SOLUTIONS

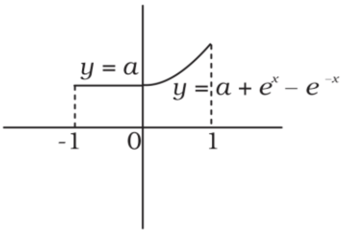
Joint Entrance Exam | IITJEE-2025

23rd JANUARY 2025 | Evening Shift

MATHEMATICS

SECTION – 1

1.(3)



$$A = a \times 1 + \int_0^1 (a + e^x - e^{-x})dx = a + a + \left[e^x\right]_0^1 + \left[e^{-x}\right]_0^1 = 2a + e + \frac{1}{e} - 2$$

$$= \frac{e^2 + (2a-2)e + 1}{e} = \frac{e^2 + 8e + 1}{e} \Rightarrow 2a - 2 = 8 \Rightarrow a = 5$$

2.(3) Reflexive

$(a_1, b_1)\ R\ (a_1, b_1)$

As $b_1 = b_2$, hence reflexive.

If symmetric $(a_1, b_1)\ R\ (a_2, b_2) \Rightarrow b_1 = b_2$ or $b_2 = b_1$

$\Rightarrow$ $(a_2, b_2)\ R\ (a, b)$ hence symmetric

If transitive $(a_1, b_1)\ R\ (a_2, b_2) \Rightarrow b_1 = b_2$

$(a_2, b_2)\ R\ (a_3, b_3) \Rightarrow b_2 = b_3$

$\Rightarrow b_1 = b_3$ and $(a_1, b_1)\ R\ (a_3, b_3)$ hence transitive.

So, R is equivalence relation. Hence statement I is true.

For Statement II.

If $(x, y)\ R\ (a, b)\ \Rightarrow y = b$ which is a horizontal line, hence statement II is false.

3.(4) $(1+x)^p (1-x)^q = ({}^pc_0 + {}^pc_1 x + {}^pc_2 x^2 + \ldots.) \times ({}^qc_0 - {}^qc_1 x + {}^qc_2 x^2 + \ldots)$

Coefficient of $x = -\,{}^pc_0\,{}^qc_1 + {}^pc_1\,{}^qc_0 \Rightarrow p - q = 1$ …(1)

Coefficient of $x^2 = {}^pc_0\,{}^qc_2 - {}^pc_1\,{}^qc_1 + {}^pc_2\,{}^qc_0 \Rightarrow \dfrac{q(q-1)}{2} - pq + \dfrac{p(p-1)}{2} = -2$

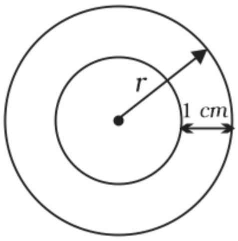
$\Rightarrow \quad (p-q)^2 - (p+q) = -4 \quad \Rightarrow \quad p + q = 5$ …(2)

From (1) and (2)

$p = 3,\ q = 2$

So, $p^2 + q^2 = 13$

4.(4)

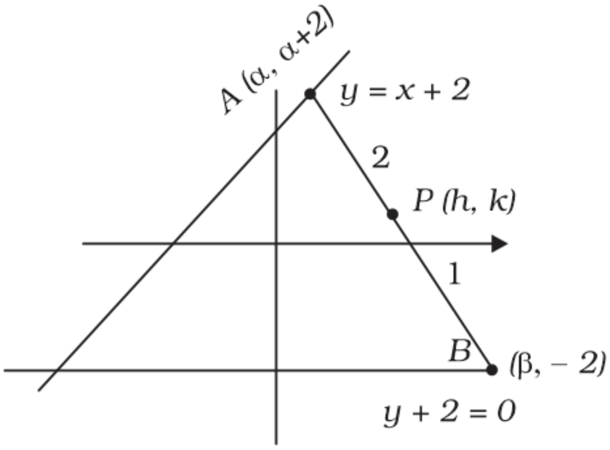


$$v = \frac{4}{3}\pi r^3 \qquad \frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt} \qquad -81 = 4\pi r^2\left(-\frac{1}{4\pi}\right)$$

$\Rightarrow \qquad r^2 = (9)^2 \Rightarrow r = 9$

Surface area of chocolate $= 4\pi(9-1)^2 = 256\,\pi$

5.(2)



$$h = \frac{2\beta + \alpha}{3}$$

$$k = \frac{-4 + \alpha + 2}{3}$$

$\Rightarrow \qquad \alpha = 3k + 2$ and $3h = 2\beta + \alpha$ $\qquad \ldots(1)$

$\Rightarrow \qquad 2\beta = 3h - \alpha \Rightarrow 2\beta = 3h - 3k - 2 \Rightarrow \beta = \dfrac{3h - 3k - 2}{2}$ $\qquad \ldots(2)$

Now, $\quad AB = 8$

$\Rightarrow \qquad (\alpha - \beta)^2 + (\alpha + 4)^2 = 64$

$\Rightarrow \qquad \dfrac{(9k - 3h + 6)^2}{4} + (3k + 6)^2 = 64$

$\Rightarrow \qquad 9(h^2 + 13k^2 - 6hk - 4h + 28k) = 76$

$\Rightarrow \qquad 9(x^2 + 13y^2 - 6xy - 4x + 28y) - 76 = 0$

So, $\quad \alpha - \beta - \gamma = 13 + 6 + 4 = 23$

6.(1) $\displaystyle\int \underset{I}{x^3} \underset{II}{\sin x}\, dx = -x^3 \cos x + \int 3x^2 \cos x\, dx$

$$= -x^3 \cos x + 3x^2 \sin x - \int 6x \sin x\, dx$$

$$= -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x + C$$

Now,

$$g(x) = -x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x$$

$$\Rightarrow \quad g'(x) = -3x^2\cos x + x^3\sin x + 6\cos x - 6\cos x$$

$$\Rightarrow \quad g'(x) = -3x^2\cos x + x^3\sin x$$

$$\Rightarrow \quad g'\left(\frac{\pi}{2}\right) = \frac{\pi^3}{8} \text{ and } g\left(\frac{\pi}{2}\right) = \frac{3\pi^2}{4} - 6$$

