VMC Vidyamandir Classes SINCE 1986

IIT JEE | MEDICAL | FOUNDATION

SOLUTIONS

Joint Entrance Exam | IITJEE-2025

28th JANUARY 2025 | Morning Shift

MATHEMATICS

SECTION – 1

1.(4) $\cos\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{33}{65}\right)$

$\cos\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{5}{12}+\tan^{-1}\frac{33}{56}\right)$

$\cos\left(\tan^{-1}\left(\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}}\right)+\tan^{-1}\frac{33}{56}\right)$

$\cos\left(\tan^{-1}\left(\frac{56}{33}\right)+\tan^{-1}\frac{33}{56}\right)$

$\cos\left(\frac{\pi}{2}\right)=0$

2.(2) $f(x)=(3a+2)x^2+\left(\frac{a+2}{a-1}\right)x+b$

$f\left(x+\frac{1}{2}\right)=f(x)+f(y)+1-\frac{2}{7}xy$ ...(i)

In (i) put $x=y=0 \Rightarrow f(0)=2f(0)+1 \Rightarrow f(0)=1$

So, $f(0)=0+0+b=-1 \Rightarrow b=-1$

In (i) put $y=-x \Rightarrow f(0)=f(x)+f(-x)+1+\frac{2}{7}x^2$

$-1=2(3a+2)x^2+2b+1+\frac{2}{7}x^2$

$-1-\left(2(3a+2)+\frac{2}{7}\right)x^2+1-2$

$a=\frac{-5}{7}$

$f(x)=\frac{-1}{7}x^2\frac{-3}{4}x-1$

$|f(x)|=\frac{1}{28}\left|4x^2+21x+28\right|$

$28\sum_{i=1}^{5}|f(i)|=675$

3.(4) $\int_0^x tf(t)dt=x^2+f(x)\ ;\ x>0$

Diff both side w.r.t.1 x

$xf(x)=x^2+f'(x)+2xf(x)$

$-xf(x)=x^2f'(x)$

$$\int \frac{f'(x)}{f(x)} = \int -\frac{1}{x}$$

$$\ln(f(x)) = -\ln x + C$$

$$f(x) = \frac{C}{x}$$

$$f(2) = 3 \Rightarrow 3 = \frac{C}{2} \Rightarrow C = 6$$

$$f(x) = \frac{6}{x}, \quad f(6) = 1$$

4.(3) $x^2 + |2x-3| - 4 = 0$

Case-1: $x \geq \frac{3}{2}$ $\quad x^2 + 2x - 3 - 4 = 0$

$x^2 + 2x - 7 = 0$

$x = 2\sqrt{2} - 1$

Case-2 : $x < \frac{3}{2}$ $\quad x^2 + 3 - 2x - 4 = 0$

$x^2 - 2x - 1 = 0$

$x = 1 - \sqrt{2}$

Sum of squences $= \left(2\sqrt{2}-1\right)^2 + \left(1-\sqrt{2}\right)^2$

$= 6\left(2-\sqrt{2}\right)$

5.(4) Case-1 5.....0

Case-2 5.....1

5.....2

5.....3

6.....0

6.....1

6.....2

7.....0

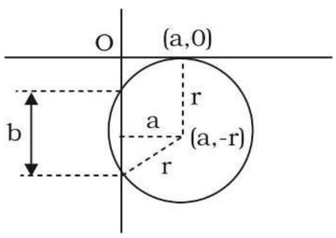
7.....1

$9 \times (8 \times 8 \times 8) = 4608$ but 50000 is not includeal, so total numebrs $4608 - 1 = 4607$

6.(4) By pytogorus

$$r^2 = a^2 + \frac{b^2}{4} = p^2$$

$$r = \sqrt{\frac{4a^2 + b^2}{2^4}}$$



Equation is $(x-a)^2 + (y-p)^2 = r^2$

$x^2 + y^2 - 2ax - 2py + \alpha^2 + \beta^2 - r^2 = 0$

Comparision $x^2 + y^2 - \alpha x + \beta y + \gamma = 0$

$-\alpha = -2a$

$\alpha = 2a$

$\beta = -2p,\ \gamma = a^2 + p^2 - r^2$

$a^2 + \frac{b^2}{4} = \frac{\beta^2}{4} \Rightarrow b^2 = \beta^2 - 4a^2$

$b^2 = \beta^2 - 4\gamma$

7.(1) $a_0 = 0,\ a_1 = \frac{1}{2}$ $\quad 2a_{n+2} = 5_{an+1} - 3a_n$

$2x^2 - 5x + 3 = 0 \Rightarrow x = 1, \frac{3}{2}$

$a_n = A(x_0^n + B(x_2)^n$

$a_n = a(1)^n + B\left(\frac{3}{2}\right)^n$

$$\left.\begin{matrix} n = 0 & 0 = A + B \\ n = 1 & \frac{1}{2} = A + \frac{3}{2} \end{matrix}\right] \begin{matrix} A = -1 \\ B = 1 \end{matrix}$$

$a_n = -1 + \left(\frac{3}{2}\right)^n$

$$\sum_{k=1}^{100} a_k = \sum_{k=1}^{100} (-1) + \left(\frac{3}{2}\right)^k$$

$$= -100 + \left(\frac{3}{2}\right) \frac{\left(\left(\frac{3}{2}\right)^{100} - 1\right)}{\left(\frac{3}{2} - 1\right)}$$

$$= -100 + 3\left[\left(\frac{3}{2}\right)^{100} - 1\right]$$

$= 3(a_{100}) - 100$

8.(2) $i^{k_1} + i^{k_2} \neq 0 \quad i^{k_1} \to 4$ option for $i, -1, -i, 1$

Total cases $\Rightarrow 4 \times 4 = 16$

Unfavorable cases $\Rightarrow i^{k_1} + i^{k_2} = 0$

$$\left.\begin{matrix} 1, -1 \\ -1.1 \\ i, -i \\ -i, i \end{matrix}\right\} \text{ 4 cases} \Rightarrow \text{probability} = \frac{16 - 4}{16} = \frac{3}{4}$$

