VMC Vidyamandir Classes SINCE 1986

IIT JEE | MEDICAL | FOUNDATION

SOLUTIONS

Joint Entrance Exam | IITJEE-2023

31st JAN 2023 | Evening Shift

PHYSICS

SECTION - 1

1.(2) $v = \sum \frac{1}{4\pi\varepsilon_0} \frac{q_i}{r_i}$

$\because$ All charges are positive

$\therefore$ Net potential cannot be zero at a point

$$\vec{E} = \sum \frac{1}{4\pi\varepsilon_0} \frac{q_i}{r_1^2} (\hat{r}_i)$$

$\vec{E}$ can be zero due to a group of positive charges as it is a vector quantity

2.(3) Current sensitivity, $I_g = \frac{NBA}{K}$; Voltage sensitivity, $V_g = \frac{NBA}{KR}$

As I_g is increased by 50%, N will be increased by 50%

$\therefore$ Resistance of the resistance will also be increased by 50%

New voltage sensitivity, $V_g' = \frac{N'BA}{KR'} = \frac{(1.5N)BA}{K(1.5R)} = \frac{NBA}{KR}$

Hence, V_g remains same

3.(2) Given, (i) Isobaric process

(ii) $\Delta Q = 735\,J$

(iii) Diatomic gas $\rightarrow C_V = \frac{5R}{2},\ C_P = \frac{7R}{2}$

$$\Delta Q = nC_P\Delta T = n \times \frac{7R}{2} \times \Delta T = 735\ J$$

$$\Delta U = nC_V\Delta T = n \times \frac{5R}{2} \times \Delta T = \frac{5}{7} \times 735\ J = 525J$$

4.(4) Thermal energy, $H = I^2Rt$

i.e. $H \propto I^2$ in same time interval

As I becomes 4 times

H will become 16 times

5.(2) $\frac{F}{A} = \frac{Y\Delta L}{L} \Rightarrow Y = \frac{FL}{A\Delta L}$

$$\therefore \quad \frac{Y_A}{Y_B} = \left(\frac{F_A}{F_B}\right) \times \left(\frac{L_A}{L_B}\right) \times \left(\frac{A_B}{A_A}\right) \times \left(\frac{\Delta L_B}{\Delta L_A}\right) = 1 \times \frac{5}{6} \times \frac{3}{2.5} \times 1 = 1$$

6.(3) Stone is rotating in the horizontal plane

$$\therefore \quad T = \frac{mv^2}{l} \quad \Rightarrow \quad v = \sqrt{\frac{Tl}{m}} \quad \Rightarrow \quad v_{max} = \sqrt{\frac{400 \times 1}{1}} = 20\ m/s$$

7.(2) $B_0 = \dfrac{\mu_0 NI}{2R}$; $L = N \times 2\pi R$

$\therefore \quad B_0 = \dfrac{\mu_0 NI}{2 \cdot \dfrac{L}{N \cdot 2\pi}} = \dfrac{\mu_0 N^2 I}{\pi L} \quad \therefore \quad B \propto N^2$

8.(3) Fact - based

9.(4) $PV^r = R$ (adiabatic process)

$\Rightarrow \quad \dfrac{P_2}{P_1} = \left(\dfrac{V_1}{V_2}\right)^r \quad \Rightarrow \quad \dfrac{16}{81} = \left(\dfrac{8}{27}\right)^r \quad \Rightarrow \quad r = \dfrac{4}{3}$

10.(2) $\dfrac{r_3}{r_2} = \dfrac{3^2}{2^2} = \dfrac{9}{4} = 2.25 \quad \left(\because\ r = 0.529\dfrac{n^2}{z}\text{Å}\right) \quad \Rightarrow \quad r_3 = 2.25\,R$

11.(3) $T = 4s$

At $t = \dfrac{3}{4}T = 3s,\ \Delta\theta = \dfrac{3\pi}{2} \quad \therefore \quad s = \sqrt{2}R$

12.(2) $K_{max} = \dfrac{1240}{350}eV - \phi_0 \geq 0 \quad \Rightarrow \quad \phi_0 \leq 3.5\,eV$ for photo-emission

13.(1) Fact – based

A – IV, B – III, C – I, D – II

14.(3) Apparent depth, $h' = \dfrac{h}{5/3} = \dfrac{3h}{5} \quad \therefore \quad \Delta h = \dfrac{2h}{5} = 30\,cm \quad \Rightarrow \quad h = 75\,cm$

15.(2) $W = mg = \dfrac{GMm}{(R+h)^2}$

$W' = W \times \dfrac{g'}{g} = W \times \dfrac{R^2}{(10R)^2} = \dfrac{W}{100}$

16.(2) $X_L = \omega L = 628 \times 5 \times 10^{-3}\Omega = 3.14\,\Omega$

17.(3) $V = \sqrt{\dfrac{Y}{\rho}} = \sqrt{\dfrac{3.2 \times 10^{11}}{8 \times 10^3}} = \sqrt{4 \times 10^7} = 6.32 \times 10^3\ m,/s$

18.(2) Fact - based

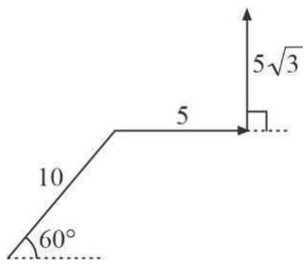
19.(2) $a = \mu g = \dfrac{20-0}{5} = 4\,m/s^2 \quad \Rightarrow \quad \mu = \dfrac{4}{10} = 0.4$

20.(2) A – III, B – I, C – IV, D – II (Fact - based)

SECTION – 2

21.(20) $A_{net} = \sqrt{(10\cos 60° + 5)^2 + (10\sin 60° + 5\sqrt{3})^2}$

$= \sqrt{10^2 + (10\sqrt{3})^2} = 20\,cm$



22.(5) $M_1 = M_2$

$$\Rightarrow \quad \rho_1 \pi R_1^2 t_1 = \rho_2 \pi R_2^2 t_2 \quad \Rightarrow \quad \left(\frac{R_1}{R_2}\right)^2 = \frac{\rho_2}{\rho_1} \times \frac{t_2}{t_1} = \frac{5}{3} \times \frac{0.5}{1} = \frac{5}{6}$$

$$\frac{I_1}{I_2} = \left(\frac{M_1}{M_2}\right)\left(\frac{R_1}{R_2}\right)^2 = \frac{5}{6} = \frac{x}{6} \quad \Rightarrow \quad x = 5$$

23.(25) $z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{80^2 + 60^2} = 100\,\Omega \quad \therefore \quad I_0 = \frac{V_0}{z} = \frac{2500}{100} = 25A$

24.(136) $E_3,\ Li^{++} = -13.6 \times \left(\frac{3}{3}\right)^2 eV = -13.6\,eV \quad \therefore \quad BE = 13.6\,eV$