So, $$8\left(g\left(\frac{\pi}{2}\right) + g'\left(\frac{\pi}{2}\right)\right) = \pi^3 + 6\pi^2 - 48$$

Hence, $\alpha + \beta - \gamma = 55$

7.(3) Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$$A\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} b \\ e \\ h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$b = e = 0, h = 1$

$$A\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 4a + b + 3c \\ 4d + e + 3f \\ 4g + h + 3i \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} 4a + 3c = 0 \\ 4d + 3f = 1 \\ 4g + 3i = 0 \end{matrix} \quad \ldots(1)$$

$$A\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{matrix} 2a + b + 2c = 1 \\ 2d + e + 2f = 0 \\ 2g + h + 2i = 0 \end{matrix} \quad \ldots *$$

By *

$d = -f$

By (1)

$4(-f) + 3f = 1 \Rightarrow f = -1 \Rightarrow a_{23} = -1$

8.(4) Let $z = 1(\cos\theta + i\sin\theta); -\pi < \theta \le \pi$

$$\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1$$

$$\Rightarrow \quad \left|\frac{z^2}{|z|^2} + \frac{(\bar{z})^2}{|z|^2}\right| = 1 \quad \Rightarrow \quad \left|z^2 + (\bar{z})^2\right| = 1$$

$$\Rightarrow \quad |\cos 2\theta + i\sin 2\theta + \cos 2\theta - i\sin 2\theta| = 1$$

$$\Rightarrow \quad |\cos 2\theta| = \frac{1}{2} \text{ where } 2\theta \in (-2\pi, 2\pi]$$

$$\Rightarrow \quad \cos 2\theta = \pm\frac{1}{2}; 2\theta \in (-2\pi, 2\pi]$$

$\Rightarrow$ 8 solutions

9.(4) $\lim_{x\to\infty}\left(\frac{2x^2-3x+5}{3x^2+5x+4}\right)\left(\frac{3x-1}{3x+2}\right)^{x/2}$

Now, $\lim_{x\to\infty}\left(\frac{2x^2-3x+5}{3x^2+5x+4}\right)$ $\left[\left(\frac{\infty}{\infty}\right)\textit{form}\right]$

$= \lim_{x\to\infty}\frac{2-\frac{3}{x}+\frac{5}{x^2}}{3+\frac{5}{x}+\frac{4}{x^2}} = \frac{2}{3}$ and $\lim_{x\to\infty}\left(\frac{3x-1}{3x+2}\right)^{x/2}$ (1^{∞}) form

$= e^{\lim_{x\to\infty}\frac{x}{2}\left(\frac{3x-1}{3x+2}-1\right)}$

$= e^{\lim_{x\to\infty}\frac{x}{2}\left(\frac{-3}{3x+2}\right)} = e^{\lim_{x\to\infty}\left(\frac{-3}{2}\times\left(\frac{1}{3+\frac{2}{x}}\right)\right)}$

$= e^{-\frac{3}{2}\times\frac{1}{3}} = \frac{1}{\sqrt{e}}$

So, $\frac{2}{3}\times\frac{1}{\sqrt{e}} = \frac{2}{3\sqrt{e}}$

10.(2) $S.D. = \left|\frac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$

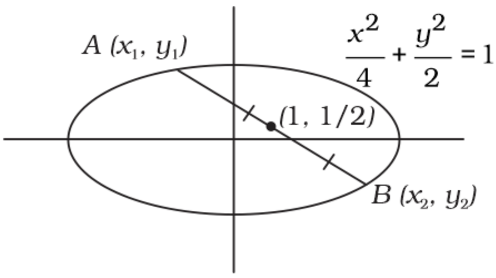
$\vec{a}_2-\vec{a}_1 = -3\hat{i}-4\hat{j}-2\hat{k}$

$\vec{b}_1\times\vec{b}_2 = 2\hat{i}-\hat{j}$

$\Rightarrow$ $S.D. = \frac{2}{\sqrt{5}}$

$\Rightarrow$ $(S.D.)^2 = \frac{4}{5} = \frac{m}{n} \Rightarrow m+n=9$

11.(1)



Equation of chord AB is $T = S_1$

$\Rightarrow$ $x+y=\frac{3}{2} \Rightarrow y=\frac{3}{2}-x$

On solving with ellipse $\frac{x^2}{4}+\frac{1}{2}\left(\frac{3}{2}-X\right)^2 = 1 \Rightarrow 6x^2-12x+1=0$

$\Rightarrow \quad x_1 + x_2 = 2$ and $x_1 x_2 = \frac{1}{6}$

$\Rightarrow \quad (x_2 - x_1)^2 = (x_1 + x_2)^2 - 4x_1x_2 = 4 - \frac{4}{6} = \frac{10}{3}$

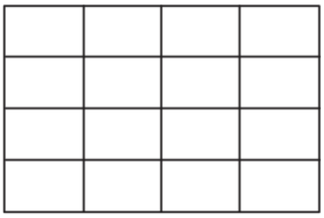
Now, $\quad y = \frac{3}{2} - x$

$(y_2 - y_1) = -1(x_2 - x_1)$

$\Rightarrow \quad (y_2 - y_1)^2 = (x_2 - x_1)^2 = \frac{10}{3}$

$AB = \sqrt{\frac{10}{3} + \frac{10}{3}} = \frac{2\sqrt{5}}{\sqrt{3}} = \frac{2\sqrt{15}}{3}$

12.(1)



P (no side common) = 1 – P (one side common)

$= 1 - \frac{3 \times 4 \times 2}{{}^{16}C_2} = 1 - \frac{24 \times 2}{16 \times 15} = 1 - \frac{1}{5} = \frac{4}{5}$

13.(3) $I = \int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx \quad \ldots(1)$

$I = \int_0^{\pi/2} \frac{\cos^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx \quad \ldots(2)$

$2I = \int_0^{\pi/2} 1\, dx = \frac{\pi}{2} \quad \Rightarrow \quad I = \frac{\pi}{4}$

Now, let $I_1 = \int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx \quad \ldots(3)$

$I_1 = \int_0^{\pi/2} \frac{(\pi / 2 - x) \cos x \sin x}{\sin^4 x + \cos^4 x} dx \quad \ldots(4)$

