SOLUTIONS

Joint Entrance Exam | IITJEE-2025

22nd JANUARY 2025 | Morning Shift

MATHEMATICS

SECTION – 1

1.(2) $z_1 = e^{-i\frac{\pi}{4}},\ z_2 = 1,\ z_3 = e^{i\frac{\pi}{4}}$

$$|z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1|^2 = \left|e^{-i\pi/4} + e^{-i\pi/4} + e^{i\pi/4} \times e^{i\pi/4}\right|^2 = \left|2\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right) + i\right|^2$$

$$= \left|\sqrt{2} + (1-\sqrt{2})i\right|^2 = 2 + (1-\sqrt{2})^2 = 5 - 2\sqrt{2}$$

$\alpha = 5,\ \beta = -2,\ \alpha^2 + \beta^2 = 29$

2.(3) $f(0) = 1,\ f(x+y) = f(x)f'(y) + f'(x)f(y)\ \forall\, x, y \in R$

Let $x = y = 0$

$$f(0) = f(0)f'(0) + f'(0)f(0) \quad \Rightarrow \quad 2f'(0) = 1 \Rightarrow f'(0) = \frac{1}{2}$$

Let $\quad y = 0 \Rightarrow f(x) = f'(x) + \frac{1}{2}f(x)$

$$\Rightarrow \quad f'(x) = \frac{1}{2}f(x) \Rightarrow f(x) = e^{x/2}$$

$$\sum_{n=1}^{100} \ln f(n) = \ln \prod_{n=1}^{100} f(n) = \ln\left(e^{\frac{1+2+\ldots+100}{2}}\right) = \frac{100 \cdot 101}{4} = 2525$$

3.(1) $e^{5(\log x)^2 + 3} = x^8,\ x > 0$

$5(\ln x)^2 + 3 = 8\ln(x)$

$5(\ln x)^2 - 8\ln(x) + 3 = 0$

$5(\ln x)^2 - 5\ln x - 3\ln x + 3 = 0$

$5\ln x(\ln x - 1) - 3(\ln x - 1) = 0$

$\ln x = \frac{3}{5}, 1;\ \ x = e^{3/5}, e^1$

$\prod x = e^{8/5}$

4.(3) $L_1 = \lambda \Rightarrow A(2\lambda+1,\ 3\lambda+2,\ 4\lambda+3)$

$L_2 = \mu \Rightarrow B(3\mu+2,\ 4\mu+4,\ 5\mu+5)$

$\overline{AB} = (3\mu - 2\lambda + 1, 4\mu - 3\lambda + 2, 5\mu - 4\lambda + 2)$

$2(3\mu - 2\lambda + 1) + 3(4\mu - 3\lambda + 2) + 4(5\mu - 4\lambda + 2) = 0$

$38\mu - 29\lambda + 16 = 0$

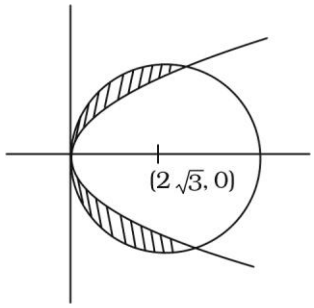
$3(3\mu - 2\lambda + 1) + 4(4\mu - 3\lambda + 2) + 5(5\mu - 4\lambda + 2) = 0$

$50\mu - 38\lambda + 21 = 0,\ \mu = \frac{-1}{6},\ \lambda = \frac{1}{3}$

$A\left(\frac{5}{3}, 3, \frac{13}{3}\right),\ B\left(\frac{3}{2}, \frac{10}{3}, \frac{25}{6}\right)$

$$\frac{x - \frac{5}{3}}{1/6} = \frac{y-3}{-1/3} = \frac{z - \frac{13}{3}}{1/6} \text{ or } \frac{x - \frac{5}{3}}{1} = \frac{y-3}{-2} = \frac{z - \frac{13}{3}}{1}$$

5.(3) $\left(x-2\sqrt{3}\right)^2+y^2=12, \quad y^2=2\sqrt{3}\,x$



Solving $\left(x-2\sqrt{3}\right)^2+2\sqrt{3}x=12$

$x^2-2\sqrt{3}x+12=12$

$x=0,\ 2\sqrt{3}$

Required area $=2\int_0^{2\sqrt{3}}\left(\sqrt{12-(x-2\sqrt{3})^2}-\sqrt{2\sqrt{3}x}\right)dx$

$=2\left[\frac{\pi}{4}\cdot 12\right]-\frac{4}{3}\sqrt{2\sqrt{3}}\left[x^{3/2}\right]_0^{2\sqrt{3}}=6\pi-\frac{4}{3}(2\sqrt{3})^2=6\pi-16$

6.(4) $y=16\left((\sec^{-1}x)^2+(\operatorname{cosec}^{-1}x)^2\right)$

Let $\sec^{-1}x=t$

$y=16\left(t^2+\left(\frac{\pi}{2}-t\right)^2\right)=16\left(2t^2-\pi t+\frac{\pi^2}{4}\right)=32\left(t^2-\frac{\pi}{2}t+\frac{\pi^2}{8}\right)=32\left(\left(t-\frac{\pi}{4}\right)^2+\frac{\pi^2}{16}\right)\in\left[2\pi^2,\ 20\pi^2\right]$

