SOLUTIONS

Joint Entrance Exam | IITJEE-2023

29th JAN 2023 | Morning Shift

PHYSICS

SECTION – 1

1.(2) Difference in frequency $=300\left(\frac{330}{330-30}\right)-300\left(\frac{330}{330+30}\right)=300\times330\left(\frac{1}{300}-\frac{1}{360}\right)=55\,Hz$

2.(4) $\lambda=\frac{\ell n2}{30}$

$N=N_0\,e^{-\lambda t}=N_0\,e^{\frac{-\ell n2}{30}\times90}=N_0\,e^{-3\ell n2}=N_0\,e^{-\ell n2^3}=N_0\,e^{-\ell n8}$

$N=\frac{N_0}{8}$ $\quad\therefore\quad$ $\frac{N}{N_0}=\frac{1}{8}$

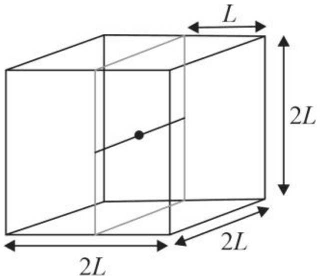
3.(2) Maximum line of sight $=\sqrt{2Rh_T}+\sqrt{2Rh_R}=\sqrt{8R80}=64\,km$

4.(2) Work done to increase the radius of soap bubble.
= Surface tension × Change in surface Area.

$=2\times10^{-2}\times2\times\left[4\pi\left(7\times10^{-2}\right)^2-4\pi\left(3.5\times10^{-2}\right)^2\right]=18.48\times10^{-4}\,J$

5.(4) Speed of light is independent of all parameters in vaccum.
But few parameters varies in medium.

6.(2) Flux through each face $\frac{q}{6\in_0}$



7.(3) First law of thermodynamics

$dQ=dU+dW$ $\quad dW$: Work done by the system.

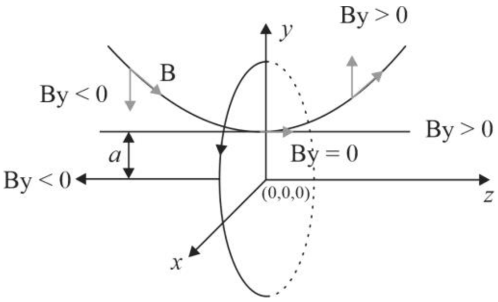
$dQ=dU-dW$ $\quad dW$: Work done on the system.

Basis of this law is 'conservation of energy'

8.(3) $\lambda_T=5500\,A^\circ$

Emission is possible for incident wavelength smaller than 5500 A° i.e. for ultra violet.

9.(4)



Remark: Direction of +ve z axis is wrong.

At $z\to\pm\infty$

By = 0

At $z=0$

By = 0

10.(1) Position (y) of first minima = $\frac{a}{2}$

$y_n = \frac{(2n-1)}{2}\frac{\lambda D}{a}$ [Position of minima above centre of screen]

For $n = 1$

$y_1 = \frac{1}{2}\frac{\lambda D}{a} = \frac{a}{2}$

$\Rightarrow \quad a^2 = \lambda D \Rightarrow a = \sqrt{\lambda D}$

$a = \sqrt{800 \times 10^{-9} \times 5 \times 10^{-2}}$

$a = 2 \times 10^{-4}\, m$

$a = 0.2\, mm$

11.(1) Energy density $\left(\frac{E}{V}\right) = \frac{[ML^2T^{-2}]}{[L^3]} = [ML^{-1}T^{-2}]$

Electric field $\left(\frac{E}{q}\right) = \frac{[MLT^{-2}]}{[AT]} = [MLT^{-3}A^{-1}]$

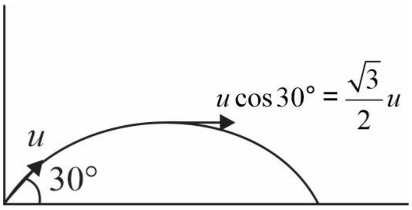
Pressure gradient $\frac{P}{L} = \frac{[ML^{-1}T^2]}{[L]} = [ML^{-2}T^{-2}]$

[Latest Heat] $Q = ML$

$L = \frac{Q}{M} = \frac{Energy}{Mass} = \frac{[ML^2T^{-2}]}{[M]}$

$= [M^0L^2T^{-2}]$

12.(4)



k = kinetic energy = $\frac{1}{2}mu^2$

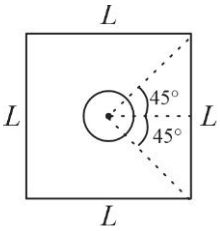
$k' = \frac{1}{2}m\left(\frac{\sqrt{3}}{2}u\right)^2$

$= \frac{1}{2}m\frac{3}{4}u^2$

$k' = \frac{3}{4}k$

$\Rightarrow \quad \frac{k}{k'} = \frac{4}{3}$

13.(3)



Magnetic field due to one side of square loop at centre

$$B=\frac{\mu_0 I}{4\pi\left(\frac{L}{2}\right)}\left(\sin 45^\circ+\sin 45^\circ\right)$$

$$B=\frac{\mu_0 I}{2\pi L}2\left|\frac{1}{\sqrt{2}}\right|$$

$$B=\frac{\mu_0 I\sqrt{2}}{2\pi L}$$

Total magnetic field = $4B=\frac{4\mu_0 I\sqrt{2}}{2\pi L}=\frac{2\sqrt{2}\mu_0 I}{\pi L}$

$$\phi=\frac{2\sqrt{2}\mu_0 I}{\pi L}\left(\pi R^2\right)=MI$$

$$M=\frac{2\sqrt{2}\mu_0 R^2}{L}$$

14.(3) $P=270\,kPa$

$T_1=27^\circ C=300k$

$T_2=36^\circ C=309k$

$PV=nRT$

$$\frac{P_1}{T_1}=\frac{P_2}{T_2}\Rightarrow P_2=\frac{P_1T_2}{T_1}=\frac{270\times 309}{300}=\frac{9\times 309}{10}$$

$$P_2=\frac{2781}{10}=278.1kPa$$

15.(4) $f\geq\frac{mv^2}{r}$

$$\frac{mv^2}{r}\leq f\leq \mu mg$$

$$v^2\leq \mu rg\leq 0.34\times 50\times 10$$

$$v^2\leq 170$$

$$v\approx 13m/s$$

16.(4) LED (light emitting diode) are

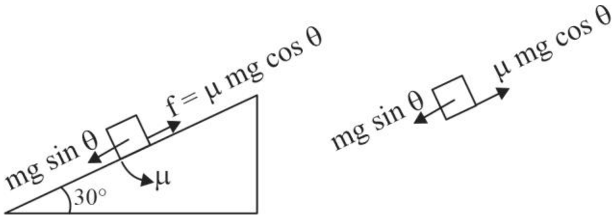
Heavily doped P-n junction.

Emits light only when it is forward biased and do not emit light in reverse bias condition.

Energy of the light emitted is equal to or slightly less than the energy gap of the semiconductor used.

As LED doesn't emit light in reverse biased. So option 4 is correct.