25.(80) $R_\alpha = \frac{u^2 \sin 2\alpha}{g},\ R_B = \frac{u^2 \sin 2\beta}{g}$

$\because \quad R_\alpha = R_\beta \quad \therefore \quad \alpha + \beta = 90° \quad \because \quad \alpha = 60°,\ \beta = 30°$

$$\therefore \quad H_1 + H_2 = \frac{u^2}{2g}(\sin^2 60° + \sin^2 30°) = \frac{u^2}{2g} = \frac{40 \times 40}{2 \times 10} = 80$$

26.(48) Fringe width, $\beta = \frac{\lambda D}{d}$

$$\beta_1 = \frac{800 \times 10^{-9} \times 7}{35 \times 10^{-5}} = 180 \times 10^{-4} = 16\,mm$$

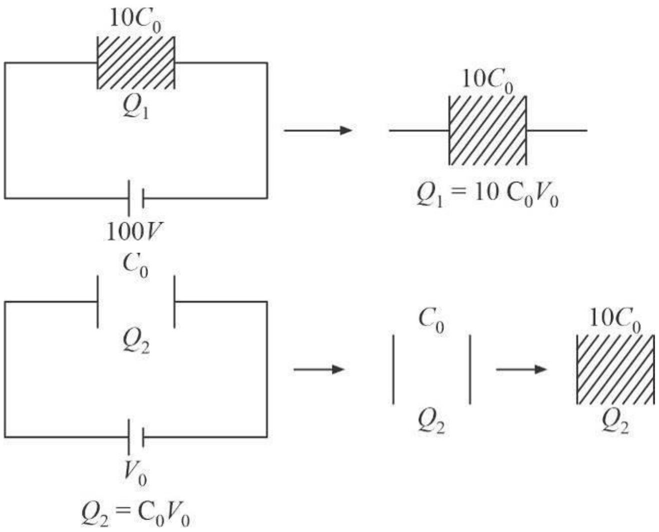
$$\beta_2 = \frac{600 \times 10^{-9} \times 7}{35 \times 10^{-9}} = 12\,mm$$

LCM (16 *mm*, 12 *mm*) = 48 *mm*

27.(55) $C_1 = C_2 = C_0 = 10\,\mu F$

$V_0 = 100\,V$

$K = 10$



$$\therefore \quad \text{Common potential} = \frac{\frac{Q_1 + Q_2}{2}}{10C_0} = \frac{\frac{11C_0V_2}{2}}{10C_0} = \frac{11}{20}V_0 = \frac{11}{20} \times 100 = 55\,V$$

28.(5) Speed of the ball just before hitting the ground, $u_0 = \sqrt{2gh}$

$\Rightarrow \quad u_0 = 20\ m/s$

Speed of the ball just after hitting the ground, $u_1 = eu_0$

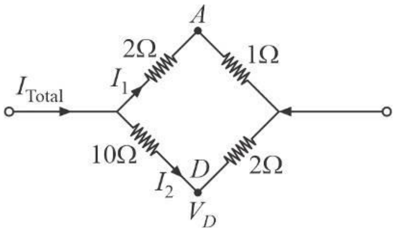
$$H_{max} = \frac{u_1^2}{2g} = \frac{e^2 \times 20^2}{20} = \left(\frac{1}{2}\right)^2 \times 20 = 5\ m$$

29.(300) $\Delta Q_{req} = ms\Delta T = 2 \times 4200 \times 50 = 42 \times 10^4\ J$

$\Delta Q_{req} = 70\%$ of $\Delta Q_{\text{consumed}}$

$\therefore \quad \Delta Q_{\text{consumed}} = \frac{42 \times 10^4 \times 10}{7} = 6 \times 10^5\ J \quad \therefore \quad t = \frac{\Delta Q_{\text{consumed}}}{\text{Power}} = \frac{6 \times 10^5}{2 \times 10^3} = 300\ s$

30.(1) At steady state, capacitor behaves like an open circuit



$$I_{\text{Total}} = \frac{6 \times 15}{3 \times 12} = \frac{5}{2} A$$

$$I_1 = \frac{12}{15} \times I_{\text{Total}} = \frac{4}{5} \times \frac{5}{2} = 2A$$

$$I_2 = \frac{1}{2} A$$

$\therefore \quad V_B = 1 \times I_1 = 2V$

$$V_D = 2 \times I_2 = 2 \times \frac{1}{2} = 1V$$

$\therefore \quad |V_B - V_D| = 1V$

CHEMISTRY

SECTION - 1

1.(2) Samarium – $Sm \rightarrow 4f^6 6s^2$

Europhium – $Su \rightarrow 4f^7 6s^2$

Terbium – $Tb \rightarrow 4f^9 6s^2$

Gadolinium – $Dd \rightarrow 4f^7 5d^1 6s^2$

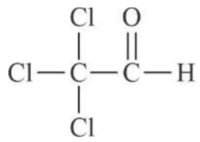
Promethium – $Pm \rightarrow 4f^5 6s^2$

2.(2) Methyl orange – pH range 3.1 – 4.5

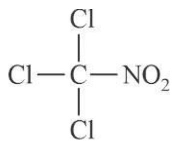
So, methyl orange changes colour in the pH range of 3.1 – 4.5.

So, it can't be used for the titration of week acid Vs strong base.

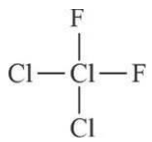
3.(4) Chloral $\rightarrow$



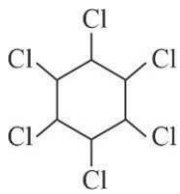
Chloropicrin $\rightarrow$



Freon – 12 $\rightarrow$



Gammaxene $\rightarrow$



4.(3) Both the statements are correct. (Fact) – NCERT.

5.(4) As per NCERT data,

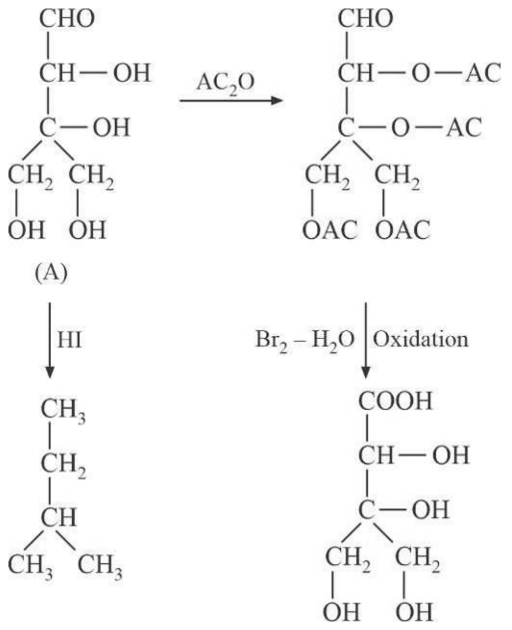
Element		$(I.E.)_1$ (in kJ/mole)
Ca	–	590
Sc	–	631
Ti	–	656

And $(I.E.)_1$ values increase from left to right in 3d-series.

