1. **Section – 1:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.
2. **Section – 2:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.

SUBJECT I: PHYSICS**MARKS: 100****SECTION-1**

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

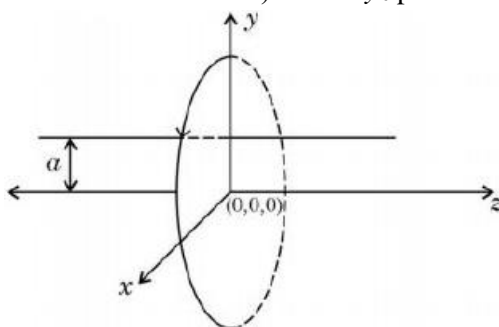
- A person observes two moving trains, 'A' reaching the station and 'B' leaving the station with equal speed of 30 m/s. If both trains emit sounds with frequency 300 Hz, (Speed of sound: 330 m/s) approximate difference of frequencies heard by the person will be:
 (1) 10 Hz (2) 55 Hz (3) 80 Hz (4) 33 Hz
- If a radioactive element having half-life of 30 min is undergoing beta decay, the fraction of radioactive element remains undecayed after 90 min. will be:
 (1) $\frac{1}{16}$ (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) $\frac{1}{8}$
- If the height of transmitting and receiving antennas are 80 m each, the maximum line of sight distance will be: Given: Earth's radius = 6.4×10^6 m.
 (1) 36 km (2) 64 km (3) 28 km (4) 32 km
- Surface tension of a soap bubble is $2.0 \times 10^{-2} \text{ Nm}^{-1}$. Work done to increase the radius of soap bubble from 3.5 cm to 7 cm will be: Take $\left[\pi = \frac{22}{7} \right]$
 (1) $5.76 \times 10^{-4} \text{ J}$ (2) $18.48 \times 10^{-4} \text{ J}$ (3) $0.72 \times 10^{-4} \text{ J}$ (4) $9.24 \times 10^{-4} \text{ J}$
- Which of the following are true?
 A. Speed of light in vacuum is dependent on the direction of propagation.
 B. Speed of light in a medium is independent of the wavelength of light.
 C. The speed of light is independent of the motion of the source.
 D. The speed of light in a medium is independent of intensity.
 Choose the correct answer from the options given below:
 (1) B and C only (2) B and D only (3) A and C only (4) C and D only
- In a cuboid of dimension $2L \times 2L \times L$, a charge q is placed at the center of the surface 'S' having area of $4L^2$. The flux through the opposite surface to 'S' is given by:
 (1) $\frac{q}{12\epsilon_0}$ (2) $\frac{q}{6\epsilon_0}$ (3) $\frac{q}{2\epsilon_0}$ (4) $\frac{q}{3\epsilon_0}$
- Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.
Assertion A: If dQ and dW represent the heat supplied to the system and the work done on the system respectively. Then according to the first law of thermodynamics $dQ = dU - dW$.
Reason R: First law of thermodynamics is based on law of conservation of energy.
 In the light of the above statements, choose the correct answer from the options given below:
 (1) Both A and R are correct but R is not the correct explanation of A
 (2) A is not correct but R is correct
 (3) Both A and R are correct and R is the correct explanation of A
 (4) A is correct but R is not correct

8. The threshold wavelength for photoelectric emission from a material is 5500 \AA . Photoelectrons will be emitted, when this material is illuminated with monochromatic radiation from a
- A. 75 W infra-red lamp B. 10 W infra-red lamp
 C. 75 W ultra-violet lamp D. 10 W ultra-violet lamp

Choose the correct answer from the options given below:

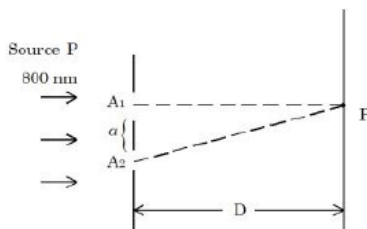
- (1) A and D only (2) B and C only (3) C and D only (4) C only

9. A single current carrying loop of wire carrying current I flowing in anticlockwise direction seen from +ve z direction and lying in xy plane is shown in figure. The plot of \hat{j} component of magnetic field (B_y) at a distance ' a ' (less than radius of the coil) and on yz plane vs z coordinate looks like:



- (1) (2)
- (3) (4)

10. In a Young's double slit experiment, two slits are illuminated with a light of wavelength 800 nm . The line joining A_1P is perpendicular to A_1A_2 as shown in the figure. If the first minimum is detected at P , the value of slits separation ' a ' will be:



- (1) 0.2 mm (2) 0.4 mm (3) 0.1 mm (4) 0.5 mm

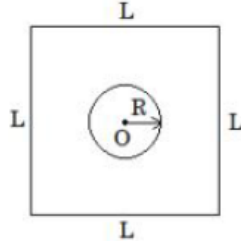
11. Match List I with List II:

List I (Physical Quantity)		List II (Dimensional Formula)	
A.	Pressure gradient	I.	$[M^0L^2T^{-2}]$
B.	Energy density	II.	$[M^1L^{-1}T^{-2}]$
C.	Electric field	III.	$[M^1L^{-2}T^{-2}]$
D.	Latent heat	IV.	$[M^1L^1T^{-3}A^{-1}]$

Choose the correct answer from the options given below:

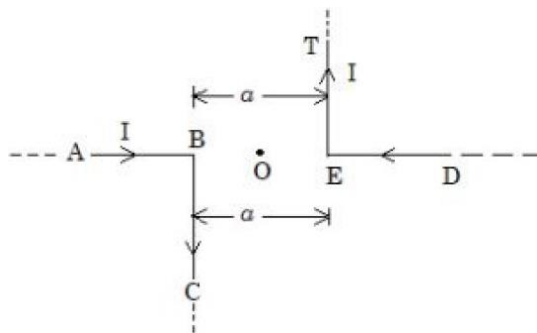
- (1) A-III, B-II, C-IV, D-I (2) A-II, B-III, C-I, D-IV
 (3) A-III, B-II, C-I, D-IV (4) A-II, B-III, C-IV, D-I

12. A stone is projected at angle 30° to the horizontal. The ratio of kinetic energy of the stone at point of projection to its kinetic energy at the highest point of flight will be:
 (1) 1 : 2 (2) 1 : 4 (3) 4 : 1 (4) 4 : 3
13. Find the mutual inductance in the arrangement, when a small circular loop of wire of radius 'R' is placed inside a large square loop of wire of side $L(L \gg R)$. The loops are coplanar and their centres coincide:



- (1) $M = \frac{2\sqrt{2}\mu_0 R}{L^2}$ (2) $M = \frac{\sqrt{2}\mu_0 R}{L^2}$
 (3) $M = \frac{2\sqrt{2}\mu_0 R^2}{L}$ (4) $M = \frac{\sqrt{2}\mu_0 R^2}{L^2}$
14. A bicycle tyre is filled with air having pressure of 270 kPa at 27°C . The approximate pressure of the air in the tyre when the temperature increases to 36°C is:
 (1) 270 kPa (2) 360 kPa (3) 278 kPa (4) 262 kPa
15. A car is moving on a horizontal curved road with radius 50 m. The approximate maximum speed of car will be, if friction between tyres and road is 0.34. [Take $g = 10 \text{ ms}^{-2}$]
 (1) 22.4 ms^{-1} (2) 17 ms^{-1} (3) 3.4 ms^{-1} (4) 13 ms^{-1}
16. Which one of the following statement is not correct in the case of light emitting diodes?
 A. It is heavily doped p-n junction.
 B. It emits light only when it is forward biased.
 C. It emits light only when it is reverse biased.
 D. The energy of the light emitted is equal to or slightly less than the energy gap of the semiconductor used.
- Choose the correct answer from the options given below:
 (1) A (2) B (3) C and D (4) C
17. A block of mass m slides down the plane inclined at angle 30° with an acceleration $\frac{g}{4}$. The value of coefficient of kinetic friction will be:
 (1) $\frac{2\sqrt{3}+1}{2}$ (2) $\frac{2\sqrt{3}-1}{2}$ (3) $\frac{1}{2\sqrt{3}}$ (4) $\frac{\sqrt{3}}{2}$
18. Ratio of thermal energy released in two resistors R and 3R connected in parallel in an electric circuit is:
 (1) 3 : 1 (2) 1 : 3 (3) 1 : 27 (4) 1 : 1
19. Two particles of equal mass 'm' move in a circle of radius 'r' under the action of their mutual gravitational attraction. The speed of each particle will be:
 (1) $\sqrt{\frac{4Gm}{r}}$ (2) $\sqrt{\frac{Gm}{4r}}$ (3) $\sqrt{\frac{Gm}{r}}$ (4) $\sqrt{\frac{Gm}{2r}}$

20. The magnitude of magnetic induction at mid point O due to current arrangement as shown in figure will be:

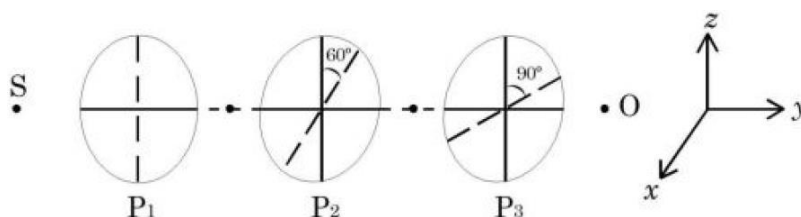


- (1) $\frac{\mu_0 I}{\pi a}$ (2) 0 (3) $\frac{\mu_0 I}{4\pi a}$ (4) $\frac{\mu_0 I}{2\pi a}$

SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be rounded off to the nearest integer.