Add (3) and (4)

$2I_1 = \frac{\pi}{2} \int_0^{\pi/2} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx$

$I_1 = \frac{\pi}{4} \int_0^{\pi/2} \frac{\tan x \sec^2 x}{1 + \tan^4 x} dx$

Let $\tan^2 x = t$

$2 \tan x \sec^2 x dx = dt$

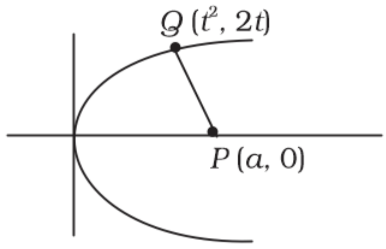
$$\tan x \sec^2 x \, dx = \frac{dt}{2}$$

$$I_1 = \frac{\pi}{8}\int_0^\infty \frac{dt}{1+t^2} \quad \Rightarrow \quad I_1 = \frac{\pi}{8}\left[\tan^{-1} t\right]_0^\infty \quad \Rightarrow \quad I_1 = \frac{\pi^2}{16}$$

14.(2) Given question of parabola $\Rightarrow y^2 = 4x$

Let the point $(a, 0)$ be P and given $PQ = 4$

Where coordinates of Q be $(t^2, 2t)$



Also, PQ is normal to parabola and we know equation of normal in parametric form:

$$y + tx = 2t + t^3$$

Now point P will satisfy this equation. Hence, we get at = $2t + t^3$

$\Rightarrow \quad a = 2 + t^2 \quad \ldots(1)$

$\therefore \quad P(2+t^2, 0)$

$\therefore \quad$ By distance formula, $PQ = \sqrt{(2+t^2-t^2)+(0-2t)} = 4$

$\Rightarrow \quad 4 + 4t^2 = 16 \Rightarrow t^2 = 3$

$\therefore \quad a = 2 + t^2$ {by equation (1)}

$\Rightarrow \quad a = 2 + 3 = 5$

And focus of parabola $\Rightarrow (1, 0)$

So, (1, 0) and (5, 0) are diameter's end point when circle passes through these points and having centre on axis of parabola i.e., x-axis.

$\Rightarrow \quad (x-1)(x-5)+(y-0)(y-0) = 0$

$\Rightarrow \quad x^2 + y^2 - 6x + 5 = 0$

15.(4) We know point $P(1, 4, 0)$ and line $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3} \Rightarrow L_2$

Now line parallel to L_2 and passing through (1, 4, 0) $\Rightarrow \frac{x-1}{1} = \frac{y-4}{2} = \frac{z-0}{3} \Rightarrow L_3$

Now, point of intersection of L_3 with line $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4} \Rightarrow L_1$ is given by Q.

So, let $\frac{x-1}{1} = \frac{y-4}{2} = \frac{z-0}{3} = \lambda$ and $\frac{x-2}{2} = \frac{y-6}{3} = \frac{2-3}{4} = \gamma$

$\Rightarrow \quad x = \lambda + 1, y = 2\lambda + 4, z = 3\lambda$ and $x = 2\gamma + 2, y = 3\gamma + 6, z = 4\gamma + 3$

$\Rightarrow \quad \lambda + 1 = 2\gamma + 2$ and $2\lambda + 4 = 3\gamma + 6$

$\Rightarrow \quad \lambda = 2\gamma + 1 \quad \Rightarrow \quad 2(2\gamma+1)+4 = 3\gamma + 6$

$\Rightarrow \quad 4\gamma + 2 + 4 = 3\gamma + 6 \Rightarrow \gamma = 0$

Hence point of intersection i.e. Q = (2, 6, 3)

$\therefore \quad$ Required distance = $PQ = \sqrt{(2-1)^2 + (6-4)^2 + (3-0)^2} = \sqrt{14}$.

16.(1) Given : $y = \left(x - y\frac{dx}{dy}\right)\sin\left(\frac{x}{y}\right); y > 0$ and $x(1) = \frac{\pi}{2}$.

$\Rightarrow \qquad y = \left(\frac{xdy - ydx}{dy}\right)\sin\left(\frac{x}{y}\right)$

$\Rightarrow \qquad y\,dy = (xdy - ydx)\sin\left(\frac{x}{y}\right)$

$\Rightarrow \qquad \frac{ydy}{y^2} = \left(\frac{xdy - ydx}{y^2}\right)\left(\sin\frac{x}{y}\right)$

$\Rightarrow \qquad \frac{dy}{y} = \left(\frac{xdy - ydx}{y^2}\right)\left(\sin\frac{x}{y}\right)$

We know $\Rightarrow \frac{dy}{y} = \sin\left(\frac{x}{y}\right)d\left(-\frac{x}{y}\right)$

$\Rightarrow \qquad \ln y = \cos\frac{x}{y} + C$

Now, $\qquad x(1) = \frac{\pi}{2} \Rightarrow \ln 1 = \cos\left(\frac{\pi}{2}/1\right) + C$

$\Rightarrow \qquad 0 = 0 + C \Rightarrow C = 0$

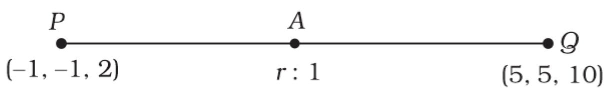
$\Rightarrow \qquad \ln y = \cos\frac{x}{y}$

Now at $y = 2 \Rightarrow \ln 2 = \cos\frac{x}{2}$ $\qquad \ldots(1)$

But $\qquad \cos x(2) = ? \qquad \Rightarrow \qquad \cos x = 2\cos^2\frac{x}{2} - 1$

$\qquad \Rightarrow \qquad \cos x = 2(\ln 2)^2 - 1$ [By (i)]

17.(1)



$\therefore \qquad$ Coordinates of $A = \frac{5r-1}{r+1}, \frac{5r-1}{r+1}, \frac{10r+2}{r+1}$.

