SOLUTIONS

Joint Entrance Exam | IITJEE-2025

24th JANUARY 2025 | Morning Shift

MATHEMATICS

SECTION – 1

1.(2) $f(x)-6f\left(\frac{1}{x}\right)=\frac{35}{3x}-\frac{5}{2}$ ...(i)

Put $\frac{1}{x}$ instead of x to get

$f\left(\frac{1}{x}\right)-6f(x)=\frac{35}{3}x-\frac{5}{2}$

$\Rightarrow 6f\left(\frac{1}{x}\right)-36f(x)=70x-15$ ...(ii)

Add (i) and (ii)

$-35f(x)=70x+\frac{35}{3x}-\frac{35}{2}$

$\Rightarrow f(x)=\frac{1}{2}-2x-\frac{1}{3x}$

$f(x)+\frac{1}{\alpha x}=\frac{1}{2}-2x+\frac{1}{x}\left(\frac{1}{\alpha}-\frac{1}{3}\right)$

$\lim_{x\to 0} f(x)+\frac{1}{\alpha x}$ exists and is equal to β

$\Rightarrow \alpha=3$ and $\beta=\frac{1}{2}$

$\Rightarrow \alpha+2\beta=4$

2.(2) With incorrect data, mean = 5.5

$\Rightarrow \sum_{i=1}^{10} x_i=55$ and $\sum_{i=1}^{10} x_i^2=371$

When correct data is used :

$\sum_{i=1}^{10} x_i=60$ and $\sum_{i=1}^{10} x_i^2=430$

$\Rightarrow$ Correct variance = $43-6^2=7$

3.(1) $S_n=\sum_{r=1}^{n}\frac{1}{r(r+1)}=\sum_{r=1}^{n}\frac{1}{r}-\frac{1}{r+1}=1-\frac{1}{n+1}=\frac{n}{n+1}$

$\Rightarrow (n+1)S_n=n$

Put $n=2025$ to get $\sqrt{2026\ S_{2025}}=45$

Sum of first 6 terms of A.P. = $9p$

$\Rightarrow p=5$

$\Rightarrow$ Difference $=25$

4.(3) $2z^2 - 3z - 2i = 0 \Rightarrow 2z^2 - 2i = 3z$

Square both sides

$\Rightarrow 4\left(z^2 - i\right)^2 = 9z^2$

$\Rightarrow 4\left(z^4 - 1 - 2iz^2\right) = 9z^2$

$\Rightarrow z^4 - 1 = z^2\left(\frac{9}{4} + 2i\right)$

Again squaring both sides

$\left(z^4 - 1\right)^2 = z^4\left(\frac{9}{4} + 2i\right)^2$

$\Rightarrow z^8 + 1 - 2z^4 = z^4\left(\frac{81}{16} - 4 + 9i\right)$

$\Rightarrow z^8 + 1 = z^4\left(\frac{49}{16} + 9i\right)$

Both α and β satisfy the above equation

$\Rightarrow \alpha^8 + 1 = \alpha^4\left(\frac{49}{16} + 9i\right)$

$\Rightarrow \alpha^{19} + \alpha^{11} = \alpha^{15}\left(\frac{49}{16} + 9i\right)$

Similarly : $\beta^{19} + \beta^{11} = \beta^{15}\left(\frac{49}{16} + 9i\right)$

$\Rightarrow \alpha^{19} + \alpha^{11} + \beta^{19} + \beta^{11} = \left(\alpha^{15} + \beta^{15}\right)\left(\frac{49}{16} + 9i\right)$

$\Rightarrow \frac{\alpha^{19} + \beta^{19} + \alpha^{11} + \beta^{11}}{\alpha^{15} + \beta^{15}} = \frac{49}{16} + 9i$

$\Rightarrow$ Answer $= 16 \times \frac{49}{16} \times 9 = 441$

5.(3) Let $m = n + 4 \Rightarrow m \neq 14$

$5^{th}, 6^{th}$ and 7^{th} terms in A.P.

$\Rightarrow 2\,{}^mC_5 = {}^mC_4 + {}^mC_6$

$\Rightarrow 2 = \frac{{}^mC_4}{{}^mC_5} + \frac{{}^mC_6}{{}^mC_5}$

$\Rightarrow 2 = \frac{5}{m-4} + \frac{m-5}{6}$

$\Rightarrow 12(m-4) = 30 + (m-4)(m-5)$

$\Rightarrow m^2 - 9m + 50 = 12m - 48$

$\Rightarrow m^2 - 21m + 98 = 0$

$\Rightarrow m = 7, 14$

$\Rightarrow m = 7$

Largest coefficient $= {}^7C_3 = {}^7C_4 = 21$

6.(4) Probability of throwing sum = 5 is 1/9

Probability of throwing sum = 8 is 5/36

Let probability of A winning = p

Then $p = \frac{1}{9} + \frac{8}{9} \times \frac{31}{36} \times p$

$\Rightarrow p\left(1 - \frac{62}{81}\right) = \frac{1}{9}$

$\Rightarrow p = \frac{9}{19}$

7.(4) The 2 focal distances of the point (α, β) are $a \pm e\alpha$, where e is the eccentricity of the ellipse.

$\Rightarrow \frac{7}{4} = a^2 - 3e^2$ ...(i)

Also $= \frac{3}{a^2} + \frac{1}{4b^2} = 1$ since $\left(\sqrt{3}, \frac{1}{2}\right)$ lies on the ellipse.

