VMC Vidyamandir Classes SINCE 1986
IIT JEE | MEDICAL | FOUNDATION

SOLUTIONS

Joint Entrance Exam | IITJEE-2025

28th JANUARY 2025 | Evening Shift

MATHEMATICS

SECTION – 1

1.(3) $P(E)=\frac{6!/2!}{6!}=\frac{1}{2}$

2.(3) $g'(x)=x\,f(x)$

$f(x)=\frac{g'(x)}{x}$

$g'\left(x^3\right)\times 3x^2=6x^5+7x^6$

$g'\left(x^3\right)=\frac{1}{3}\left(6x^3+7x^4\right)$

$f\left(x^3\right)=\frac{g'\left(x^3\right)}{x^3}=2+\frac{7}{3}x$

$\sum_{r=1}^{15} f\left(r^3\right)=30+\frac{7}{3}\times\frac{15\times 16}{2}$

$=30+280=310$

3.(4) $\left(x^2-3x+2\right)+i(2x-2)=0$

$(x-1)(x-2+2i)=0$

$(x-1)(x-2+2i)=0$

$x=1$ and $x=2-2i$

$\alpha\gamma+\beta\delta=2$

4.(4) $2[x]+1\geq 1$ and $2[x]+1\leq -1$

$x\geq 0$ and $x<0$

$x\in(-\infty,\infty)$

5.(1) From hyperbola

$y^2=4\left(\frac{x^2}{9}-1\right)$ **...(i)**

Using (i) in equation of circle $x^2+y^2-8x=0$

$x^2+4\left(\frac{x^2}{9}-1\right)-8x=0$

$\frac{13x^2}{9}-8x-4=0$

$13x^2-72x-36=0$

$13x^2-78x+6x-36=0$

$13x(x-6)+6(x-6)=0$

$x=\frac{-6}{13}, x=6$

$y^2=4\left(\frac{-6}{13\times 9}-1\right)$ no y exist.

For $x=6 \rightarrow y^2=4\left(\frac{36}{9}-1\right)=4\times 3=12$

Centroid of triangle $PAB=\left(\frac{6+6+h}{3},\frac{k}{3}\right)$ $(h,\ k)$ is any point on given line $2x-3y+4=0$

$x=4+\frac{h}{3}, y=\frac{k}{3}$

$3(x-4)=h, k=3y$ lies on $2x-3y+4=0$

$6(x-4)-3\times 3y+4=0 \Rightarrow 6x-9y=20$

6.(3) Let $x^{\frac{1}{4}}=t$

$x=t^4 \Rightarrow dx=4t^3dt$

$f(x)=\int\frac{4t^3}{t(t+1)}dt=4\int\frac{t^2-1+1}{(t+1)}dt$

$=4\int(t-1)dt+4\int\frac{dt}{t+1}$

$=4\left(\frac{t^2}{2}-t\right)+4\log|t+1|+C$

$=4\left(\frac{\sqrt{x}}{2}-x^{\frac{1}{4}}\right)+4\log\left(x^{\frac{1}{4}}+1\right)+C$

$f(0)=-6 \Rightarrow C=-6, f(1)=4\log 2-8=2(\log 2-4)$

7.(1) $S_{2025}=4\sum_{n=1}^{2025}\left(\frac{1}{n+2}-\frac{1}{n+3}\right)$

Substituting $n=1,2,......2025$

$S_{2025}=4\left(\frac{1}{3}-\frac{1}{2028}\right)$

$=\frac{4\times 2025}{3\times 2028}$

$=\frac{675}{567}$

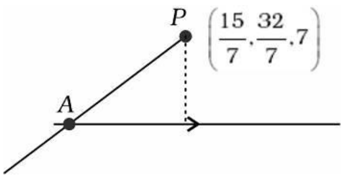
$507S_{2025}=675$

8.(4) Equation of line along $\overrightarrow{AP}$

$$\frac{x-\frac{15}{7}}{1}=\frac{y-\frac{32}{7}}{4}=\frac{Z-7}{7}=\lambda$$

$P\left(\lambda+\frac{15}{7},4\lambda+\frac{32}{7},7\lambda+7\right)$ lies on given line, $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$

So, $7\left(\frac{\lambda+22}{7}\right)=3(7\lambda+12)\Rightarrow\lambda=-1$ So $P=\left(\frac{8}{7},\frac{4}{7},0\right)$



$AP=\sqrt{1+16+49}$

$=\sqrt{66}$

9.(4) Solving both curves

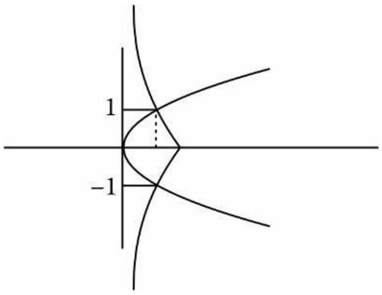
$x(1+2x)-1=0$ $\qquad\qquad x=\frac{1}{1+y^2}>0$

$2x^2+x-1=0$

$2x^2+2x-x-1=0$

$2x(x+1)-1(x+1)=0$

$x=\frac{1}{2},x=-1$ (not possible)