21. In a meter bridge experiment the balance point is obtained if the gaps are closed by 2Ω and 3Ω . A shunt of $X\Omega$ is added to 3Ω resistor to shift the balancing point by 22.5 cm. The value of X is ____.
22. A radioactive element ${}_{92}^{242}X$ emits two α -particles, one electron and two positrons. The product nucleus is represented by ${}_{P}^{234}Y$. The value of P is _____.
23. As shown in the figure, three identical polaroids P_1 , P_2 and P_3 are placed one after another. The pass axis of P_2 and P_3 are inclined at angle of 60° and 90° with respect to axis of P_1 . The source S has an intensity of $256 \frac{W}{m^2}$. The intensity of light at point O is _____ $\frac{W}{m^2}$.

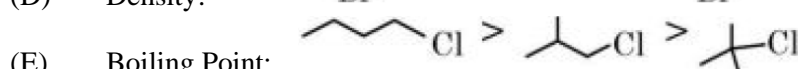
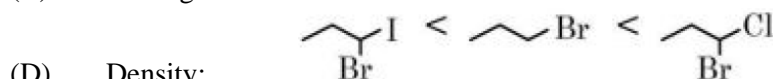
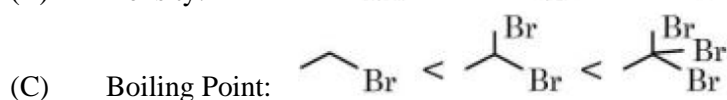
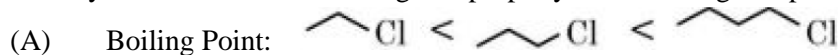


24. Two simple harmonic waves having equal amplitudes of 8 cm and equal frequency of 10 Hz are moving along the same direction. The resultant amplitude is also 8 cm. The phase difference between the individual waves is ____ degree.
25. A point charge $q_1 = 4q_0$ is placed at origin. Another point charge $q_2 = -q_0$ is placed at $x = 12$ cm. Charge of proton is q_0 . The proton is placed on x axis so that the electrostatic force on the proton is zero. In this situation, the position of the proton from the origin is ____ cm.
26. A solid sphere of mass 2 kg is making pure rolling on a horizontal surface with kinetic energy J. The velocity of centre of mass of the sphere will be _____ ms^{-1} .
27. A certain elastic conducting material is stretched into a circular loop. It is placed with its plane perpendicular to a uniform magnetic field $B = 0.8T$. When released the radius of the loop starts shrinking at a constant rate of 2 cms^{-1} . The induced emf in the loop at an instant when the radius of the loop is 10 cm will be ____ mV.
28. A 0.4 kg mass takes 8s to reach ground when dropped from a certain height 'P' above surface of earth. The loss of potential energy in the last second of fall is _____ J. (Take $g = 10 \text{ m/s}^2$)
29. A body cools from 60°C to 40°C in 6 minutes. If, temperature of surroundings is 10°C . Then, after the next 6 minutes, its temperature will be _____ $^\circ\text{C}$.
30. A tennis ball is dropped on to the floor from a height of 9.8 m. It rebounds to a height 5.0 m. Ball comes in contact with the floor for 0.2s. The average acceleration during contact is _____ ms^{-2} . (Given $g = 10 \text{ ms}^{-2}$)

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. Identify the correct order for the given property for following compounds.



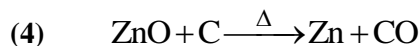
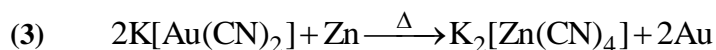
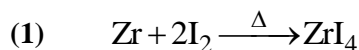
(1) (B), (C) and (D) only

(2) (A), (C) and (E) only

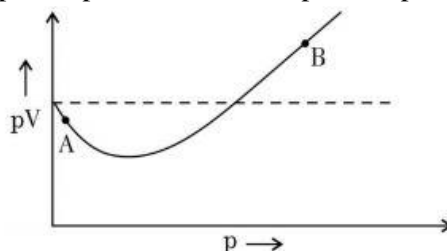
(3) (A), (B) and (E) only

(4) (A), (C) and (D) only

2. The reaction representing the Mond process for metal refining is _____.



3. For 1 mol of gas, the plot of pV vs. p is shown below. p is the pressure and V is the volume of the gas



What is the value of compressibility factor at point A?

(1) $1 + \frac{a}{RTV}$

(2) $1 + \frac{b}{V}$

(3) $1 - \frac{a}{RTV}$

(4) $1 - \frac{b}{V}$

4. Match List I with List II.

List I Antimicrobials		List II Names	
(A)	Narrow Spectrum Antibiotic	(I)	Furacin
(B)	Antiseptic	(II)	Sulphur dioxide
(C)	Disinfectants	(III)	Penicillin G
(D)	Broad spectrum	(IV)	Chloramphenicol

Choose the correct answer from the options given below:

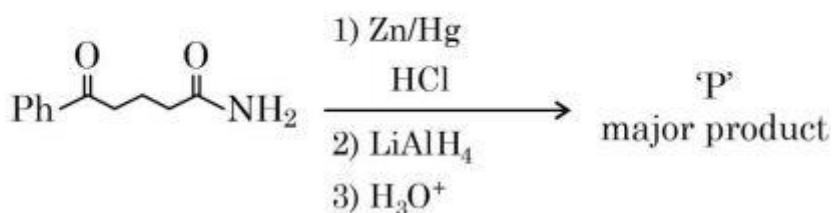
(1) (A) – I, (B) – II, (C) – IV, (D) – III

(2) (A) – III, (B) – I, (C) – IV, (D) – II

(3) (A) – II, (B) – I, (C) – IV, (D) – III

(4) (A) – III, (B) – I, (C) – II, (D) – IV

5. Compound that will give positive Lassaigne's test for both nitrogen and halogen is:
 (1) $\text{CH}_3\text{NH}_2 \cdot \text{HCl}$ (2) NH_4Cl
 (3) $\text{NH}_2\text{OH} \cdot \text{HCl}$ (4) $\text{N}_2\text{H}_4 \cdot \text{HCl}$
6. During the borax bead test with CuSO_4 , a blue green colour of the bead was observed in oxidising flame due to the formation of:
 (1) CuO (2) Cu (3) $\text{Cu}(\text{BO}_2)_2$ (4) Cu_3B_2
7. "A" obtained by Ostwald's method involving air oxidation of NH_3 , upon further air oxidation produces "B". "B" on hydration forms an oxoacid of Nitrogen along with evolution of "A". The oxoacid also produces "A" and gives positive brown ring test. Identify A and B, respectively.
 (1) NO, NO_2 (2) $\text{NO}_2, \text{N}_2\text{O}_5$ (3) $\text{NO}_2, \text{N}_2\text{O}_4$ (4) $\text{N}_2\text{O}_3, \text{NO}_2$
8. The standard electrode potential (M^{3+} / M^{2+}) for V, Cr, Mn & Co are -0.26 V , -0.41 V , $+1.57 \text{ V}$ and $+1.97 \text{ V}$, respectively. The metal ions which can liberate H_2 from a dilute acid are;
 (1) Mn^{2+} and Co^{2+} (2) V^{2+} and Mn^{2+}
 (3) Cr^{2+} and Co^{2+} (4) V^{2+} and Cr^{2+}
9. The magnetic behavior of Li_2O , Na_2O_2 and KO_2 , respectively, are:
 (1) Paramagnetic, paramagnetic and diamagnetic
 (2) Diamagnetic, paramagnetic and diamagnetic
 (3) Paramagnetic, diamagnetic and paramagnetic
 (4) Diamagnetic, diamagnetic and paramagnetic
10. Chiral complex from the following is:
 Here en = ethylene diamine
 (1) $\text{cis} - [\text{PtCl}_2(\text{en})_2]^{2+}$ (2) $\text{cis} - [\text{PtCl}_2(\text{NH}_3)_2]$
 (3) $\text{trans} - [\text{PtCl}_2(\text{en})_2]^{2+}$ (4) $\text{trans} - [\text{Co}(\text{NH}_3)_4\text{Cl}_2]^+$
11. The major product 'P' for the following sequence of reactions is:



- (1) $\text{Ph}-\overset{\text{OH}}{\text{C}}-\text{CH}_2-\text{CH}_2-\text{CH}_2-\overset{\text{OH}}{\text{C}}-\text{NH}_2$ (2) $\text{Ph}-\overset{\text{OH}}{\text{C}}-\text{CH}_2-\text{CH}_2-\text{CH}_2-\text{OH}$
 (3) $\text{Ph}-\text{CH}_2-\text{CH}_2-\text{CH}_2-\text{CH}_2-\text{NH}_2$ (4) $\text{Ph}-\overset{\text{OH}}{\text{C}}-\text{CH}_2-\text{CH}_2-\text{CH}_2-\text{NH}_2$

12. Which of the following salt solutions would coagulate the colloid solution formed when FeCl_3 is added to NaOH solution, at the fastest rate?
 (1) 10 mL of $0.1 \text{ mol dm}^{-3} \text{Na}_2\text{SO}_4$ (2) 10 mL of $0.15 \text{ mol dm}^{-3} \text{CaCl}_2$
 (3) 10 mL of $0.2 \text{ mol dm}^{-3} \text{AlCl}_3$ (4) 10 mL of $0.1 \text{ mol dm}^{-3} \text{Ca}_3(\text{PO}_4)_2$

13. The shortest wavelength of hydrogen atom in Lyman series is λ . The longest wavelength in Balmer series of He^+ is:

- (1) $\frac{36\lambda}{5}$ (2) $\frac{5}{9\lambda}$ (3) $\frac{9\lambda}{5}$ (4) $\frac{5\pi}{9}$

14. The correct order of hydration enthalpies is

- (A) K^+ (B) Rb^+ (C) Mg^{2+}
 (D) Cs^+ (E) Ca^{2+}

Choose the correct answer from the options given below:

- (1) $\text{C} > \text{A} > \text{E} > \text{B} > \text{D}$ (2) $\text{C} > \text{E} > \text{A} > \text{D} > \text{B}$
 (3) $\text{E} > \text{C} > \text{A} > \text{B} > \text{D}$ (4) $\text{C} > \text{E} > \text{A} > \text{B} > \text{D}$

15. The bond dissociation energy is highest for:

- (1) Cl_2 (2) F_2 (3) Br_2 (4) I_2

16. Number of cyclic tripeptides formed with 2 amino acids A and B is:

- (1) 2 (2) 4 (3) 5 (4) 3

17. Which of the given compounds can enhance the efficiency of hydrogen storage tank?

- (1) Li / P_4 (2) Di-isobutylaluminium
 (3) NaNi_5 (4) SiH_4

18. The increasing order of pK_a for the following phenols is:

- (A) 2, 4 - Dinitrophenol (B) 4 - Nitrophenol
 (C) 2, 4, 5 - Trimethylphenol (D) Phenol
 (E) 3-Chlorophenol

Choose the correct answer from the option given below:

- (1) (A), (E), (B), (D), (C) (2) (C), (E), (D), (B), (A)
 (3) (C), (D), (E), (B), (A) (4) (A), (B), (E), (D), (C)

19. Match List I with List II.

List I		List II	
Reaction		Reagents	
(A)	Hoffmann Degradation	(I)	Conc. KOH, Δ
(B)	Clemenson reduction	(II)	$\text{CHCl}_3, \text{NaOH} / \text{H}_3\text{O}^{\oplus}$
(C)	Cannizaro reaction	(III)	Br_2, NaOH
(D)	Reimer-Tiemann Reaction	(IV)	Zn-Hg/HCl

Choose the correct answer from the options given below:

- (1) (A) – II, (B) – I, (C) – III, (D) – IV (2) (A) – III, (B) – IV, (C) – I, (D) – IV
 (3) (A) – III, (B) – IV, (C) – II, (D) – I (4) (A) – II, (B) – IV, (C) – I, (D) – III

20. Correct statement about smog is:

- (1) Classical smog also has high concentration of oxidizing agents
 (2) Both NO_2 and SO_2 are present in classical smog
 (3) NO_2 is present in classical smog
 (4) Photochemical smog has high concentration of oxidizing agent

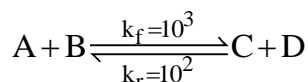
SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be rounded off to the nearest integer.

21. Solid Lead nitrate is dissolved in 1 litre of water. The solution was found to boil at 100.15°C . When 0.2 mol of NaCl is added to the resulting solution, it was observed that the solution froze at -0.8°C . the solubility product of PbCl_2 formed is _____ $\times 10^{-6}$ at 298 K. (Nearest integer)

Given : $K_b = 0.5 \text{ K kg mol}^{-1}$ and $K_f = 1.8 \text{ K kg mol}^{-1}$. Assume molality to be equal to molarity in all cases.

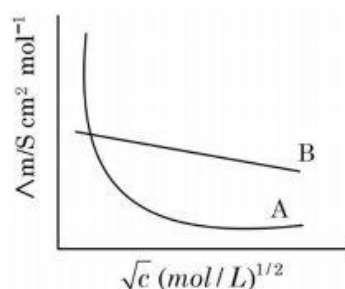
22. Consider the following reaction approaching equilibrium at 27°C and 1 atm pressure



The standard Gibb's energy change ($\Delta_r G^{\theta}$) at 27°C is (-) _____ kJ mol^{-1}

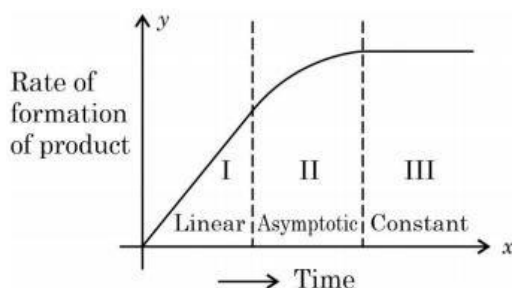
(Nearest integer) (Given: $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$ and $\ln 10 = 2.3$)

23. Following figure shows dependence of molar conductance of two electrolytes on concentration. Λ_m° is the limiting molar conductivity.



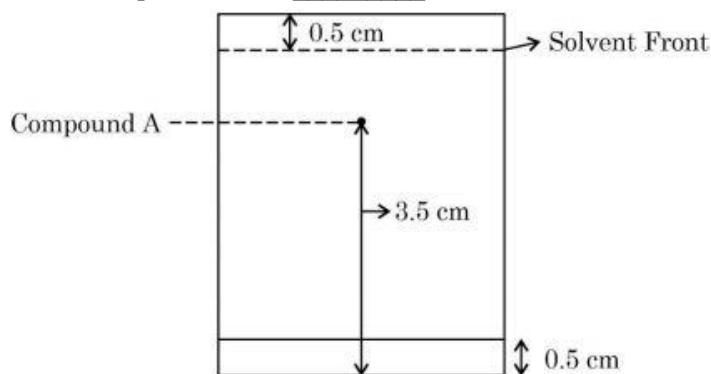
The number of incorrect statement(s) from the following is _____

- (A) Λ_m° for electrolyte A is obtained by extrapolation
 (B) For electrolyte B, Λ_m vs \sqrt{c} graph is a straight line with intercept equal to Λ_m°
 (C) At infinite dilution, the value of degree of dissociation approaches zero for electrolyte B.
 (D) Λ_m° for any electrolyte A or B can be calculated using λ° for individual ions
24. For certain chemical reaction $\text{X} \rightarrow \text{Y}$, the rate of formation of product is plotted against the time as shown in the figure. The number of correct statement/s from the following is _____ .