Also, we know, $(\overrightarrow{OQ}.\overrightarrow{OA}) - \frac{1}{5}\left(\left|\overrightarrow{OP} \times \overrightarrow{OA}\right|\right)^2 = 10 \qquad \ldots(1)$

So, $\qquad \overrightarrow{OQ}.\overrightarrow{OA}$

$$= 5\left(\frac{5r-1}{r+1}\right) + 5\frac{(5r-1)}{r+1} + 10\left(\frac{10r+2}{r+1}\right)$$

$$= \frac{5}{r+1}(5r - 1 + 5r - 1 + 20r + 4)$$

$$= \frac{5}{r+1}(30r + 2) \qquad \ldots(2)$$

And $\quad \left(\left|\overrightarrow{OP} \times \overrightarrow{OA}\right|\right)^2 = \left\| \begin{matrix} i & j & k \\ -1 & -1 & 2 \\ \frac{5r-1}{r+1} & \frac{5r-1}{r+1} & \frac{10r+2}{r+1} \end{matrix} \right\|^2$

$$= \frac{r^2}{(r+1)^2} 800 \qquad \ldots(3)$$

Now putting (ii) and (iii) in equation (i) we get

$$\frac{5}{r+1}(30r+2) - \frac{1}{5}\frac{r^2 800}{(r+1)^2} = 10$$

$$\Rightarrow \quad \frac{10}{r+1}(15r+1) - \frac{r^2}{(r+1)^2}(160) = 10$$

$$\Rightarrow \quad \frac{15r+1}{r+1} - \frac{16r^2}{(r+1)^2} = 1 \Rightarrow 2r^2 - 14r = 0 \Rightarrow r = 7, r \neq 0$$

18.(4) $A = \{(x,y) \in R \times R : |x+y| \geq 3\}$ and $B = \{x, y \in R \times R :) |x| + |y| \leq 3\}$.

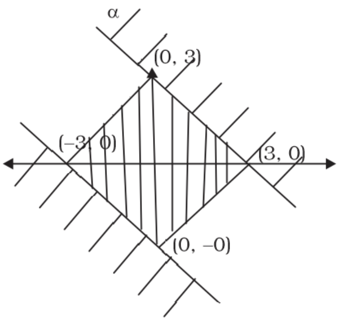
For Set A $|x+y| \geq 3$

$\Rightarrow \quad x + y \geq 3$ or $\quad x + y \leq -3$

For Set B $|x| + |y| \leq 3$

So, $\quad C = \{(3,0),(0,3),(-3,0),(0,-3)\}$

$$\sum |x+y| = 3+3+3+3 = 12$$



19.(3) Given, $f(x) = 6 + 16\cos x.\cos\left(\frac{\pi}{3} - x\right).\cos\left(\frac{\pi}{3} + x\right).\sin 3x.\cos 6x$ and $x \in R$.

We know, $\frac{1}{4}\cos 3x = \cos\left(\frac{\pi}{3} - x\right)\cos x \cos\left(\frac{\pi}{2} + x\right)$

$$\Rightarrow \quad f(x) = 6 + 16\left(\frac{1}{4}\cos 3x\right)\sin 3x.\cos 6x$$

$$= 6 + 4\cos 3x \sin 3x \cos 6x \qquad [\because 2\sin\theta\cos\theta = \sin 2\theta]$$

$$= 6 + 2\sin 6x \cos 6x$$

$\Rightarrow \quad f(x) = 6 + \sin 2x$ and range of $\sin\theta$ is $(-1, 1)$.

$\therefore \quad f(x)$ range $\Rightarrow (5, 7)$ given (α, β)

$\therefore \quad \alpha = 5, \beta = 7$

$\therefore \quad$ Distance of $(5, 7)$ from $3x + 4x + 12 = 0$

$$= \left|\frac{3 \times 5 + 4 \times 7 + 12}{\sqrt{3^2 + 4^2}}\right| = \frac{15 + 28 + 12}{5} = 11$$

20.(2) Given, $\quad x + y + z = 6$

$$x + 2y + 5z = 9$$

$x + 5y + \lambda z = \mu$ has no solution

$\therefore \quad \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 1 & 5 & \lambda \end{vmatrix} = 0 \quad \ldots(i)$

and $\quad \Delta' = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 9 \\ 1 & 5 & \mu \end{vmatrix} \neq 0 \quad \ldots(ii)$

Expanding (i) we get,

$$1(2\lambda - 25) - 1(\lambda - 5) + 1(5 - 2) = 0$$

$\Rightarrow \quad \lambda = 17$

and expanding (ii), we get

$$1(2\mu - 45) - 1(\mu - 9) + 6(5 - 2) \neq 0$$

$\Rightarrow \quad \mu \neq 18$

SECTION – 2

21.(474) Given common difference = $d = \frac{3}{2}$, Let first term = a

Also, we know, $S_{11} = 88$

$\Rightarrow \quad \frac{11}{2}[2a + 10d] = 88$

$\Rightarrow \quad 2a + 10 \times \frac{3}{2} = 16$

$\Rightarrow \quad a = \frac{1}{2}$

Hence roots of Quadratic $3x^2 - px + q = 0$

are $\quad \alpha = T_{10} = a + 9d = \frac{1}{2} + 9 \times \frac{3}{2} = 14$

and $\quad \beta = T_{11} = a + 10d = \frac{1}{2} + 10 \times \frac{3}{2} = \frac{31}{2}$

Now $\quad \frac{p}{3} = \alpha + \beta = 14 + \frac{31}{2} \Rightarrow P = \frac{177}{2}$

$\frac{q}{3} = \alpha\beta = 14 \times \frac{31}{2} \Rightarrow q = 651$

$\therefore \quad q - 2p = 651 - 2\left(\frac{177}{2}\right) = 651 - 177 = 474$

22.(8788) 8, 21, 34, 47, ...320 A.P.

$320 = 8 + (n - 1)13$

$\Rightarrow \quad \boxed{n = 25}$

Now, $\sigma^2 = \left(\frac{n^2-1}{12}\right)d^2 = \frac{624}{12}\times 169 = 8788$

23.(31) We know, $\alpha+\beta = a$ and $\alpha\beta = -b$

$\therefore \quad P_b = \alpha^6 - \beta^6 = \alpha^6 - \alpha\beta^5 + \beta\alpha^5 - \beta^6 - \alpha^5\beta + \alpha\beta^5$

$= (\alpha+\beta)(\alpha^5 - \beta^5) - \alpha\beta(\alpha^4 - \beta^4)$

$\Rightarrow \quad p_6 = aP_5 - bP_4$

Similarly, we can write $\Rightarrow P_5 = aP_4 + bP_3$.