9.(2) $R = \{(x, y),\ x + y$ is even $x, y \in z\}$

Reflexive $x + x = 2x \Rightarrow$ even

Symmeric of $x + y$ is even then $y + x$ is also even

Transitive of $x + y$ is even and $y + z$ is even then $(x + z)$ is also even

So, relation is an equivalence relation

10.(3) $A(x,y,z)$ Let $P(0,3,2)$, $Q(2,0,3)$, $R(0,0,1)$

$AP = AQ = AR$

$x^2+(y-3)^2+(z-2)^2=(x-2)^2+y^2+(z-3)^2=x^2+y^3+(z-1)^2$

In xy plane $z = 0$

$x^2-4x+4+y^2+9=x^2+y^2+1$

$x=3$

$9+y^2-6y+9+4=x^2+y^2+1$

$A(3,2,0)$ also $B(1,4,-1)$ & $C(2,0,-2)$

$AB=\sqrt{4+4+1}=3$

$AC=\sqrt{1+4+4}=3$

$BC=\sqrt{1+16+1}=\sqrt{18}$

$AB = AC$

Isosceles Δ & $AB^2+AC^2=BC^2$ right angle triangle

Area of $\Delta ABC=\frac{1}{2}\times b\times h$

$=\frac{1}{2}\times 3\times 3=\frac{9}{2}$

Only S_1 is true.

11.(2) $\frac{{}^nC_r}{{}^nC_{r-1}}=2 \Rightarrow n-3r=-1$

$\frac{{}^nC_{r+1}}{{}^nC_r}=\frac{70}{56} \Rightarrow 4n-9r=5$

$n=8,\quad r=3$

$C(1,0)$

Let centroid be (h, k)

$h=\frac{4\cos t+2\sin t+1}{3}$

$k=\frac{4\sin t-2\cos t}{3}$

$(3x-1)^2+(3y)^2=20$

$\therefore\ \alpha=20$

12.(2) $OA=|z_1|=\sqrt{11}$

$OB=|z_2|=\frac{|z_1|}{\sqrt{3}}=\sqrt{\frac{11}{3}}$

$z_2=\frac{1}{\sqrt{3}}\left(\sqrt{3}+2\sqrt{2}i\right)e^{i\frac{\pi}{6}}$

$|z_2-z_1|=\sqrt{\frac{11}{3}}$

13.(2) Let A.P. be $a, a+d, a+2d, ...$

$$a+(m-1)d=\frac{1}{25}$$

$$a+24d=\frac{1}{20}$$

$$20\sum_{r=1}^{25} t_r=13$$

$$2a+24d=\frac{13}{250}$$

$$a=\frac{1}{500},\ d=\frac{1}{500},\ m=20$$

$$5m\sum_{r=m}^{2m} t_r$$

$$100[t_{20}+t_{21}+.....+t_{40}]=126$$

14.(4)

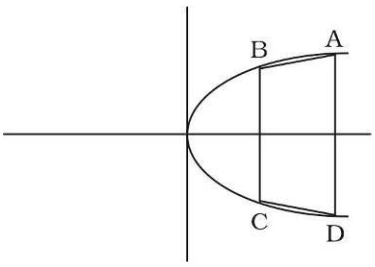
X	0	1	2
$P(X)$	$\frac{^7C_2}{^{10}C_2}$	$\frac{^7C_1 \cdot {^3C_1}}{^{10}C_2}$	$\frac{^3C_2}{^{10}C_2}$

$$\text{Variance} = \left[\frac{^7C_2}{^{10}C_2}0^2+\frac{^7C_1 \cdot {^3C_1}}{^{10}C_2}1^2+\frac{^3C_2}{^{10}C_2}2^2\right]-\left[\frac{^7C_2}{^{10}C_2}(0)+\frac{^7C_1 \cdot {^3C_1}}{^{10}C_2}(1)+\frac{^3C_2}{^{10}C_2}(2)\right]^2$$

$$=\frac{28}{75}$$

15.(1) $AC=\frac{25}{4}$

$$a(t_1-t_2)^2=\frac{25}{4}$$



$$t_1-t_2=\frac{5}{2}$$

$$t_1t_2=-1$$

$$t_1=2 \quad t_2=-\frac{1}{2}$$

$$A\equiv(4,4)$$

$$C\equiv\left(\frac{1}{4},-1\right)$$

$$\text{Area }=\frac{75}{4}$$

16.(4) $f(x)+f(1-x)=\dfrac{2^x}{2^x+\sqrt{2}}+\dfrac{2^{1-x}}{2^{1-x}+\sqrt{2}}=1$

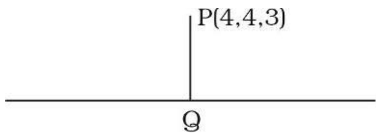
$f\left(\dfrac{1}{82}\right)+f\left(\dfrac{2}{82}\right)+.....+f\left(\dfrac{40}{81}\right)+......+f\left(\dfrac{80}{82}\right)+f\left(\dfrac{81}{82}\right)$

$=40+f\left(\dfrac{1}{2}\right)$

$=40+\dfrac{1}{2}$

$=\dfrac{81}{2}$

17.(1) $Q\equiv(2\lambda+1,\ \lambda+2,\ 3\lambda+1)$



drs of PQ

$(2\lambda-3, \lambda-2,\ 3\lambda-2)$

$2(2\lambda-3)+1(\lambda-2)+3(3\lambda-2)=0$

$\lambda=1$

$Q\equiv(3,3,4)$

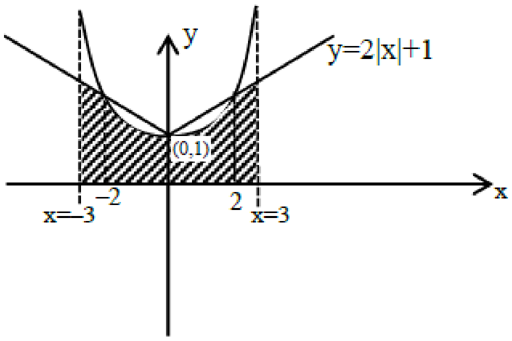
$\dfrac{\alpha+4}{2}=3 \qquad \dfrac{\beta+4}{2}=3 \qquad \dfrac{\gamma+3}{2}=4$