Put $b^2 = a^2(1-e^2)$ and we get : $3 + \frac{1}{4\left(1-e^2\right)} = a^2$

Using (i), we get

$3 + \frac{1}{4\left(1-e^2\right)} = \frac{7}{4} + 3e^2$

$\Rightarrow 3(1-e^2) + \frac{1}{4\left(1-e^2\right)} = \frac{7}{4}$

Let $t = 1 - e^2$

$\Rightarrow 3t + \frac{1}{4t} = \frac{7}{4}$

$\Rightarrow 12t^2 + 1 = 7t$

$\Rightarrow t = \frac{1}{3}, \frac{1}{4}$

$\Rightarrow e_1^2 = \frac{2}{3}$ and $e_2^2 = \frac{3}{4}$

$\Rightarrow e_2 - e_1 = \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{3} = \frac{3 - 2\sqrt{2}}{2\sqrt{3}}$

8.(1) For infinitely many solutions

Using Cramer's rule $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$\Delta_2 = 0 \Rightarrow \mu = 53$

$\Delta_3 = 0 \Rightarrow \lambda = -2$

$\Rightarrow \mu - 2\lambda = 57$

9.(1) $\lim_{x\to 0} \frac{1}{\sin x}\left(\sqrt{2\cos^2 x+3\cos x}-\sqrt{\cos^2 x+\sin x+4}\right)$

$= \lim_{x\to 0} \frac{\left(2\cos^2 x+3\cos x-\left(\cos^2 x+\sin x+4\right)\right)}{\sin x\left(\sqrt{2\cos^2 x+3\cos x}+\sqrt{\cos^2 x+\sin x+4}\right)}$

$= \lim_{x\to 0} \frac{\cos^2 x-1+3\cos x-3-\sin x}{2\sqrt{5}\sin x}$

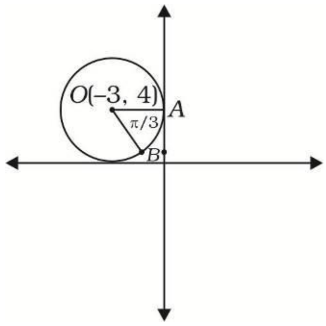
$= \lim_{x\to 0} \cdot \frac{1}{2\sqrt{5}\sin x}\left(-\sin^2 x-\sin x-6\sin^2\frac{x}{2}\right)=\frac{-1}{2\sqrt{5}}$

10.(2) C is the image of the circle $(x-1)^2+(y+2)^2=3^2$ in the line $2x-3y+5=0$

Note : C is the circle with same radius and with centre being the image of (1, –2) in the $2x-3y+5=0$

New Centre O : (–3, 4)

Since length of arc AB is $\frac{1}{6}$ of perimeter of C.



$\angle AOB=\frac{2\pi}{6}=\frac{\pi}{3}$

$\Rightarrow (\alpha,\beta)=\left(-3+3\cos\frac{\pi}{3}, 4-3\sin\frac{\pi}{3}\right)$

$\beta-\sqrt{3}\alpha=4-\frac{3\sqrt{3}}{2}+\frac{3\sqrt{3}}{2}=4$

11.(4) Let $t=x^2-9x+11$

$(x^2-9x+11)^2-(x^2-9x+20)=3$

$\Rightarrow t^2-t-12=0$

$\Rightarrow t=-3,4$

$\Rightarrow x^2-9x+14=0 \Rightarrow x=2,7$

AND $x^2-9x+7=0 \Rightarrow x=$ 2, 7 irrational roots as discriminant = 53 which isn't a perfect square

$\Rightarrow$ Product of rational roots = 14

12.(1) $f(x)=\frac{4.2^x+16}{2.2^{2x}+16.2^x+32}=\frac{2\left(2^x+4\right)\times 2}{2(2^x+4)^2}=\frac{2}{2^x+4}$

$=\frac{1}{2}\left(\frac{4}{2^x+4}\right)$

AND $f(4-x)=\frac{1}{2}\left(\frac{2^x}{2^x+4}\right)$

$\Rightarrow f(x)+f(4-x)=\frac{1}{2}$

AND $f(2)=\frac{1}{4}$

$$S = 8\sum_{r=1}^{59} f\left(\frac{r}{15}\right) \quad \ldots(i)$$

$$S = 8\sum_{r=1}^{59} f\left(\frac{60-r}{15}\right) = 8\sum_{r=1}^{59} f\left(4-\frac{r}{15}\right) \quad \ldots(ii)$$

Adding (i) and (ii)

$$2S = 8\sum_{r=1}^{59} f\left(\frac{r}{15}\right) + f\left(4-\frac{r}{15}\right) = 8\sum_{r=1}^{59}\frac{1}{2}$$

$$\Rightarrow S = 4\times\frac{59}{2} = 118$$

13.(1) $\vec{c}\perp\vec{b}$ and $\vec{c}$ is coplanar to $\vec{a}$ and $\vec{b}$

$$\Rightarrow \vec{c}\perp\left(\vec{a}\times\vec{b}\right)$$

$$\Rightarrow \vec{c} = t\vec{b}\times\left(\vec{a}\times\vec{b}\right) \text{ for some } t\in R$$

$$\Rightarrow \vec{c} = t\left(\left|\vec{b}\right|^2\vec{a} - \left(\vec{a}\cdot\vec{b}\right)\vec{b}\right) = t\left(11\vec{a}-2\vec{b}\right)$$

$$\vec{c}\cdot\vec{a} = 5 \Rightarrow t\left(11\left|\vec{a}\right|^2 - 2\vec{a}\cdot\vec{b}\right) = 5 \Rightarrow t = \frac{1}{30}$$

$$\Rightarrow \vec{c} = \frac{1}{30}\left(11\vec{a}-2\vec{b}\right)$$

$$\left|\vec{c}\right| = \frac{1}{30}\sqrt{121\left|\vec{a}\right|^2 + 4\left|\vec{b}\right|^2 - 44\vec{a}\cdot\vec{b}} = \frac{1}{30}\sqrt{121\times14+44-88}$$

$$= \frac{1}{30}\sqrt{1650} = \sqrt{\frac{165}{90}} = \sqrt{\frac{11}{6}}$$

14.(1) Let P be the point of intersection $P : (-1 + 2a, 2 + 3a, 1 + 4a)$

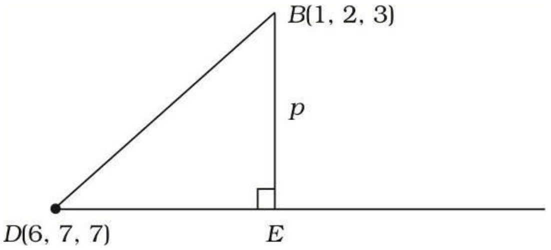
Also, P lies on second line

$$\Rightarrow \frac{2a+1}{3} = \frac{3a-1}{2} = \frac{4a-3}{1} \Rightarrow a = 1$$

$$\Rightarrow P : (1,5,5)$$

$$\Rightarrow PQ = \sqrt{125} = 5\sqrt{5}$$

15.(2) Area $\left(\Delta ABC\right) = \frac{1}{2}\times p\times AC = 3p$



Where p is the length of $\perp r$ from B on AC.