For $x=\frac{1}{2}$ $y^2=1$

Considering area along y axis

$$2\int_0^1\left(f(y)-g(y)\right)dy$$

Where $f(y)=\frac{1}{1+y^2},g(y)=\frac{y^2}{2}$

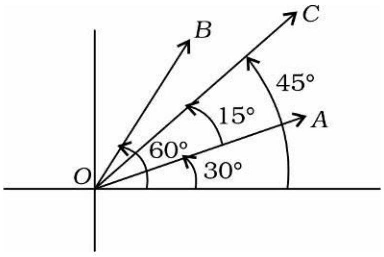
$$=2\int_0^1\left(\frac{1}{1+y^2}-\frac{y^2}{2}\right)dx=2\left[\tan^{-1}y-\frac{y^3}{3\times2}\right]_0^1$$

$$=2\left[\frac{\pi}{4}-\frac{1}{6}\right]=\frac{\pi}{2}-\frac{1}{3}$$

10.(2) Equation of angle bisector of OA and OB is

$x-y=0$

Distance of $C(a,1-a)$ from $x-y=0$



$$\left|\frac{a-(1-a)}{\sqrt{2}}\right|=\frac{9}{\sqrt{2}}$$

$2a-1=9$ or $2a-1=-9$

$a=5$ or $a=-4$ $\qquad\Rightarrow\qquad$ Sum of values of $a=1$

11.(3) $f(x) = -x^2 + 1$ (Codomain = $(-\infty, 1)$) and range is subset of codomain.

$-k^2 + 1 = -2k$

$k^2 - 2k - 1 = 0$ (let α, β are 2 values of k)

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 4 + 2 = 6$

12.(1) $m_1 = \frac{1}{2}, m_2 = -1$

$$\frac{m - \frac{1}{2}}{1 + \frac{m}{2}} = \frac{m+1}{1-m} \Rightarrow (2m-1)(1-m) = (m+1)(2+m)$$

$$\frac{2m-1}{2+m} = \frac{m+1}{m-1} \Rightarrow 2m^2 + 3m - 1 = 3m + 2 + m^2$$

$3m^2 = -3$

$m^2 = -1$ (no values of m exist)

$2m^2 - 3m + 1 = m^2 + 2 + 3m$

$m^2 - 6m - 1 = 0$

Sum of values of $m = 6$

13.(2) $\vec{a}$ = component of $\vec{a} \perp$ to $\vec{b}$ + component of $\vec{a}$ parallel to $\vec{b} = 4\hat{i} + \hat{j} - 3\hat{k}$

$\alpha^2 + \beta^2 + \gamma^2 = 16 + 1 + 9 = 26$

14.(1) $\int_0^2 x.f'(x) = 6$

$[x F(x)]_0^2 - \int_0^2 F(x)dx = 6$ also $F(2) = 2f(2) = 2$ (using $F(x) = xf(x)$)

$2F(2) - \int_0^2 F(x)dx = 6 \Rightarrow \int_0^2 F(x)dx = -2$

$\int_0^2 x^2.F''(x)dx = 40$

$\left[x^2 F'(x)\right]_0^2 - 2\int_0^2 x F'(x)dx = 40$

$4F'(2) - 2 \times 6 = 40$

$F'(2) = 13 \Rightarrow \quad F'(2) + \int_0^2 F(x)dx = 13 - 2 = 11$

15.(3) $\dfrac{1}{\sin\frac{\pi}{6}}\displaystyle\sum_{r=1}^{13}\dfrac{1\times\sin\frac{\pi}{6}}{\sin\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right).\sin\left(\frac{\pi}{4}+r\frac{\pi}{6}\right)}=2\sum_{r=1}^{13}\left(\cot\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)-\cot\left(\frac{\pi}{4}+r\frac{\pi}{6}\right)\right)$

$=2\left(\cot\frac{\pi}{4}-\cot\left(\frac{\pi}{4}+\frac{13\pi}{6}\right)\right)$

$=2\left(1-\cot 75^\circ\right)$

$=2\left(1-\left(2-\sqrt{3}\right)\right)$

$=2\left(\sqrt{3}-1\right)$

$=2\sqrt{3}-2=a\sqrt{3}+b$

So, $a^2+b^2=8$

16.(1) $P.P^T=I$ (Orthogonal matrix)

$B=PA\ P^T$

$P^T B.P=A$

$P^T B^{10} P=A^{10}=C$

$A^2=A.A=\begin{bmatrix}\frac{1}{\sqrt{2}} & -2\\ 0 & 1\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}} & -2\\ 0 & 1\end{bmatrix}$

$A^2=\begin{bmatrix}\left(\frac{1}{\sqrt{2}}\right)^2 & -2-\sqrt{2}\\ 0 & 1\end{bmatrix}, A^3=\begin{bmatrix}\left(\frac{1}{\sqrt{2}}\right)^3 & -\\ 0 & 1\end{bmatrix}$, ... $A^{10}=\begin{bmatrix}\left(\frac{1}{\sqrt{2}}\right)^{10} & -\\ 0 & 1\end{bmatrix}$ (Only diagonal elements required)

Sum of diagonal elements of $C=\frac{1}{32}+1=\frac{33}{32}$

17.(2) Number of rational terms in expansion of

$\left(3^{\frac{1}{4}}+4^{\frac{1}{3}}\right)^{12}$

$T_{r+1}={}^{12}C_r\,(3)^{\frac{12-r}{4}}.(4)^{r/3}$

For rational terms $\rightarrow r=0,12$

$q=$ sum of rational terms $=(3)^3.(4)^0+1.(3)^0.(4)^4$

$=27+256=283$

Also Binomial coff of T_r, T_{r+1} and T_{r+2} in Binomial expansion of $(a+b)^{12}$ are in G.P.