$\alpha=2 \quad \beta=2 \quad \gamma=5$

$\alpha+\beta+\gamma=9$

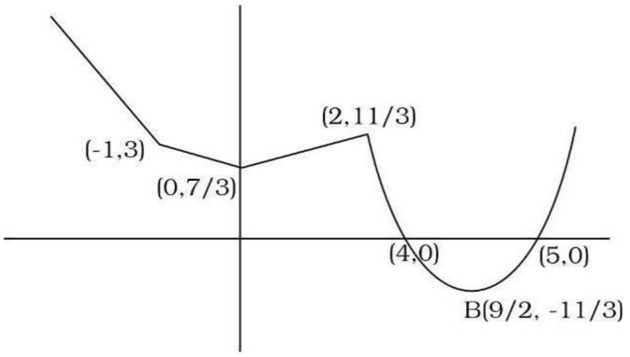
18.(4) Area $=2\left[\displaystyle\int_0^2(x^2+1)\,dx+\int_2^3(2x+1)\,dx\right]$

$=\dfrac{64}{3}$



19.(3) $\left(0,\dfrac{7}{3}\right)$ & $\left(\dfrac{9}{2},\dfrac{-11}{72}\right)$ are points of local minima

Sum $=\dfrac{157}{72}$



20.(4) $I = \int_{-\pi/2}^{\pi/2} \frac{96x^2 \cos^2 x}{1+e^x} dx$

$= \int_0^{\pi/2} 96x^2 \cos^2 x \, dx$

$= 48 \int_0^{\pi/2} x^2 (1+\cos 2x) \, dx$

$= 48 \left(\frac{x^3}{3}\right)\Big|_0^{\pi/2} + 48 \int_0^{\pi/2} x^2 \cos 2x \, dx$

$= 16\left[\frac{\pi^3}{8}\right] + 48\left[\frac{x^2 \sin 2x}{2}\Big|_0^{\pi/2} - \int_0^{\pi/2} (2x)\left(\frac{\sin 2x}{2}\right) dx\right]$

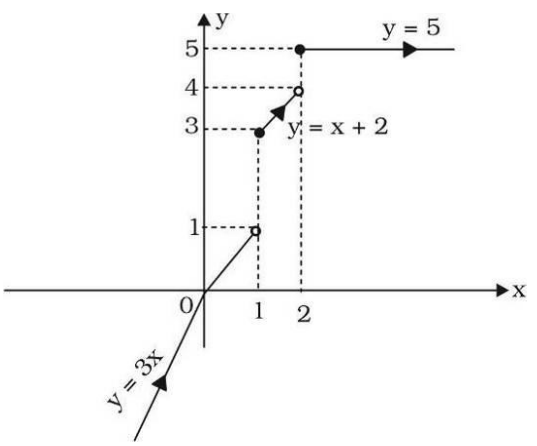
$= 2\pi^3 - 12\pi$

$\Rightarrow (\alpha+\beta)^2 = 100$

SECTION – 2

21.(5) $f(x) = \begin{cases} 3x, & x < 0 \\ \min\{1+x+[x],\ x+2[x]\}, & 0 \le x \le 2 \\ 5. & x > 2 \end{cases}$

$f(x) = \begin{cases} 3x, & x < 0 \\ x, & 0 \le x < 1 \\ x+2, & 1 \le x < 2 \\ 5, & x = 2 \\ 5. & x > 2 \end{cases}$



$f(x)$ is not cont. at $x = 1, 2$

$f(x)$ is not diff. at $x = 0, 1, 2$

$\alpha = 2,\ \beta = 3$

$\alpha + \beta = 5$

22.(5) $\sum_{r=1}^{6}(-3)^{r-1}\times {}^{12}C_{2r-1}=\sum_{r=1}^{6}(-1)^{r-1}\left(\sqrt{3}\right)^{2r-2}\times {}^{12}C_{2r-1}$

$=\frac{1}{\sqrt{3}}\sum_{r=1}^{6}(-1)^{r-1}\left(\sqrt{3}\right)^{2r-1}\times {}^{12}C_{2r-1}$

$=\frac{1}{\sqrt{3}}\left[{}^{12}C_1\sqrt{3}-{}^{12}C_3\left(\sqrt{3}\right)^3+{}^{12}C_5\left(\sqrt{3}\right)^5.....-{}^{12}C_{11}\left(\sqrt{3}\right)^{11}\right]$

Now, $(1+x)^{12}={}^{12}C_0+{}^{12}C_1x+{}^{12}C_2x^2+....+{}^{12}C_{12}x^{12}$

Put $x=\sqrt{3}i$

$\left(1+\sqrt{3}i\right)^{12}={}^{12}C_0+{}^{12}C_1\sqrt{3}i-{}^{12}C_2\left(\sqrt{3}\right)^2-{}^{12}C_3\left(\sqrt{3}\right)^3+.....$

$\left(2\cdot e^{i\frac{\pi}{3}}\right)^{12}=\left({}^{12}C_0-{}^{12}C_2\left(\sqrt{3}\right)^2+.....\right)+i\left({}^{12}C_1\sqrt{3}-{}^{12}C_3\left(\sqrt{3}\right)^3+......\right)$

$2^{12}\cdot e^{4\pi i}=\left({}^{12}C_0-{}^{12}C_2\left(\sqrt{3}\right)^2+.....\right)+i\left({}^{12}C_1\sqrt{3}-{}^{12}C_3\left(\sqrt{3}\right)^3+...\right)$