7.(3) $a_1a_5=28;\ a\cdot ar^4=28$

$a^2r^4=28\Rightarrow ar^2=\sqrt{28}=2\sqrt{7}$

$a_2+a_4=29\Rightarrow ar+ar^3=29$

$\Rightarrow\quad ar(1+r^2)=29\quad\Rightarrow\quad\frac{ar(1+r^2)}{ar^2}=\frac{29}{2\sqrt{7}}$

$\Rightarrow\quad 2\sqrt{7}r^2-29r+2\sqrt{7}=0$

$\Rightarrow\quad 2\sqrt{7}r^2-28r-r+2\sqrt{7}=0$

$\Rightarrow\quad 2\sqrt{7}r(r-2\sqrt{7})-(r-2\sqrt{7})=0$

$\Rightarrow\quad r=2\sqrt{7},\frac{1}{2\sqrt{7}}$

Increasing G.P. $\Rightarrow r=2\sqrt{7}$ & $a=\frac{1}{2\sqrt{7}}$

$a_6=ar^5=(2\sqrt{7})^4=784$

8.(4) $A=\{1,2,\ldots,10\}$

$B=\left\{\frac{m}{n}: m,n\in A, m<n, gcd(m,n)=1\right\}$

$=\left\{\begin{array}{l}\frac{1}{2},\frac{1}{3},\ldots,\frac{1}{10},\frac{2}{3},\frac{2}{5},\frac{2}{7},\frac{2}{9},\frac{3}{4},\frac{3}{5},\frac{3}{7},\frac{3}{8},\frac{3}{10},\\ \frac{4}{5},\frac{4}{7},\frac{4}{9},\frac{5}{6},\frac{5}{7},\frac{5}{8},\frac{5}{9},\frac{6}{7},\frac{7}{8},\frac{7}{9},\frac{7}{10},\frac{8}{9},\frac{9}{10}\end{array}\right\}$

$|B|=9+4+5+3+4+1+4+1=31$

9.(4)

	X
HHH	0
HHT	1
HTH	1
THH	0
HTT	1
THT	1
TTH	0
TTT	0

X :	0	1
$P(X) =$	$\frac{1}{2}$	$\frac{1}{2}$

$$\mu = \Sigma p_i x_i = \frac{1}{2}$$

$$\sigma^2 = \Sigma p_i x_i^2 - \left(\Sigma p_i x_i\right)^2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$64(\mu + \sigma^2) = 64\left(\frac{1}{2} + \frac{1}{4}\right) = 48$$

10.(2) Centroid of given $\Delta = \left(\frac{1+3+2}{3}, \frac{3+1+4}{3}\right) = \left(2, \frac{8}{3}\right)$

Its image about $x + 2y = 12$ is (α, β), then

$$\frac{\alpha - 2}{1} = \frac{\beta - \frac{8}{3}}{2} = \frac{-2\left(2 + \frac{16}{3} - 2\right)}{5} = \frac{-32}{15}$$

$$\alpha = 2 - \frac{32}{15} = \frac{-2}{15}, \beta = \frac{8}{3} - \frac{64}{15} = \frac{-24}{15} = \frac{-8}{5}$$

$$15(\alpha - \beta) = 15\left(\frac{-2}{15} + \frac{8}{5}\right) = 22$$

11.(1) {1, 2, 3} × {1, 2, 3} = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}

Equivalence relations

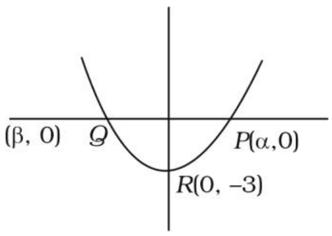
{(1, 1), (2, 2), (3, 3)} Identity relation

{(1, 1), (2, 2), (3, 3) , (a, b), (b, a)} 3 such relations

{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 2)} universal relation

12.(2) $y = x^2 + px - 3$

Centre ΔPQR of lies an $\perp$ bisector of PQ $\Rightarrow$ $x = \frac{\alpha + \beta}{2}$



$$\frac{\alpha + \beta}{2} = -1 \Rightarrow \alpha + \beta = -2 \Rightarrow p = 2$$

$$\alpha = 1, \beta = -3$$

$$ar\Delta PQR = \frac{1}{2} \times 4 \times 3 = 6$$

13.(3) $y^2dx + \left(x - \frac{1}{y}\right)dy = 0,\ x(1) = 1,\ x\left(\frac{1}{2}\right) = ?$

$$\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3} \Rightarrow IF = e^{\int \frac{1}{y^2}dy} = e^{\frac{-1}{y}}$$

$$x.e^{\frac{-1}{y}} = \int e^{\frac{-1}{y}}.\frac{1}{y^3}dy + C$$

$$-\frac{1}{y} = t \Rightarrow -\int e^t t\,dt = -\left[te^t - e^t\right]$$

$$x = \frac{1}{y} + 1 + ce^{\frac{1}{y}}$$

$$c = -\frac{1}{e}$$

$$x = \frac{1}{y} + 1 - e^{\frac{1}{y}-1}$$

$$x\left(\frac{1}{2}\right) = 2 + 1 - e = 3 - e$$

14.(4)

4W 6B

$$P(1-B\,|\,2=B) = \frac{m}{n} \Rightarrow \frac{P\left(2=B|\,1=B\right)\cdot P(1=B)}{P\left(2=B\right)}$$

$$\Rightarrow \frac{\frac{5}{9}\times\frac{6}{10}}{\frac{5}{9}\times\frac{6}{10}+\frac{4}{10}\times\frac{6}{9}} = \frac{30}{30+24} = \frac{30}{54} = \frac{5}{9}$$