So, ${}^{12}C_{r-1}+{}^{12}C_{r+1}=2\times{}^{12}C_r$

$$\frac{1}{(r-1)!(13-r)!}+\frac{1}{(r+1)!(11-r)!}=\frac{2}{r!(12-r)!}$$

$$2r^2-24r+156=26r-2r+26-2r^2$$

$$4r^2-48r+130=0$$

No integral value of r . $(p=0)$

$p+q=283$

18.(1) $P\left(\frac{B_2}{W}\right)=\dfrac{\frac{1}{3}\times\frac{4}{10}}{\frac{1}{3}\times\frac{6}{10}+\frac{1}{3}\times\frac{5}{10}+\frac{1}{3}\times\frac{4}{10}}$

$=\frac{4}{15}$

19.(4) $T=S_1$ (Using chord whose midpoint is given w.r. to point $\left(\sqrt{2},\frac{4}{3}\right)$ for ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$)

$\sqrt{2}x+3y=6$ solving with $\frac{x^2}{9}+\frac{y^2}{4}=1$

$$\frac{x^2}{9}+\frac{\left(\frac{-\sqrt{2}x+6}{3}\right)^2}{4}=1$$

$$6x^2-12\sqrt{2}x=0$$

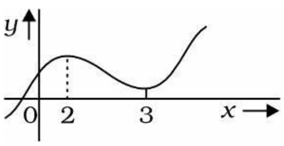
$$x=0,x=2\sqrt{2}$$

Length of chord = $|x_2-x_1|\sqrt{1+m^2}$

$=2\sqrt{2}\times\frac{\sqrt{11}}{3}=\frac{2\sqrt{22}}{3}=\frac{2\sqrt{\alpha}}{3}\Rightarrow\alpha=22$

20.(1) $f(x)=2x^3-15x^2+36x+7$

$f'(x)=6x^2-30x+36=6(x-2)(x-3)$



$f(0)=7, f(2)=35, f(3)=34$

$f(x)\in[7,35]\rightarrow$ number of integral points in range = 29

$g(x)=1-\frac{1}{1+x^{2025}}$ $g(x)\in[0,1)$ $\rightarrow$ number of integral points in range = 1

Number of integral points in S

$=n(s)=30$

SECTION – 2

21.(20) $(n-2)180^\circ=\frac{n}{2}\left[2\times219^\circ+(n-1)\times-6^\circ\right]$ (sum of interior angles = $(n-2)\pi$)

$(n-2)180^\circ=n\left[-3n+222^\circ\right]$

$3n^2 - 42n - 360 = 0$

$n^2 - 14n - 120 = 0$

$(n-20)(n+6) = 0$

$(n-20)(n+6) = 0$

$n = 20$ and $n = -6$ (not possible)

22.(1) $f(x) = \sum_{r=0}^{\infty}\left(\tan\frac{x}{2^r} - \tan\frac{x}{2^{r+1}}\right)$ substituting $r = 0, 1, 2......\infty$

$f(x) = \tan x$, So $\lim_{x\to 0} \frac{e^x - e^{f(x)}}{x - f(x)}$

$= \lim_{x\to 0} e^{\tan x}\left(\frac{e^{x-\tan x} - 1}{x - \tan x}\right) = 1$

23.(64) Coff of x^{15} from $\left(x^0 + x^1 + ...x^9\right)^3$

Coff of x^{15} from $\left(\frac{1-x^{10}}{1-x}\right)^3$

$\left(1-x^{10}\right)^3 (1-x)^{-3}$

$= \left({}^3C_o - {}^3C_1 x^{10} + ...\right)(1-x)^{-3}$

$= 1 \times coff\ x^{15}$ from $(1-x)^{-3} - 3 \times$ coff of x^5 from $(1-x)^{-3}$

$= {}^{17}C_{15} - 3 \times {}^{7}C_5$

$= 73$ (number less than 212 = (096, 069, 087, 078, 159, 195, 168, 186, 177))

So number of natural numbers between 212 and 999.

$= 73 - 9 = 64$

24.(4) Let $\sin^{-1}\frac{x}{2} = t$

$\frac{1}{\sqrt{4-x^2}}dx = dt$

$\frac{dy}{dt} + ty = t^3 \to I.F = e^{t^2/2}$

$y.e^{\frac{\left(\sin^{-1}\frac{x}{2}\right)^2}{2}} = \int t^3.e^{\frac{t^2}{2}}dt = \int \underset{I}{t^2}\left(\underset{II}{t\,e^{\frac{t^2}{2}}}\right)dt$ (applying integration by parts)

$= t^2.e^{\frac{t^2}{2}} - \int 2t.e^{\frac{t^2}{2}}dt$

$= \left(\sin^{-1}\frac{x}{2}\right)^2.e^{\frac{\left(\sin^{-1}\frac{x}{2}\right)^2}{2}} - 2e^{\frac{\left(\sin^{-1}\frac{x}{2}\right)^2}{2}} + C$

At $x = 2$ $y(2) = \frac{\pi^2 - 8}{4} \to C = 0$, $y(0) = -2, (y(0))^2 = 4$

25.(14) Image of focus (1, 0)