$\Rightarrow\ {}^{12}C_1\sqrt{3}-{}^{12}C_3\left(\sqrt{3}\right)^3+......=0$

$\alpha=1+\sum_{r=1}^{6}(-3)^{r-1}\times {}^{12}C_{2r-1}=1+0=1$

$\frac{|12\alpha-2|}{\sqrt{\alpha^2+3}}=\frac{10}{2}=5$

23.(1613) $S=\{-3,\ -2,\ -1,\ 1,\ 2\}$

$n(S_1)=5\times5\times5\times5\times5\times5\times1\times1\times1=5^6$

$n(S_2)=0$ $\{\because$ All diagonal elements = 0 but $0\notin S$

$n(S_3)=3!\times5^6+\frac{3!}{2}\times5^6+\frac{3!}{2}\times5^6=12\times5^6$

{Diagonal elements $(-3,1,2)$ $(-2,1,1)$ $(-1,-1,2)$}

$n(S_1\cap S_3)=12\times5^3\times1^3$

$n(S_1\cup S_2\cup S_3)=n(S_1)+n(S_3)-n(S_1\cap S_3)$ $\{\because n(S_2)=0\}$

$=5^6+12\times5^6-12\times5^3$

$=5^3(125+12\times125-12)$

$=125\times1613$ $\Rightarrow$ $\alpha=1613$

24.(6) $\vec{d}=\vec{a}\times\vec{b}$

$\vec{d}=-\hat{i}+\hat{j}\ \Rightarrow\ |\vec{d}|=\sqrt{2}$

$|\vec{c}-2\vec{a}|^2=8$

$|\vec{c}|^2+4|\vec{a}|^2-4\vec{c}\cdot\vec{a}=8$

$|\vec{c}|^2+4(3)-4|\vec{c}|=8$ $\{\because\ \vec{a}\cdot\vec{c}=|\vec{c}|\}$

$|\vec{c}|^2-4|\vec{c}|+4=0$

$\Rightarrow\ |\vec{c}|=2$

$$\left|\vec{d}\times\vec{c}\right| = \left|\vec{d}\right|\left|\vec{c}\right|\sin\frac{\pi}{4} = \sqrt{2}\times 2\times\frac{1}{\sqrt{2}} = 2$$

$$\vec{d}\cdot\vec{c} = \left|\vec{d}\right|\left|\vec{c}\right|\cos\frac{\pi}{4} = 2$$

$$\Rightarrow \left(\vec{a}\times\vec{b}\right)\cdot\vec{c} = 2 \qquad \Rightarrow \left[\vec{a}\,\vec{b}\,\vec{c}\right] = 2$$

$$\Rightarrow \left[\vec{a}\,\vec{b}\,\vec{c}\right]^2 = 4$$

$$\Rightarrow \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{b}\cdot\vec{a} & \vec{c}\cdot\vec{a} \\ \vec{a}\cdot\vec{b} & \vec{b}\cdot\vec{b} & \vec{c}\cdot\vec{b} \\ \vec{a}\cdot\vec{c} & \vec{b}\cdot\vec{c} & \vec{c}\cdot\vec{c} \end{vmatrix} = 4$$

$$\Rightarrow \begin{vmatrix} 3 & 5 & 2 \\ 5 & 9 & \vec{b}\cdot\vec{c} \\ 2 & \vec{b}\cdot\vec{c} & \vec{c}\cdot\vec{c} \end{vmatrix} = 4$$

$$\Rightarrow \vec{b}\cdot\vec{c} = 4 \text{ or } \frac{8}{3}$$

$$\Rightarrow \left|10 - 3\vec{b}\cdot\vec{c}\right| = 2$$

$$\left|10 - 3\vec{b}\cdot\vec{c}\right| + \left|\vec{d}\times\vec{c}\right|^2 = 2 + 2^2 = 6$$

25.(54) $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$

$$\frac{x^2}{a_1^2} + \frac{y^2}{b_1^2} = 1$$

$$a_1 = 3,\ b_1 = 2,\ e = \frac{\sqrt{5}}{3}$$

$$a_2 = 2,\ b_2^2 = a_2^2\left(1 - e^2\right)$$

$$b_2^2 = 4\times\frac{4}{9}$$

$$b_2 = \frac{4}{3}$$

$$a_2 = 2,\ b_2 = \frac{4}{3}$$

Similarly $a_3 = \frac{4}{3},\ b_3 = \frac{8}{9}$

$$A_1 = \pi a_1 b_1 = 6\pi$$

$$A_2 = \pi a_2 b_2 = \frac{8\pi}{3}$$

$$A_3 = \pi a_3 b_3 = \frac{32}{27}\pi$$

$\Rightarrow A_1, A_2, A_3, \ldots\ldots$ G.P. with $r = \frac{4}{9}$

$$\Rightarrow \sum_{i=1}^{\infty} A_i = \frac{6\pi}{1 - \frac{4}{9}} = \frac{54\pi}{5} \Rightarrow \frac{5}{\pi}\sum_{i=1}^{\infty} A_i = 54$$

PHYSICS

SECTION – 1

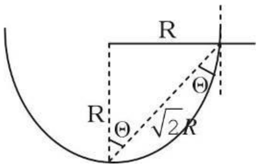
26.(2) Truth table

A	B	Y
1	0	0
0	1	1
1	1	0
0	0	1

$Y = \bar{A}$

27.(1) Apply snell's law

$\mu \sin\theta = 1 \sin 90°$



$\mu = \frac{R}{\sqrt{2}R} = 1$

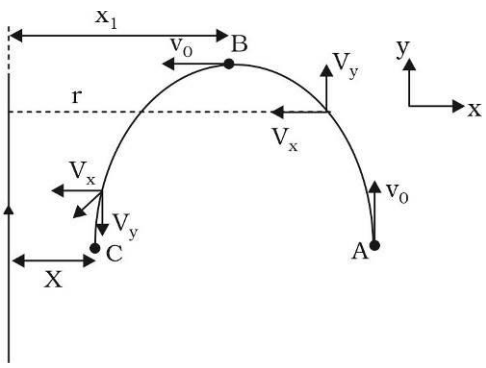
$\mu = \sqrt{2}$

28.(3) $A \to B$

$\vec{V} = -v_x\hat{i} + v_y\hat{j}$

$\vec{B} = \frac{\mu_0 I}{2\pi r}(-\hat{k})$

$\vec{F} = q\left(\vec{v} \times \vec{B}\right) = \frac{\mu_0 Iq}{2\pi r}\left[-v_x\hat{j} - v_y\hat{i}\right]$