Hence, Assertion (A) and Reason (R) both are correct. And reason is not the correct explanation of assertion.

6.(4) $C_5H_{10}O_5 \rightarrow DU = x + 1 - \frac{y}{2}$

$$= 5 + 1 - \frac{10}{2} = 1$$



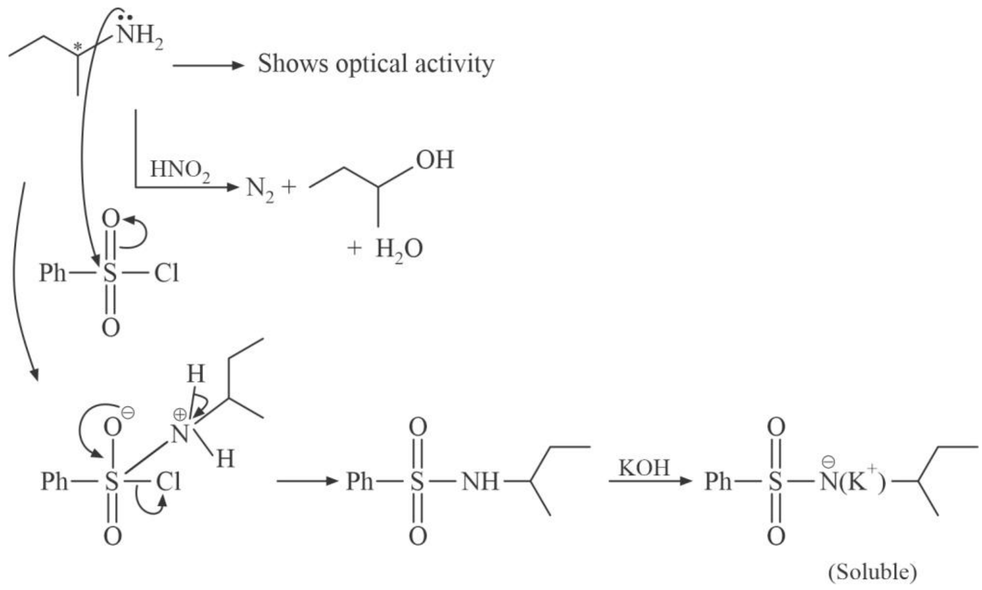
7.(1) $C_xH_y + \left(x+\frac{y}{4}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$

$x+\frac{y}{4}=11$ … (i)

$\frac{y}{2}=4$ … (ii)

$\Rightarrow$ $y=8$ $\Rightarrow$ $x=9$ $\Rightarrow$ the compound is C_9H_8

8.(1)



9.(2) Physisorption – $(20-40)$ kJ mole^{-1}

Chemisorption – Single layer adsorption

$N_2(g)+3H_2(g)\xrightarrow{Fe(s)}2NH_3(g) \rightarrow$ Heterogeneous catalysis

Analytical application or adsorption – chromatography

10.(4) $CO_{2(g)}+H_2O_{(l)} \rightleftharpoons H_2CO_{3(aq)}$

11.(2) Copper gauze

12.(3) Antiseptics are Chloroxylenol, terpineol, Bithional

So, answer is 3

13.(2) $$Na_2B_4O_7 \cdot 10H_2O \xrightarrow{\Delta} \underset{\text{(Anhydrous)}}{Na_2B_4O_7} \xrightarrow{740^\circ} \underbrace{\underset{\text{(Sodium metaborate)}}{2NaBO_2} + \underset{\text{(Boric anhydride)}}{B_2O_3}}_{\text{Glasy mass}}$$

$$CuSO_4 + B_2O_3 \rightarrow CuO \cdot B_2O_3 + SO_3$$

$$\downarrow$$

$$\underset{\text{(Blue)}}{Cu(BO_2)_2}$$

14.(1) $A \rightarrow 3s$

$B \rightarrow 4s$

$C \rightarrow 3p$

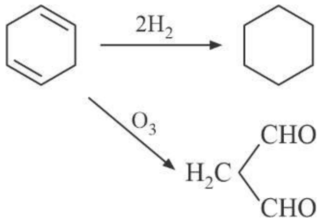
$D \rightarrow 3d$

3d > 4s > 3p > 3s

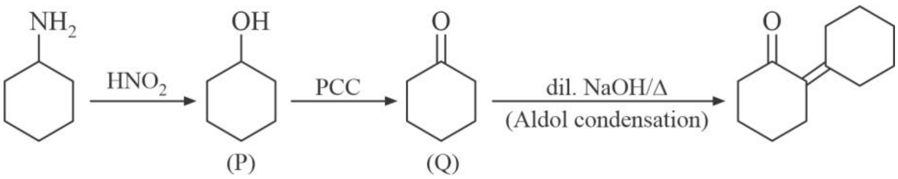
$\downarrow$

D > B > C > A

15.(2)



16.(1)



17.(4) **(A)** $\Delta T_b = i \times K_b \times m$

$i = 2$ for NaCl

$i = 1$ for urea

(B) is correct

(C) Osmosis always takes place from hypotonic to hypertonic solution.

(D) $M = \dfrac{10dx}{M_0}$

$\Rightarrow \quad 4.09 = \dfrac{10 \times d \times 32}{98} \quad \Rightarrow \quad d \approx 1.26 \text{ gm/ml}$

(E) When KI solution is added to $AgNO_3$ solution, positively charged sol resultants due to adsorption of Ag^+ ions from dispersion medium.

(B) and (D) correct

18.(2) Calcium plays important roles in neuromuscular function, interneuronal transmission, cell membrane integrity and blood coagulation. [From NCERT]

19.(2) Lewis acid character of $BI_3 > BBr_3 > BCl_3 > BF_3$ due to back bonding.

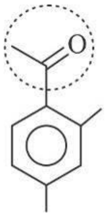
20.(2) Van Arkel method is used to purify Zirconium and titanium.