In line $x + y + 4 = 0$

Is $(-4, -5)$

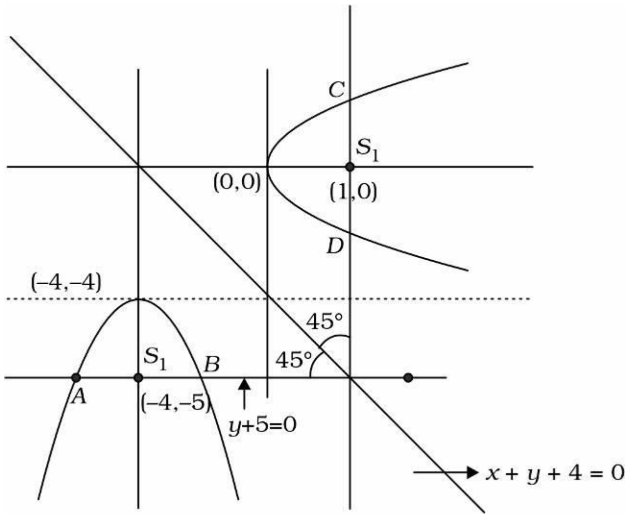
$d = AB$ is latus rectum,

length of L.R. = $2 \times 2 = 4$

a = Area of Δ SAB will be same

as that of Δ $S_1 CD = \frac{1}{2} \times 4 \times 5 = 10$

$a + d = 10 + 4 = 14$



PHYSICS

SECTION – 1

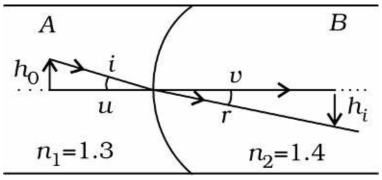
26.(2) For magnification,

By Snells Law,

$1.3\sin i = 1.4\sin r$

Angles $'i'\&'r'$ are small $\Rightarrow$ $\sin i \simeq \tan i$

$\sin r \simeq \tan r$



$\Rightarrow \quad 1.3 \times \frac{h_0}{u} = 1.4 \times \frac{h_i}{v} \Rightarrow v = \frac{14}{13}\frac{h_i}{h_0}u = 28\,cm$

By $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R} \Rightarrow \frac{1.4}{28} - \frac{1.3}{(-13)} = \frac{(1.4-1.3)}{R} \Rightarrow R = \frac{2}{3}cm$

27.(4) Since magnification is -3, so image is inverted and enlarged so image is far and object is closer

$\Rightarrow \quad |v - u| = 20$ and $v = 3u$

$\Rightarrow \quad 2u = 20 \Rightarrow u = 10cm \Rightarrow v = 30cm$

Using sign convention in Mirror formula

$\frac{1}{-30} + \frac{1}{-10} = \frac{2}{R} \Rightarrow R = -15\,cm$

28.(1) By Bohr's Theory,

Speed : $v \propto \frac{z}{n}$ & radius : $r \propto \frac{n^2}{z}$

$\Rightarrow \quad$ Angular frequency : $\omega = \frac{v}{r}$

$\omega \propto \frac{z^2}{n^3}$

But $\omega = 2\pi v \qquad v$ = frequency

$\Rightarrow \quad$ Frequency : $v \propto \frac{1}{n^3}$

29.(1) Initial Position vector : $\vec{r}_i = 3\hat{i} + 4\hat{j} \quad m$

Final position vector : $\vec{r}_f = 6\hat{i} + 10\hat{j} \quad m$

Displacement : $\vec{s} = \vec{r}_f - \vec{r}_i = 3\hat{i} + 6\hat{j}$

Work done by Force $(2\hat{i} + 3\hat{j})N, W = (2\hat{i} + 3\hat{j}).(3\hat{i} + 6\hat{j}) = 24\,Joule$

Average Power $\vec{P} = \frac{W}{t} = \frac{24}{4} = 6W$

Now acceleration: $\vec{a} = \frac{\vec{F}}{m} = \frac{2\hat{i} + 3\hat{j}}{4}$

Velocity at $t = 4\,\text{sec.}: \vec{v} = \vec{u} + \vec{a}t = 2\hat{i} + 3\hat{j}$

$u = 0$

Now Instantaneous Power : $\vec{F}.\vec{v} = (2\hat{i} + 3\hat{j}).(2\hat{i} + 3\hat{j}) = 13W$

Ratio: $\frac{\bar{P}}{P} = \frac{6}{13}$

30.(3) Magnetic field at O, $B_o = \frac{\mu_0 I}{4\pi a} + \frac{\mu_0 I}{4\pi a}\left(\frac{3\pi}{2}\right)$

$= \frac{\mu_0 I}{4\pi a}\left[1 + \frac{3\pi}{2}\right]$

31.(1)

32.(2) Initially: $F_B - Mg = Ma$ ...(i)

Let mass 'm' to be dropped

$\Rightarrow \quad F_B - (M - m)g = (M - m)3a$ ...(ii)

(ii) – (i) $\Rightarrow \quad Mg - (M - m)g = (M - m)3a - Ma$

$Mg - Mg + mg = 3Ma - 3ma - Ma$

$\Rightarrow \quad mg = 2Ma - 3ma$

$\Rightarrow \quad m(g + 3a) = 2Ma \Rightarrow m = \frac{2Ma}{(g + 3a)}$

33.(4) KE of Translation = $n\left[.3\left(\frac{1}{2}RT\right)\right]$

$= \frac{50}{44} \times 3 \times \frac{1}{2} \times 8.3 \times 290$

$= 4102.8\,Joule$

34.(1) Compton Effect, (Theoretical Question)

35.(3) $\hat{B} \times \hat{s} = \hat{E}$

(Unit vectors)

$\hat{s}$ unit vector along the direction of propagation of wave.