17.(3)



$$F_{net} = mg\sin\theta - \mu mg\cos\theta$$

$$ma = mg\sin\theta - \mu mg\cos\theta$$

$a = g\sin\theta - \mu g\cos\theta$ and given $a = \frac{g}{4}$

$$\Rightarrow \quad \frac{g}{4} = g\left[\sin 30^\circ - \mu\cos 30^\circ\right]$$

$$\frac{1}{4} = \frac{1}{2} - \frac{\mu\sqrt{3}}{2}$$

$$\mu = \frac{1}{2\sqrt{3}}$$

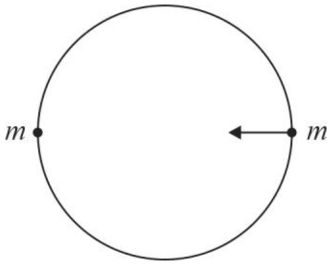
18.(1) Thermal energy or heat loss = i^2Rt

$= \frac{V^2}{R}t$ (in time t)

When in parallel, potential will be same.

So $\frac{E_1}{E_2} = \frac{V^2/R_1}{V^2/R_2} = \frac{R_2}{R_1} = \frac{3R}{R} = \frac{3}{1}$

19.(2)



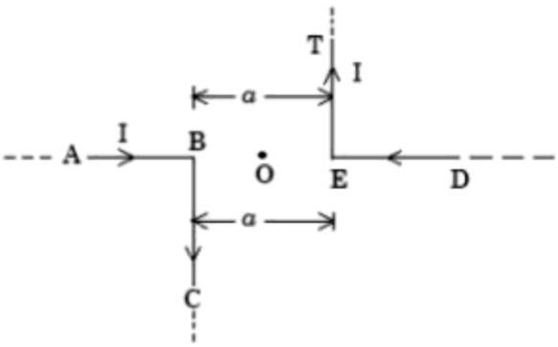
Gravitational force = centripetal force.

$$\frac{Gm^2}{(2r)^2} = \frac{mv^2}{r}$$

$$\frac{Gm}{4r} = v^2$$

$$v = \sqrt{\frac{Gm}{4r}}$$

20.(1)



Magnetic field at Point O = B (due to AB + due to BC + due to DE + due to ET)

$$= 0 + \frac{\mu_0 I}{4\pi\left(\frac{a}{2}\right)} \odot + 0 + \frac{\mu_0 I}{4\pi\left(\frac{a}{2}\right)} \odot = \frac{\mu_0 I}{2\pi a} + \frac{\mu_0 I}{2\pi a} \text{ (outward)}$$

$$B = \frac{\mu_0 I}{\pi a}$$

SECTION – 2

21.(2) When shunt is not connected

$$\frac{R_1}{R_2} = \frac{\ell}{100-\ell} \Rightarrow \frac{2}{3} = \frac{\ell}{10-\ell} \Rightarrow \ell = 40\,cm$$

When shunt is connected to 3Ω resistance

$$\frac{R_1}{R_2} = \frac{\ell}{100-\ell} \Rightarrow \frac{2}{\frac{3r}{3+r}} = \frac{40+22.5}{100-(40+22.5)}$$

Net value of 3Ω resistance will decrease $\Rightarrow \frac{2(3+r)}{3r} = \frac{62.5}{37.5} = \frac{25}{15}$

$$90 + 30r = 75r$$

$$r = \frac{90}{45} = 2\Omega$$

22.(87) 2α withdrawal assures $\rightarrow$ Reduced atomic mass of $2 \times 4U = 8\,units$

$1e^-$ withdrawal assures $\rightarrow$ 1 atomic no. increase

$2e^+$ withdrawal assures $\rightarrow$ 2 atomic no decrease

Hence final atomic mass will be

$242 - 8 = 234$

Hence final atomic no. will be $\rightarrow 92 - (4) + 1 - 2 = 87$

23.(24) Unpolarized light $(I_0) \xrightarrow{P_1} \frac{I_0}{2}$

Polaroid $(P_1)\left(\frac{I_0}{2}\right) \xrightarrow[\theta b/w P_1 + P_2 (60^\circ)]{P_2} \frac{I_0}{2}\cos^2 60^\circ = \frac{I_0}{8}$

Polaroid $P_2\left(\frac{I_0}{8}\right) \xrightarrow[\theta b/w P_2 + P_3 (90-60^\circ = 30^\circ)]{P_3} \frac{I_0}{8}\cos^2 30^\circ = \frac{3I_0}{32}$

Given $I_0 = 256\, w/m^2$

So $\frac{3I_0}{32} = \frac{3}{32} \times 256 = 24\, w/m^2$

24.(120)

Resultant amplitude is given by:

$\therefore \quad R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$8 = \sqrt{8^2 + 8^2 + 2 \times 8^2 \cos\theta}$

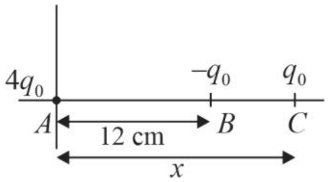
$8^2 = 8^2 + 8^2 + 128\cos\theta$

$\frac{-64}{128} = \cos\theta$

$\cos\theta = -\frac{1}{2}$

$\theta = 120°$

25.(24)



Force between B and C will be towards x^- and force on c due to A will be towards x^+

For equilibrium

$|F_{CA}| = |F_{CB}|$

$\frac{K(4q_0)q_0}{x^2} = \frac{Kq_0q_0}{(12-x)^2}$

$\frac{4}{x^2} = \frac{1}{(12-x)^2}$

$\frac{2}{x} = \pm\frac{1}{(12-x)} \Rightarrow 24 - 2x = \pm x \quad \Rightarrow \quad x = 24\,cm$ and $x = 8\,cm$

We will take $x = 24\,cm$ according to our situation.

26.(40) Kinetic energy of sphere in pure rolling is given as

$K_T + K_R = 2240J$

$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = 2240J$

$\frac{1}{2}mv^2 + \frac{1}{2}\frac{2}{5}mR^2\frac{v^2}{R^2} = 2240$ [For pure rolling $v = R\omega$]

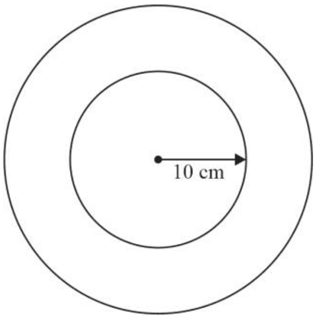
$\frac{1}{2}mv^2\left[1 + \frac{2}{5}\right] = 2240$

$\frac{1}{2} \times 2 \times v^2 \times \frac{7}{5} = 2240$

$v^2 = \frac{2240 \times 5}{7} = 1600$

$v = 40$

27.(10)



$B = 0.8T$

$\frac{dR}{dt} = 2\,cm/s$

Flux linked to coil at any time *t*,

$\phi = BA = 0.8\pi R^2$

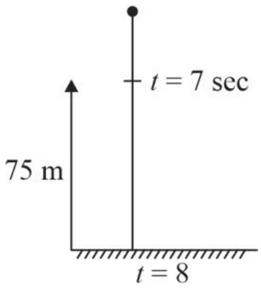
Induced emf, $E = \left|\frac{-d\phi}{dt}\right| = 0.8\pi \times \frac{dR^2}{dt}$

$= 0.8\pi \times (2R)\frac{dR}{dt}$

$E = 0.8 \times 3.14 \times 2 \times 0.1 \times 2 \times 10^{-2}$

$E = 10 \times 10^{-3} V = 10\,mV$

28.(300)



Distance travelled in 8th second

$S_{8th} = u + \frac{a}{2}(2n-1)$

$S_{8th} = 0 + \frac{10}{2}(2 \times 8 - 1)$

$S_{8th} = 5 \times 15 = 75\,m$

Loss of potential energy in last second of motion

$\Delta u = mgh = 0.4 \times 10 \times 75 = 300\ J$

29.(28) Using concept of Newton's law of cooling.