SECTION – 2

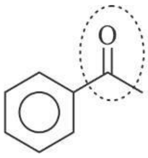
21.(25) $\Lambda = K \times \frac{1000}{M}$

$$= \frac{1}{5\times10^{-3}} \times \frac{1000}{0.8} = \frac{10^6 \times 10}{5\times8} = 25\times10^4\ \Omega^{-1}\ cm^2\ mol^{-1}$$

22.(3)



(Methyl ketones)
(+ve iodoform tert)



(Methyl ketones)
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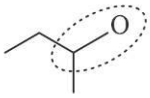


(2° – OH)
(+ve iodoform tert)

23.(2)

Element		**Ionization Enthalpy**
Li	–	520 kJ
K	–	419 kJ
Rb	–	403 kJ

Stable super oxide $\rightarrow KO_2$ and RbO_2

24.(480)

$[Ti(H_2O)_6]^{3+}$

For octahedral complex:

$CFSE = 0.4 \times \Delta_0$

$\because \quad \Delta_0 = h\nu = \frac{hc}{\lambda}$

So, CFSE $= 0.4 \times \frac{hc}{\lambda}$

$$\Rightarrow \quad \frac{96\times10^3}{6\times10^{23}} = \frac{64\times10^{-34}\times3\times10}{\lambda}$$

$$\lambda = 480\times10^{-9} = 480\ nm$$

Answer = 480 nm

25.(173)

Given,

$$\Delta H_f(CCl_4) = -10^5 \text{ kJ/mol}$$

$$\Delta H_f(H_2O) = -242 \text{ kJ/mol}$$

$$\Delta H_f(CO_2) = -394 \text{ kJ/mol}$$

$$\Delta H_f(HCl) = -92 \text{ kJ/mol}$$

Given Reaction,

$$CCl_4(g) + 2H_2O(g) \rightarrow CO_2(g) + 4HCl(g)$$

$$\Delta H_r = \Delta H_P - \Delta H_R$$

$$\Delta H_r = (-394 + 4\times -92) - (-105 + 2\times -242)$$

$$= (-394 - 368) - (-105 - 484)$$

$$= 589 - 394 - 368 = -173 \text{ kJ/mol}$$

26.(10) $\underset{C-S}{AgCl(s)} \longrightarrow \underset{S}{Ag^+} + \underset{S}{Cl^{\ominus}}$

$$KSP = S^2$$

$$\Rightarrow \quad S = 1.434\times 10^{-3}\,\frac{g}{l}$$

Molar mass of AgCl = 107.9 + 35.5 = 143.4 g

$$\Rightarrow \quad S = \frac{1.434}{143.4}\times 10^{-3} \quad \Rightarrow \quad 10^{-5}M$$

So, $KSP = (10^{-5})^2$

$$KSP = 10^{10}$$

–log KSP = 10

27.(227)

$$\underset{\underset{12g}{\downarrow}}{2C(s)} + \underset{\underset{48g}{\downarrow}}{O_2} \rightarrow 2CO(g)$$

$$\eta_C = \frac{12}{2} = 1\text{ mole} \qquad \eta_{O_2} = \frac{48}{32} = 1.5\text{ mole}$$

Hence, C(s) is limiting reagent

So, C(s) ———— CO(g)

2 ———— 2

1 ———— 1

So, moles of CO produced is 1 mole

So, volume at STP is 22.7 $\rightarrow$ 227

28.(17) Given,

$$K = 20\,\text{min}^{-1}$$

$$t_{1/2} = \frac{\ln 2}{20}$$

$$= \frac{2.303 \times \log 2}{20} = \frac{2.303 \times 0.301}{20}$$

$$\frac{1}{32} = \left(\frac{1}{2}\right)^n$$

$n = 5$, So 5 half = lines have passed

So, time required

$$t = 5 \times t_{1/2}$$

$$= \frac{5 \times 2.303 \times 0.301}{20} = 17 = 17 \times 10^{-2}\ \text{min} = 17$$

29.(59) $M_{0.83}O_1 \times 100 \rightarrow M_{83}\ O_{100}$

$$M \begin{cases} +2 \longrightarrow x \\ +3 \longrightarrow (83 - x) \end{cases}$$

Now we will apply charge balanced,

Net negative = net positive

$$2 \times x + 3(83 - x) = 100 \times 2$$

$$\Rightarrow \quad 2x + 249 - 3x = 200 \quad \Rightarrow \quad -x = -49$$

$$x = 49$$

So, number of +2 cation = 49

and number of +3 cation = 34

% of +2 cation $= \frac{49}{83} \times 100 = 59\%$

30.(5)

Compound	**Hybridisation**	**Shape**
XeF_2	sp^3d	Linear
I_3^+	sp^3	V-shape
C_3O_2	sp	Linear
I_3^-	sp^3d	Linear
CO_2	sp	Linear
SO_2	sp^2	V-shape
$BeCl_2$	sp	Linear
$\overset{\Theta}{B}Cl_2$	sp^2	V-shape

MATHEMATICS

SECTION - 1

1.(1) $e^{4x}+8e^{3x}+13e^{2x}-8e^{x}+1=0$

Say $e^{x}=t>0$

$t^2+8t^3+13t^2-8t+1=0$

$(t^2+4t-1)^2=t^2$ or $t^2+3t-1=0$ or $t^2+5t-1=0$

Therefore two values of $0<t<1$ which satisfy the equation which correspond to 2 negative values of x

2.(2) $(3y^2-5x^2)ydx+2x(x^2-y^2)dy=0$

$$\frac{dy}{dx}=\frac{(3y^2-5x^2)y}{2x(y^2-x^2)}$$

Say $y=tx$

$$\frac{dy}{dx}=t+\frac{xdt}{dx}$$

$$t+\frac{xdt}{dx}=\frac{3t^3-5t}{2t^2-2}$$

$$\frac{xdt}{dx}=\frac{t^3-3t}{2t^2-2}$$

$$\frac{dx}{x}=\frac{2(t^2-1)dt}{t^3-3t}$$

Say $t^3-3t=u$

$3(t^2-1)dt=du$

$$\frac{dx}{x}=\frac{2}{3}\frac{du}{u}$$

$$\ln x=\frac{2}{3}\ln u+\ln C$$

$x=Cu^{2/3}$

$x=C(t^3-3t)^{2/3}$

$x^3=C(t^3-3t)^2$

$$x^3=C\left(\frac{y^3}{x^3}-\frac{3y}{x}\right)^2$$

$$x^3=C\left(\frac{y^3-3x^2y}{x^3}\right)^2$$

$$x^9 = C(y^3 - 3x^2 y)^2$$

$$y(1) = 1$$

$$1 = C(1-3)^2 \Rightarrow C = \frac{1}{4}$$

$$2^9 = \frac{1}{4}(y^3 - 12y)^2$$

$$(y^3 - 12y)^2 = 2^{11} \qquad \Rightarrow \qquad \left|y^3 - 12y\right| = 32\sqrt{2}$$

3.(1) $$= \lim_{x\to\infty} \frac{2\left((3x+1)^3 + {}^6C_2(3x+1)^2(3x-1) + {}^6C_4(3x+1)(3x-1)^2 + (3x-1)^3\right)\cdot x^3}{2\left(x^6 + {}^6C_2 x^4(x^2-1) + {}^6C_4 x^2(x^2-1)^2 + (x^2-1)^3\right)}$$

$$= \frac{27 + 15\cdot 27 + 15\cdot 27 + 27}{1 + 15 + 15 + 1} = 27$$

4.(2) $$\int_0^{\alpha} \frac{x}{\sqrt{x+\alpha} - \sqrt{x}}\, dx = \int_0^{\alpha} (\sqrt{x+\alpha} + \sqrt{x})\, x dx = \int_0^{\alpha} \frac{(\sqrt{x+\alpha})\cdot x dx}{\alpha} + \int_0^{\alpha} \frac{x^{9/2}}{\alpha}\, dx$$