Now putting value of P_3, P_4, P_5 in above two equations and solving for a and b, we set.

$\Rightarrow \quad -3\sqrt{7}ia - 5\sqrt{7}ib = 11\sqrt{7}i \quad \Rightarrow \quad \boxed{-3a-5b=11}$ ...(i)

and $\quad 45\sqrt{7}i = 11\sqrt{7}ia - 3\sqrt{7}ib$

$\Rightarrow \quad \boxed{45 = 11a - 3b}$ ...(ii)

$\therefore \quad a = 3$ and $b = -4 \quad \therefore \quad \alpha+\beta = 3, \alpha\beta = 4$

Hence $\left|\alpha^4+\beta^4\right| = \left|(\alpha^2+\beta)^2 - 2\alpha^2\beta^2\right| = \left|\left((\alpha+\beta)^2 - 2\alpha\beta\right)^2 - 2\alpha^2\beta^2\right| = \left|(9-8)^2 - 2\times 16\right| = |1-32| = 31$

24.(17280) Number of ways.

Case I = If all boys sit together = $\underline{|5}\times\underline{|5} = 14400$

$\square\, G_1 \,\square\, G_2 \,\square\, G_3 \,\square\, G_4$

Case II = If no two boys sit together

$\therefore$ Number of ways $= \underline{|4}\times\underline{|5} = 2880$

$B_1 \,\square\, B_2 \,\square\, B_3 \,\square\, B_4 \,\square\, B_5$

$\therefore$ Total number of ways = Case I + Case II

= 14400 + 2880 = 17280

25.(15) We know equation of parabola $\Rightarrow y^2 = 4x + 16$

$\Rightarrow y^2 = 4(x+4)$

Hence, Focus of parabola = (–3, 0)

And (–3, 0) = centre of circle of radius 5.

$\therefore$ equation of circle $\Rightarrow (x+3)^2 + y^2 = 25 \Rightarrow x^2 + y^2 + 6x = 16$

and circle passes through point of intersection of lines $3x - y = 0$ and $x + \lambda y = 4$

$\Rightarrow \quad x = \frac{4}{3\lambda+1}, y = \frac{12}{3\lambda+1}$ Now putting x, y in equation of circle

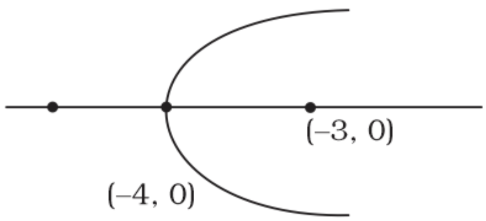
We set $\left(\frac{4}{3\lambda+1}+3\right)^2 + \left(\frac{12}{3\lambda+1}\right)^2 = 25 \Rightarrow \frac{(4+9\lambda+3)^2+144}{(3\lambda+1)^2} = 25$

$\Rightarrow (9\lambda+7)^2 + 144 = 25(3\lambda+1)^2$

$\Rightarrow 81\lambda^2 + 49 + 126 + 144 = 225\lambda^2 + 25 + 150\lambda$

$\Rightarrow \lambda = \frac{-7}{6}, 1$ as $\frac{-7}{6} < 1 \therefore \lambda_1 = \frac{-7}{6}$ and $\lambda_2 = 1$

$\therefore \; 12\lambda_1 + 29\lambda_2 = 12\left(-\frac{7}{6}\right) + 29(1) = -14 + 29 = 15$



PHYSICS

SECTION – 1

26.(3) $dQ = mcdT$

$$dS = \frac{dQ}{T}$$

$$\int dS = \int_{T_1}^{T_2} \frac{mcdT}{T} \qquad \text{(as } C = 1\,J\,kg^{-1}K^{-1}\text{)}$$

$$\Delta S = m\left[\ln T\right]_{T_1}^{T_2}$$

$$= m\left[\ln T_2 - \ln T_1\right]$$

$$= m\ln\frac{T_2}{T_1}$$

27.(3) $p = q2a$

$$= 4\times10^{-6}\times2\times9\times10^{-2} = 72\times10^{-8}$$

$$= 7.2\times10^{-7}\,C-m$$

$$W = -pE\left(\cos\theta_f - \cos\theta_i\right)$$

$$W = -7.2\times10^{-7}\times10^{4}\left(\cos180^\circ - \cos0^\circ\right)$$

$$= 14.4\times10^{-3}J = 14.4\,mJ$$

28.(2) $KE = \frac{1}{2}mu^2$

$$KE' = \frac{1}{2}m(u\cos60)^2$$

$$KE' = \frac{1}{2}mu^2\cos^2 60^\circ$$

$$KE' = \frac{KE}{4}$$

29.(3) Focal length of concave mirror does not depend on the medium surrounding it.

30.(4) $\frac{hc}{\lambda} = \phi_0 + KE_{\max}$

$$\frac{1242eVnm}{\lambda} = (2.14+2)eV$$

$$\lambda = \frac{1242nm}{4.14} = 300\,nm$$

31.(2) $5N = kx_1$

$$7N = kx_2$$

$$F = k(5x_1 - 2x_2)$$

$= 5kx_1 - 2kx_2$

$= 5 \times 5N - 2 \times 7N$

$= 25N - 14N = 11N$

32.(2) $\theta = 5t^2 - 8t$

$\omega = \dfrac{d\theta}{dt} = 10t - 8$

$\alpha = \dfrac{d\omega}{dt} = 10$

Power $= \tau\omega = I\alpha\omega$

$= \dfrac{1}{2}MR^2 \times 10(10t - 8) = 60MR^2$

33.(4) $I_g = 20\,mA \qquad G = 30\Omega$

$S = \dfrac{I_g}{I - Ig}G = \dfrac{20\,mA}{3A - 20\,mA} \times 30$

$\dfrac{30}{X} = \dfrac{20 \times 10^{-3}}{3 - 20 \times 10^{-3}} \times 30$

$X = \dfrac{3000 - 20}{20} = 149\Omega$

34.(2) Magnetic moment $= [M^0 L^2 T^0 A]$

Torsional constant $= [M^1 L^2 T^{-2}]$

Magnetic field $= [M^1 L^0 T^{-2} A^{-1}]$

Permeability of free space $= [M^1 L^1 T^{-2} A^{-2}]$

35.(1) $B_z = \dfrac{E_y}{C} = \dfrac{9.3}{3 \times 10^8}T$

$= 3.1 \times 10^{-8}T$

36.(3) $P_1 + \dfrac{1}{2}\rho(0)^2 = P_2 + \dfrac{1}{2}\rho v^2$

$P_1 - P_2 = \dfrac{1}{2}\rho v^2$

$v^2 = \dfrac{2(P_1 - P_2)}{\rho}$

$v \propto \sqrt{P_1 - P_2}$

37.(4) BE per nucleon is more than 8.5 MeV/nucleon for nuclei having mass number between 30 to 70 & nuclear force is a small range force.