$\frac{T_2 - T_1}{t} = k\left(\frac{T_1 + T_2}{2} - T_0\right)$

$\frac{60-40}{6} = k\left(\frac{60+40}{2} - 10\right)$

$\Rightarrow \quad \frac{20}{6} = k(50-10)$ …(i)

Again $\frac{40-T}{6} = k\left(\frac{40+T}{2} - T_0\right)$ ...(ii)

$$\frac{(ii)}{(i)} = \frac{\frac{40-T}{6}}{\frac{20}{6}} = \frac{\frac{40+T}{2} - 10}{40}$$

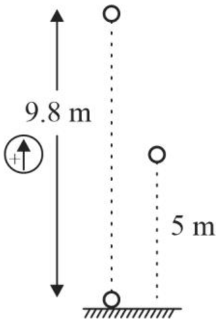
$$\Rightarrow \quad \frac{40-T}{20} = \frac{40+T-20}{2\times 40}$$

$$160 - 4T = 20 + T$$

$$5T = 140$$

$$T = \frac{140}{5} = 28°C$$

30.(120)



Given $t = 0.2\text{sec}$

Velocity before hitting the surface

$|u| = \sqrt{2gh} = \sqrt{2\times 10\times 9.8} = 14\,m/s$ So $u = -14\,m/s$

Velocity after hitting the surface

$|v| = \sqrt{2gh'} = \sqrt{2\times 10\times 5} = 10\,m/s$ So $v = 10\,m/s$

Avg acceleration

$$a_{avg} = \frac{v-u}{t} = \frac{10-(-14)}{0.2}$$

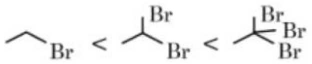
$$a_{avg} = \frac{24}{0.2} = 120\,m/s^2$$

CHEMISTRY

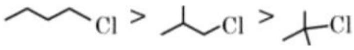
SECTION – 1

1.(2) (A) Boiling point: increase in molar mass

(C) Boiling point: increase in halogen atom



(E) Boiling point: Branching inversely proportional to boiling point



2.(2) Mond process for refining of nickel in based on the principle that nickel is heated in presence of carbon monoxide to form nickel tetracarbonyl which is volatile compound.

$$Ni + 4CO \xrightarrow{330-350K} Ni(CO)_4$$

Nickel tetra carbonyl

Nickel tetra carbonyl is decomposed by subjecting it to a higher temperature $(450-470K)$ to obtain pure nickel metal

$$Ni(CO)_4 \xrightarrow{450-470K} \underset{\text{Pure Nickel}}{Ni} + 4CO$$

3.(3) At low pressure b = 0

The vander waal equation reduced to

$$\left(P + \frac{a}{V^2}\right)V = RT$$

$$PV + \frac{aV}{V^2} = RT$$

Divide whole equation by RT

$$\frac{PV}{RT} + \frac{a}{RTV} = \frac{RT}{RT}$$

$$Z = 1 - \frac{a}{RTV}$$

4.(4) Narrow spectrum antibiotics – Penicillin G.

Antiseptic – Furacin

Disinfectant – Sulphur dioxide

Broad spectrum antibiotic – Chloramphenicol

5.(1) Positive nitrogen in lassaigne's test C – N Bond should be present

$CH_3NH_2.HCl$

6.(3) Borax Bead Test

$$Na_2B_4O_7,10H_2O \xrightarrow{\Delta} Na_2B_4O_7 \xrightarrow{\Delta} NaBO_2 + B_2O_3$$

$$B_2O_3 + CuSO_4 \longrightarrow \underset{\text{Blue green}}{Cu(BO_2)_2}$$

7.(1) $$4NH_3 + 5O_2 \longrightarrow 4\underset{A}{NO} + 6H_2O$$

$$2NO + O_2 \longrightarrow 2NO_2$$

$$2\underset{B}{NO_2} + H_2O \longrightarrow HNO_3 + HNO_2$$

8.(4) Standard reduction potential of hydrogen in 0.0V so metal ion which has less standard reduction potential then hydrogen liberate hydrogen from dilute acids are:

V^{2+} and Cr^{2+}

9.(4)

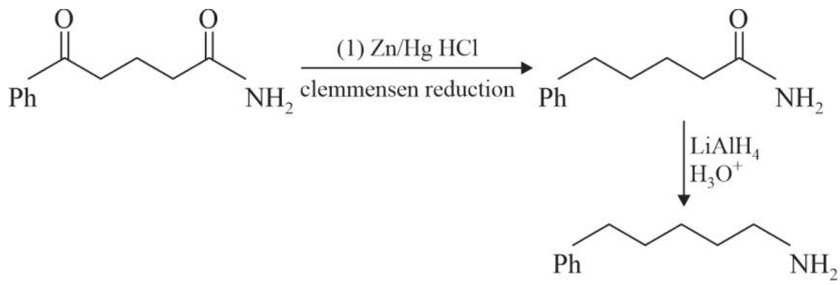
Li_2O	Na_2O_2	KO_2
↓	↓	↓
Lithium monooxide	Sodium peroxide	Potessium superoxide
O^{2-}	O_2^{2-}	O_2^{-}

Acc to molecular orbital theory
Monooxide = Diamagnetic
Peroxide = Diamagnetic
Saperoxide = Paramagnetic

10.(1) $\text{cis}-\left[PtCl_2(en)_2\right]^{2+}$

No symmetry optically active.

11.(3)



12.(3) $-ve$ sol in formed, according to the hardy Schulze Rule. Most coagulation carry the maximum positive charge so for negative sol., $AlCl_3$ is best for coagulation.

13.(3) Shortest wavelength for hydrogen atom in lyman series.

$$\frac{1}{\lambda_1}=R\left\{\frac{1}{n_1^2}-\frac{1}{n_2^2}\right\}z^2$$

$n_1=1$

$n_2=2$

$$\frac{1}{\lambda_1}=R\left\{\frac{1}{1^2}-\frac{1}{\infty^2}\right\}\times 1^2 \Rightarrow \frac{1}{\lambda_1}=R$$

Longest wavelength for Balmer series in He^+

$$\frac{1}{\lambda_2}=R\left\{\frac{1}{n_1^{\,2}}-\frac{1}{n_2^{\,2}}-\frac{1}{n_2^{\,2}}\right\}z^2$$

$n_1=2$

$n_2=3$

$$\frac{1}{\lambda_2}=R\left\{\frac{1}{2^2}-\frac{1}{3^2}\right\}\times 4$$

$$\frac{1}{\lambda_2}=\frac{5R}{9}$$

Ratio of $\lambda_1:\lambda_2$

$$\frac{\lambda_1}{\lambda_2}=\frac{\frac{1}{R}}{\frac{5}{9R}}=\lambda_2=\frac{9}{5}\lambda_1$$

$$\lambda_2=\frac{9}{5}\lambda$$

14.(4) Hydration enthalpy $\propto \dfrac{1}{\text{size of cation}}$

$C > E > A > B > D$

$Mg^{2+} > Ca^{2+} > K^{+} > Rb^{+} > Cs^{+}$

15.(1) Bond dissociation energy of halogen molecule

$Cl_2 > Br_2 > F_2 > I_2$

Maximum bond dissociation energy = Cl_2 (chlorine)