Equation of AC $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$

Length of $BD = \sqrt{66}$

Length of projection of BD on $AC = DE = \sqrt{17}$

$\Rightarrow p = BE = 7$

$\Rightarrow$ Area = 21

16.(4) Rearranging terms in given differential equation,

We get :

$$\frac{dy}{dx}+\frac{x}{1+x^2}y=\frac{5x^2}{\sqrt{1+x^2}}$$

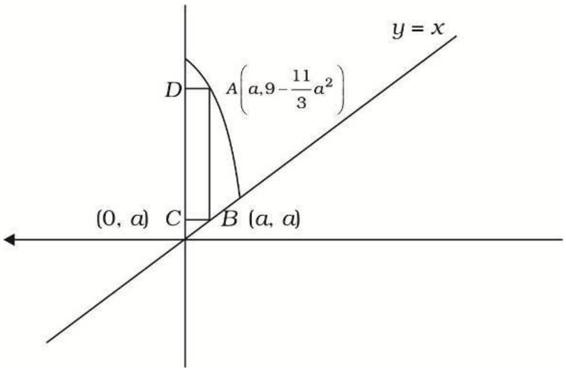
$$\text{I. F.}=\sqrt{1+x^2}$$

$$\Rightarrow y\sqrt{1+x^2}=\frac{5}{3}x^3 \quad [\text{Using } y(0)=0]$$

$$\Rightarrow y(x)=\frac{5x^3}{3\sqrt{1+x^2}}$$

Put $x=\sqrt{3}$ to get: $y\left(\sqrt{3}\right)=\frac{5\sqrt{3}}{2}$

17.(1) One side of the rectangle will be on y-axis



Let the one side parallel to it be : $x=a$

Hence, area of rectangle $=a\left(9-\frac{11}{3}a^2-a\right)$

To maximise $f(a)=a\left(9-\frac{11}{3}a^2-a\right)$

Put $f'(a)=9-11a^2-2a=0$ to get : $a=-1,\frac{9}{11}$

Hence, maximum value of area $=\frac{9}{11}\left(9-\frac{27}{11}-\frac{9}{11}\right)=\frac{567}{121}$

18.(2) $I(m,n)+I(m+1,n-1)=\int_0^1 x^{m-1}(1-x)^{n-1}+x^m(1-x)^{n-2}\,dx$

$$=\int_0^1 x^{m-1}(1-x)^{n-2}(x+1-x)dx=I(m,n-1)$$

Put $m=9$ and n 14 to get answer $=I(9,13)$

19.(1) Using concurrency condition, we get :

$33=2\alpha-\lambda \qquad \ldots(\text{i})$

Also mid point of $(1,2)$ and $\left(\frac{57}{13},-\frac{40}{13}\right)$

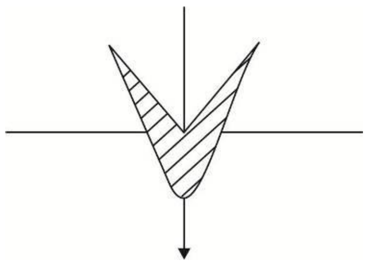
Lies on $2x-3y+\lambda=0$

$$\Rightarrow \frac{70}{13}+\frac{21}{13}+\lambda=0$$

$$\Rightarrow \lambda=-7\Rightarrow \alpha=13\Rightarrow |\alpha\lambda|=91$$

20.(1) Let $t=(x+2)$

$\Rightarrow t^2-2\le y\le |t| \quad$ [Both functions are even]



$$=2\int_0^2 |t|-t^2+2dt=\int_0^2 2t-2t^2+4dt=\frac{20}{3}$$

SECTION – 2

21.(44) $X^T AX = 0$ for all (3×1) matrices X

$\Rightarrow$ A must be a skew symmetric matrix (Please understand and remember this result)

Proof : Let $A = [a_{ij}]_{3\times3}$ and $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$X^T AX = [xyz]\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$= a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + xy(a_{12} + a_{21}) + yz(a_{23} + a_{32}) + xz(a_{13} + a_{31}) = 0$

$\forall x, y, z \in R$

$\Rightarrow$ All coefficients of the above polynomial in x, y, z are zero.

$\Rightarrow a_{11} = a_{22} = a_{33} = 0$

AND $a_{21} = -a_{12}$

$a_{32} = -a_{23}$

$a_{31} = -a_{13}$

$\Rightarrow$ A is a skew symmetric matrix

$A\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ -8 \end{bmatrix}$ ...(*i*)

$A\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}$ ...(*ii*)

(i) – (ii) gives us : $A\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ -3 \end{bmatrix}$

$\Rightarrow$ 2nd column of A is $\begin{bmatrix} -1 \\ 0 \\ -3 \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} 0 & -1 & k \\ 1 & 0 & 3 \\ -k & -3 & 0 \end{bmatrix}$

Using $A\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}$

we get : $k = 2$

$\Rightarrow A + I = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 3 \\ -2 & -3 & 1 \end{bmatrix}$

$\Rightarrow |A + I| = 15$

$\Rightarrow \det\ 2(A + I) = 2^3 \times 3 \times 5$

$\Rightarrow \det\left(adj\left(2\left(A + I\right)\right)\right) = 2^6 3^2 5^2$

$\Rightarrow \alpha = 6, \beta = 2, \gamma = 2$ and $\alpha^2 + \beta^2 + \gamma^2 = 44$