Say $\quad x + \alpha = t^2$

$\Rightarrow \quad dx = 2t\, dt$

$$= 2\int_{\sqrt{\alpha}}^{\sqrt{2\alpha}} t^2\cdot(t^2 - \alpha)dt + \frac{2}{5}\alpha^{5/2} = \frac{2}{5}\left[t^5\right]_{\sqrt{\alpha}}^{\sqrt{2\alpha}} - \frac{2\alpha}{3}\left[t^3\right]_{\sqrt{\alpha}}^{\sqrt{2\alpha}} + \frac{2}{5}\alpha^{5/2}$$

$$= \frac{2}{5}\left[\alpha^{5/2}(5^{5/2} - 1)\right] - \frac{2}{3}[\alpha^{5/2}\cdot 2^{3/2} - \alpha^{5/2}] + \frac{2}{5}\alpha^{5/2}$$

$$= \frac{2}{5}\cdot 2^{5/2}\cdot\alpha^{5/2} - \frac{2}{3}\alpha^{5/2}\cdot 2^{3/2} + \frac{2}{3}\alpha^{5/2} = \frac{16 + 20\sqrt{2}}{15}$$

$$\Rightarrow \qquad 2\alpha^{5/2}\left(\frac{3\sqrt{2}}{5} - \frac{4\sqrt{2}}{3} + \frac{1}{3}\right) = \frac{16 + 20\sqrt{2}}{15} \Rightarrow \qquad \alpha^{5/2}\left(\frac{4\sqrt{2} + 5}{15}\right) = \frac{8 + 10\sqrt{2}}{15}$$

$$\alpha^{3/2} = \frac{8 + 10\sqrt{2}}{4\sqrt{2} + 5} = (2)^{3/2}$$

5.(1) $$\alpha x^2 + \beta x + \sin^{-1}((x-3)^2 + 1) + \cos^{-1}((x-3)^2 + 1) = 0 \qquad \Rightarrow \qquad x = 3$$

$$9\alpha + 3\beta + \frac{\pi}{2} = 0$$

And $\quad \alpha - \beta = \dfrac{\pi}{2}$

$$3\alpha - 3\beta = \frac{2\pi}{2}$$

$$12\alpha = \pi$$

$$\alpha = \frac{\pi}{12}$$

6.(2) $\phi'(x)=-\frac{1}{2x}\phi(x)+\frac{1}{\sqrt{x}}(4\sqrt{2}\sin x-3\phi'(x))$

$$\phi'(x)\left(1+\frac{3}{\sqrt{x}}\right)+\frac{1}{2x}\phi(x)=\frac{1}{\sqrt{x}}4\sqrt{2}\sin x$$

$$\phi'(x)(\sqrt{x}+3)+\frac{1}{2\sqrt{x}}\phi(x)=4\sqrt{2}\sin x$$

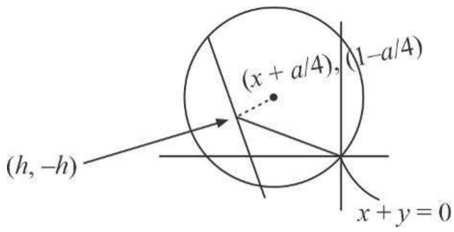
Now, $\phi\left(\frac{\pi}{4}\right)=0$

$$\phi'\left(\frac{\pi}{4}\right)\left(\sqrt{\frac{\pi}{4}}+3\right)=4$$

$$\phi'\left(\frac{\pi}{4}\right)=\frac{4}{3+\sqrt{\frac{\pi}{4}}}=\frac{8}{6+\sqrt{\pi}}$$

7.(1) Circle is $x^2+y^2-\frac{(1+a)}{2}-\frac{(1-a)}{2}y=0$

$$\frac{\frac{1+a}{2}-h}{\frac{1-a}{2}+h}\cdot\frac{\frac{1+a}{4}-h}{\frac{1-a}{4}+h}=-1$$



$$\frac{(1+a-2h)}{(1-a+2h)}\cdot\frac{(1+a-4h)}{(1-a+4h)}=-1$$

$$(1+a)^2+8h^2-6h(1+a)=-(1-a)^2-8h^2-6h(1-a)$$

$$16h^2-12ha+2(1+a^2)=0$$

$$8h^2-6ha+1+a^2=0$$

$$D>0$$

$$36a^2-4(8)(1+a^2)>0$$

$$4a-4\cdot 8>0$$

$$a^2>8$$

$$a^2\in(8,\ \infty)$$

8.(1) Say $y=\frac{x^2+2x+1}{x^2-8x+12}$

$$x^2y-8xy+12y=x^2+2x+1$$

$$x^2(y-1)-2x(4y+1)+12y-1=0$$

$$D\geq 0$$

$$4(4y+1)^2-4(y-1)(12y-)\geq 0$$

$(4y+1)^2-(y-1)(12y-1)\geq 0$

$4y^2+21y\geq 0$

$y(4y+21)\geq 0$

$y\in(-\infty,-21/4]\cup[0,\infty)$

9.(4) $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$

$\vec{a}\times(\vec{r}\times\vec{b})=\vec{a}\times(\vec{c}\times\vec{b})$

$(\vec{a}\cdot\vec{b})\vec{r}-(\vec{a}\cdot\vec{r})\vec{b}=(\vec{a}\cdot\vec{b})\vec{c}-(\vec{a}\cdot\vec{c})\vec{b}$

$5\vec{r}-0=5(5\hat{i}-3j+3k)-8(\hat{i}-j+2k)$

$5\vec{r}=+7\hat{i}-7j-k$

$25||\vec{r}||=289+49+1=339$

10.(2) $z=\dfrac{i-1}{\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)}=(-1+i)\left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)$

$=\sqrt{2}\,e^{i\frac{3\pi}{4}}\cdot e^{-1\frac{\pi}{3}}=\sqrt{2}\,e^{i\frac{5\pi}{12}}=\sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$