16.(2) Let A $NH_2-H_2C-\overset{\overset{\displaystyle O}{\|}}{C}-OH$

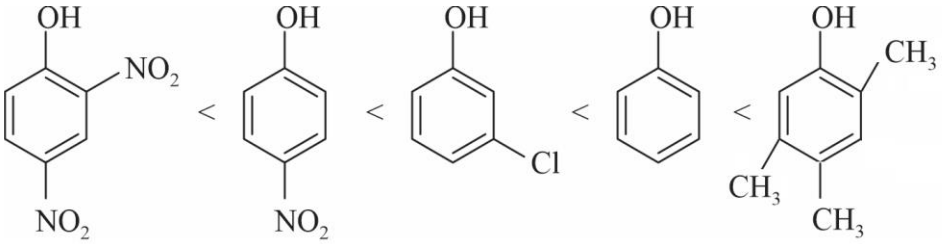
B $NH_2-(CH_2)_2-\overset{\overset{\displaystyle O}{\|}}{C}-OH$

No. of possible arrangements of cyclic tripeptides

AAB, ABA, BBA, BAB

17.(3)

18.(4) $PKa \propto \dfrac{1}{\text{acidity}}$



19.(2)

20.(4) Photochemical smog has high concentration of oxidizing agents and it can be controlled by controlling the release of NO_2, O_3 etc.

SECTION – 2

21.(13) For $Pb(NO_3)_2$: $Pb(NO_3)_2 \rightarrow Pb^{2+} + 2NO_3^-$

$\Delta T_b = i \times K_b \times m$

$0.15 = 3 \times 0.5 \times m$

$m = 0.1$

$$Pb(NO_3)_2 + 2NaCl \rightarrow PbCl_2 + 2NaNO_3$$

	$Pb(NO_3)_2$	$2NaCl$	$PbCl_2$	$2NaNO_3$
Initial concen.	0.1	0.2	0	0
After Reaction	0	0	0.1	0.2

$\Delta T_f = i \times K_f \times m$

$0.8 = (3s + 0.4) \times 1.8$

[S = Solubility of $PbCl_2$]

$\Rightarrow$ $S = 0.0148$

$K_{sp} = 4S^3$

$= 4 \times (0.0148)^3$

$=13\times10^{-6}$

22.(6) $K_c = 10^3/10^2 = 10$

$\Delta G^\circ = -RT\log K_c = -8.3\times300\times\ell n10\,J = -6\,kJ$

23.(2) The molar conductivity at infinite dilution of strong electrolyte can be obtained by extrapolating the curve $\wedge m$ versus $\sqrt{c}$.

The degree of dissociation approaches to one at infinite dilution.

Statements A and C are incorrect.

24.(2) For Reaction: x $\rightarrow$ y

Initial conc. C_o O

At time t $C_o - C$ C

Now $C_t = C_o\, e^{-kt}$

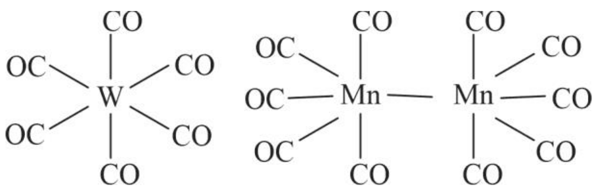
$(C_o - C) = C_o\, e^{-kt}$

$C = C_o\left(1 - e^{-kt}\right)$

Statements B and D are correct

25.(6) $R_f = \dfrac{\text{Distance travelled by solute}}{\text{Distance travelled by solvent}} = \dfrac{3}{5} = 6\times10^{-1}$

26.(0)



27.(3) Moles of hydrocarbon = $\dfrac{17\times10^{-3}}{136} = 0.125\times10^{-3}$

Moles of $H_2(g) = \dfrac{8.4}{22400} = 0.375\times10^{-3}$

Moles of hydrocarbon : moles of H_2

$1:3$

Therefore no. of double bond = 3 in hydrocarbon.

28.(3) ICl_4^-, BrF_3, NO_2^+ do not have odd no. of electrons.

29.(2)

$H_2O \rightleftharpoons H_2 + \dfrac{1}{2}O_2$

$1-\alpha \quad \alpha \quad \dfrac{\alpha}{2}$

$$K_c = \frac{\alpha.\left(\frac{\alpha}{2}\right)^{1/2}}{(1-\alpha)}$$

$\Rightarrow \quad 2\times10^{-3} = \dfrac{(\alpha)^{3/2}}{\sqrt{2}} \Rightarrow (\alpha)^{3/2} = 2\sqrt{2}\times10^{-3}$

$\Rightarrow \quad (\alpha)^3 = 8\times10^{-6} \Rightarrow \quad \alpha = 2\times10^{-2}$

30.(5) $\left[\overset{-}{O}H\right] = 10^{-2}$

$\left[Ca(OH)_2\right] = \frac{1}{2}\times10^{-2} = \frac{\text{moles of } Ca(OH)_2}{0.1}$

$\Rightarrow \quad \text{Moles of } Ca(OH)_2 = 0.5\times10^{-3}$

$\Rightarrow \quad \text{m moles of } Ca(OH)_2 = 0.5 = 5\times10^{-1}$

MATHEMATICS

SECTION – 1

1.(3) $2^2+2p+q=0 \qquad q+4=-2p$

$$L=\lim_{x\to 2p^+}\frac{1-\cos\left(x^2-4px+q^2+8q+16\right)}{(x-2p)^4}$$

$$\lim_{x\to 2p^+}\frac{1-\cos\left[(x-2p)^2+(q+4)^2-4p^2\right]}{(x-2p)^4}$$

Now, $x\to 2p+h$

$$\Rightarrow \quad L=\lim_{h\to 0}\frac{1-\cos\left[h^2+4p^2-4p^2\right]}{h^4}$$

$$\Rightarrow \quad L=\lim_{h\to 0}\frac{2\sin^2\left(h^2/2\right)}{h^4}=\lim_{h\to 0}2\left(\frac{\sin^2 h^2/2}{4.\left(\frac{h^2}{2}\right)^2}\right)=\frac{2}{4}=\frac{1}{2}$$

$[L]=[1/2]=0$

2.(4)

3.(3) $y=f(x), y(1)=e$

$y(x+1)dx-x^2dy=0$

$y(x+1)dx=x^2dy$

$\frac{x+1}{x^2}dx=\frac{1}{y}dy$

$\left(\frac{1}{x}+\frac{1}{x^2}\right)dx=\frac{1}{y}dy$

$\ell nx-\frac{1}{x}=\ell ny+c$

$\Rightarrow -1=1+c\Rightarrow c=-2 \ \{\because y(1)=e\}$

$\ell nx-\frac{1}{x}=\ell ny-2$

$y=e^{\ell nx}.e^{-1/x}.e^2$

$y=xe^{-1/x}.e^2$

$\lim_{x\to 0^+} xe^{-1/x}.e^2=0$

4.(2) $\begin{vmatrix} \lambda & \mu & 4 \\ -2 & 4 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 0$

$\lambda(4+6) - \mu(-2+4) + 4(-6-8) = 0$

$10\lambda - 2\mu = 56$

$5\lambda - \mu = 28$ ...(i)

Now, $\frac{\vec{a}.\vec{b}}{|\vec{b}|} = \sqrt{54}$

$\Rightarrow \left|\frac{-2\lambda + 4\mu - 8}{\sqrt{4+16+4}}\right| = \sqrt{54}$