$\left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right) \times (\hat{k}) = \frac{\sqrt{3}}{2}(\hat{i} \times \hat{k}) + \frac{1}{2}(\hat{j} \times \hat{k}) = \left(\frac{-\sqrt{3}}{2}\hat{j} + \frac{1}{2}\hat{i}\right)$

So direction of electric field is $\left(\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right)$

And phase of $\vec{E}$ and $\vec{B}$ is same

$|\vec{E}_0| = C|\vec{B}_0|$

So, $\vec{E} = \left(\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right) 30c\sin\left[\omega\left(t - \frac{z}{c}\right)\right]$

36.(4) Vapour density $= \dfrac{mol.\,\omega t}{2}$

$$V_{r.m.s} = \sqrt{\frac{3RT}{m}}$$

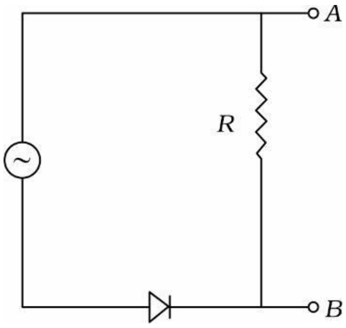
At constant temperature $\rightarrow \dfrac{V_1}{V_2} = \sqrt{\dfrac{m_2}{m_1}} = \sqrt{\dfrac{d_2}{d_1}} = \sqrt{\dfrac{25}{4}}$

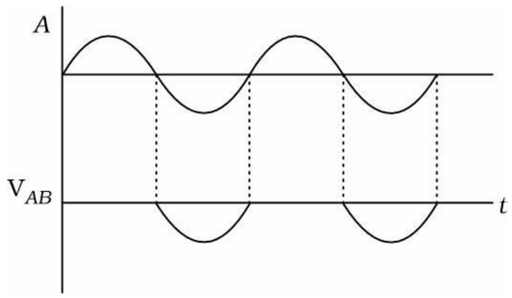
$$= \frac{5}{2}$$

37.(3) Potential $= \dfrac{1}{2}B\omega R^2 = \dfrac{1}{2}(0.4)(10\pi)(0.2)^2$

$$= 0.2512\,volt$$

38.(3) If A is at higher potential than B so diode will be reverse biased and when A is at lower potential than diode is forward biased.



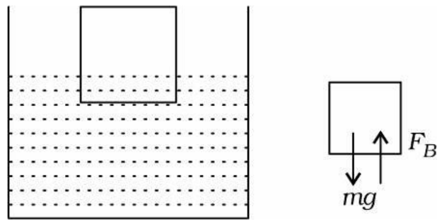


39.(3) At equilibrium $F_B = mg$

$$V_I \rho_\ell g = mg$$

$$V_I(1)g = 400g$$

$$V_I = 400\,cm^3$$



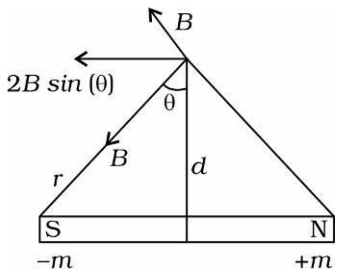
Volumes outside $= 1000 - 400 = 600\,cm^3$

40.(2) $B_{Net} = 2B\sin(\theta)$

$$= \frac{\mu_0}{4\pi}\frac{m}{r^2}\frac{L}{2(r)}$$

$$B_{Net} = \frac{\mu_0}{4\pi}\frac{mL}{r^3}$$

$$\left(\frac{dB}{B}\times 100\right) = \frac{dL}{L}\times 100 + 3\left(\frac{dr}{r}\right)\times 100$$



$\therefore \quad r = \sqrt{\dfrac{L^2}{4} + d^2}$ For error in r, $\dfrac{dr}{r}\times 100 = 1\%$

So, error in B_{net} $\left(\dfrac{dB}{B}\times 100\right) = 1 + 3 = 4\%$

41.(2) $C = 1\mu F$

$V = 20\,volt$

$d = 1\,\mu m$

Energy density (u)

$= \frac{1}{2}\varepsilon_0 E^2$

We know, $E = \frac{V}{d} = \frac{20}{1\times10^{-6}} = 20\times10^{6}\ V/m$

$u = \frac{1}{2}\times8.85\times10^{-12}\times\left(20\times10^{6}\right)^2$

$u = \frac{1}{2}\times8.85\times10^{-12}\times4\times10^{14}$

$u = 2\times8.85\times100 = 17.7\times10^{2}$

$u = 1.77\times10^{3} J/m^3$

$\Rightarrow \quad u = 1.8\times10^{3} J/m^3$

42.(2) $V = \sqrt{\frac{2GM}{R}}$

$M_e = 8M_p, R_e = 2R_p, V_e = 11.2\,km/s$

Escape velocity for earth $(V_e) = \sqrt{\frac{2GMe}{Re}}$

Escape velocity for planet $(V_p) = \sqrt{\frac{2GM_p}{R_p}}$

$\Rightarrow \quad \frac{V_p}{V_e} = \sqrt{\frac{2GM_p}{R_p}\times\frac{R_e}{2G\,M_e}} = \sqrt{\frac{M_p}{M_e}\times\frac{R_e}{R_p}}$

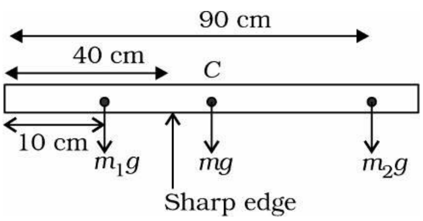
$\Rightarrow \quad \frac{V_p}{V_e} = \sqrt{\frac{1}{8}\times2} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$\Rightarrow \quad V_p = \frac{V_e}{2} = \frac{11.2}{2} = 5.6\,km/s$

43.(3) $m = 250\,gm$

$m_1 = 400\,gm$

$m_2 = ?$



So, torque about sharp edge = 0

$\Rightarrow \quad m_1g\times30 = mg\times10 + m_2g\times50$

$\Rightarrow \quad 400\times3 = 250 + 5m_2$

$\Rightarrow \quad 1200 - 250 = 5m_2$

$\Rightarrow \quad 950 = 5m_2 \qquad \Rightarrow \qquad m_2 = \frac{950}{5} \Rightarrow m_2 = 190gm$

44.(2) Here, the particle is moving in straight line.