38.(3) Both diodes are forward biased

$I = \dfrac{5}{20} + \dfrac{5}{20} = 0.5\,A$

39.(1) $\mu = \dfrac{\sin\left(\dfrac{A+\delta_{min}}{2}\right)}{\sin\dfrac{A}{2}}$

$\sqrt{3} = \dfrac{\sin A}{\sin\dfrac{A}{2}} = 2\cos\dfrac{A}{2}$

$\cos\dfrac{A}{2} = \dfrac{\sqrt{3}}{2} \Rightarrow \dfrac{A}{2} = 30°$

$A = 60°$

40.(4) $\dfrac{I_{max}}{I_{min}} = \dfrac{(E_1+E_2)^2}{(E_1-E_2)^2} = \dfrac{9}{4}$

$\dfrac{E_1+E_2}{E_1-E_2} = \dfrac{3}{2} \Rightarrow 2E_1 + 2E_2 = 3E_1 - 3E_2$

$E_1 = 5E_2$

Given that

$E \propto$ slit width

$\dfrac{E_1}{E_2} = \dfrac{xd}{d} = 5$

$\Rightarrow \quad x = 5$

41.(4) Work $= W_{AB} + W_{BC} + W_{CD}$

$= +P_0V_0 + 0 + (-4P_0V_0)$

$= -3P_0V_0$

42.(3) From Kepler's third law

$T^2 \propto r^3$

$\dfrac{T_2}{T_1} = \left(\dfrac{r_2}{r_1}\right)^{3/2} = \left(\dfrac{1}{9}\right)^{3/2} = \dfrac{1}{27}$

$\dfrac{T_2}{27} = \dfrac{1}{27} \Rightarrow T_2 = 1\,\text{day}$

43.(1) Velocity of wave is

$$= \frac{-\text{coefficient of } t}{\text{coefficient of } x} = \frac{-600}{20\times10^{-3}} = -30\times10^3\ \text{mm/s} = -30\ \text{m/s}$$

44.(3) $U = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1q_2}{r}$

$= 9\times10^9 \times \dfrac{7\times10^{-6}\times -4\times10^{-6}}{14\times10^{-2}} = -18\times10^{-1}J = -1.8J$

45.(4) $E = \alpha^3 e^{-\beta t}$

$\ln E = 3\ln\alpha - \beta t$

$$\frac{\Delta E}{E} = 3\frac{\Delta\alpha}{\alpha} - \beta\Delta t$$

$$\frac{\Delta E}{E}\times 100\% = 3\left(\frac{\Delta\alpha}{\alpha}\times 100\%\right) + \beta\left(\frac{\Delta t}{t}\times 100\%\right)t$$

$$= 3\times 1.2\% + 0.3\times 1.6\%\times 5\%$$

$$= 3.6\% + 2.4\% = 6\%$$

SECTION – 2

46.(2190) $P_{in} = P_0 + h\rho g + \frac{2S}{r}$

$$P_{in} - P_0 = 0.2\times 10^3\times 10 + \frac{2\times 0.095}{10^{-3}} = 2000 + 190 = 2190\,Nm^{-2}$$

47.(100) $q = CV$

$$\frac{dq}{dt} = C\frac{dV}{dt}$$

$$0.25\times 10^{-3} = 2.5\times 10^{-6}\frac{dV}{dt}$$

$$\frac{dV}{dt} = 100\,V/m$$

48.(16) In steady state current $= \frac{5}{10+15} = \frac{1}{5}A$

Potential difference across the capacitor $= \frac{1}{5}\times 10 = 2V$

$q = CV = 8\mu F\times 2V = 16\mu C$

49.(2) Resonance angular frequency $= \frac{1}{\sqrt{LC}}$

$$= \frac{1}{\sqrt{25\times 10^{-9}\times 100\times 10^{-3}}} = \frac{10^6}{50} = 2\times 10^4\ \text{rad/s}$$

50.(3) $L = \frac{M}{2}vr$

$$= \frac{M}{2}\sqrt{\frac{GM}{r}}r$$

$$= \frac{M}{2}\sqrt{GMr} = \frac{M}{2}\sqrt{GM\frac{4R}{3}}$$

$$= M\sqrt{\frac{GMR}{3}}$$

CHEMISTRY

SECTION – 1

51.(3) $\Delta G = \Delta H - T\Delta S$

For a reaction to be spontaneous $\Delta G = -ve$

(B) $\Delta G = \underset{+ve}{\Delta H} - T\underset{+ve}{\Delta S}$

ΔG will be –ve only at high T

(C) $\Delta G = \underset{-ve}{\Delta H} - T\underset{-ve}{\Delta S}$

ΔG will be -ve only at low T

52.(4) With increase in temperature K_w increases So, degree of dissociation of water increases

i.e., $[H^+]$ and $[OH^-]$ will increase, pH and pOH will decrease.

53.(1) α - Helix and β – pleated sheets belongs to secondary structure of protein.