$a_x = -\frac{\mu_0 Iq}{2\pi m} \cdot \frac{v_y}{r}$

$a_y = -\frac{\mu_0 Iq}{2\pi m} \cdot \frac{v_x}{r}$

$\frac{v_x dv_x}{dr} = -\frac{\mu_0 Iq}{2\pi m} \frac{v_y}{r}$

$\frac{v_x dv_x}{v_y} = -\frac{\mu_0 Iq}{2\pi m} \frac{dr}{r}$

$$\int_0^{v_0} \frac{v_x dv_x}{\sqrt{v_0^2 - v_x^2}} = -\frac{\mu_0 Iq}{2\pi m}\int_a^{x_1} \frac{dr}{r}$$

Let $z^2 = v_0^2 - v_x^2$

$2zdz = -2v_x dv_x$

$zdx = -v_x dv_x$

$\frac{v_x dv_x}{\sqrt{v_0^2 - v_x^2}} = \frac{-zdz}{z} = -dz$

then integral becomes

$$-\int_{v_0}^{0} dz = -\frac{\mu_0 Iq}{2\pi m}\ln\frac{x_1}{a}$$

$$v_0 = -\frac{\mu_0 Iq}{2\pi m}\ln\frac{x_1}{a}$$

$$x_1 = a\,e^{\frac{2\pi m v_0}{\mu_0 Iq}} \quad \text{...(i)}$$

For $B \to C$

$$\vec{v} = -v_x\hat{i} - v_y\hat{j}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r}(-\hat{k})$$

$$\vec{F} = q(\vec{v}\times\vec{B}) = \frac{\mu_0 Iq}{2\pi q}(-v_x\hat{j} + v_y\hat{i})$$

$$a_x = \frac{\mu_0 Iq}{2\pi m}\frac{v_y}{r} \qquad\qquad a_y = -\frac{\mu_0 Iq}{2\pi m}\frac{v_x}{r}$$

$$\frac{v_x dv_x}{dr} = \frac{\mu_0 Iq}{2\pi m}\frac{v_y}{r}$$

$$\int_{v_0}^{0}\frac{v_x dv_x}{\sqrt{v_0^2 - v_x^2}} = \frac{\mu_0 Iq}{2\pi m}\int_{x_1}^{x}\frac{dr}{r}$$

$$\frac{\mu_0 Iq}{2\pi m}\ln\frac{x}{x_1} = -\int_{0}^{v_0} dz = -v_0$$

$$x = x_1 e^{-\frac{2\pi m v_0}{\mu_0 Iq}} \quad \text{...(ii)}$$

From equaiton (i) and (ii)

$$X = ae^{-\frac{4\pi m v_0}{\mu_0 Iq}}$$

29.(2) $v = -\int \vec{E}.d\vec{r} = \int \frac{2k\lambda}{r}dr = 2k\lambda \ln r + c$

Net potential due to all wire

$$v = 2k\lambda\ln\sqrt{x^2+y^2} + 2k\lambda\ln\sqrt{y^2+z^2} + 2k\lambda\ln\sqrt{z^2+x^2} + c$$

For v = c

$$\sqrt{(x^2+y^2)(y^2+z^2)(z^2+x^2)} = c$$

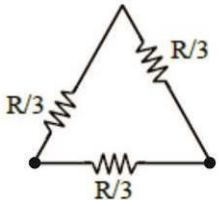
$$\therefore\ (x^2+y^2)(y^2+z^2)(z^2+x^2) = c$$

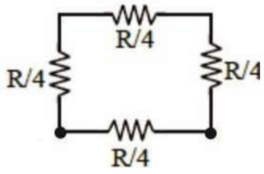
Where c = constant

30.(1) $V_{rms} \propto \sqrt{T}$

31.(2) Theoretical

32.(4)





$$R_1 = \frac{\frac{2R}{3} \times \frac{R}{3}}{\frac{2R}{3} + \frac{R}{3}}$$

$$R_2 = \frac{\frac{3R}{4} \times \frac{R}{4}}{\frac{3R}{4} + \frac{R}{4}}$$

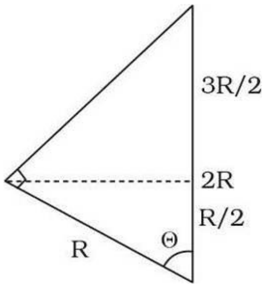
$$R_1 = \frac{2R}{9}$$

$$R_2 = \frac{3R}{16}$$

$$\frac{R_1}{R_2} = \frac{32}{27}$$

33.(2) $V_T = \frac{2}{9} R^2 \frac{g}{\eta}(d - \rho)$

34.(2)



Potential energy of spring $= \frac{1}{2} K(R)^2$

Potential energy of mass $= mg \times \frac{3R}{2}$

Enerty conservation $= \frac{1}{2} KR^2 + \frac{3mgR}{2} = \frac{1}{2} mv^2$

$$v = \sqrt{3Rg + \frac{KR^2}{m}}$$

35.(2) $\eta = 1 - \frac{T_L}{T_H}$

$$\eta_{12} = 1 - \frac{273}{473} = \frac{200}{473}$$

$$\eta_1 = 1 - \frac{373}{473} = \frac{100}{473}$$

$$\eta_2 = 1 - \frac{273}{373} = \frac{100}{273}$$

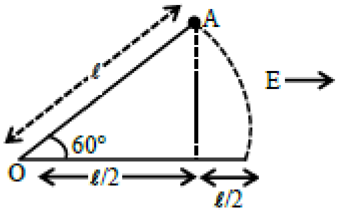
$$\eta_{12} < \eta_1 + n_2$$

36.(1) $W_{all} = \Delta k$

$W_e = k_f - k_i$

$qE\frac{l}{2} = \frac{1}{2}mv^2 - 0$

$v = \sqrt{\frac{qEl}{m}}$

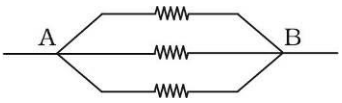


37.(4) $R_e > 4000$ for turbulence flow.