15.(3) $S(1,14),\ S'(1,-12),\ A(1,6)$

Centre $\equiv (1,1) \Rightarrow ae = 13,\ a = 5$

$$b^2 = 169 - 25 = 144$$

$$\frac{2b^2}{a} = \frac{2.144}{5} = \frac{288}{5}$$

16.(3) $f(x) = 7\tan^6 x\sec^2 x - 3\tan^2 x\sec^2 x$

$$I_1 = \int_0^{\pi/4} f(x)dx = \int_0^1 (7t^6 - 3t^2)\,dt = 1 - 1 = 0$$

$$I_2 = \int_0^{\pi/4} x(7\tan^6 x - 3\tan^2 x)\sec^2 x\,dx$$

$$\left|x\left(\tan^7 x - \tan^3 x\right)\right|_0^{\pi/4} - \int_0^{\pi/4}\left(\tan^7 x - \tan^3 x\right)dx = \int_0^{\pi/4}\left(\tan^3 x - \tan^7 x\right)dx$$

$$= \int_0^{\pi/4}\tan^3 x\left(1 - \tan^2 x\right)\sec^2 x dx = \int_0^1 t^3\left(1 - t^2\right)dt = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$$

17.(4) $S_n = \frac{(2n-1)(2n+1)(2n+3)(2n+5)}{64}$

$T_n = S_n - S_{n-1} = \frac{1}{64}\left[\begin{matrix}(2n-1)(2n+1)(2n+3)(2n+5) \\ -(2n-3)(2n-1)(2n+1)(2n+3)\end{matrix}\right]$

$= \frac{1}{64}(2n-1)(2n+1)(2n+3)\ (8) \ = \frac{(2n-1)(2n+1)(2n+3)}{8}$

$8\sum_{1}^{\infty}\frac{1}{(2n-1)(2n+1)(2n+3)} = 8\left(\frac{1}{1.3.5}+\frac{1}{3.5.7}+...\right) = 2\left(\frac{1}{1.3}\right) = \frac{2}{3}$

18.(1) Middle is M, 2 Letters before and after M

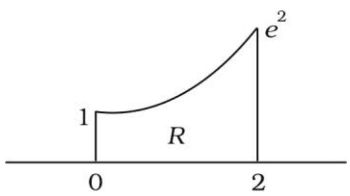
M is 13th letter $\Rightarrow$ 2 letters from first 12 (A to L) & 2 from last 13 (N to Z)

$\Rightarrow \quad {}^{12}C_2 \times {}^{13}C_2 \quad \Rightarrow \frac{12.11}{2}\times\frac{13.12}{2} \Rightarrow 66\times 78 = 5148$

19.(2) $f(x+y) = f(x).f(y)\ \forall x, y \in R$

$f'(0) = 4a \qquad f''(x) - 3af'(x) - f(x) = 0,\ a > 0$

$R = \{(x,y) \mid 0 \le y \le f(ax), 0 \le x \le 2\}$



$f(x+y) = f(x).(y) \Rightarrow f(x) = k^x$

$f'(0) = k^0, \ln k = 4a \Rightarrow k = e^{4a}$

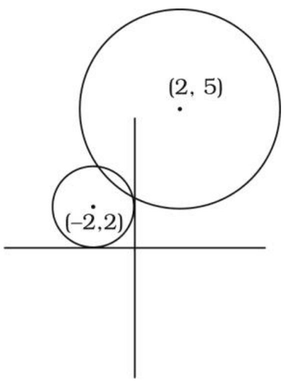
$f(x) = e^{4ax}$

$16a^2 - 12a^2 - 1 = 0 \Rightarrow a = \frac{1}{2} \quad \Rightarrow \quad f(x) = e^{2x}$

$R = \left\{(x,y) : 0 \le y \le e^x, 0 \le x \le 2\right\}$

$\int_0^2 e^x dx = e^2 - 1$

20.(4)



$|r_1 - r_2| < d < r_1 + r_2$

$|r - 2| < \sqrt{16+9} < r + 2$

$|r - 2| < 5 < r + 2$

$3 < r < 7 \Rightarrow \alpha = 3, \beta = 7$

$3\beta - 2\alpha = 21 - 6 = 15$

SECTION – 2

21.(216)

For intersection point $y = 0 \Rightarrow x = 4;\ z = -1$

$\Rightarrow\ \alpha = -1 + 4 = 3$

Let $P(2\lambda + 2, 0, 3\lambda - 4)$

$\overrightarrow{AP} = (2\lambda + 1, -1, 3\lambda - 3)\ = (2\lambda + 1) + 0(-1) + 3(3\lambda - 3) = 0$

$13\lambda - 7 = 0\ \Rightarrow \lambda = \dfrac{7}{13}$

$P\left(\dfrac{40}{13}, 0, -\dfrac{31}{13}\right)$

$$26\alpha PB^2 = 26 \times 3 \times \left(\left(\frac{40}{13} - 4\right)^2 + 0^2 + \left(\frac{31}{13} - 1\right)^2\right) = 26.3\frac{144 + 324}{169} = 216$$

22.(34) $f(x) = \begin{cases} -3ax^2 - 2, & x < 1 \\ a^2 + bx, & x \ge 1 \end{cases}$

LHL = RHL $\Rightarrow\ -3a - 2 = a^2 + b$ ...(1)