- (A) Over all order of this reaction is one
 (B) Order of this reaction can't be determined
 (C) In region I and III, the reaction is of first and zero order respectively
 (D) In region-II, the reaction is of first order
 (E) In region-II, the order of reaction is in the range of 0.1 to 0.9.

25. Following chromatogram was developed by adsorption of compound 'A' on a 6 cm TLC glass plate. Reterdation factor of the compound 'A' is _____ $\times 10^{-1}$.



26. The sum of bridging carbonyls in $W(CO)_6$ and $Mn_2(CO)_{10}$ is _____ .
27. 17 mg of a hydrocarbon (M.F. $C_{10}H_{16}$) takes up 8.40 mL of the H_2 gas measured at $0^\circ C$ and 760 mm of Hg. Ozonolysis of the same hydrocarbon yields
- $$CH_3 - \underset{\text{O}}{\underset{\parallel}}{C} - CH_3, \quad H - \underset{\text{O}}{\underset{\parallel}}{C} - H, \quad H - \underset{\text{O}}{\underset{\parallel}}{C} - CH_2 - CH_2 - \underset{\text{O}}{\underset{\parallel}}{C} - \underset{\text{O}}{\underset{\parallel}}{C} - H$$
- The number of double bond/s present in the hydrocarbon is _____ .
28. The number of molecules or ions from the following, which do not have odd number of electrons are _____.
- (A) NO_2 (B) ICl_4^- (C) BrF_3
- (D) ClO_2 (E) NO_2^+ (F) NO
29. Water decomposes at 2300 K $H_2O(g) \rightarrow H_2(g) + \frac{1}{2}O_2(g)$
- The percent of water decomposing at 2300 K and 1 bar is _____ (Nearest integer).
- Equilibrium constant for the reaction is 2×10^{-3} at 2300 K.
30. Millimoles of calcium hydroxide required to produce 100 mL of the aqueous solution of pH 12 is $x \times 10^{-1}$. The value of x is _____ (Nearest integer).
- Assume complete dissociation.

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. Let $x = 2$ be a root of the equation $x^2 + px + q = 0$ and

$$f(x) = \begin{cases} 1 - \cos(x^2 - 4px + q^2 + 8q + 16) & , x \neq 2p \\ 0 & , x = 2p \end{cases}$$

Then $\lim_{x \rightarrow 2p^+} [f(x)]$, where $[.]$ denotes greatest integer function, is:

- (1) -1 (2) 2 (3) 0 (4) 1

2. Fifteen football players of a club-team are given 15 T-shirts with their names written on the backside. If the players pick up the T-shirts randomly, then the probability that at least 3 players pick the correct T-shirts is:

- (1) $\frac{2}{15}$ (2) $\frac{5}{24}$ (3) $\frac{5}{36}$ (4) $\frac{1}{6}$

3. Let $y = f(x)$ be the solution of the differential equation $y(x+1)dx - x^2dy = 0$, $y(1) = e$. Then $\lim_{x \rightarrow 0^+} f(x)$ is equal to:

- (1) $\frac{1}{e^2}$ (2) $\frac{1}{e}$ (3) 0 (4) e^2

4. If the vectors $\vec{a} = \lambda\hat{i} + \mu\hat{j} + 4\hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} + \hat{k}$ are coplanar and the projection of \vec{a} on the vector \vec{b} is $\sqrt{54}$ units, then the sum of all possible values of $\lambda + \mu$ is equal to:

- (1) 0 (2) 24 (3) 6 (4) 18

5. Let $A = \left\{ (x, y) \in \mathbb{R}^2 : y \geq 0, 2x \leq y \leq \sqrt{4 - (x-1)^2} \right\}$ and

$$B = \left\{ (x, y) \in \mathbb{R} \times \mathbb{R} : 0 \leq y \leq \min \left\{ 2x, \sqrt{4 - (x-1)^2} \right\} \right\}.$$

Then the ratio of the area of A to the area of B is:

- (1) $\frac{\pi}{\pi-1}$ (2) $\frac{\pi}{\pi+1}$ (3) $\frac{\pi-1}{\pi+1}$ (4) $\frac{\pi+1}{\pi-1}$

6. Let $f(\theta) = 3 \left(\sin^4 \left(\frac{3\pi}{2} - \theta \right) + \sin^4(3\pi + \theta) \right) - 2(1 - \sin^2 2\theta)$ and $S = \left\{ \theta \in [0, \pi] : f'(\theta) = -\frac{\sqrt{3}}{2} \right\}$.