11.(2) $a_1+6d=3$

$a_1(a_1+3d)=a_1a_4$

$(3-6d)(3-3d)=a_1a_4=18d^2-27d+9=9(2d^2-3d+1)$

For this to be min $d=\dfrac{3}{4}$

$a_7=3-6d=3-\dfrac{9}{2}=-\dfrac{3}{2}$

Now, $S_n=0\Rightarrow\quad a_7+a_n=0$

$a_7+a_7+(n-1)\dfrac{3}{4}=0$

$-3+(n-1)\dfrac{3}{4}=0$

$\dfrac{s}{4}(n-1)=3$

$n-1=4$

$n=5$

$5!-4a_{35}$

$a_{35}=-\dfrac{3}{2}+34\cdot\dfrac{3}{4}=-\dfrac{3}{2}+\dfrac{51}{2}=24$

$120-4.24=24$

12.(4) $2\alpha+9\beta+8y=0$

$10\alpha+3\beta+4\gamma=0$

$8\alpha+8\beta+8\gamma=0$ $\quad$ $42\beta+36\gamma=0$

$\Rightarrow \quad \alpha+\beta+\gamma=0$ $\quad$ $7\beta+6\gamma=0$

$2\alpha+4\beta+3\gamma=5$

$2\beta+\gamma=5$ and $7\beta+6\gamma=0, 12\beta+6\gamma=30$

$6\alpha+7\beta+7\gamma=3\beta+\gamma$

$\Rightarrow \quad 5\beta=30, \beta=6$

$\Rightarrow \quad \gamma=-7$

$3\beta+\gamma=18-7=11$

13.(2) $(1,-4)\in S$ but $(-4,1)\notin S$

So, S is not symmetric however T is symmetric.

14.(4) Not solution

15.(3) $8.2+3\alpha_1+5\alpha_2=0$

$3\alpha_1+5\alpha_2=-16$

and $\dfrac{-12}{\alpha_1}=1\Rightarrow \alpha_1=-12$

$\alpha_2=4$

Plane, $8x-12y+4z+12=0$

$2x-3y+z+3=0$

$$\left|\frac{2(-2)-3(3)+(-4)+3}{\sqrt{4+9+1}}\right|=\sqrt{14}$$

16.(4) $2ae=2\sqrt{2}$

$ae=\sqrt{2}$ $\quad$ $e=\sqrt{2}$

$a=1$

$b^2=a^2(e^2-1)=2-1$

$b^2=1$

Length of L.R. $=\dfrac{2b^2}{a}=\dfrac{2}{1}=2$

17.(2) Mean of class A = 40

Standard deviation $=\alpha$

$\dfrac{\sum x_i^{\,2}}{100}-40^2=\alpha^2$ $\quad$ … (i) $\quad$ $\sum x_i=4000$ $\quad$ … (iii)

Mean of class B = 55

Standard deviation $=30-\alpha$

$$\frac{\sum y_i^2}{n}-55^2=(30-\alpha)^2 \quad \ldots \text{(ii)} ; \qquad \sum y_i^0=55n \quad \ldots \text{(iv)}$$

Mean of combined class $=50: \dfrac{\sum x_i+\sum y_i}{n+100}=500 \qquad \dfrac{400+55n}{n+100}=50$

$$\frac{\sum(x_i^2+y_i^2)}{n+100}-50^2=350 \qquad 4000+55n=50n+5000$$

$$\frac{100(40^2+\alpha^2)+n[(30-\alpha)^2+55]}{n+100}=350+50^2 \qquad 5n=1000$$

$$n=200$$

$$\frac{40^2+\alpha^2+2[(30-\alpha)^2+55^2]}{3}=350+50^2$$

$$1600+\alpha^2+2[100+\alpha^2-60\alpha+3025]=1050+7500$$

$$3\alpha^2-120\alpha+1600+1800+6050=8550$$

$$3\alpha^2-120\alpha+900=0$$

$\alpha^2-40\alpha+30° \quad \Rightarrow \quad \alpha=10 \quad$ or $\quad \alpha=30 \quad$ (Not possible)

$\therefore$ Sum of variances $=\alpha^2+(30-\alpha)^2=100+400=500$

18.(2) $y=x^2-x+1+[x^2-x+1] \qquad \because \quad \{x\}=x-[x]$

$$t=x^2-x+1=\left(x-\frac{1}{2}\right)^2+\frac{3}{4}$$

$\because \quad x\in[-1,2]$

$$x-\frac{1}{2}\in\left[-\frac{3}{2},\frac{3}{2}\right]$$

$$\left(x-\frac{1}{2}\right)^2\in\left[0,\frac{9}{4}\right]$$

$$\left(x-\frac{1}{2}\right)^2+\frac{3}{4}\in\left[\frac{3}{4},3\right]$$

$$\left[\left(x-\frac{1}{2}\right)^2+\frac{3}{4}\right]\in\{0,1,3\}$$

$$\left[0\left(x-\frac{1}{2}\right)^2+\frac{3}{4}\right]\in\left[\frac{1}{4},1\right]$$

$$\left(x-\frac{1}{2}\right)^2$$

$$y=\left[\frac{3}{4},\ 1\right]+0$$

$$y_{\min}=\frac{3}{4}$$

Answer = $\frac{3}{4}$

19.(2) Equation of plane is $2(x-2)+a(y-a)+4(z-4)=0$

$$2x+ay+4z-(20+a^2)=0 \qquad \ldots \text{(i)}$$

$$2x+ay+4z=20+a^2$$

$$\frac{x}{\frac{20+a^2}{2}}+\frac{y}{\frac{20+a^2}{a}}+\frac{2}{\frac{20+a^2}{4}}=1$$

$$A\left(\frac{20+a^2}{2},0,0\right),\ B\left(0,\frac{20+a^2}{a},0\right),\ C\left(0,0,\frac{20+a^2}{a}\right)$$

Volume of $OABC=\frac{1}{6}\left[\frac{20+a^2}{2}\ \frac{20+a^2}{a}\ \frac{20+a^2}{a}\right]=\frac{1}{24a}(20+a^2)^3=144$

$$(20+a^2)^3=a(12\times2^2\times12\times2)=a(12^3\cdot2^2)$$

For a = 2 it is satisfied

$\therefore$ Equation of plane is $2x+2y+4z=24$

$$x+y+2z=12$$

Which is not satisfying by option (2)

20.(4) Equation of plane

$$2(x-1)-3(y+1)-6(z+5)=0$$

$$2x-3y-6z-35=0$$

Distance (3, –2, 2) is $=\left|\frac{6+6-12-35}{7}\right|=5$

SECTION – 2

21.(45) $\dfrac{\frac{\lfloor 2n+1}{\lfloor n+2}}{\frac{\lfloor 2n-1}{\lfloor n-1}}=\dfrac{11}{21}$

$$\frac{\lfloor 2n+1}{\lfloor n+2}\cdot\frac{\lfloor n-1}{\lfloor 2n-1}=\frac{11}{21}$$