So, distance = Displacement

Area under the (v-t) curve and t-axis = Displacement

Displacement = $\frac{1}{2}\times 10\times 2+10\times 2$

$=10+20$

$=30m$

45.(1) At $t=0 \Rightarrow x=x_0$ & $p=p_0$

$\omega \Rightarrow$ Given

We know, $x=A\sin(\omega t+\theta)$ ...(i)

$v=A\omega\cos(\omega t+\theta)$

$p=mv=mA\omega\cos(\omega t+\theta)$ ...(ii)

$\Rightarrow \quad x_0=A\sin\theta$ and $p_0=mA\omega\cos\theta$

$$\Rightarrow \quad \frac{x_0}{p_0}=\frac{A\sin\theta}{mA\omega\cos\theta}=\frac{1}{m\omega}\tan\theta$$

$$\Rightarrow \quad m\omega\left(\frac{x_0}{p_0}\right)=\tan\theta$$

$$\Rightarrow \quad \theta=\tan^{-1}\left(\frac{m\omega x_0}{p_0}\right)$$

$$\Rightarrow \quad {x_0}^2=A^2\sin^2\theta \text{ and } {p_0}^2=m^2\omega^2A^2\cos^2\theta$$

$$\Rightarrow \quad \frac{{x_0}^2}{A^2}=\sin^2\theta \text{ and } \frac{{p_0}^2}{m^2\omega^2A^2}=\cos^2\theta$$

$$\Rightarrow \quad \frac{{x_0}^2}{A^2}+\frac{{p_0}^2}{m^2\omega^2A^2}=1$$

$$\Rightarrow \quad \frac{1}{A^2}\left({x_0}^2+\frac{{p_0}^2}{m^2\omega^2}\right)=1$$

$$\Rightarrow \quad A=\sqrt{{x_0}^2+\frac{{p_0}^2}{m^2\omega^2}}$$

SECTION – 2

46.(54) For minimum after every 12 sec., Destructive interference should happen through thin film during evaporation.

By $2\mu d\cos r=(2n-1)\frac{\lambda}{2}$ for minima in interference by transmission through thin films.

Let Δd be thickness of film to be evaporated between consecutive minima.

$\Rightarrow \quad 2\mu\,\Delta d\cos 0^\circ=\lambda$

$$\Delta d = \frac{\lambda}{2\mu}$$

Consider uniform rate of evaporation,

$\Rightarrow$ Rate of evaporation = $\frac{\pi r^2 \Delta d}{t}$

$$= \frac{\pi\left(1.8\times10^{-2}\right)^2 \times 560\times10^{-9}}{12\times2\times1.4}$$

$$= 54\times\pi\times10^{-13}\, m^3 / \text{sec}$$

47.(12) $U = -\vec{P}.\vec{E} = -PE\cos\theta$

$\omega = U_f - U_i \;\; \omega = U_f - U_i$

$= -6\times10^{-6}\times10^{6}\cos180^\circ - \left(-6\times10^{-6}\times10^{6}\times\cos0^\circ\right)$

$= 6 + 6 = 12\,J$

48.(50) $\ell = 10\,cm = 100\,mm$

$V = \ell^3 = (100)^3\, mm^3 = 10^6 mm^3$

$$B = \frac{-P}{\Delta V / V}$$

$$\frac{\Delta V}{V} = \frac{-P}{B} = \frac{-7\times10^6}{1.4\times10^{11}} = \frac{-1}{2\times10^4}$$

$$\Delta V = \frac{-1}{2\times10^4}\times V = \frac{-1}{2\times10^4}\times10^6 mm^3$$

$\Delta V = -50\,mm^3$

(– sign indicates decrease in volume)

49.(1) Height of $\Delta = K_1 t$ (As velocity is constant)

Base of $\Delta = K_2 t$ (As $\frac{b}{h}$ = constant)

Area of $\Delta = \frac{1}{2}bh = \frac{1}{2}(K_2 t)(K_1 t) = K_3 t^2$

$$\varepsilon = \frac{-d\phi}{dt} = \frac{-d}{dt}\left(BA\cos0^\circ\right)$$

$$= -B\frac{dA}{dt}$$

$$= -B\times\frac{d}{dt}K_3 t^2$$

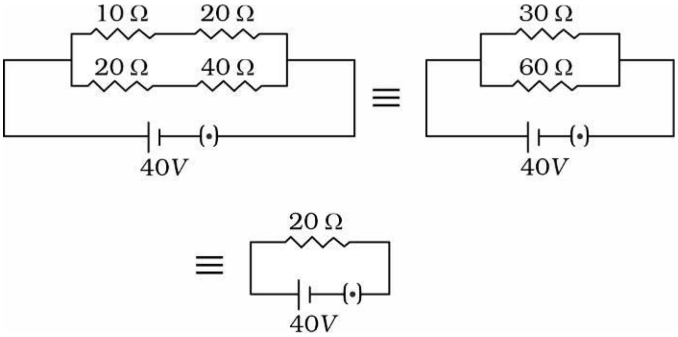
$$= -B\times K_3\times 2t$$

$= K_4 t \qquad \therefore \qquad \varepsilon \propto t^1$

50.(2) As circuit is Wheatstone bridge circuit:

$$\frac{10}{R}=\frac{20}{40}$$

$R=20\,\Omega$



$I=2A$

CHEMISTRY

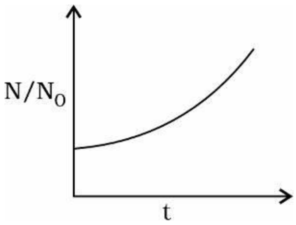
SECTION – 1

51.(2) N_0 = Initial number of Bacteria

$N =$ Number of Bacteria at any time 't'