54.(3) Bronze $\rightarrow$ Cu, Sn

Brass $\rightarrow$ Cu, Zn

UK silver coin $\rightarrow$ Cu, Ni

Stainless steel $\rightarrow$ Fe, Cr, Ni, C

55.(2) $\underset{(A)}{PbS} \xrightarrow{HNO_3} Pb(NO_3)_2 \xrightarrow{H_2SO_4} \underset{(B)}{PbSO_4} \xrightarrow[3.\,K_2CrO_4]{1.\text{ Ammonium acetate}\; 2.\text{ Acetic acid}} \underset{(C)\text{ yellow}}{PbCrO_4}$

56.(1) $\Delta G° = -nFE°$

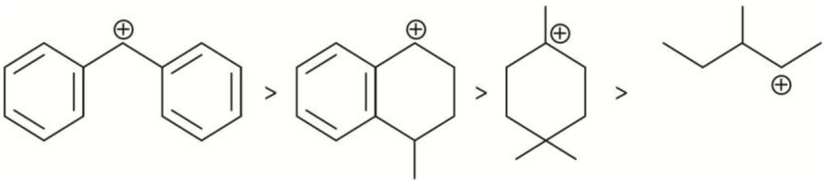
Option 1 $\Rightarrow$ $\Delta G° = -2 \times F \times \{0.8 + 0.76\} = -3.12F$

Option 2 $\Rightarrow$ $\Delta G° = -20 \times F \times \{-2.37 + 0.76\} = +3.22F$

Option 3 $\Rightarrow$ $\Delta G° = -2 \times F\{-2.37 - 0.8\} = +6.34F$

Option 4 $\Rightarrow$ $\Delta G° = -2 \times F\{0.8 - 0.34\} = -0.92F$

57.(2) Solvolysis follows S_N1 i.e. stability of carbocation



58.(2) For liquid solution of two liquids 1 and 2

$\Rightarrow \quad P_1 = P_T y_1 = P_1^{\circ} X_1$

$\Rightarrow \quad \dfrac{P_T}{X_1} = \dfrac{P_1^{\circ}}{y_1}$

$\Rightarrow \quad \dfrac{P_2^{\circ} + X_1\left(P_1^{\circ} - P_2^{\circ}\right)}{X_1} = \dfrac{P_1^{\circ}}{y_1}$

$$\Rightarrow \quad \frac{P_2^\circ}{X_1} + \left(P_1^\circ - P_2^\circ\right) = \frac{P_1^\circ}{y_1}$$

$$\Rightarrow \quad \frac{1}{X_1} = \left(\frac{P_1^\circ}{P_2^\circ}\right)\frac{1}{y_1} + \frac{P_2^\circ - P_1^\circ}{P_2^\circ}$$

Slope $= \frac{P_1^\circ}{P_2^\circ}$ and intercept $= \frac{P_2^\circ - P_1^\circ}{P_2^\circ}$

59.(4) $$X_2Y(g) \rightleftharpoons X_2(g) + \frac{1}{2}Y_2(g)$$

Moles at equilibrium $\quad 1-x \qquad x \qquad \frac{x}{2}$

$$K_P = \frac{\left(P_{X_2}\right)\left(P_{Y_2}\right)^{\frac{1}{2}}}{\left(P_{X_2Y}\right)}$$

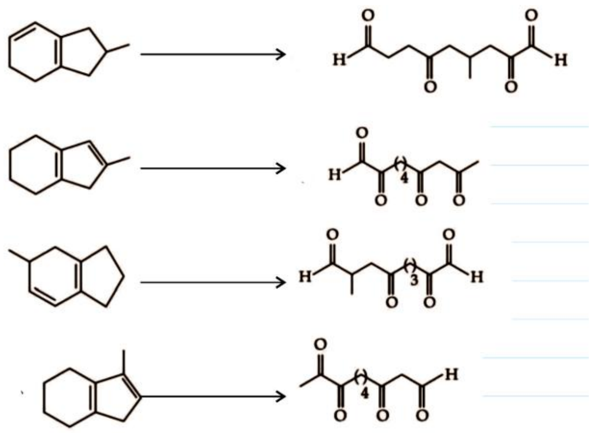
$$K_P = \frac{\left(\frac{x}{1+\frac{x}{2}} \times P\right)\left(\frac{x}{2\left(1+\frac{x}{2}\right)} \times P\right)^{\frac{1}{2}}}{\left(\frac{1-x}{1+\frac{x}{2}} \times P\right)}$$

Assuming x to be very small

$$K_P = \frac{x^{\frac{3}{2}}\,P^{\frac{1}{2}}}{(2)^{\frac{1}{2}}} \quad \Rightarrow \quad x^{\frac{3}{2}} = K_P \times \sqrt{\frac{2}{P}}$$

$$\Rightarrow \quad x = \left(K_P^2 \times \frac{2}{P}\right)^{\frac{1}{3}}$$

60.(3) Correct match of ozonolysis product are:



61.(3) A plot of $\sqrt{v}$ vs atomic number is a straight line. So both statements are false.

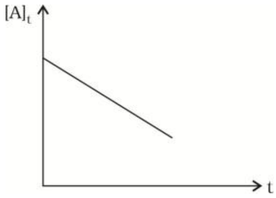
62.(3) Boiling point $\propto$ molecular mass

Boiling point $\propto$ intermolecular H-bonding.

63.(4) For zero order reaction $A \rightarrow P$

Rate = k

$[A]_t = [A]_0 - kt$



64.(4) $\Rightarrow \quad P° - P \propto X_{solute}$

$\Rightarrow \quad 10 \propto 0.2$

So, $\quad 20 \propto 0.4$

$X_{solvent} = 1 - 0.4 = 0.6$

65.(3) $\underset{}{K_2Cr_2O_7} \xrightarrow[-H_2O]{KOH} \underset{[A]}{K_2CrO_4} \xrightarrow[-H_2O]{H_2SO_4} \underset{[B]}{K_2Cr_2O_7} + K_2SO_4$

66.(4) Hydrate of formaldehyde is stable due to small substituents hydrate of chloral is stable because of -I effect.

67.(1) Order of melting point of group-14

$$\begin{array}{llllll} & C > & Si > & Ge > & Pb > & Sn \\ z = & 6 & 14 & 32 & 82 & 50 \end{array}$$

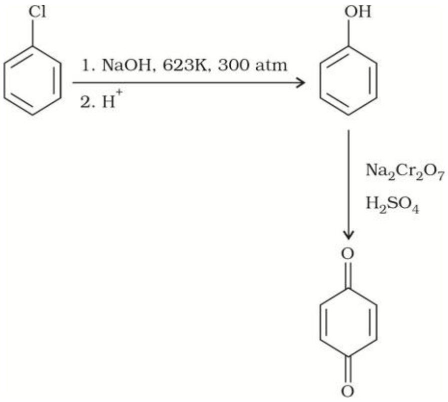
68.(1) For a shell total number of orbitals $= n^2$

Magnetic quantum numbers have values $(-l \text{ to} + l)$ including zero.