Tangent at a point on the stream line gives the direction of net velocity of the flow. If the 2 steam lines intersect each other its signifies 2 directions of velocity which cannot be possible.

38.(1) $\frac{1}{r_{eg}} = \frac{1}{r/3} + \frac{1}{r/3} + \frac{1}{r/3}$

$r_{eq} = \frac{r}{9}$



39.(2) Theoretical

40.(2) $U_1 = \frac{1}{2}\varepsilon E_1^2 V_1$

$U_2 = \frac{1}{2}\varepsilon E_2^2 V_2$

$U_1 = U_2$

$E_1^2 V_1 = E_2^2 V_2$

$E_1^2 \times \pi\left(\frac{d_1}{2}\right)^2 \times l = E_2^2 \times \pi\left(\frac{d_2}{2}\right)^2 \times l$

$E_1^2 d_1^2 = E_2^2 d_2^2$

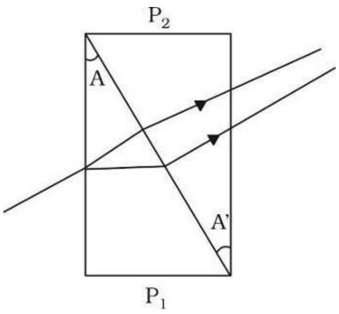
$E_2 = \left(\frac{d_1}{d_2}\right)E_1$

$E_2 = 200\sin(\omega t - kx)$

41.(2) $A' = -\left(\frac{A-1}{A'-1}\right)A$

$A' = -\frac{(1.54-1)}{(1.72-1)} \times 4°$

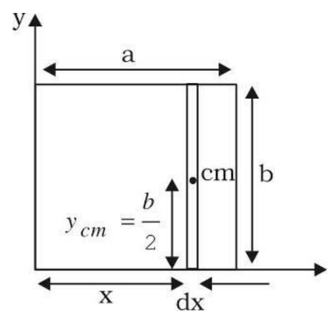
$A' = -3°$



42.(2) $m = \int dm = \int_0^a \frac{\sigma_0 x}{ab} \times bdx$

$m = \frac{\sigma_0 a}{2}$

$x_{cm} = \frac{\int dmx}{\int dm} = \frac{\int_0^a \left(\frac{\sigma_0 x}{ab} \times bdx\right)x}{\frac{\sigma_0 a}{2}}$



$$x_{cm} = \frac{2\int_0^a x^2 dx}{a^2} = \frac{2}{3}a$$

$$x_{cm} = \frac{2}{3}a,\ y_{cm} = \frac{b}{2}$$

43.(3) Energy of proton = Energy of photon of wavelength λ.

$$\frac{1}{2}m_p V_p^2 = \frac{hc}{\lambda} = E \text{ or } \lambda = \frac{hc}{E}$$

de broglie wavelength

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda_p = \frac{h}{\sqrt{2m_p E}}$$

$$\frac{\lambda_p}{\lambda} = \frac{\frac{h}{\sqrt{2m_p E}}}{\frac{hc}{E}}$$

$$\frac{\lambda_p}{\lambda} = \frac{1}{C}\sqrt{\frac{E}{2m_p}}$$

44.(3) $n \to P + \bar{e} + \bar{\nu}$

45.(4) From graph

$q_2 > q_1$

$C_2 V > C_1 V$

$C_2 > C_1$

$$U = \frac{1}{2}CV^2$$

$$\frac{U_1}{U_2} = \frac{C_1}{C_2} \qquad \left\{\frac{C_1}{C_2} < 1\right\}$$

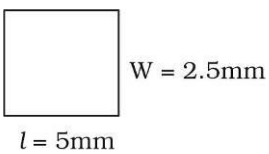
$U_2 > U_1$

SECTION – 2

46.(3) Least count $= \frac{0.75}{15} = 0.05\,mm$

$A = lW$

$$\frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta w}{w} = \frac{0.05}{5} + \frac{0.05}{2.5}$$



$$\frac{\Delta A}{A} = \frac{3}{100}$$

47.(0) Torque required to twist a cylinder is given by

$$\tau = \frac{1}{2}\frac{\pi\eta r^4}{\ell}\theta \Rightarrow \frac{\eta}{\tau} = \frac{2\ell}{\pi r^4\theta} \qquad \Rightarrow \qquad \left[\frac{\eta}{\tau}\right] = M^{\circ}L^{-3}T^{\circ}$$