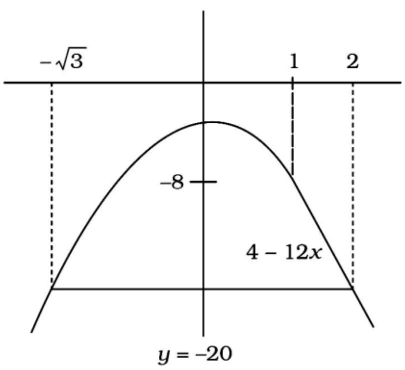
LHD = $RHD\ \Rightarrow -6a = b$ ...(2)

$a^2 - 3a + 2 = 0$

$a = 1, 2\ \Rightarrow\ a = 2\ (a > 1)$

$b = -12$

$f = \begin{cases} -6x^2 - 2 & x < 1 \\ 4 - 12x & x \ge 1 \end{cases}$

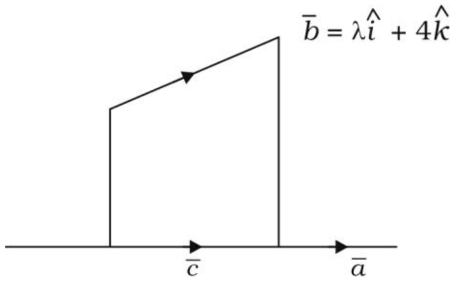


$$= \int_{-\sqrt{3}}^{1} \left(-6x^2 - 2 + 20\right) dx + \frac{1}{2} \times 1 \times 12$$

$$= \left|-2x^3 + 18x\right|_{-\sqrt{3}}^{1} + 6$$

$$= 16 - (6\sqrt{3} - 18\sqrt{3}) + 6 = 22 + 12\sqrt{3}$$

23.(16) $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$



$$\vec{c} = \frac{\lambda+8}{1+4+4}(\hat{i} + 2\hat{j} + 2\hat{k})$$

$$(\bar{a}+\bar{c}) = 7 \Rightarrow \left(\frac{\lambda+8}{9}+1\right)^2 + \left(\frac{\lambda+8}{9}\cdot 2+2\right)^2 + \left(\frac{\lambda+8}{9}\cdot 2+2\right)^2 = 49$$

$$(\lambda+17)^2 + (2\lambda+16+18)^2 + (2\lambda+16+18)^2 = 49\times 81$$

$$(\lambda+17)^2 + (2\lambda+34)^2 + (2\lambda+34)^2 = 49\times 81$$

$$\lambda+17 = \pm 7\times 3 = \pm 21$$

$$\lambda = 21-17, -21-17$$

$$= 4, -38 \Rightarrow \lambda = 4 \quad (\lambda > 0)$$

$$\bar{c} = \frac{4}{3}(\hat{i} + 2\hat{j} + 2\hat{k})$$

$$\bar{b} = 4\hat{i} + 4\hat{k}$$

$$\vec{b}\times\bar{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & 4 \\ \frac{4}{3} & \frac{8}{3} & \frac{8}{3} \end{vmatrix} = \frac{16}{3}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 1 & 2 & 2 \end{vmatrix} = \frac{16}{3}\left(-2\hat{i} - \hat{j} + 2\hat{k}\right)$$

$$\text{area} = |\bar{b}\times\bar{c}| = 16$$

24.(2035)

$$\sum_{r=0}^{5} \frac{{}^{11}C_{2r+1}}{2r+2} = \frac{m}{n}$$

$$\Rightarrow \sum_{r=0}^{5} \frac{2r+2}{12}\,\frac{{}^{12}C_{2r+2}}{2r+2}$$

$$= \frac{1}{12}\sum_{r=0}^{5} {}^{12}C_{2r+2} = \frac{1}{12}\left({}^{12}C_2 + {}^{12}C_4 + {}^{12}C_6 + {}^{12}C_8 + {}^{12}C_{10} + {}^{12}C_{12}\right) = \frac{1}{12}\left(2^{11}-1\right) = \frac{2047}{12} = \frac{m}{n}$$

$$m - n = 2047 - 12 = 2035$$

25.(34) $A_{3\times 3}, |A| = -2$

$$|3adj(-6adj(3A))| = 2^{m+n}.3^{mn}, m > n$$

$$3^3\ |(-6)^2\ adj(3^2 adjA)|$$

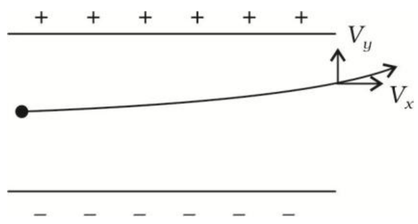
$$3^3(-6)^6((3^2)^2)^3\ |adj(adjA)|$$

$$3^3\cdot 6^6\cdot 3^{12}\ |A|^4 = 3^3\cdot 2^6\cdot 3^6.3^{12}\cdot 2^4$$

$$\Rightarrow \quad 2^{10}\ 3^{21} \Rightarrow m = 7, n = 3$$

PHYSICS

SECTION – 1



26.(1) $t = \frac{S_x}{V_x} = \frac{0.1}{10^6} = 10^{-7} s$

$$V_y = a_y t = \frac{qE}{m} t = \frac{1.6 \times 10^{-19} \times 910}{9.1 \times 10^{-31}} \times 10^{-7} = 16 \times 10^6 \, m/s$$

27.(1) It is a combination of lens and mirror.