69.(1)

$Fe^{+6} = [Ar]3d^2$
$Mn^{+3} = [Ar]3d^4$
$Fe^{+3} = [Ar]3d^5$
$Cr^{+2} = [Ar]3d^4$
$Ni^{+4} = [Ar]3d^6$

70.(4)



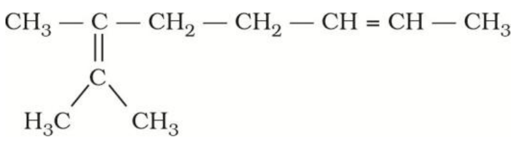
SECTION – 2

71.(200)

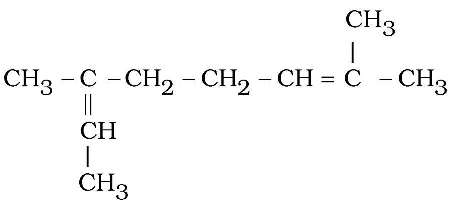
$$\Delta_f H^\circ = \Delta_{sub} H^\circ + \Delta_i H^\circ + \frac{1}{2} B \cdot E \text{ of } X_2 + \Delta_{eg} H^\circ + \Delta_{Lattice} H^\circ$$

$$\text{B.E of } X_2 = 2\{800 + 300 - 400 - 500 - 100\} = 200\,\text{kJ mol}^{-1}$$

72.(27) The compound X can be:

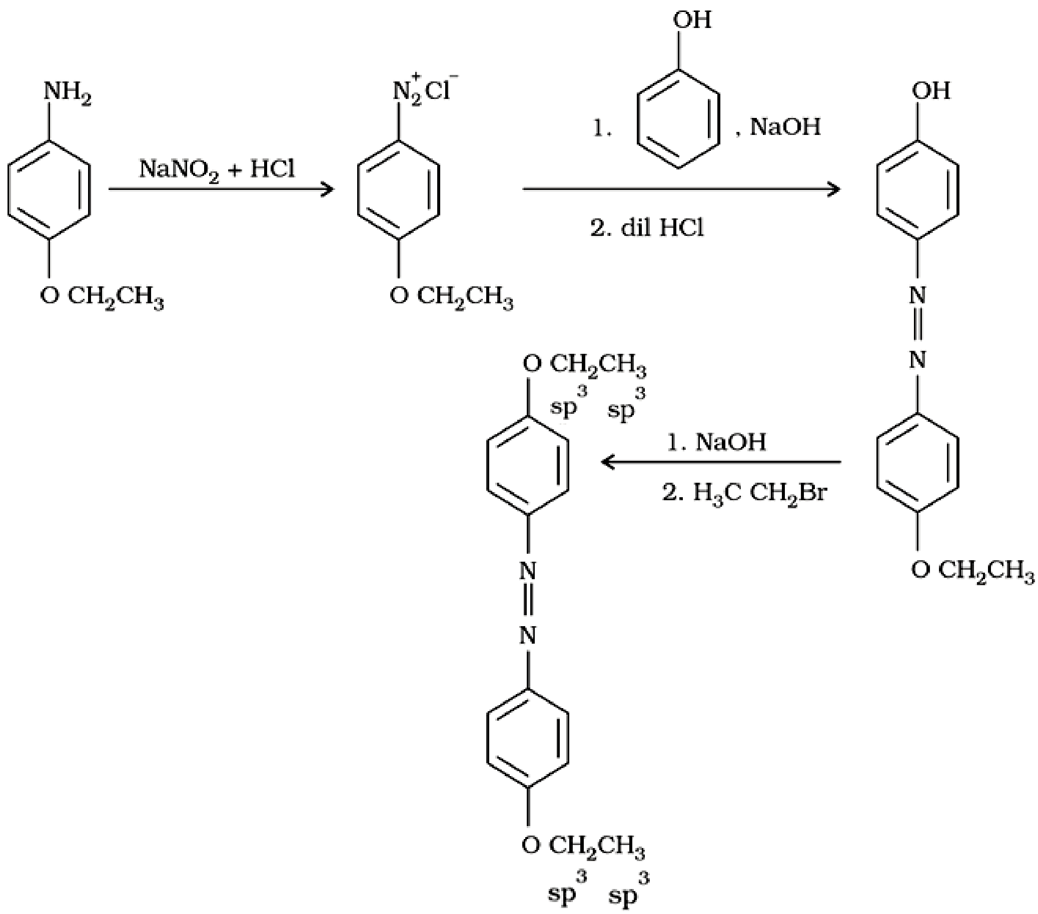


OR



Total number of σ bonds in any of the X is 27.

73.(4)



74.(153)

$$4Al + 3O_2 \rightarrow 2Al_2O_3$$

$$n_{Al} = \frac{81}{27} = 3\,\text{moles}$$

$$n_{O_2} = \frac{128}{32} = 4\,\text{moles}$$

Limiting reagent is Al.

$$n_{Al_2O_3} = \frac{2}{4} \times n_{Al}$$

$$n_{Al_2O_3} = \frac{1}{2} \times 3 = \frac{3}{2}$$

$$m_{Al_2O_3} = \frac{3}{2} \times 102 = 153\,g.$$

75.(100)

$$\text{Organic compound} \xrightarrow{\text{Combustion}} \underset{0.9\,g}{H_2O}$$

$$n_{H_2O} = \frac{0.9}{18} = \frac{1}{20}$$

$$n_H \text{ in } H_2O = \frac{1}{20} \times 2 = 0.1$$

n_H in organic compound = 0.1

$m_H = 0.1g$ in 0.01 mole of X

Wt. of H atom in 1 mole compound $= \frac{0.1}{0.01} = 10\,g.$

$$10 = \frac{10}{M} \times 100 \Rightarrow M = 100\,g$$