22.(14) Let $\tan^{-1}\alpha = \theta \Rightarrow \alpha = \tan\theta$

AND $\cot^{-1}\beta = \phi \Rightarrow \cot\phi = \beta$

$\sec^2\theta = 1+\alpha^2$ AND $\text{cosec}^2\phi = 1+\beta^2$

$\Rightarrow \alpha^2+\beta^2 = 34$ and $\alpha+\beta = 8$

$\Rightarrow \alpha = 3, \beta = 5$ and $\alpha^2+\beta = 14$

23.(19) $(x+2)^2\left(2f(x)-3\right) = 10\int_0^x (t+2)f(t)dt$

RHS = 0 for $x = 0 \Rightarrow f(0) = \frac{3}{2}$

Differentiating both sides

$2f'(x)(x+2)^2 + (2f(x)-3)2(x+2) = 10(x+2)f(x)$

$\Rightarrow f'(x)(x+2) + 2f(x) - 3 = 5f(x)$

$\Rightarrow f'(x)(x+2) = 3\left(f(x)+1\right)$

$\Rightarrow \frac{f'(x)}{\left(f(x)+1\right)} = \frac{3}{x+2}$

$\Rightarrow (f(x)+1) = C(x+2)^3$

Put $f(0) = \frac{3}{2}$, we get $C = \frac{5}{16}$

$\Rightarrow f(x)+1 = \frac{5}{16}(x+2)^3$

$\Rightarrow f(2) = 19$

24.(125) Number of 3-digit numbers = 9000

Number of 3-digit numbers divisible by 6 = 150

Number of 3-digit numbers divisible by 36 = 25

Final answer = 150 – 25 = 125

25.(5120) For any ordered pair (x, y), x can be chosen in 10 ways.

For a given prime $p_i \in S, i \in \{1, 2 \ldots 10\}$

Number of elements $y \in A$ such that pi/y are exactly $2^9 = 512$ because elements of A are of the type

$\prod_{j=1}^{10} p_j^{a_j}$ where $a_j \in [0, 1]$ (Not all can be zero together)

For p_i/y, y must've $a_i = 1$ {Power of p_i}

For the other 9 values of a_j, we've exactly 2 options to choose from 0 or $1 = 2^9$

Hence, $10 \times 2^9 = 5120$

PHYSICS

SECTION – 1

26.(3) $\because$ Pressure at depth (h = 20 cm) below the liquid surface

$$= P_{atm} + \rho gh$$

$$= P_{atm} + 10^3 \times 10 \times 0.2 = P_{atm} + 2000$$

$\because$ Pressure inside air-bubble = $2100 + P_{atm}$

$$P_{excess} = (P_{atm} + 2100) - (P_{atm} + 2000)$$

$$= 100 = \frac{2T}{r}$$

$$\Rightarrow T = 0.05$$

27.(3) Time – period $\propto$ (Radius)$^{3/2}$

$$\Rightarrow \quad T_1 \propto R^{3/2}$$

$$\Rightarrow \quad T_2 \propto (1.03)^{3/2} R^{3/2}$$

$$\Rightarrow \quad \frac{T_2}{T_1} = (1.03)^{1.5} = 1.045$$

$$\Rightarrow \quad \frac{T_2 - T_1}{T_1} = 4.5\%$$

28.(2) If liquid drop of radius = R breaks in n droplets.

$$\Rightarrow \quad \frac{4}{3}\pi R^3 = n.\frac{4}{3}\pi r^3 \Rightarrow r = \frac{R}{n^{1/3}}$$

$\Rightarrow$ Work done to break it = $\Delta U = n.4\pi r^2 - 4\pi R^2 = 4\pi R^2 \left(n^{1/3} - 1\right)$

$\because$ For $n = 27$; $\Delta U = 10J$

Then for $n = 64$; $\Delta U = xJ$

$$\Rightarrow \quad \frac{x}{10} = \frac{64^{1/3} - 1}{27^{1/3} - 1} = \frac{4-1}{3-1} = \frac{3}{2}$$

$$\Rightarrow \quad x = 15J$$

29.(1) $\because$ $y = \dfrac{32.3 \times 1125}{27.4}$

S.F in (32.3) = 3

S.F in (1125) = 4

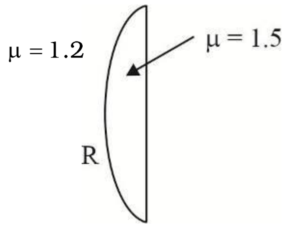
S.F in (274) = 3

$\Rightarrow$ y must have 3 S.F.

$\Rightarrow$ y = 1326.18 (convert it into 3 S.F.)

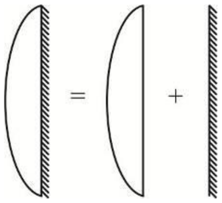
= 1330

30.(2) Finding focal length of lens:



$$\frac{1}{f_{lens}} = \left(\frac{1.5}{1.2} - 1\right)\left(\frac{1}{R}\right) = \frac{1}{4R} \Rightarrow f_{lens} = 4R$$

Finding focal length of modified mirror:



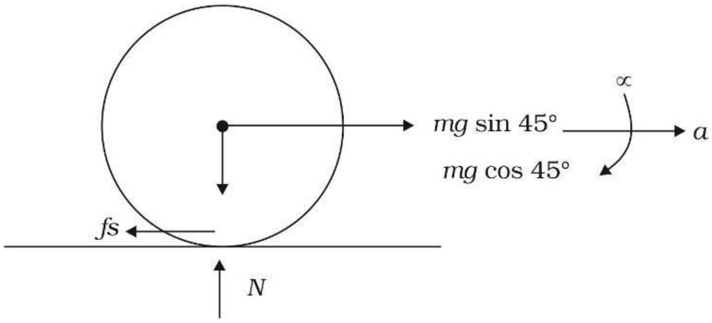
$$\frac{1}{f_{mirror}} = \frac{1}{f_{plane\,mirror}} - \frac{2}{f_{lens}} = \frac{1}{\infty} - \frac{2}{4R}$$