48.(11) $y = \frac{nDA}{d}$

$$10\times10^{-3} = \frac{10\times D\times 600\times10^{-9}}{d}$$

$$\frac{D}{d} = \frac{1}{6}\times10^4$$

$$y_{10} = 10\left(\frac{D}{d}\right)660\times10^{-9}$$

$$= 10\times\frac{1}{6}\times10^4\times660\times10^{-9}$$

$$y_{10} = 11\,mm$$

49.(16) $I_1 = \frac{M_1R_1^2}{2}$

$$I_2 = \frac{M_2R_2^2}{2}$$

$$\frac{I_1}{I_2} = \frac{M_1}{M_2}\times\left(\frac{R_1}{R_2}\right)^2$$

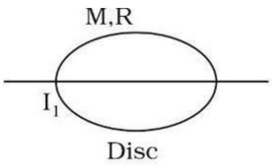
$$M_1 = \sigma\pi R_1^2$$

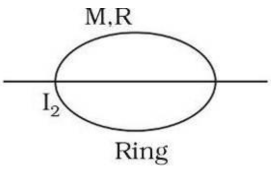
$$M_2 = \sigma\pi\left(2R_1\right)^2$$

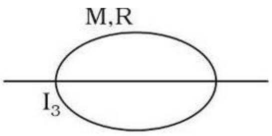
$$\frac{I_1}{I_2} = \frac{\sigma\pi R_1^2}{4\sigma\pi R_1^2}\times\left(\frac{R_1}{2R_1}\right)^2$$

$$\frac{I_1}{I_2} = \frac{1}{16}$$

50.(4)







$I_1 = \frac{MR^2}{4}$ $\qquad I_2 = \frac{MR^2}{2}$ $\qquad I_3 = \frac{2}{5}MR^2$

$I_1 = 2.5I_2$ $\qquad I_3 = \frac{2}{5}M\left(\sqrt{5}r\right)^2$

$\frac{MR^2}{4} = 2.5\times\frac{Mr^2}{2}$ $\qquad I_3 = 2Mr^2$

$R = \sqrt{5}r$ $\qquad I_3 = 4I_2$

CHEMISTRY

SECTION – 1

51.(1) Ni^{2+} gives violet colour in non-luminous flame under hot conditions in Borax Bead Test.

52.(2) A, B, D are correct statement.

(A). $t_{1/2} \propto (A_0)^{1-n}$

$$\frac{(t_{1/2})_2}{(t_{1/2})_1} \propto \frac{(A_0)_2^{1-n}}{(A_0)_1^{1-n}}$$

$$\frac{200}{100} \propto \left(\frac{0.1}{0.025}\right)^{1-n}$$

$$2 = 4^{1-n}$$

$$2 = 2^{2(1-n)}$$

$$1 = 2 - 2n$$

$$n = \frac{1}{2}$$

Order of reaction is $\frac{1}{2}$.

(B). $\frac{t_{1/2}}{200} = \left(\frac{1}{0.1}\right)^{1-n}$

$$\frac{t_{1/2}}{200} = (10)^{1/2}$$

$$t_{1/2} = 200\sqrt{10} \text{ min}$$

(D). $\frac{800}{200} = \left(\frac{A_0}{0.1}\right)^{1-\frac{1}{2}}$

$$4 = (10A_0)^{1/2}$$

$$16 = 10A_0$$

$$A_0 = 1.6M$$

53.(3) ClF_3 is T-shape. Number of lone pair present in equitorial position is 2.

n = 2

number of unpaired electrons in

$V^{3+} : 2$

$Ti^{3+} : 1$

$Cu^{2+} : 1$

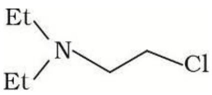
$Ni^{2+} : 2$

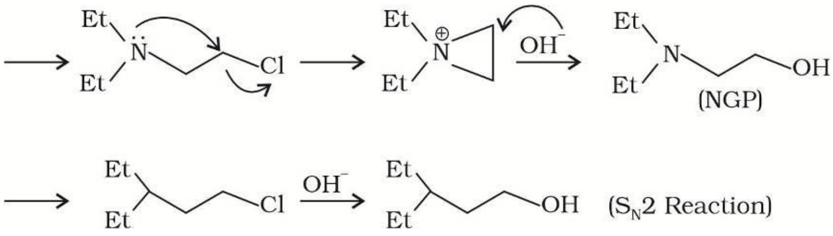
$Ti^{2+} : 2$

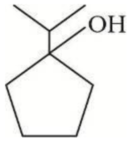
54.(4) Both statements are correct.

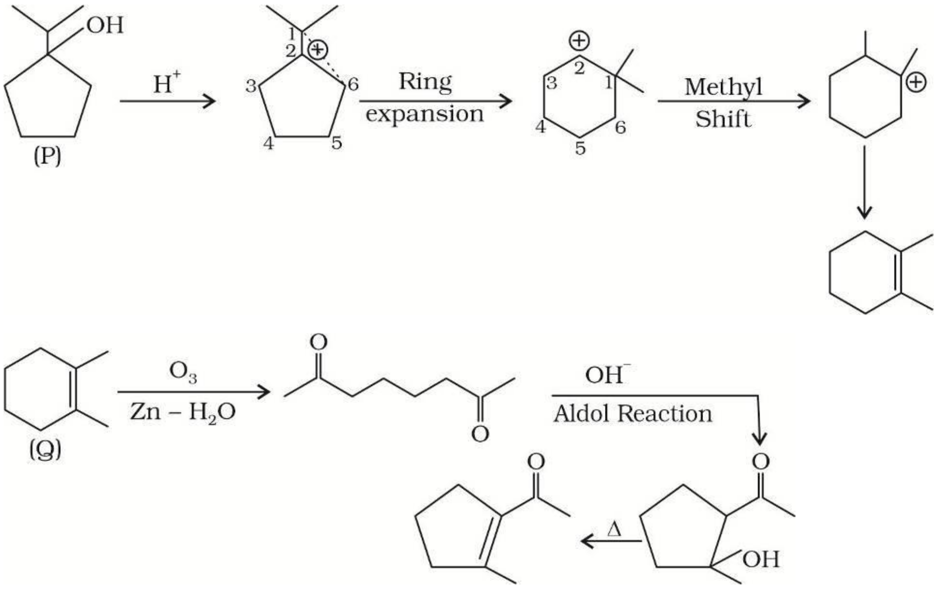
55.(1) Carboxylic acid and Picric acid reacts with $NaHCO_3$ to give CO_2.

A, C, D give $NaHCO_3$ Test.