Since Bacterial growth $\Rightarrow$ $N = N_0 e^{\lambda t}$

$\therefore$ graph of $\frac{N}{N_0}$ v / s t will be



52.(4) (I) According to the law of octaves, the elements were arranged in the increasing order of their atomic weights.

Ref.: Classification of elements and periodicity (NCERT) Sec. No. 3.2.

(II) Meyer observed a change in length of repeated pattern upon plotting physical properties of certain elements against their respective atomic weight.

Ref.: Classification of elements and periodicity (NCERT) Sec. No. 3.2.

53.(1)

(A)	Starch	$\rightarrow$	Glucose
(B)	Cane sugar	$\rightarrow$	Glucose + Fructose
(C)	Milk sugar	$\rightarrow$	Galactose + Glucose
(D)	Amylopectin	$\rightarrow$	Glucose
(E)	Amylose	$\rightarrow$	Glucose

Ref.: Biomolecules (NCERT) Sec. No. 10.1.3 and 10.1.4

54.(4) Sublimation

Ref.: Organic chemistry – Some Basic Principles and Techniques. (NCERT) Sec. No. 8.8.1

55.(3) (A) Primary amines give diazonium salt when treated with $NaNO_2$ in acidic medium.

(B) Aliphatic and aromatic amines on heating with $CHCl_3$ and ethanolic KOH form carbylamines.

(C) Only primary amines gives carbyl-amine test.

(D) Benzene sulphonyl chloride is known as Hinsberg's reagent.

(E) Tertiary amines do not react with benzenesulfonyl chloride.

56.(1)

The language of this question is a bit controversial.

The last line of the question says that the cell is immersed in another solution having equal mole fraction of glucose and water. The above statement can be interpretated in two forms.

Interpretation 1:

$\chi_{\text{Glucose}} = \chi_{H_2O} = 0.5$ (inside the solution)

$$\therefore \quad \frac{\omega}{\omega}\% = \frac{180}{180+18} \times 100 = 91\%$$

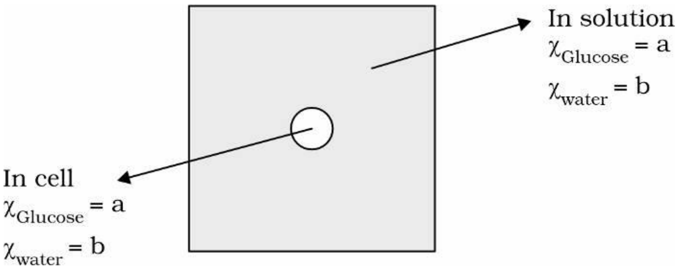
As the concentration in the solution is very high as compared to the cell.

Hence the cell should shrink, but for this interpretation there is no matching option given.

Interpretation 2:

Mole fraction of glucose in the living cell is equal to the mole fraction of glucose in the solution and same will be true for water.

The below diagram explains this interpretation



The cell will show no change in volume since solution is 0.9% (w/w),

For this interpretation the correct option should be 1.

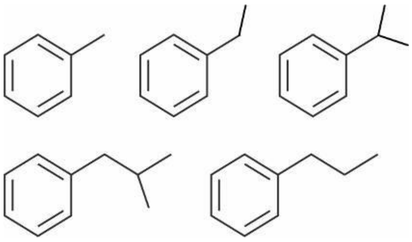
57.(3) $\left[CoF_6\right]^{3-}$ - sp^3d^2

$\left[NiCl_4\right]^{2-}$ - sp^3

$\left[Co\left(NH_3\right)_6\right]^{3+}$ - d^2sp^3

$\left[Ni(CN)_4\right]^{2-}$ - dsp^2

58.(1) To give Benzoic acid the compound must have at least 1 α hydrogen



59.(3) Energy of atomic orbitals of single electron system depends upon n value.

$$\left.\begin{array}{l} 1s < 2p < 3d < 4s \\ 1s < 2s = 2p < 3s = 3p \end{array}\right\} \text{Correct}$$

Hence C and D are incorrect.

60.(1) Given 75% by mass means

75 g of nitric acid in 100 grams of solution

$\Rightarrow$ 30 grams of nitric acid in $\frac{100}{75} \times 30$ grams of solution = 40 grams of solution

As we know: $M = d \times V$

$$\therefore \quad V = \frac{M}{d} = \frac{40}{1.25} = 32 \text{ ml}$$

61.(4) (A) Sucrose - $\alpha 1 - \beta 2$

(B) Matlose - $\alpha 1 - 4$

(C) Lactose - $\beta 1 - 4$

(D) Amylopectine - $\alpha 1 - 4$ and $\alpha 1 - 6$

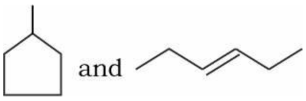
Refer (NCERT) Sec. No. 10.1.3 and 10.1.4

62.(3) V_2O_5 reacts with alkalies to give VO_4^{3-}.