$\Rightarrow \quad f_{mirror} = -2R$

$\because \quad$ As per question, 2R = 0.2 meter

$\Rightarrow \quad R = 0.1\,m$

31.(4) $\tau_{Bottom} = I_{Bottom} \,.\, \alpha$

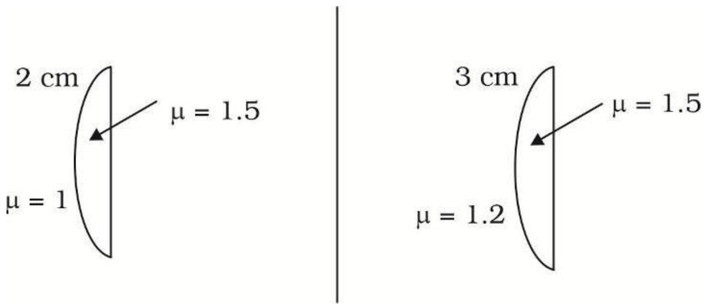


$\Rightarrow \quad mg\sin 45 \times r = \left(\frac{mr^2}{2} + mr^2\right).\alpha$

$\Rightarrow \quad \alpha = \frac{2g\sin 45°}{3r}$

$\Rightarrow \quad a_{cm} = \alpha.r = \frac{2g}{3} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{3}g$

32.(3)



$\because \quad \frac{1}{f_1} = (1.5 - 1)\left(\frac{1}{2}\right)$ $\qquad \because \quad \frac{1}{f_2} = \left(\frac{1.5}{1.2} - 1\right)\left(\frac{1}{3}\right)$

$\Rightarrow \quad f_1 = 4\ cm$ $\qquad \Rightarrow \quad \frac{1}{f_2} = \frac{0.3}{1.2} \times \frac{1}{3}$ $\qquad \Rightarrow \quad f_2 = 12cm$

$\therefore \quad \frac{f_1}{f_2} = \frac{1}{3}$

33.(1) In 12.5 sec, particle executed 6 complete oscillation and $\frac{1}{4}th$ oscillation.

So, displacement = d = (Zero for 6 complete oscillation) + A(for $1/4^{th}$ oscillation)

$\Rightarrow \quad d = 1\ cm$

So, distance = D = (4A for one oscillation) × 6 + A (for $1/4^{th}$ oscillation)

= 24 A + A = 25 A = 25 cm

$\Rightarrow \quad \frac{D}{d} = 25$

34.(4) $f_{old} = \frac{1}{2.5}m = \frac{100}{2.5}cm = \frac{1000}{25}cm$

$f_{new} = \frac{1}{2.6}m = \frac{1000}{26}cm$

$\because \frac{\Delta f}{f_{old}} = \frac{\frac{1000}{25} - \frac{1000}{26}}{\frac{1000}{25}} = 1 - \frac{25}{26} \cong 0.04$

35.(3) Charge particle will do projectile motion.

Initial momentum = $p_0 = mV_0$

So, initial de-broglie wave-length = $\lambda_0 = \frac{h}{p_0} = \frac{h}{mV_0}$

Velocity after time $t = \sqrt{V_0^2 + (at)^2}$

Where $a = \frac{eE_0}{m}$

$\Rightarrow \quad V = \sqrt{v_0^2 + \frac{e^2 {E_0}^2}{m^2} t^2}$

$\Rightarrow \quad p = mv = m\sqrt{v_0^2 + \frac{e^2 {E_0}^2 t^2}{m^2}} = mv_0\sqrt{1 + \frac{e^2 {E_0}^2 t^2}{m^2 {V_o}^2}}$

$\Rightarrow \quad \lambda = \frac{\lambda_0}{\sqrt{1 + \frac{e^2 {E_0}^2 t^2}{m^2 {v_0}^2}}}$

36.(2) Work-done $= \int_{x=0}^{1} F dx = \int_0^1 (\alpha + \beta x^2) dx$

$= \alpha + \frac{\beta}{3} = 5$

$\because \quad \alpha = 1 \Rightarrow \beta = 12$

37.(3) **(A)** – Solar cell has broad junction area compared to photo-diode to absorb more light to produce emf. So, it is wrong statement.

(B) – Solar cell creates electromotive force when exposed to sun-light, and does it without usage of external voltage. So, it is correct.

(C) – LED are highly doped (wrong statement)

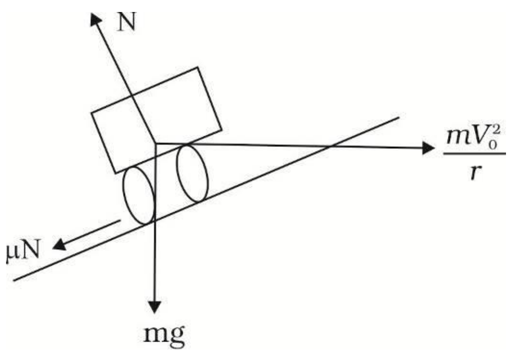
(D) – In LED when we increase forward current light intensity increases but this increase is non-linear but after some critical forward current, LED goes in saturation where its photonic efficiency decreases due to heat production at the junction, so in saturation zone light intensity decreases if current is increased further. (So, it is wrong)

(E) – LED emits light in forward bias only (So, it is correct)

38.(3) Along horizontal, $N\sin\theta + \mu N\cos\theta = \frac{mV_0{}^2}{r}$ ...(i)

Along vertical, $N\cos\theta = \mu\ N\sin\theta + mg$ ...(ii)

Solving (i) and (ii), we get, $\mu = \frac{v_0^2 - rg\tan\theta}{rg + v_0^2\tan\theta}$



39.(4) $\because$ Capacitance = C = $\frac{\epsilon_0\, lb}{d}$

(A) l becomes 10 times and d becomes $\frac{1}{3}rd$ $\Rightarrow$ C becomes 30 times

(B) d becomes 10 times $\Rightarrow$ C becomes $\frac{1}{10}th$ times.

(C) l is doubled, b becomes 5 times $\Rightarrow$ C becomes 10 times.