$\Rightarrow 4\mu - 2\lambda = 8 + 2\sqrt{6}.3\sqrt{6}$

$\Rightarrow 2\mu = \lambda + 22$ ...(ii)

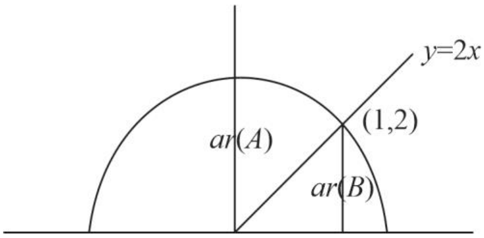
Now from equation (i) and (ii)

$\mu = \frac{46}{3}, \lambda = \frac{26}{3}$

$\therefore \lambda + \mu = \frac{26}{3} + \frac{46}{3} = \frac{72}{3}$

= 24

5.(3)



$[r = 2]$

$\frac{area(A)}{area(B)} = \frac{\frac{\pi}{2}r^2 - ar(B)}{ar(B)}$

$= \frac{2\pi - ar(B)}{ar(B)}$ ...(i)

$ar(B) = \frac{1}{2} \times 1 \times 2 + \int_1^3 \sqrt{4-(x-1)^2}\,dx$

$= 1 + \left(\frac{x-1}{2}\sqrt{4-(x-1)^2} + \frac{4}{2}\sin^{-1}\left(\frac{x-1}{2}\right)\right)_1^3$

$= 1 + \left(0 + \frac{4}{2}.\frac{\pi}{2}\right) - (0+0)$

$= 1 + \pi$

$\therefore \frac{area(A)}{area(B)} = \frac{2\pi - 1 - \pi}{1+\pi} = \frac{\pi - 1}{\pi + 1}$

6.(4) $f(\theta)=3\left(\sin^4(3\pi/2-\theta)+\sin^4(3\pi+\theta)\right)-2\left(1-\sin^2 2\theta\right)$

$f(\theta)=3\left(\cos^4\theta+\sin^4\theta\right)-2\left(1-\sin^2 2\theta\right)$

$f(\theta)=3-\frac{3}{2}\sin^2 2\theta-2+2\sin^2\theta$

$f(\theta)=1+\frac{\sin^2 2\theta}{2}$

$f'(\theta)=\frac{2\sin(2\theta)(\cos 2\theta).2}{2}=\sin 4\theta=-\frac{\sqrt{3}}{2}$

$4\theta=\pi+\pi/3, 2\pi-\pi/3, 3\pi+\pi/3, 4\pi-\pi/3$

$\theta=\pi/3, 5\pi/12, 5\pi/6, 11\pi/12$

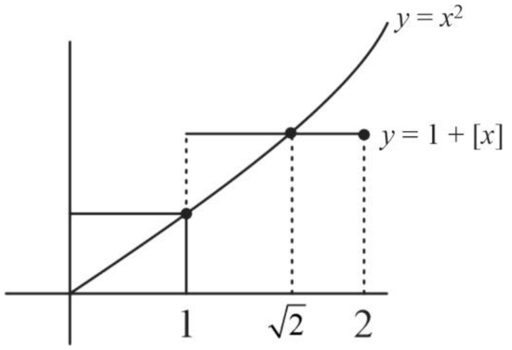
$4\beta=\frac{\pi}{3}+\frac{5\pi}{12}+\frac{5\pi}{6}+\frac{11\pi}{12}=\frac{30\pi}{12}=\frac{5\pi}{2}$

$\beta=\frac{5\pi}{8}$

$f(\beta)=1+\frac{\sin^2\left(\frac{5\pi}{4}\right)}{2}=1+\frac{\frac{1}{2}}{2}=1+\frac{1}{4}$

$=\frac{5}{4}$

7.(1)



$f(x)=\begin{cases} 1+[x] & 0\le x\le\sqrt{2} \\ x^2 & \sqrt{2}<x\le 2 \end{cases}$

$\Rightarrow \int_0^2 f(x)dx=\int_0^{\sqrt{2}}(1+[x])dx+\int_{\sqrt{2}}^2 x^2 dx$

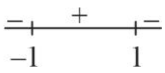
$=\int_0^1 1dx+\int_1^{\sqrt{2}}2dx+\left[\frac{x^3}{3}\right]_{\sqrt{2}}^2$

$=1+2\left(\sqrt{2}-1\right)+\frac{1}{3}\left(8-2\sqrt{2}\right)$

$=\frac{4\sqrt{2}+5}{3}$

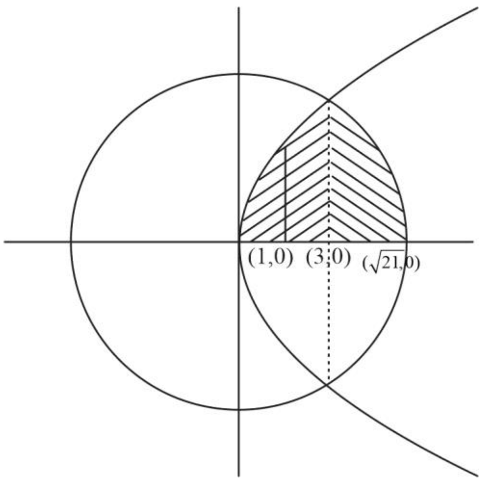
8.(1) $f(x)=\frac{x^2+2x+1}{x^2+1}=1+\frac{2x}{x^2+1}$

$$f'(x)=\frac{2\left(x^2+1\right)-2x(2x)}{\left(x^2+1\right)^2}=\frac{2-2x^2}{\left(x^2+1\right)^2}=\frac{-2\left(x^2-1\right)}{\left(x^2+1\right)^2}$$



Therefore $f(x)$ is one-one in $[1,\infty)$ but not in $(-\infty,\infty)$

9.(1)



$x^2+y^2=21$ ...(i)

$y^2=4x$ ...(ii)

From these two equations

$x=3$

Now, $A_1=2\int_1^3 2\sqrt{x}dx$

$=4\left[\frac{2}{3}\left(x^{3/2}\right)_1^3\right]$

$=\frac{8}{3}\left(3\sqrt{3}-1\right)$

$$A_2=2\int_3^{\sqrt{21}}\sqrt{21-x^2}=2\left[\frac{1}{2}x\sqrt{21-x^2}+\frac{1}{2}\times 21\sin^{-1}\left(\frac{x}{\sqrt{21}}\right)\right]_3^{\sqrt{21}}$$

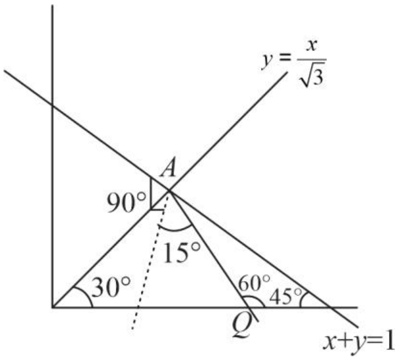
$=0+21\times\frac{\pi}{2}-3\times 2\sqrt{3}-21\sin^{-1}\frac{\sqrt{3}}{\sqrt{7}}$

$=21\cos^{-1}\sqrt{3/7}-6\sqrt{3}=21\sin^{-1}2/\sqrt{7}-6\sqrt{3}$

$\Delta=A_1+A_2=\frac{8}{3}\left(3\sqrt{3}-1\right)+21\sin^{-1}2/\sqrt{7}-6\sqrt{3}$