$$\frac{(2n+1)2n}{(n+2)(n+1)n}=\frac{11}{21}$$

$$\frac{4n+2}{n^2+3n+2}=\frac{11}{21}$$

$\Rightarrow \quad 84n+42=11n^2+33n+22$

$11n^2-51n-20=0$

$11n^2-55n+4n-20=0$

$11n(n-5)+2n(n-5)$

$(n-5)(11n+2)=0$

$n=5$

$n^2+n+15=25+5+15=45$

22.(3) $2(\vec{a}\times\vec{b})-3\vec{c}\times\vec{a}=0$

$+3\vec{a}\times\vec{c}=0$

$\vec{a}\times(2\vec{b}+3\vec{c})=0$

$\vec{a}\,11(2\vec{b}+3\vec{c})$

$\vec{a}=\lambda(2\vec{b}+3\vec{c})$

$\therefore \quad \left(\frac{2(2\vec{b}+3\vec{c})\times\vec{c}}{2(2\vec{a}+3\vec{c})\cdot\vec{b}}\right)^2$

$$\left(\frac{2(\vec{b}\times\vec{c})}{2b^2+3\vec{c}\cdot\vec{b}}\right)^2=\frac{2bc\sin 120^\circ}{2\times\frac{1}{4}+3(2)\left(\frac{1}{2}\right)\cos 120^\circ}=\left(\frac{2\left(\frac{1}{2}\right)(2)\left(+\frac{\sqrt{3}}{2}\right)}{\frac{1}{2}+3\left(-\frac{1}{2}\right)}\right)=(\sqrt{3})^2=3$$

23.(5) $\left|adj(2adj(2A^{-1}))\right|=2^{84}$

$\left|2(adj(2A^{-1}))\right|^{n-1}=2^{84}$

$2^{n(n-1)}\left|adj(2A^{-1})\right|^{n_1}=2^{84}$

$2^{n(n-1)}\left|2A^{-1}\right|^{(n-1)^2}=2^{84}$

$2^{n(n-1)}\,2^{n(n-1)^2}\left|A^{-1}\right|^{(n-1)^2}=2^{84}$

$2^{n(n-1)}\,2^{n(n-1)^2}\left|A\right|^{-(n-1)^2}=2^{84}$

$$2^{n(n-1)+n(n-1)^2-(n-1)^2} = 2^{84}$$

$$n(n-1)+(n-1)^2(n-1)=84$$

$$(n-1)(n+n^2-2n+1)=84$$

$$(n-1)\,(n^2-n+1)=84$$

Let $n=4$

$$3(13)\times 4(21)$$

$$n=5$$

24.(125)

$$|2x-1| \le y \le |x^2-x|$$

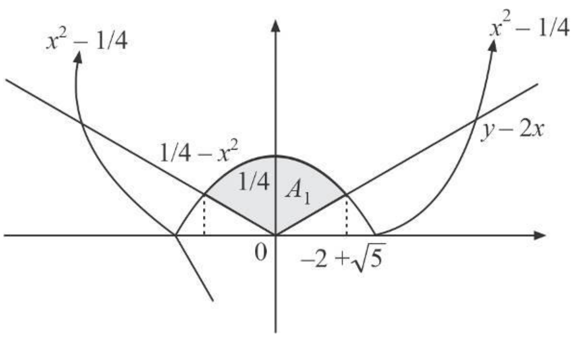
$$2\left|x-\frac{1}{2}\right| \le y \le \left|\left(x-\frac{1}{2}\right)^2-\frac{1}{4}\right| \qquad x\in[0,1] \qquad x-\frac{1}{2}=t$$

$$2|x| \le y \le \left|x^2-\frac{1}{4}\right| \qquad t\in\left[-\frac{1}{2},\frac{1}{2}\right]$$

$$-x^2+\frac{1}{4}=2x$$

$$x^2+2x-\frac{1}{4}=0$$

$$x=\frac{-2\pm\sqrt{5}}{2}=\frac{-2+\sqrt{5}}{2}$$



$$\int_0^{\frac{\sqrt{5}-2}{2}} \left(\frac{1}{4}-x^2-2x\right)dx$$

$$\left(\frac{x}{4}-\frac{x^3}{3}-x^2\right)\frac{\sqrt{5}-2}{2} \qquad \alpha=\frac{\sqrt{5}-2}{2}$$

$$\frac{\alpha}{4}-\frac{\alpha^3}{3}-\alpha^2 \qquad \alpha^2=\frac{9-4\sqrt{5}}{4}$$

$$\frac{\alpha}{4}-\frac{\alpha^3}{3}-\alpha^2$$

$$\frac{\alpha}{12}(3-4\alpha^2)-\alpha^2$$

$$\frac{1}{12}\left(\frac{\sqrt{5}-2}{2}\right)[3-9+4\sqrt{5}]-\left(\frac{9-4\sqrt{5}}{4}\right)$$

$$\frac{1}{2}(\sqrt{5}-2)(-3+2\sqrt{5})-\frac{3-4\sqrt{5}}{4}$$

$$\frac{1}{2}[16-7\sqrt{5}]-\frac{9-4\sqrt{5}}{4}=\frac{46-7\sqrt{5}-27+12\sqrt{5}}{12}$$

$$A=\frac{5\sqrt{2}-11}{6}\Rightarrow(6A+11)^2=125$$

25.(98) $${}^9C_r\left(\frac{x^{5/2}}{2}\right)^{9-r}\left(-\frac{4}{x^l}\right)^r$$

$${}^9C_r\cdot\frac{(-4)^r}{2^{9-r}}\cdot x^{\frac{45-5r}{2}-lr}$$

$$\frac{45-5r}{2}-lr=0$$

$$45-5r=2lr$$

$$r=\frac{45}{5+2l}$$

$${}^9C_{\frac{45}{5+2l}}\cdot\frac{(-4)^{\frac{45}{5+2l}}}{2^{9-\frac{45}{5+2l}}}=-84$$

Let $l=2$

$${}^9C_5\cdot\frac{(-4)^5}{2^{9-5}}=-\,{}^9C_4\cdot\frac{2^{10}}{2^4}=2^3$$

$$l=5$$

$${}^9C_3\frac{(-4)^3}{2^{9-3}}=-\frac{9\cdot8\cdot7}{1\cdot2\cdot3}\cdot\frac{2^6}{2^6}$$

$$-84$$

Coefficient of x^{-15} is $2^\alpha\cdot\beta$

$$\frac{45-5r}{2}-\frac{4r}{1}$$

$$\frac{45-5r}{2}-5r=-3\times5$$

$$45-15r=-30$$

$$75=15r$$

$$r=5$$

$${}^9C_5\frac{(-4)^5}{2^4}=-\,{}^9C_4\cdot\frac{2^{10}}{24}=-\frac{9\cdot8\cdot7\cdot6}{1\cdot2\cdot3\cdot4}\cdot2^6=-63\cdot2^7$$

$$\alpha=7;\ \beta=-63$$

$$|2l-\beta|=|7(5)+63|=|35+63|=98$$

26.(204)

1st Solution

$a+b+c+d = \text{Prime} = \{3,5,7,11\}$

$a,b,c,d \in \{0,2,3,4\}$

a, b, c d are entries of matrix A

Using multinomial theorem

$$(1+x+x^2+x^3+x^4)^4 = \left(\frac{1-x^5}{1-x}\right)^4 = (1-x^5)^4(1-x)^{-4} = (1-4x^5+6x^{10})(1-x)^{-4}$$