(D) l becomes $\frac{1}{3}rd$, d becomes $\frac{10}{3}$ times $\Rightarrow$ C becomes $\frac{1}{10}th$

(E) l becomes $\frac{5}{2}$ times, b becomes 2 times, d becomes $\frac{1}{3}rd$

$\Rightarrow$ C becomes 10 times

40.(1) $\because$ Pressure $\propto$ Temperature

$\Rightarrow$ Volume is constant.

(A) Work done = Zero $\Rightarrow$ (A) is correct.

(B) $\Delta Q = \Delta U$ $(\because$ Process is isochoric$)$ $\Rightarrow$ (B) is wrong.

(C) Volume is constant $\Rightarrow$ (C) is wrong.

(D) As pressure increases so, temperature increases

$\Rightarrow$ $\Delta U = nC_V\Delta T$ is positive $\Rightarrow$ (D) is correct.

(E) Process is iso-choric $\Rightarrow$ (E) is correct.

41.(3) $E_A - E_C = \frac{hc}{\lambda_1}$ where $\lambda_1 = 2000\ \mathring{A}$

$E_B - E_C = \frac{hc}{\lambda_2}$ where $\lambda_2 = 6000\ \mathring{A}$

$\Rightarrow \quad \frac{1}{\lambda} = \frac{1}{\lambda_1} - \frac{1}{\lambda_2} \Rightarrow \lambda = \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1} = \frac{6000 \times 2000}{4000} = 3000\mathring{A}$

42.(3) Energy stored = Energy – Density × Volume

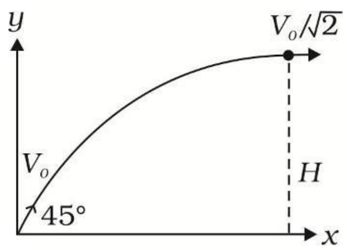
$= \frac{1}{2} \epsilon_0 E^2 Ad$

43.(1) $I = I_A \sin\omega t + I_B \cos\omega t = \sqrt{I_A^2 + I_B^2}\left(\frac{I_A}{\sqrt{I_A^2 + I_B^2}}\sin\omega t + \frac{I_B}{\sqrt{I_A^2 + I_B^2}}\cos\omega t\right)$

$\Rightarrow \quad I = \sqrt{I_A^2 + I_B^2}\ \sin(\omega t + \phi)$

$\Rightarrow \quad \text{r.m.s value} = \frac{I_{max}}{\sqrt{2}} = \frac{\sqrt{I_A^2 + I_B^2}}{\sqrt{2}}$

44.(4) Particle about origin $\vec{L} = \frac{-mV_0}{\sqrt{2}} H\hat{k}$



Where $H = \frac{V_0^{\ 2} \sin^2 45°}{2g}$

$\Rightarrow \quad \vec{L} = \frac{-mV_0^{\ 3}}{4\sqrt{2}}\hat{k}$

45.(1) For matching, $n_1\beta_1 = n_2\beta_2$

$\Rightarrow \quad n_1\frac{\lambda_1 D}{d} = n_2\frac{\lambda_2 D}{d}$

$\Rightarrow \quad n_1\lambda_1 = n_2\lambda_2$

$\Rightarrow \quad n_1 \times 480 = n_2 \times 600$

$\Rightarrow \quad 4n_1 = 5n_2$

Minimum value of $n_1 = 5$.

SECTION – 2

46.(35) Pitch of screw guage = 1 mm

Number of circular division = 100

$\Rightarrow$ Least count of screw guage $= \dfrac{\text{Pitch}}{\text{Number of circular division}} = \dfrac{1mm}{100} = 0.01\,mm$

If pitch is increased 75% i.e., new pitch = 1.75 mm

and if number of circular division reduced 50% i.e., new number of circular division = 50

then new least count = $\dfrac{1.75\,mm}{50} = 0.035$

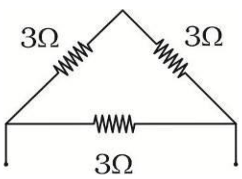
47.(48) Data insufficient as distance of charge from loop is not given.

48.(8) $B = \dfrac{\mu_0 I}{4\pi(a/2)}(2.\sin 45°)\times 4$ where a = side-length of square

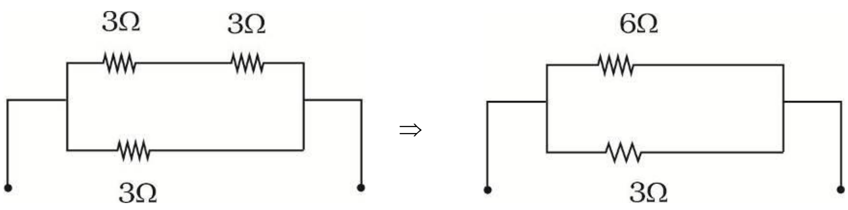
$= \dfrac{\mu_0.5}{2\pi a}.\sqrt{2}\times 4 = \dfrac{10\sqrt{2}\mu_0}{\pi a}$

$= \dfrac{10\sqrt{2}\mu_0}{\pi}\times\sqrt{2} = \dfrac{20}{\pi}.4\pi\times 10^{-7} = 80\times 10^{-7}$

49.(2)



$R_{eq} = \dfrac{6\times 3}{6+3} = \dfrac{18}{9} = 2\Omega$



50.(15) For iso-baric process, heat added $= \Delta Q = nC_P\Delta T = E_1$

For iso-baric process, change in internal energy $= \Delta U = nC_p\Delta T = 1$

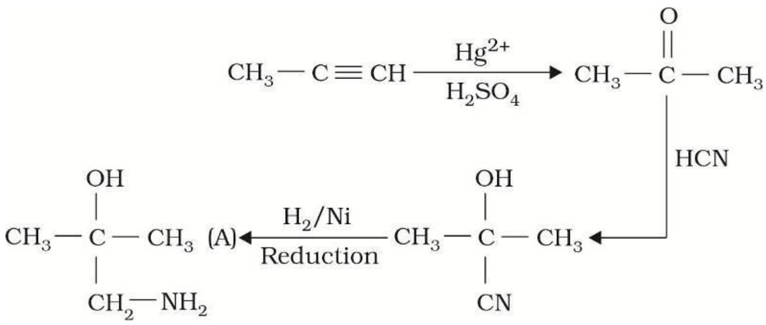
$\Rightarrow \dfrac{E_1}{E_2} = \dfrac{C_P}{C_V} = \dfrac{5}{3} = \dfrac{15}{9}$