$\frac{1}{2}\left(\Delta-21\sin^{-1}\frac{2}{\sqrt{7}}\right)=\sqrt{3}-\frac{4}{3}$

10.(4) Equation of line $y = mx$



$y = (\tan 30)x$

$y = \frac{x}{\sqrt{3}}$

$x + y = 1$ …(i)

For $A \to x + \frac{x}{\sqrt{3}} = 1$

$x = \frac{\sqrt{3}}{1+\sqrt{3}}$

$y = \frac{1}{1+\sqrt{3}}$

$A\left(\frac{\sqrt{3}}{1+\sqrt{3}}, \frac{1}{1+\sqrt{3}}\right)$

For line AQ, $y - \frac{1}{1+\sqrt{3}} = \sqrt{3}\left(x - \frac{\sqrt{3}}{1+\sqrt{3}}\right)$

At Q, $-\frac{1}{1+\sqrt{3}} = \sqrt{3}\left(x - \frac{\sqrt{3}}{1+\sqrt{3}}\right)$

$x = -\frac{1}{\sqrt{3}(1+\sqrt{3})} + \frac{\sqrt{3}}{1+\sqrt{3}}$

$= \frac{1}{1+\sqrt{3}}\left(-\frac{1}{\sqrt{3}} + \sqrt{3}\right)$

$= \frac{2}{\sqrt{3}(1+\sqrt{3})}$

$= \frac{\sqrt{3}-1}{\sqrt{3}}$

$= \frac{(1+\sqrt{3})(\sqrt{3}-1)}{\sqrt{3}(1+\sqrt{3})} = \frac{2}{3+\sqrt{3}}$

11.(3) $A^2 = 3A + \alpha I$

$A^4 = 21A + \beta I$

$A^4 = A^2.A^2 = (3A + \alpha I)(3A + \alpha I) = 9A^2 + 6\alpha A + \alpha^2 I = 21A + \beta I$

Again using $A^2 = 3A + \alpha I$

$9(3A + \alpha I) + 6\alpha A + \alpha^2 I = 21A + \beta I$

$(27 + 6\alpha)A + (9\alpha + \alpha^2)I = 21A + \beta I$

Now comparing, $27 + \alpha.6 = 21 \Rightarrow 6\alpha = -6 \Rightarrow \alpha = -1$

$9\alpha + \alpha^2 = \beta$ ...(ii)

$\beta = 9 \times (-1) + 1 = -8$

12.(3) Let $z_1 = a + ib$ and $z_2 = c + id$

$\text{Re}(z_1 z_2) = 0$ (given)

$ac - bd = 0$

$ac = bd$ ...(i)

$\text{Re}(z_1 + z_2) = 0$

$a + c = 0$ ...(ii)

$c = -a$ ...(iii)

From equation (i) and (iii)

$bd = -a^2 < 0$ (which is less than zero)

Therefore we can say imaginary part of z_1 & z_2 are of opposite sign.

13.(3) $f(x) = x + \frac{a}{a^2 - 4}\sin x + \frac{b}{a^2 - 4}\cos x$

$$x + \frac{a}{a^2 - 4}\sin x + \frac{b}{a^2 - 4}\cos x = x + \int_0^{a/2} \sin(x + 4) f(y)\,dy$$

$$\frac{a\sin x + b\cos x}{a^2 - 4} = \int_0^{a/2} \sin(x + y) f(y)\,dy$$

$$\frac{a\sin x + b\cos x}{a^2 - 4} = \int_0^{a/2} (\sin x\cos y + \cos x\sin y) + (y)\,dy$$

$$\frac{a\sin x + b\cos x}{a^2 - 4} = \sin x\int_0^{a/2} \cos y\, f(y)\,dy + \cos x\int_0^{a/2} \sin y\, f(y)\,dy$$

$$\frac{a}{a^2 - 4} = \int_0^{a/2} \cos y\, f(y)\,dy; \frac{b}{a^2 - 4} = \int_0^{a/2} \sin y\, f(y)\,dy$$

$$\frac{b}{\pi^2 - 4} = \int_0^{\pi/2}\left[y\sin y + \frac{a}{\pi^2 - 4}\left(\frac{1 - \cos 2y}{2}\right) + \frac{b}{\pi^2 - 4}\left(\frac{\sin 2y}{2}\right)\right]dy$$

$$= \left(y(-\cos y) + \sin y + \frac{a}{2(\pi^2-4)}\left(y - \frac{\sin 2y}{2} \right) - \frac{b}{2(\pi^2-4)} \frac{\cos 2y}{2} \right)_0^{\pi/2}$$

$$\frac{b}{\pi^2-4} = \left[1 + \frac{a\pi}{4(\pi^2-4)} + \frac{2b}{4(\pi^2-4)} \right]$$

$$\frac{b}{2(\pi^2-4)} = 1 + \frac{a\pi}{4(\pi^2-4)}$$

$$2b - a\pi = 4(\pi^2-4) \qquad \ldots(\text{i})$$

$$\frac{a}{\pi^2-4} = \int_0^{\pi/2} \left[y\cos y + \frac{a}{\pi^2-4}\frac{\sin 2y}{2} + \frac{b}{\pi^2-4}\left(\frac{1+\cos 2y}{2} \right) \right] dy$$

$$= y(\sin y) + \cos y + \frac{a}{2(\pi^2-4)}\left(\frac{-\cos 2y}{2} \right) + \frac{b}{2(\pi^2-4)}\left(y + \frac{\sin 2y}{2} \right) \Bigg]_0^{\pi/2}$$

$$= \left[\frac{\pi}{2} + \frac{a}{4(\pi^2-4)} + \frac{b\pi}{4(\pi^2-4)} \right] - \left[1 - \frac{a}{4(\pi^2-4)} \right]$$

$$\frac{a}{2(\pi^2-4)} = \left(\frac{\pi}{2} - 1 \right) + \frac{b\pi}{4(\pi^2-4)}$$

$$2a - b\pi = 2(\pi-2)(\pi^2-4) \qquad \ldots(\text{ii})$$

By equation (i) and (ii)

$$2b = -8\pi(\pi-1) + 4(\pi^2-4)$$

$$2b = 4\pi^2 + 8\pi - 16$$

$$b = -2\pi^2 + 4\pi - 8 \text{ and } a = -4(2\pi-2)$$

$$\therefore \qquad a + b = -2\pi^2 + 4\pi - 8 - 8\pi + 8$$

$$a + b = -2\pi(\pi+2)$$

14.(2) $\alpha x + 2y + z = 1$

$$2\alpha x + 3y + z = 1$$

$$3x + \alpha y + 2z = \beta$$

$$\Delta = \begin{bmatrix} \alpha & 2 & 1 \\ 2\alpha & 3 & 1 \\ 3 & \alpha & 2 \end{bmatrix} = 0$$

$$\alpha(6-\alpha) - 2(4\alpha-3) + 1(2\alpha^2-9) = 0$$

$$6\alpha - \alpha^2 - 8\alpha + 6 + 2\alpha^2 - 9 = 0$$

$\alpha^2-2\alpha-3=0$

$\alpha^2-3\alpha+\alpha-3=0$

$\alpha(\alpha-3)+1(\alpha-3)=0$

$\alpha=3,-1$

$$\Delta_1=\begin{bmatrix}1&2&1\\1&3&1\\\beta&\alpha&2\end{bmatrix}$$

$=1(c-\alpha)-2(2-\beta)+1(\alpha-3\beta)$ ...(i)

$$\Delta_2=\begin{bmatrix}\alpha&1&1\\2\alpha&1&1\\3&\beta&2\end{bmatrix}$$

$=\alpha(2-\beta)-1(2\alpha-3)+1(2\alpha\beta-3)$ ...(ii)

$$\Delta_3=\begin{bmatrix}\alpha&2&1\\2\alpha&3&1\\3&\alpha&\beta\end{bmatrix}=\alpha(3\beta-\alpha)-2(2\alpha\beta-3)+1\left(2\alpha^2-9\right)$$