Fef.: (NCERT) D and F Block Sec. No. 4.4.1

63.(2) For cyclic process

$\Delta U = 0$ (As internal energy is a state function)

64.(1)  Are isomeric compounds.

1° and 2° amines are functional group isomers.

Both statement 1 and statement II are true.

65.(4) $r = k[A][B]$

$r' = k[3A][3B]$

$r' = 9k[A][B]$

$r' = 9k[A][B]$

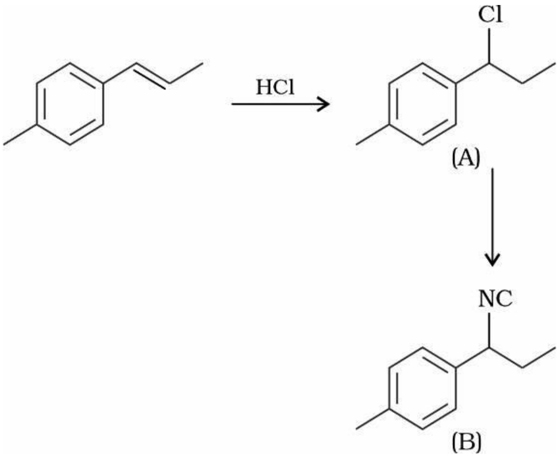
$r' = 9r$

66.(1) $CH_3-C\equiv CH \xrightarrow[H_2]{Pd/C} \underset{(A)}{CH_3-CH=CH_2} \xrightarrow{(i)\,O_3,\ (ii)\,Zn,H_2O} \underset{(C)}{HCHO} + \underset{(B)}{CH_3CHO}$

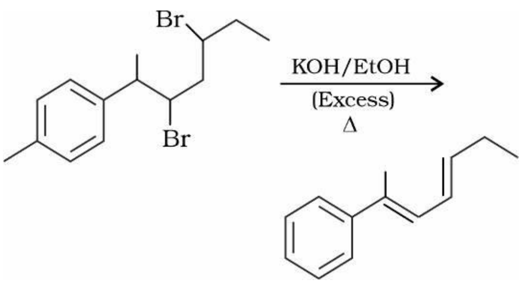
67.(1) $\underset{K_{sp}=4\times10^{-53}}{HgS} < \underset{K_{sp}=8\times10^{-28}}{PbS} < \underset{K_{sp}=5\times10^{-13}}{AgBr} < \underset{K_{sp}=5.5\times10^{-6}}{Ca(OH)_2}$

Refer to NCERT Table No. 6.9

68.(3)



69.(2)



70.(4)

$(NH_4)_2S$	-	Yellow
PbS	-	Black
CuS	-	Blue
As_2S_3	-	Yellow
As_2S_5	-	Yellow

SECTION – 2

71.(48) $C(s)+O_2(g)\rightarrow CO_2(g),\quad \Delta H_f=-393.5\,kJ\,mol^{-1}$ ...(i)

$H_2(g)+\frac{1}{2}O_2(g)\rightarrow H_2O(\ell),\quad \Delta H_f=-286\,kJ\,mol^{-1}$ ...(ii)

$C_6H_6+\frac{15}{2}O_2\rightarrow 6CO_2+3H_2O,\ \Delta H_c=-3267\,kJ\,mol^{-1}$ ...(iii)

$6C(s)+3H_2(g)\rightarrow C_6H_6(\ell),\quad \Delta H_f=?$

$6(i)+3(ii)-(iii)$

$=6(-393.5)+3(-286)-(-3267)$

$=-2361-858+3267$

$=48\,kJ\,mol^{-1}$

72.(5) $\overset{+3}{Mn_2}O_3,\ \overset{+2}{Ti}O,\ \overset{+2}{V}O$

Mn_2O_3 has the strongest oxidizing power.

$=\sqrt{n(n+2)}$ BM

$Mn=[Ar]3d^54s^2$

$Mn^{+3}=[Ar]3d^44s^0 \quad\Rightarrow\quad n=4$

$\mu=\sqrt{4(4+2)}$

$=4.81\,BM$

$\simeq 5\,BM$

73.(6) O_2 = Paramagnetic

O_2^+ = Paramagnetic

O_2^- = Paramagnetic

NO = Paramagnetic

NO_2 = Paramagnetic

CO = Diamagnetic

$k_2[NiCl_4]$ = Paramagnetic

$[Co(NH_3)_6]Cl_3$ = Diamagnetic

$k_2[Ni(CN)_4]$ = Diamagnetic

74.(2) Final moles of OH^- ions present in 600 mL

$= \frac{10^{-2}}{1000} \times 600$

$= 0.6 \times 10^{-2}$ moles

$\therefore$ moles of OH^- formed due to electrolysis $\approx 0.6 \times 10^{-2}$

At cathode

$H_2O + 2\bar{e} \rightarrow H_2 + 2OH^-$

$2f \equiv 2$ moles of OH^-

$\therefore$ 2 moles of OH^- ions are produced from

= 2f charge

$\therefore$ 0.6×10^{-2} moles of OH^- are produced from

$= \frac{2F}{2} \times 0.6 \times 10^{-2}$

$= 0.6\,F \times 10^{-2}$

$0.6 \times 96500 \times 10^{-2} = i \times t$

$i = \frac{0.6 \times 96500 \times 10^{-2}}{5 \times 60} \Rightarrow i = 1.93A$

$\approx 2A$

75.(15) Phosphorous Atomic No. 15

Refer to NCERT Sec. No. 7.1.7