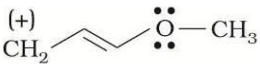
CHEMISTRY

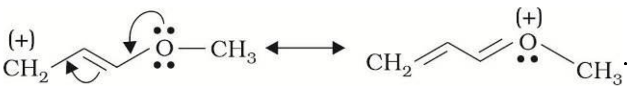
SECTION – 1

51.(2) $MnO_2 \xrightarrow[\text{Oxidized with air or } KNO_3]{\text{Fused with KOH}} K_2MnO_4 \xrightarrow[\text{Oxidation}]{\text{Electrolytic}} KMnO_4$

52.(2)



53.(1)  is most stable because of the maximum stabilization due to +M effect of $-OCH_3$ group

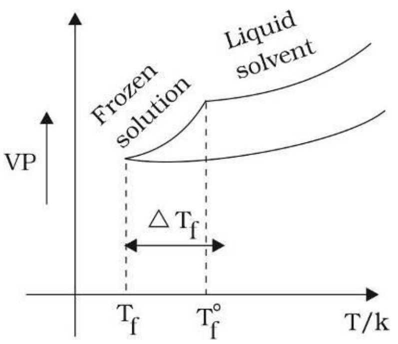


54.(2) Statements A, C and E are incorrect. Statements B and D are correct.

55.(4) Tb^{4+} is the strongest oxidizing agent out of the given ions as Lanthanides having +4 oxidation state tend to revert to the common +3 state by acting as oxidizing agents.

56.(3) The large difference between the melting and boiling points of oxygen and sulphur may be explained on the basis of their atomicity. Oxygen exists as O_2. Sulphur exists as S_8.

57.(2) Correct Plot:



58.(1) For a chemical reaction to be spontaneous,

$\Delta G < O$

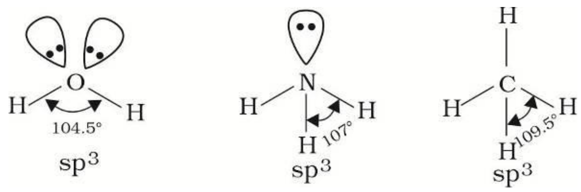
$\Rightarrow \Delta H - T\Delta S < O$

For an endothermic reaction

$\Delta H = +ve$

$\Rightarrow \Delta S = +ve$

59.(3)



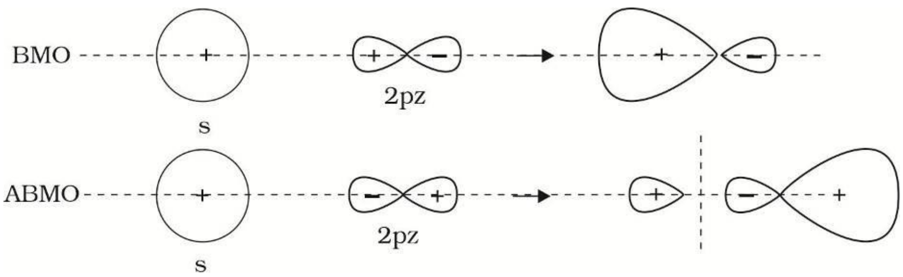
Order of dipole moment: $H_2O > NH_3 > CH_4$

NH_3 is a Lewis base.

$$\underset{\text{base}}{NH_3} + \underset{\text{acid}}{H_2O} \rightleftharpoons NH_4^+ + OH^-$$

(A), (B) and (C) are correct.

60.(2) Only 2s and 2pz orbitals can lead to formation of MO's along z-axis.



61.(2)

$$N_2O_{5(g)} \longrightarrow 2NO_2 + \frac{1}{2}O_2$$

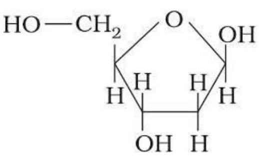
	N_2O_5	$2NO_2$	$\frac{1}{2}O_2$
Initial Pressure	P	O	O
Final Pressure	$P-\frac{P}{2}$	P	$\frac{P}{4}$
	$\frac{P}{2}$		

Final Pressure $= \frac{P}{2} + P + \frac{P}{4} = \frac{7}{4}P$

62.(4)

Ribose present in DNA

⇓



β — D — 2 — Deoxyribose

→ It is pentose sugar

→ It is in D configuration

→ It is a reducing sugar

→ It is not present in pyranose form

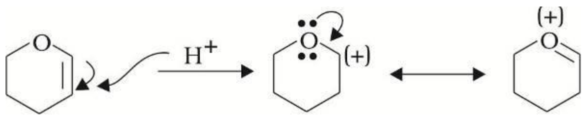
→ It is in β – anomeric form

(A), (C) and (D) are correct.

63.(1) Both statements (I) and (II) are correct.

Highly polar medium decreases the nucleophilicity of charged Nu^-, thereby, decreasing the rate of reaction in SN_2 mechanism, while polar medium increases the rate of SN_2 mechanism in case of a neutral nucleophile by stabilizing the transition state.

64.(3) 'Q' reacts fastest with HBr due to formation of the most stable carbocation.