If $4\beta = \sum_{\theta \in S} \theta$, then $f(\beta)$ is equal to:

- (1) $\frac{11}{8}$ (2) $\frac{9}{8}$ (3) $\frac{3}{2}$ (4) $\frac{5}{4}$

7. Let $[x]$ denote the greatest integer $\leq x$. Consider the function $f(x) = \max\{x^2, 1+[x]\}$. Then the value of the integral $\int_0^2 f(x)dx$ is:
- (1) $\frac{5+4\sqrt{2}}{3}$ (2) $\frac{1+5\sqrt{2}}{3}$ (3) $\frac{4+5\sqrt{2}}{3}$ (4) $\frac{8+4\sqrt{2}}{3}$
8. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x) = \frac{x^2+2x+1}{x^2+1}$. Then:
- (1) $f(x)$ is one-one in $[1, \infty)$ but not in $(-\infty, \infty)$
 (2) $f(x)$ is one-one in $(-\infty, \infty)$
 (3) $f(x)$ is many-one in $(-\infty, -1)$
 (4) $f(x)$ is many-one in $(1, \infty)$
9. Let Δ be the area of the region $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 21, y^2 \leq 4x, x \geq 1\}$. Then $\frac{1}{2} \left(\Delta - 21 \sin^{-1} \frac{2}{\sqrt{7}} \right)$ is equal to:
- (1) $\sqrt{3} - \frac{4}{3}$ (2) $\sqrt{3} - \frac{2}{3}$ (3) $2\sqrt{3} - \frac{2}{3}$ (4) $2\sqrt{3} - \frac{1}{3}$
10. A light ray emits from the origin making an angle 30° with the positive x -axis. After getting reflected by the line $x + y = 1$, if this ray intersects x -axis at Q , then the abscissa of Q is:
- (1) $\frac{\sqrt{3}}{2(\sqrt{3}+1)}$ (2) $\frac{2}{(\sqrt{3}-1)}$ (3) $\frac{2}{3-\sqrt{3}}$ (4) $\frac{2}{3+\sqrt{3}}$
11. Let α and β be real numbers. Consider a 3×3 matrix A such that $A^2 = 3A + \alpha I$. If $A^4 = 21A + \beta I$, then:
- (1) $\alpha = 4$ (2) $\alpha = 1$ (3) $\beta = -8$ (4) $\beta = 8$
12. For two non-zero complex numbers z_1 and z_2 , if $\operatorname{Re}(z_1 z_2) = 0$ and $\operatorname{Re}(z_1 + z_2) = 0$, then which of the following are possible?
- A. $\operatorname{Im}(z_1) > 0$ and $\operatorname{Im}(z_2) > 0$ B. $\operatorname{Im}(z_1) < 0$ and $\operatorname{Im}(z_2) > 0$
 C. $\operatorname{Im}(z_1) > 0$ and $\operatorname{Im}(z_2) < 0$ D. $\operatorname{Im}(z_1) < 0$ and $\operatorname{Im}(z_2) < 0$
- Choose the correct answer from the options given below:
- (1) A and B (2) A and B (3) B and C (4) B and D
13. Let $f(x) = x + \frac{a}{\pi^2 - 4} \sin x + \frac{b}{\pi^2 - 4} \cos x, x \in \mathbb{R}$ be a function which satisfies $f(x) = x + \int_0^{\pi/2} \sin(x+y)f(y)dy$. Then $(a+b)$ is equal to:
- (1) $-\pi(\pi-2)$ (2) $-2\pi(\pi-2)$ (3) $-2\pi(\pi+2)$ (4) $-\pi(\pi+2)$