$$\frac{1}{f_{eq}} = \frac{1}{f_m} - \frac{2}{f_l} = \frac{2}{-R} - 2(\mu - 1)\left(\frac{2}{R}\right) = \frac{-4\mu + 2}{R}$$

$\therefore \qquad f_{eq} = \frac{-R}{2(2\mu - 1)}$

$\therefore \qquad x = 2\left|f_{eq}\right| = \frac{R}{2\mu - 1}$

28.(1) Mass of removed portion $= \frac{M}{4}$

$$I = \left(\frac{MR^2}{2}\right) - \left[\frac{\left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2}{2} + \left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2\right] = \frac{MR^2}{2} - \frac{3}{32} MR^2 = \frac{13}{32} MR^2$$

29.(4) Total heat required $= mS_i(10) + mL_f + mS_w(100) + mL_v + mS_s(10)$

$= 21 + 335 + 418 + 2250 + 19.2 = 3043$

30.(2) If we imagine 3 more cubes, the edge will be shared by 4 cubes

$\therefore \qquad Q_{enclosed} = \frac{\lambda\left(\frac{a}{2}\right)}{4} = \frac{\lambda a}{8}$

$\therefore \qquad \phi = \frac{Q_{enc}}{\epsilon_0} = \frac{\lambda a}{8 \epsilon_0}$

31.(1) $F_1 = \frac{GMm}{(2R)^2} = \frac{GMm}{4R^2}$

Mass of removed portion $= \frac{M}{27}$

Force due to removed portion

$$F = \frac{G\left(\frac{M}{27}\right)m}{\left(\frac{4R}{3}\right)^2} = \frac{GMm}{48R^2}$$

$$F_2 = F_1 - F = \frac{11}{48}\frac{GMm}{R^2}$$

$$\frac{F_1}{F_2} = \frac{12}{11}$$

32.(3) $V_A = \sqrt{5gl}$

Energy conservation between A & B :

$$\frac{1}{2}m(5gl) = K_B + mg\frac{l}{2} \quad \Rightarrow \quad K_B = 2mgl$$

Energy conservation between A and C:

$$\frac{1}{2}m(5gl) = K_C + mg\left(\frac{3l}{2}\right) \quad \Rightarrow \quad K_C = mgl$$

$$\therefore \quad \frac{K_B}{K_C} = 2$$

33.(1) Let length of closed organ pipe be l_1.

Its fundamental frequency, $f_1 = \frac{1}{4l_1}\sqrt{\frac{B_1}{\rho_1}}$

Let length of open organ pipe be l_2 Its fundamental frequency $f_2 = \frac{1}{2l_2}\sqrt{\frac{B_2}{\rho_2}}$

Given $9f_1 = 4f_2$

$$9\frac{1}{4l_1}\sqrt{\frac{B_1}{\rho_1}} = 4\frac{1}{2l_2}\sqrt{\frac{B_2}{\rho_2}}$$

$$\frac{l_2}{l_1} = \frac{8}{9}\sqrt{\frac{\rho_1}{\rho_2}\frac{B_2}{B_1}} = \frac{8}{9}\times\frac{1}{4} = \frac{2}{9}$$

$$l_2 = \frac{20}{9}\,cm$$

34.(3) $P = e\sigma AT^4$

$$\frac{P_2}{P_1} = \frac{A_2}{A_1}\left(\frac{T_2}{T_1}\right)^4 = \left(\frac{r_2}{r_1}\right)^2\left(\frac{T_2}{T_1}\right)^4 = (4)^2\left(\frac{1}{2}\right)^4 = 1$$

$$\Rightarrow \quad P_2 = P_1 = E$$

35.(2) For forward biased, voltage of p side should be greater than voltage of n side.

36.(4) $E_{eq} = \dfrac{\frac{E_1}{r_1} + \frac{E_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}$

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

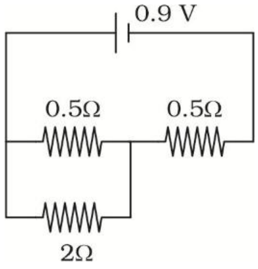
37.(4) Magnetic field of a solenoid is

$$B = \mu_0 nI$$

$$\frac{B}{\mu_0} = nI$$

$$\therefore \quad \left[\frac{B}{\mu_0}\right] = [nI] = L^{-1}A$$

38.(1) Circuit can be redrawn as,



$$R_{eq} = \frac{2 \times 0.5}{2.5} + 0.5 = 0.9\Omega$$

$$\therefore \qquad I = \frac{0.9}{0.9} = 1A$$

39.(4) $\lambda = \frac{h}{mV}$

As $V \propto \frac{1}{n} \Rightarrow \lambda \propto n$

$$\frac{\lambda_1}{\lambda_4} = \frac{1}{4}$$

40.(1) Figure width $= \frac{\lambda D}{d}$

As $c = \lambda f$

In denser medium, c will be less and hence λ is less

$\therefore$ Fringe width reduces

41.(4) $E_{photon} = \frac{1240}{550} = 2.25\,eV$

42.(4) Vernier constant or least count of vernier calliper $= 1MSD - 1VSD$

43.(4) Initial charge $Q_1 = CV$

Final charge $Q_2 = KCV$

Extra charge $= Q_2 - Q_1 = (K-1)CV = CV = 4mC$

$$U_1 = \frac{1}{2}CV^2$$

$$U_2 = \frac{1}{2}KCV^2$$

$$\Delta U = U_2 - U_1 = \frac{1}{2}(K-1)CV^2 = \frac{1}{2}CV^2 = 0.2\ J$$

44.(3) $\frac{1}{f_1} = \left(\frac{4}{3} - 1\right)\left(0 - \frac{1}{-|R_1|}\right) = \frac{1}{3|R_1|}$

$$\frac{1}{f_2} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{-|R_1|} - \frac{1}{-|R_2|}\right) = \frac{1}{2}\left(\frac{1}{|R_2|} - \frac{1}{|R_1|}\right)$$

$$\frac{1}{f_3} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{-|R_2|} - 0\right) = -\frac{1}{3|R_2|}$$