For $\alpha=-1\ \&\ \beta=2$

$\Delta_1=\Delta_2=\Delta_3=0$

$\therefore$ Incorrect statement is it has no solution for $\alpha=-1$ and for all $\beta\in R$

15.(2)

$x\to 0$	1	2	3
$p(x)\to\dfrac{{}^7C_4}{{}^{10}C_4}$	$\dfrac{{}^3C_1\times{}^7C_4}{{}^{10}C_4}$	$\dfrac{{}^3C_2\times{}^7C_2}{{}^{10}C_4}$	$\dfrac{{}^7C_1}{{}^{10}C_4}$

$\mu=\sum xiPi$

$\mu=\frac{6}{5}$ (by calculating)

Variance = $V=\sum x_i^2Pi-(\mu)^2$

$=\sum x_i^2Pi-\frac{36}{25}$

$\sigma^2=\frac{14}{25}$

Then $10\left(\mu^2+6^2\right)=10\left(\frac{36}{25}+\frac{14}{25}\right)=\frac{10\times 50}{25}=20$

16.(4) Tangent at (x_1,y_1)

$$xx_1+yy_1-3\left(\frac{x+x_1}{2}\right)+5(y+y_1)-15=0$$

Tangent at $(4,-11)$

$$4x-11y-\frac{3}{2}(x+4)+5(y-11)-15=0$$

$$\frac{5}{2}x-6y-6-55-15=0$$

$$\frac{5x}{2}-6y-76=0$$

$$5x-12y-152=0 \qquad \ldots\text{(i)}$$

Now, Tangent at $(8,-5)$

$$8x-5y-\frac{3}{2}(x+8)+5(y-5)-15=0$$

$$\frac{13x}{2}-12-25-15=0$$

$$x=8$$

AB : $y+11=\frac{6}{4}(x-4)$

$4y+44=6x-24$

$6x-4y=68 \Rightarrow 3x-2y-34=0$

Now, at $x=8$ $40-12y-152=0$

$12y=-112$

$y=-28/3$

$C(8,-28/3)$

$$r=\frac{|24+56/3-34|}{\sqrt{13}}=\frac{|56-30|}{3\sqrt{13}}=\frac{2\sqrt{13}}{3}$$

17.(2) $f(x)=\dfrac{\log_{(x+1)}(x-2)}{e^{2\log_e x}-(2x+3)}=\dfrac{\log_{(x+1)}(x-2)}{x^2-2x-3}$

Now $x+1\neq 1 \Rightarrow x\neq 0$

$x-2>0 \Rightarrow x>2$

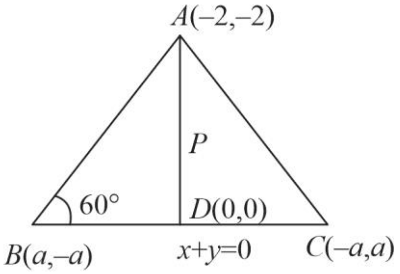
$x+1>0 \Rightarrow x>-1$

$x^2-2x-3\neq 0 \Rightarrow (x-3)(x+1)\neq 0$

$\Rightarrow \quad x\neq 3,-1$

$\therefore \quad x\in(2,\infty)-\{3\}$

18.(1) Let co-ordinates of B and C are



$(a,-a)\,\&\,(-a,a)$ respectively

And A lies on the perpendicular bisector of BC.

The co-ordinate of $D(0,0)$

Equation of line $AD \Rightarrow y=x$ and A passing through line

$y-2x=2$,

Therefore $A(-2,-2)$

$$P=AD=\left|\frac{-2-2}{\sqrt{2}}\right|=2\sqrt{2}$$

$$\Delta ABC=\frac{P^2}{\sqrt{3}}=\frac{8}{\sqrt{3}}$$

19.(1) If $\{(p\vee q)\wedge(\sim p)\vee r\}\to\{(-q)\vee r\}$ is false (fallacy)

Then $\{(p\vee q)\wedge(\sim p)\vee r\}$ must be true and $\{(-q)\vee r\}$ must be false.

$\therefore$ $(\sim q)\vee r$ is false then $\Rightarrow$ $\sim q$ is false and r is false

Means q is true and r is false ...(i)

$\{(p\wedge q)\wedge((\sim p)\vee r)\}$ is true means $(p\vee q)\&(\sim p\vee r)$

Both must be true

$(\sim p\vee r)\Rightarrow$ true then p is false

$p\to$ false

$q\to$ true

$r\to$false

20.(1) $14x^2-31x+3\lambda=0;\alpha,\beta$

$35x^2-53x+4\lambda=0;\alpha,\gamma$

For common root α

$$\frac{\alpha^2}{-124\lambda+159\lambda}=\frac{-\alpha}{56\lambda-105\lambda}=\frac{1}{-742+1085}$$

$$\frac{\alpha^2}{35\lambda}=\frac{\alpha}{49\lambda}=\frac{1}{343}$$

$$\alpha=\frac{49\lambda}{343}=\frac{\lambda}{7};\alpha^2=\frac{35\lambda}{343}=\frac{5\lambda}{49}$$

$$\frac{\lambda^2}{49}=\frac{5\lambda}{49}\Rightarrow\lambda=0,5$$

$\lambda\neq 0$

Now, if $\lambda=5$ then

$$\alpha=\frac{5}{7},\beta=\frac{3}{2},\gamma=\frac{4}{5}$$

Then $\frac{3\alpha}{\beta}=3\times\frac{5}{7}\times\frac{2}{3}=\frac{10}{7}$

$$\frac{4\alpha}{\gamma}=4\times\frac{5}{7}\times\frac{5}{4}=\frac{25}{7}$$

Equation x^2-x (sum of roots) + product of roots = 0

$49x^2-245x+250=0$

SECTION – 2

21.(2) $\vec{A}=\vec{a}-\vec{b}+\vec{c}$

$\vec{B}=\lambda\vec{a}-3\vec{b}+4\vec{c}$

$\vec{C}=-\vec{a}+2\vec{b}-3\vec{c}$

$\vec{D}=2\vec{a}-4\vec{b}+6\vec{c}$

$\overrightarrow{AB}=(\lambda-1)\vec{a}-2\vec{b}+3\vec{c}$

$\overrightarrow{AC}=-2\vec{a}+3\vec{b}-4\vec{c}$

$\overrightarrow{AD}=\vec{a}-3\vec{b}+5\vec{c}$

AB, *AC* and *AD* are coplanar,

$$\begin{vmatrix} \lambda-1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix}=0$$

$(\lambda-1)(15+12)+2(-10+4)+3(6-3)=0$

$(\lambda-1)3-12+9=0$

$3(\lambda-1)=3$

$\lambda=2$

22.(32)