14. Consider the following system of equations
- $$\begin{aligned} \alpha x + 2y + z &= 1 \\ 2\alpha x + 3y + z &= 1 \\ 3x + \alpha y + 2z &= \beta \end{aligned}$$
- for some $\alpha, \beta \in \mathbb{R}$. Then which of the following is NOT correct.
- (1) It has a solution for all $\alpha \neq -1$ and $\beta = 2$
 (2) It has no solution for $\alpha = -1$ and for all $\beta \in \mathbb{R}$
 (3) It has no solution if $\alpha = -1$ and $\beta \neq 2$
 (4) It has no solution for $\alpha = 3$ and for all $\beta \neq 2$
15. Three rotten apples are mixed accidentally with seven good apples and four apples are drawn one by one without replacement. Let the random variable X denote the number of rotten apples. If μ and σ^2 represent mean and variance of X , respectively, then $10(\mu^2 + \sigma^2)$ is equal to:
- (1) 25 (2) 20 (3) 30 (4) 250
16. Let the tangents at the points $A(4, -11)$ and $B(8, -5)$ on the circle $x^2 + y^2 - 3x + 10y - 15 = 0$, intersect at the point C . Then the radius of the circle, whose centre is C and the line joining A and B is its tangents, is equal to:
- (1) $\frac{3\sqrt{3}}{4}$ (2) $2\sqrt{13}$ (3) $\frac{2\sqrt{13}}{3}$ (4) $\sqrt{13}$
17. The domain of $f(x) = \frac{\log_{(x+1)}(x-2)}{e^{2\log_e x} - (2x+3)}$, $x \in \mathbb{R}$ is:
- (1) $(-1, \infty) - \{3\}$ (2) $(2, \infty) - \{3\}$ (3) $\mathbb{R} - \{3\}$ (4) $\mathbb{R} - \{-1, 3\}$
18. Let B and C be the two points on the line $y + x = 0$ such that B and C are symmetric with respect to the origin. Suppose A is a point on $y - 2x = 2$ such that $\triangle ABC$ is an equilateral triangle. Then, the area of the $\triangle ABC$ is:
- (1) $\frac{8}{\sqrt{3}}$ (2) $\frac{10}{\sqrt{3}}$ (3) $3\sqrt{3}$ (4) $2\sqrt{3}$
19. If p, q , and r are three propositions, then which of the following combination of truth values of p, q and r makes the logical expression $\{(p \vee q) \wedge ((\sim p) \vee r)\} \rightarrow ((\sim q) \vee r)$ false?
- (1) $p = F, q = T, r = F$ (2) $p = T, q = F, r = T$
 (3) $p = T, q = F, r = F$ (4) $p = T, q = T, r = F$
20. Let $\lambda \neq 0$ be a real number. Let α, β be the roots of the equation $14x^2 - 31x + 3\lambda = 0$ and α, γ be the roots of the equation $35x^2 - 53x + 4\lambda = 0$. Then $\frac{3\alpha}{\beta}$ and $\frac{4\alpha}{\gamma}$ are the roots of the equation:
- (1) $49x^2 - 245x + 250 = 0$ (2) $49x^2 + 245x + 250 = 0$
 (3) $7x^2 + 245x - 250 = 0$ (4) $7x^2 - 245x + 250 = 0$

SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be rounded off to the nearest integer.

21. Let \vec{a} , \vec{b} and \vec{c} be three non-zero non-coplanar vectors. Let the position vectors of four points A, B, C and D be $\vec{a}-\vec{b}+\vec{c}$, $\lambda\vec{a}-3\vec{b}+4\vec{c}$, $-\vec{a}+2\vec{b}-3\vec{c}$ and $2\vec{a}-4\vec{b}+6\vec{c}$ respectively. If \vec{AB} , \vec{AC} and \vec{AD} are coplanar, then λ is equal to _____ .
22. If all the six digit numbers $x_1x_2x_3x_4x_5x_6$ with $0 < x_1 < x_2 < x_3 < x_4 < x_5 < x_6$ are arranged in the increasing order, then the sum of the digits in the 72th number is _____ .
23. Five digit numbers are formed using the digits 1, 2, 3, 5, 7 with repetitions and are written in descending order with serial numbers. For example, the number 77777 has serial number 1. Then the serial number of 35337 is _____ .
24. Let the equation of the plane P containing the line $x+10 = \frac{8-y}{2} = z$ be $ax+by+3z=2(a+b)$ and the distance of the plane P from the point $(1, 27, 7)$ be c . Then $a^2 + b^2 + c^2$ is equal to _____ .
25. Let a_1, a_2, a_3, \dots be a GP of increasing positive numbers. If the product of fourth and sixth terms is 9 and the sum of fifth and seventh terms is 24, then $a_1a_9 + a_2a_4a_9 + a_5 + a_7$ is equal to _____ .
26. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function that satisfies the relation $f(x+y) = f(x) + f(y) - 1$, $\forall x, y \in \mathbb{R}$. If $f'(0) = 2$, then $|f(-2)|$ is equal to _____ .
27. Let the co-ordinates of one vertex of ΔABC be $A(0, 2, \alpha)$ and the other two vertices lie on the line $\frac{x+\alpha}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. For $\alpha \in \mathbb{Z}$, if the area of ΔABC is 21 sq. units and the line segment BC has length $2\sqrt{21}$ units, then α^2 is equal to _____ .
28. Let the coefficient of three consecutive terms in the binomial expansion of $(1+2x)^n$ be the ratio 2 : 5 : 8. Then the coefficient of the term, which is in the middle of these three terms, is _____ .
29. Suppose f is a function satisfying $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{N}$ and $f(1) = \frac{1}{5}$. If $\sum_{n=1}^m \frac{f(n)}{n(n+1)(n+2)} = \frac{1}{12}$, then m is equal to _____ .
30. If the co-efficient of x^9 in $\left(\alpha x^3 + \frac{1}{\beta x}\right)^{11}$ and the co-efficient of x^{-9} in $\left(\alpha x - \frac{1}{\beta x^3}\right)^{11}$ are equal, then $(\alpha\beta)^2$ is equal to _____ .