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} = \frac{1}{3|R_1|} + \frac{1}{2|R_2|} - \frac{1}{2|R_1|} - \frac{1}{3|R_2|} = \frac{1}{6|R_2|} - \frac{1}{6|R_1|}$$

$$P = \frac{1}{f_{eq}} = -\frac{1}{6}\left(\frac{1}{|R_1|} - \frac{1}{|R_2|}\right)$$

45.(2) Variation of resistivity with temperature should be small.

SECTION – 2

46.(5) $T=\frac{2u\sin\theta}{g}=\frac{2(60)(0.5)}{10}=6s$

$h_0=u\sin\theta t-\frac{1}{2}gt^2=30-5=25$

Vertical velocity at $t=2$

$V_y=30-(10)(2)=10$

$h_1=10(1)-\frac{1}{2}(10)(1)=5$

47.(8) Speed of car $=90\,km/hr=25\,m/s$

Applying mirror formula

$\frac{1}{v}+\frac{1}{-24}=\frac{1}{1}\Rightarrow\frac{1}{v}=1+\frac{1}{24}=\frac{25}{24}\quad\Rightarrow\qquad V=\frac{24}{25}m/s$

$V_i=\frac{-v^2}{u^2}V_0=-\left(\frac{1}{25}\right)^2(25)=\frac{-1}{25}m/s$

$$a_i=\frac{dV_i}{dt}=-V_0\left[\frac{u^2\frac{d}{dt}(v^2)-v^2\frac{d}{dt}(u^2)}{u^4}\right]=-V_0\left[\frac{2v}{u^2}V_i-\frac{2v^2}{u^3}V_0\right]=-V_0\left[\frac{2v}{u^2}\left(\frac{-v^2}{u^2}V_0\right)-\frac{2v^2}{u^3}V_0\right]$$

$$=2V_0^2\frac{v^2}{u^3}\left(\frac{v}{u}+1\right)=2(25)^2\frac{\left(\frac{24}{25}\right)^2}{(-24)^3}\left(\frac{\left(\frac{24}{25}\right)}{-24}+1\right)=2\left(\frac{-1}{24}\right)\left(\frac{24}{25}\right)=\frac{-2}{25}=-0.08\,m/s^2$$

48.(90) $\vec{V}_A=\frac{d\vec{r}_A}{dt}=2t\hat{i}+3n\hat{j}+2\hat{k}$

$\vec{V}_B=\frac{d\vec{r}_B}{dt}=2\hat{i}-2t\hat{j}+4p\hat{k}$

At $t=1$, $\vec{V}_A=2\hat{i}+3n\hat{j}+2\hat{k}$ and $\vec{V}_B=2\hat{i}-2\hat{j}+4p\hat{k}$

$\vec{V}_A\cdot\vec{V}_B=0\;\Rightarrow\;4-6n+8p=0\;\Rightarrow\;3n-4p=2\quad\ldots\text{(i)}$

$|\vec{V}_A|=|\vec{V}_B|\Rightarrow4+9n^2+4=4+4+16p^2$

$\Rightarrow\qquad 3n=\pm4p\qquad\ldots\text{(ii)}$

From (i) and (ii)

$6n=2\;\Rightarrow\;n=\frac{1}{3}$

$p=\frac{-3n}{4}=-\frac{1}{4}$

Angular momentum of particle A w.r.t. position of particle B is

$$\vec{L} = \vec{r}_{AB} \times m_A \vec{V}_A$$

$$\vec{V}_A = 2\hat{i} + \hat{j} + 2\hat{k}$$

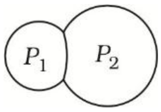
$$\vec{r}_{AB} = \vec{r}_A - \vec{r}_B = \left(1\hat{i} + 1\hat{j} + 2\hat{k}\right) - \left(2\hat{i} - \hat{j} - \hat{k}\right) = -\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\therefore \quad \vec{L} = (-\hat{i} + 2\hat{j} + 3\hat{k}) \times (2\hat{i} + \hat{j} + 2\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 3 \\ 2 & 1 & 2 \end{vmatrix} = \hat{i} + 8\hat{j} - 5\hat{k}$$

$$\left|\vec{L}\right| = \sqrt{1^2 + 8^2 + 5^2} = \sqrt{90}$$

49.(4) $P_1 = P_0 + \dfrac{4\sigma}{r_1}$

$$P_2 = P_0 + \frac{4\sigma}{r_2}$$



$$P_1 - P_2 = \frac{4\sigma}{r}$$

$$\Rightarrow \quad \frac{1}{r_1} - \frac{1}{r_2} = \frac{1}{r}$$

$$\frac{1}{2} - \frac{1}{4} = \frac{1}{r} \Rightarrow r = 4\,cm$$

50.(40) $H_1 + H_2 = H_3$

$$\frac{k_1 A(100 - \theta)}{l} + \frac{k_2 A(100 - \theta)}{l} = \frac{k_3 2A(\theta - 0)}{l}$$

$$60(100 - \theta) + 120(100 - \theta) = 270\theta$$

$$18000 = 450\theta \Rightarrow \theta = 40$$

CHEMISTRY

SECTION – 1

51.(4) Aliphatic aldehydes and α -hydroxy ketone gives positive test for Fehling's Test. Compound (C), (D) and (E) gives positive Fehling test.