56.(2)  show NGP reaction, so undergo faster hydrolysis than S_N2 reaction.



57.(2) P is 



58.(3) D-glucose pentaacetate do not reacts with 2, 4 DNP.

59.(4) D and E will have same energy due to same (n + l) value.

60.(1) $HA \rightleftharpoons H^+ + A^- \quad K_a$

$1 - x \qquad x \qquad x$

$$K_a = \frac{x}{1-x} \times (H^+)$$

$$pK_a = -\log\frac{x}{1-x} + pH$$

$$pH - pK_a = \log\left(\frac{x}{1-x}\right)$$

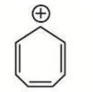
61.(1) 2nd statement is incorrect. Size of Mg > Be

Correct order is: Mg > Al > Si > Be

62.(2) $CH_3CH_2CH_2Br \xrightarrow{AgCN} CH_3CH_2CH_2NC$

$CH_3CH_2CH_2Br \xrightarrow{AgNO_2} CH_3CH_2CH_2NO_2$

63.(4) Tropylium carbocation is most stable due to Aromaticity.

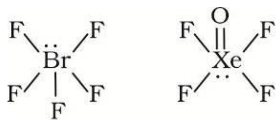

Aromatic

$> (Ph)_3 - \overset{\oplus}{C} > (Ph)_2 - \overset{\oplus}{CH} > CH_3CH_2\overset{\oplus}{CH} - CH_3$

3°Benzylic 2°Benzylic 2°

64.(3)
- Both Acetaldehyde and Acetone give Iodoform Test due to presence of methyl carbonyl group.
- Both Acetaldehyde and Acetone give Aldol condensation due to presence of α Hydrogen.

65.(3) BrF_5 and $XeOF_4$ are square Pyramidal due to 5σ bond and 1 1p.



66.(3)

A. $CH_4(g) + O_2(g) \longrightarrow CO_2(g) + H_2O(l)$ Combustion reaction

B. $2NaH(s) \xrightarrow{\Delta} 2Na(s) + H_2(g)$ Decompostion reaction

C. $V_2O_5(s) + 5Ca(s) \xrightarrow{\Delta} 2V(s) + 5CaO(s)$ Displacement reaction

D. $2H_2O_2(aq) \xrightarrow{\Delta} 2H_2O(l) + O_2(g)$ Disproportionation reaction

67.(3) $H_2O(s) \rightleftharpoons H_2O(l)$

If pressure of system is increased 2 times keeping temperature constant, ice disappear completely.

68.(4) $\Delta T_f = k_f\,m$

$$0.4 = k_f\left(\frac{1}{256} \times \frac{1000}{50}\right)$$

$k_f = 5.12\,K\,Kg\,mol^{-1}$

69.(4)
- Both $K_2Cr_2O_7$ and $KMnO_4$ can oxidise I^-, S^{2-}, Fe^{2+} in acidic medium.
- Only $KMnO_4$ oxidise I^- to IO_3^- and $S_2O_3^{2-}$ to SO_4^{2-} in faintly alkaline or neutral medium.

70.(2) Overall order of IE: In < Al < Sn < Pb < Ge
Maximum IE is of Ge. Most stable oxidation state of Ge is + 4
Maximum IE is of In. Most stable oxidation state of In is + 3
($E°In^{3+}/In = -0.34$ V, $E°In^{+}/In = -0.18$ V)
∴ Most stable oxidation state of In is + 3

SECTION – 2

71.(150) $\Lambda_m = \dfrac{k \times 1000}{M}$ $k = \dfrac{x}{R}$

For $\sqrt{0.15}M$; $\Lambda_m = 100$ $100 = \dfrac{x}{100} \times \dfrac{1000}{(0.15)^2}$ ...(i)

For $\sqrt{0.1}M$; $\Lambda_m = 150$ $150 = \dfrac{x}{R} \times \dfrac{1000}{(0.1)^2}$ ...(ii)

Solving (i) and (ii), $R = 150\Omega$

72.(125) $M = \frac{10xd}{M^\circ}$

$= \frac{10 \times 70 \times 1.25}{70}$

$= 12.5 = 125 \times 10^{-1}$

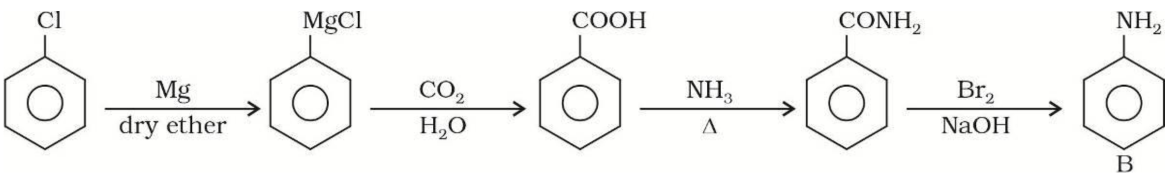
73.(1655)

	Mass	**Mole**	**Minimum Ratio**	
C	14.5	1.2	1	2
H	1.8	1.8	1.5	3
O	19.24	1.2	1	2
Cl	64.46	1.8	1.5	3

Empirical formula $C_2H_3Cl_3O_2$

Mass = 165.5 = 1655×10^{-1}g

74.(93)



$11.25\,mg$

$M^\circ = 112.5$

mmol = 0.1

mmol of Aniline = 0.1 ; M° = 93

mass in mg = 9.3 = 93×10^{-1}mg

75.(466) ΔH°

$\frac{1}{2}H_2(g) \rightarrow H(g)$	$220\,kJmol^{-1}$
$\frac{1}{2}O_2(g) \rightarrow O(g)$	$250\,kJmol^{-1}$
$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(g)$	$-242\,kJmol^{-1}$

$\Delta H_f^\circ = -242 = BE(H-H) + \frac{1}{2}BE(O=O) - 2(O-H)$

$= -242 = 440 + 250 - 2x$

$x = 466\,kJmol^{-1}$