$\because$ Coefficient of x^r in $(1-x)^{-n}$ is ${}^{n+r-1}C_r$

Coefficient of x^3 $\Rightarrow$ $1\times {}^{4+3-1}C_3x^3 = {}^6C_3 = 20$

Coefficient of x^5 $\Rightarrow$ $1\times {}^{4+5-1}C_5x^4 = {}^8C_5x^5 = 56x^5$

$-4x^5\times 1 = -4x^5$

$\therefore$ Coefficient of $x^5 = 52$

Coefficient of x^7 $\Rightarrow$ $1\times {}^{4+7-1}C_7x^7 = {}^{10}C_3x^7 = 120x^7$

$-4x^5\times {}^{4+2-1}C_2x^2 = 40x^7$

$\therefore$ Coefficient of $x^7 = 80$

Coefficient of x^{11} $\Rightarrow$ $1\times {}^{4+11-1}C_{11}x^{11} = {}^{14}C_3x^{11} = 364x^{11}$

$-4x^5\times {}^{4+6-1}C_6x^6 = -4\cdot {}^9C_3x^6 = -336x^{11}$

$6x^{10}\times {}^{4+1-1}C_1x^1 = 24x^{11}$

$\therefore$ Coefficient of $x^{11} = 52$

Answer = 20 + 52 + 80 + 52 = 204

2nd Solution

Let the matrix be $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Now, $a_{11}, a_{12}, a_{21}, a_{22} \in \{0,1,2,4\}$ such that … (A)

$a_{11}+a_{12}+a_{21}+a_{22} = p$ where $p\in\{3,5,7,11\}$ … (B)

We are looking for total number non-negative solutions of (B) under the constraint (A).

Total number of solutions

$$= {}^{p+3}C_3 - {}^4C_1({}^{p-2}C_3) + {}^4C_2({}^{p-7}C_3)$$

$$= ({}^6C_3) + ({}^8C_3 - {}^4C_1\cdot 1) + ({}^{10}C_3 - {}^4C_1\cdot {}^5C_3) + ({}^{14}C_3 - {}^4C_1\cdot {}^9C_3 + {}^4C_2\cdot {}^4C_1) = 204$$

27.(6952)

$$(1^2+3\cdot 5^2+5\cdot 9^2+\ldots+15\cdot 29^2) - 2(1\cdot 3^2+2\cdot 7^2+3\cdot 11^2+\ldots 8\text{ terms})$$

$$\sum_{n=1}^{8}(2n-1)(4n-3)^2 - 2\sum_{n=1}^{7} n(4n-1)^2$$

$$\sum_{n=1}^{8}(2n-1)(16n^2-24n+9) - 2\sum_{n=1}^{7} n(16n^2-8n+1)$$

$$\sum_{n=1}^{8}(32n^3-64n^2+42n-9)-2\sum(16n^3-8n^2+n) \quad \because \quad \sum n^3=\left(\frac{n(n+1)}{2}\right)^2$$

$$\sum n^2=\frac{n(n+1)(2n+1)}{6}$$

$$\sum n=\frac{n(n+1)}{2}$$

After simplifying at we get 6952

28.(10) $|x-y|\le a \qquad x\ y\in[0,60]$

Total area $=60\times 60$

$=3600$

$\because |x-y|\le a \Rightarrow -a\le x-y\le a$

$x-y\ge -a \quad : \quad x-y\le a$

$y=x+a \qquad x-y=a\Rightarrow\ y=x-a$

Favorable Area = Total $-(A_1+A_2)$

$=3600-\left(2\times\frac{1}{2}\times(60-a)(60-a)\right)$

$=3600-(60-a)^2$

$prob=\frac{3600-(60-a)^2}{3600}=\frac{21}{36}$

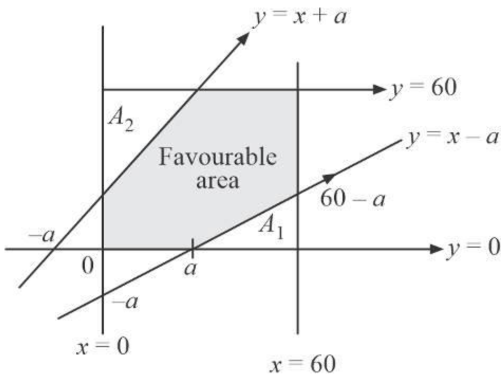
$1-\frac{(60-a)^2}{(60)^2}=\frac{11}{36}$

$\frac{25}{36}=\left(\frac{60-a}{60}\right)^2$

$\frac{60-a}{60}=\frac{5}{6}$

$60-a=50$

$a=10$



29.(146) Tangent is $\quad cy=a(x+b)\,;\, y^2=2ab\Rightarrow \quad c^2=2ab$

$cy=ax+ab$

$-ax+cy=ab$

$\Rightarrow \quad -\frac{x}{b}+\frac{yc}{ab}=1$

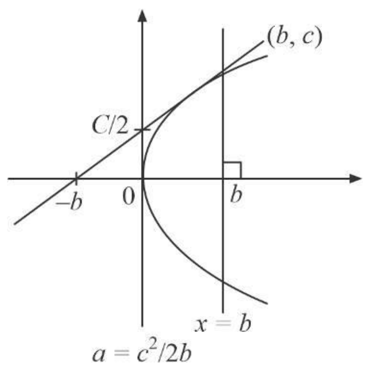
$-\frac{x}{b}+\frac{y}{c/2}=1$

Area $=\frac{1}{2}\times 2b\times 2c=2ac=16$

$bc=8\times 2$

$b=2\,;\,c=8\Rightarrow a=1/6$

$b=2\,;\,c=8\Rightarrow a=1/6$



$b=4; c=4 \Rightarrow a=2$

$b=8; c=1 \Rightarrow a=xx$

$b=16; c=1 \Rightarrow a=x$

$b=1; c=16 \Rightarrow a=128$

Ans. 16 +2 + 128 = 146

30.(5040)

$$T_{r+1} = {}^9C_r\left(\frac{4x}{5}\right)^{9-r}\left(\frac{5}{2x^2}\right)^r$$

$$= {}^9C_r 4^{9-r}\cdot\frac{5^r}{2^r\cdot 5^{9-r}}\cdot x^{9-3r}$$

$9-3r=-6$

$3r=15$

$r=5$

$$T_6 = \frac{{}^9C_5\cdot 4^4}{5^4}\cdot\frac{5^5}{2^5}\cdot x^{-6} = \frac{9\cdot 8\cdot 7\cdot 6}{1\cdot 2\cdot 3\cdot 4}\cdot 2\cdot 5 = 9\cdot 8\cdot 7\cdot 2\cdot 5 = 72\times 70 = 5040$$