52.(4) Halide that form stable carbocation readily undergoes S_N1 reaction.

$$CH_3-O-CH_2-Cl \xrightarrow[-Cl^{\ominus}]{} CH_3-\ddot{O}-\overset{\oplus}{C}H_2 \leftrightarrow CH_3-\overset{\oplus}{O}=CH_2$$ [Resonance stabilized carbocation]

Sterically hindered halide is not suitable substrate for S_N2 reaction. Neopentyl substrate $\left((CH_3)_3CCH_2Cl\right)$ is not good substrate for S_N2 reaction.

53.(4) $CO_2(g)+C(s) \rightleftharpoons 2CO(g)$

0.5 atm	x mol	0
(0.5 – p) atm	(x – a)mol	2p atm

Total equilibrium pressure = 0.5 + p = 0.8

$\Rightarrow \quad p = 0.3$

$$K_p = \frac{(0.3\times 2)^2}{0.2} = 1.8\,\text{atm}$$

54.(4) $Al^{3+} + 3e^- \rightarrow Al$

$$\text{Eq. of Al} = \frac{W}{E} = \frac{I\times t}{96500}$$

$$\frac{W}{27}\times 3 = \frac{2\times 30\times 60}{96500}$$

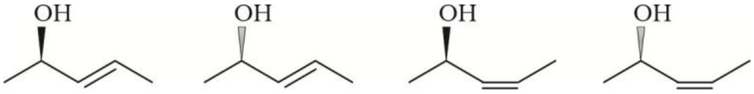
W = 0.3357 g

55.(3) $\underset{1\,\text{mole}}{CH_3-C\equiv C-H} + \underset{\text{Excess}}{Na} \rightarrow CH_3-C\equiv CNa + \underset{1/2\,\text{mole}}{1/2H_2}$

$\underset{1\,\text{mole}}{CH_3-C\equiv C-H} + NaNH_2 \rightarrow CH_3-C\equiv CNa + \underset{22400\,\text{mL}}{NH_3}$

$\frac{4}{40}$ mole $\qquad\qquad \frac{4}{40}\times 22400 = 2240\,\text{mL}$

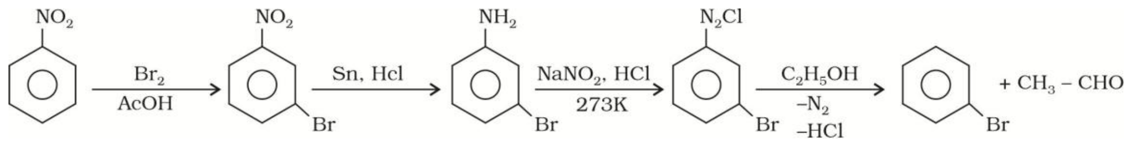
56.(2) Radio active decay is nuclear reaction and it is independent of external factor pressure and temperature. Decay constant is independent of temperature.

57.(3) 

58.(1)
- **A.** $Al^{3+} < Mg^{2+} < Na^+ < F^- \Rightarrow$ Order of ionic radii ; (A) – IV
- **B.** $B < C < O < N \Rightarrow$ order of ionization enthalpy ; (B) – I
- **C.** $B < Al < Mg < K \Rightarrow$ order of metallic character ; (C) – II
- **D.** $Si < P < S < Cl \Rightarrow$ Order of electronegativity ; (D) – III

59.(3) In case of octahedral complex central atom having configuration $t_{2g}^3\,e_g^2$ will have CFSE, Δ_0 equal to zero.

$\Delta_0 = +0.4\times 3 - 0.6\times 2 = 0$

60.(3) 

61.(4) Electronegativity of Al is more than electronegativity of Mg.

62.(1) Ascorbic acid is a vitamin (vit C).

63.(2) Principal functional group is –COOH (-oic acid) and principal chain possess six carbon atoms hence its IUPAC name is 6- Methoxycarbonyl-2,5-dimethylhexanoic acid.

64.(3) For thermally insulated closed vessel q = 0, liquid is stirred mechanically hence w > 0.

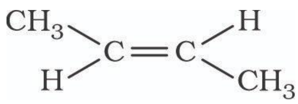
$\Delta U = q + w$; $q = 0$; $\Delta U > 0$; $w > 0$

65.(1) $r = \frac{a_0 n^2}{z}$; For He^+ z = 2 and n = 2 (first excited state)

$= \frac{a_0 \times 2^2}{2} = 2a_0$

66.(4) **A.** Propene does not show geometrical isomerism due to absence of stereogenic bond.

B. In trans isomer similar groups are on opposite sides of double bond.



C. Dipole moment of cis-but-2-ene is non zero while dipole moment of trans-but-2-ene is zero due to symmetrical structure.

D. 2-methylbut-2-ene does not shows geometrical isomerism.

E. Trans isomer has higher melting point than cis isomer due to formation of more close packed arrangement.

Statements (A), (D) and (E) are incorrect. Statements (B) and (C) are correct.

67.(4) $\Delta T_b = i \times k_b \times m$; For dilute solution $M \approx m$.