65.(4) $Cr(OH)_{3(s)} \rightleftharpoons \underset{s}{Cr^{3+}}(aq) + \underset{3s}{3OH^-}(aq)$

$Ksp = s(3s)^3$

$Ksp = 27s^4$

$S = \sqrt[4]{\frac{KSP}{27}} = \sqrt[4]{\frac{1.6\times10^{-30}}{27}}$

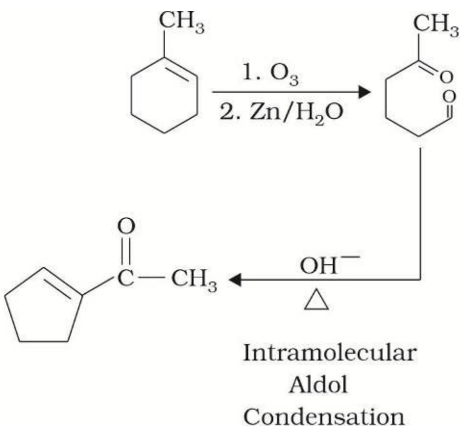
66.(2)

$[Co(NH_3)_5Cl]Cl_2 \longrightarrow [Co(NH_3)_5Cl]^{2+} + 2Cl^-$

3 ions in water

$[Co(NH_3)_5Cl]Cl_2 \xrightarrow{AgNO_3}$ 2 moles of AgCl ppt

67.(3)



68.(3) $R_1 : Ag^+ + e^- \rightarrow Ag \quad E_1^\circ = xV$

$R_2 : Fe^{2+} + 2e^- \rightarrow Fe \quad E_2^\circ = yV$

$R_3 : Fe^{3+} + 2e^- \rightarrow Fe \quad E_3^\circ = zV$

$R : Fe^{2+}(aq) + Aq^+(aq) \rightarrow Fe^{3+}(aq) + Ag(s)\ E^\circ = ?$

$R = R_1 + R_2 - R_3$

$\Delta^\circ G = \Delta^\circ G_1 + \Delta^\circ G_2 - \Delta^\circ G_3$

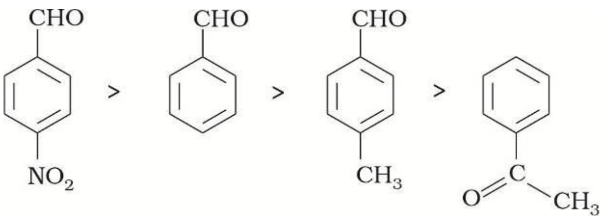
$-nFE^\circ = -n_1FE_1^\circ - n_2FE_2^\circ + n_3FE_3^\circ$

$1\times E^\circ = x + 2y - 3z$

$E^\circ = (x + 2y - 3z)V$

69.(1) Aromatic aldehydes are more reactive towards Nu^- addition as compared to aromatic ketones due to steric factors. EWG group increases the rate of reaction while EDG group decreases the rate of reaction.

Order of Nu^- addition



70.(2)

- Dumas method is used for estimation of Nitrogen.
- Statement II is incorrect.

SECTION – 2

71.(962) Initial pressure $P = \frac{nRT}{V}$

$$= \left(\frac{37.8}{108}\right) \times \frac{0.082 \times 500}{1}$$

$$= 14.35 \text{ bar}$$

	$2N_2O_{5(g)} \rightleftharpoons$	$2N_2O_{4(g)}$ +	$O_{2(g)}$
Initial pressure	14.35	O	O
Pressure at equilibrium	$14.35 - X$	X	X / 2

Total pressure $= 14.35 + \frac{X}{2} = 18.6$

$$\Rightarrow X = 8.6$$

	$2N_2O_{5(g)} \rightleftharpoons$	$2N_2O_{4(g)}$ +	$O_2(g)$
Pressure at equilibrium	14.36−8.6 =5.75	8.6	4.3

$$Kp = \frac{(PN_2O_4)^2(PO_2)}{(P(N_2O_5)^2}$$

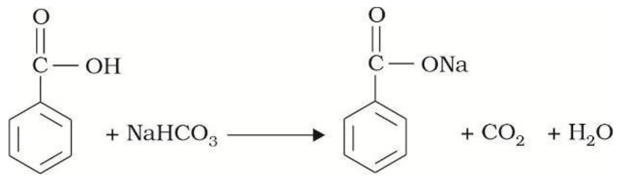
$$= \frac{(8.6)^2 \times 4.3}{(5.75)^2}$$

$$= 961.8$$

$$= 961.8 \times 10^{-2}$$

$$= 962 \times 10^{-2}$$

72.(61)



n (Benzoic acid) $= n(CO_2)$

$$nCO_2 = \frac{11.2}{22.4} = 0.5$$

$\therefore$ n benzoic acid = 0.5

mass $= 0.5 \times 122$ g

$= 61$ g

73.(700) $\frac{1}{2}X_2 + \frac{5}{2}Y_2 \rightleftharpoons XY_5$

$$\Delta^\circ S = S^\circ_m(XY_5) - \frac{1}{2}S^\circ_m X_2 - \frac{5}{2}S^\circ_m Y_2$$

$$= 110 - \frac{70}{2} - 50 \times \frac{5}{2} = -50 \text{ J}$$

$$T_{eq} = \frac{\Delta^\circ l \wedge}{\Delta^\circ S} = \frac{-35 \times 10^3}{-50} = 700 \text{ K}$$

74.(420) $Fe_3O_4 + 4CO \longrightarrow 3Fe + 4CO_2$

$$nFe_3O_4 = \frac{2.320 \times 10^3}{232} = 10^4 \text{ moles} \quad ; \quad nCO = \frac{280 \times 10^3}{28} = 10^4 \text{ moles}$$

Limiting reagent : CO

$$\therefore \quad nFe = \frac{3}{4}nCO = \frac{3}{4} \times 10^4 = 7500 \text{ moles}$$

$$nFe = \frac{7500 \times 56}{1000}\text{Kg} = 420 \text{ Kg}$$

75.(3) Cu^{2+}, Fe^{3+} and Zn^{2+} give characteristic ppt with $K_4[Fe(CN)_6]$

$$Cu^{2+} \xrightarrow{K_4[Fe(CN)_6]} \underset{\text{Chocolate brown ppt}}{Cu_2[Fe(CN)_6]}$$

$$Fe^{3+} \xrightarrow{K_4[Fe(CN)_6]} \underset{\text{Prussion blue ppt}}{Fe_4[Fe(CN)_6]_3}$$

$$Zn^{2+} \xrightarrow{K_4[Fe(CN)_6]} Zn_2[Fe(CN)_6]$$

Or

$K_2Zn_3[Fe(CN)_6]$

Bluish white ppt