For NaCl i = 2 and for Urea i = 1

Boiling point $\propto i \times m$

$10^{-2} M\,NaCl > 10^{-3} M\,NaCl > 10^{-4} M\,NaCl > 10^{-4} M\,Urea$

68.(4) $\underset{\text{Conc.}}{2H_2SO_4} \xrightarrow{\text{Electrolysis}} H_2 + H_2S_2O_8$

69.(4) $[NiCl_4]^{2-}$ Tetrahedral (Cl^- is weak field ligand): oxidation state of Ni is +2.

$[Ni(CO)_4]$ Tetrahedral (CO is strong field ligand): oxidation state of Ni is zero.

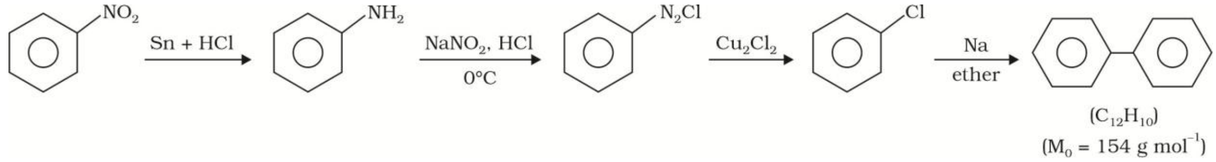
70.(2) $_{63}Eu \Rightarrow 4f^7 6s^2$ and $Eu^{2+} \Rightarrow 4f^7$; $_{64}Gd \Rightarrow 4f^7 5d^1 6s^2$ and $Gd^{3+} \Rightarrow 4f^7$

$Eu^{3+} \Rightarrow 4f^6$

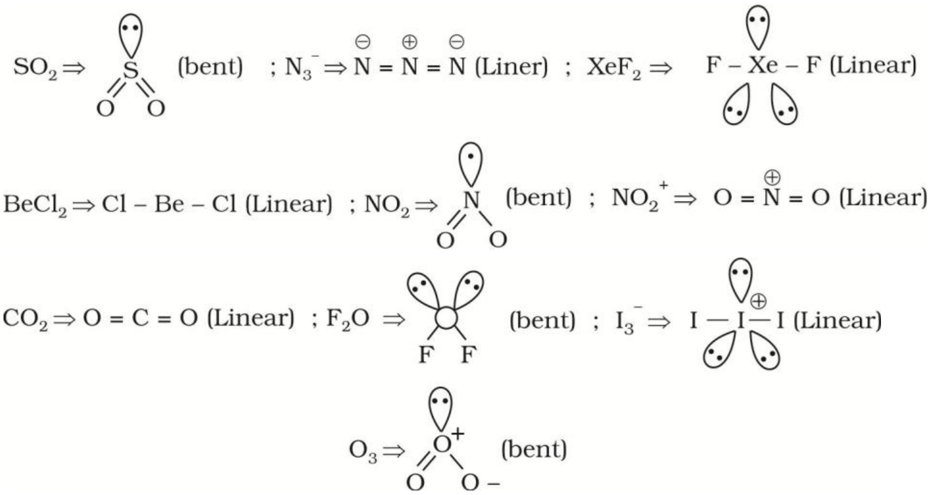
$_{65}Tb \Rightarrow 4f^9 6s^2$ and $Tb^{3+} \Rightarrow 4f^8$

$_{65}Sm \Rightarrow 4f^6 6s^2$ and $Sm^{2+} \Rightarrow 4f^6$

71.(154)



72.(6)



73.(45) Let volume of CO_2 be V mL.

$Ca(OH)_2 + CO_2 \rightarrow CaCO_3 + H_2O$

$Ca(OH)_2 + 2HCl \rightarrow CaCl_2 + H_2O$

Mole of $Ca(OH)_2$ taken = x

Mole of $Ca(OH)_2$ unreached $= \frac{1}{2} \times$ mole of HCl used $= \frac{1}{2} \times 0.1 \times \frac{40}{100} = \frac{x}{2}$

Mole of $Ca(OH)_2$ reacted with $CO_2 = \frac{x}{2}$; mole of $CO_2 = \frac{x}{2}$

$$x = \frac{1}{2} \times 0.1 \times \frac{40}{1000} \times 2 = 0.004$$

$$V_{CO_2} = \frac{n_{CO_2} RT}{P} = \frac{0.004 \times 0.0821 \times 273}{2 \times 1} = 0.0448 L$$

$= 44.82 \text{ mL} = 45 \text{ mL}$

74.(20) Mole of Cl in organic compound = Mole of AgCl

$$\frac{W}{35.5} = \frac{143.5}{143.5} \times \frac{1}{1000}$$

$$\% \text{ of Cl} = \frac{143.5}{143.5} \times \frac{35.5}{1000} \times \frac{1000}{180} \times 100 = 19.72 \approx 20$$

75.(69) $\log K = \log A - \frac{E_a}{2.303RT}$

$$\Rightarrow \quad \log K = \log 10^{20} - \frac{191.48 \times 1000}{2.303 \times 8.314 \times 1000}$$

$$\Rightarrow \quad \log K = 20 - 10 \quad \Rightarrow \quad K = 10^{10}$$

$$t_{50\%} = \frac{0.693}{K} = \frac{0.693}{10^{10}} \text{sec}$$

$$= \frac{0.693}{10^{10}} \times \frac{1}{10^{-12}}$$

= 69.3 picoseconds = 69 picoseconds