Digit = $1,2,3,4,5,6,7,8,9$

$$\left.\begin{array}{l} \underline{1}\,\underline{2}\,\underline{3}\,\_\_\_ \Rightarrow {}^6C_3=20 \\ \underline{1}\,\underline{2}\,\underline{4}\,\_\_\_ \Rightarrow {}^5C_3=10 \\ \underline{1}\,\underline{2}\,\underline{5}\,\_\_\_ \Rightarrow {}^4C_3=4 \\ \underline{1}\,\underline{2}\,\underline{6}\,\underline{7}\,\underline{8}\,\underline{9} \Rightarrow 1 \end{array}\right] 35$$

$$\left.\begin{array}{l} 134\,\_\_\_ \Rightarrow {}^5C_3=10 \\ 135\,\_\_\_ \Rightarrow {}^4C_3=4 \\ \underline{1}\,\underline{3}\,\underline{6}\,\underline{7}\,\underline{8}\,\underline{9} \Rightarrow 1=1 \end{array}\right] 15$$

$$\left.\underline{1}\,\underline{4}\,\underline{5}\,\_\_\_ \Rightarrow {}^4C_3=4\right] \rightarrow 4$$

$$\left.\begin{array}{l} \underline{1}\,\underline{4}\,\underline{6}\,\underline{7}\,\underline{8}\,\underline{9} \Rightarrow 1=1 \\ \underline{1}\,\underline{5}\,\underline{6}\,\underline{7}\,\underline{8}\,\underline{9} \Rightarrow 1 \end{array}\right] \rightarrow 2$$

$\underline{2}\,\underline{3}\,\underline{4}\,\_\_\_ \Rightarrow {}^5C_3=10$

$235\,\_\_\_ \Rightarrow {}^4C_3=4$

71^{th} number 2 3 6 7 8 9

72^{th} number 2 4 5 6 7 8

Sum of digit $\Rightarrow$ $2+4+5+6+7+8=32$

23.(1436) Digits, $\rightarrow$ 1, 2, 3, 5, 7

Leading digit 5 or 7 $\Rightarrow ------\Rightarrow 2\times5^4=1250$

$37---\Rightarrow 5^3=125$

$357---\Rightarrow 5^2=25$

$355--\Rightarrow 5^2=25$

$353\_\_\Rightarrow 10$

$35337\Rightarrow 1$

$1250+125+50+10+1=1436$

24.(355) $L=\frac{x+10}{1}=\frac{y-8}{-2}=\frac{z-0}{1}$

$p=a(x-2)+b(y-2)+3z=0$

$\therefore$ Plane containing the line,

$a(-10-2)+b(8-2)+0=0$

$-12a+6b=0$

$a=\frac{b}{2}$ ...(i)

Also, $a-2b+3=0$

$a-4a+3=0$

$a=1;b=2$

Now equation of plane, $(x-2)+(2y-4)+3z=0$

$x+2y+3z-6=0$ ...(ii)

Perpendicular distance, $C=\frac{|1+54+21-6|}{\sqrt{1+4+9}}=\frac{70}{\sqrt{14}}=5\sqrt{14}$

$a^2+b^2+c^2=1+4+25\times 14=355$

25.(60) Let first term a; common ration = r

$a_4=ar^3, a_6=ar^5, a_5=ar^4, a_7=ar^6$

Given $ar^3.ar^5=9$ and $ar^4+ar^6=24$

$a^2r^8=9$ $\quad$ $ar^4(1+r^2)=24$

$ar^4=3$ ...(ii) $\quad$ $1+r^2=8$

$r^2=7$ ...(i)

$\therefore$ $a=\frac{3}{49}$

Value of $a_1a_9+a_2a_4a_9+a_5+a_7$

$aar^8+arar^3ar^8+ar^4+ar^6$

$a^2r^8+a^3+r^{12}+ar^4+ar^6$

$\frac{3^2}{(49)^2}\times 7^4+\frac{3^3}{49^3}\times(7)^6+3+\frac{3}{49}\times 7^3$

$9+27+3+21=60$

26.(3) $f(x+y)=f(x)+f(y)-1$ ...(i)

$\forall, x, y \in R$

Put $x=0, y=0$

$f(0+0)=f(0)+f(0)-1$

$f(0)=1$ ...(ii)

Differentiate equation (i) with respect to *x*,

$f'(x+y)=f'(x)$

$f'(y)=f'(0)$

$f'(y)=2$

$f(y)=2y+c$

$\therefore \quad f(x)=2x+c$

$x=0$

$c=1$

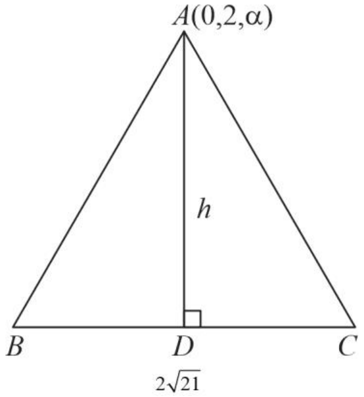
$f(x)=2x+1$

$f(-2)=-4+1$

$|f(-2)|=3$

27.(9) Equation of $BC: \dfrac{x+\alpha}{5}=\dfrac{y-1}{2}=\dfrac{z+4}{3}$

Area of triangle = $\dfrac{1}{2}\times base \times height$



$21=\dfrac{1}{2}\times 2\times\sqrt{21}\times h$

$h=\sqrt{21}$

$\therefore$ Point of D are $5\lambda-\alpha, 2\lambda+1, 3\lambda-4$

$AD=5\lambda-\alpha-0, 2\lambda+1-2, 3\lambda-4-\alpha$

$\Rightarrow \quad 5\lambda-\alpha, 2\lambda-1, 3\lambda-4-\alpha$

$\therefore \quad 5(5\lambda-\alpha)+2(2\lambda-1)+3(3\lambda-4-\alpha)=0$

$25\lambda-5\alpha+4\lambda-2+9\lambda-12-3\alpha=0$

$19\lambda-4\alpha-7=0$

$$\lambda = \frac{4\alpha+7}{19}$$

Now point of $D\left(\frac{\alpha+35}{19}, \frac{8\alpha+33}{99}, \frac{12\alpha-55}{19}\right)$

$A(0,2,\alpha)$

Now, $\left(\frac{\alpha+35}{19}\right)^2 + \left(\frac{8\alpha+33}{19}-2\right)^2 + \left(\frac{12\alpha-55}{19}-\alpha\right)^2 = 21$

$(\alpha+35)^2 + (8\alpha-5)^2 + (7\alpha+55)^2 = 21 \times 361$

$114\alpha^2 + 760\alpha + 4275 - 7581 = 0$

$114\alpha^2 + 760\alpha - 3306 = 0$

$6\alpha^2 + 40\alpha - 1742 = 0$

$\alpha = \frac{-20 \pm \sqrt{400+1044}}{6} = \frac{-20 \pm 38}{6} = -\frac{58}{6} \, or \, 3$

$\alpha = 3$

$\alpha^2 = 9$

28.(1120) Let the term are T_r, T_{r+1}, T_{r+2}

$T_r = {}^nC_{r-1}(2x)^{r-1} \Rightarrow {}^nC_{r-1}(2)^{r-1}$ ...(i)

$T_{r+1} = {}^nC_r(2x)^r \Rightarrow {}^nC_r 2^r$ ...(ii)

$T_{r+2} = {}^nC_{r+1}(2x)^{r+1} \Rightarrow {}^nC_{r+1}(2)^{r+1}$ ...(iii)

Now, $\frac{{}^nC_{r-1}2^{r-1}}{{}^nC_r r^r} = \frac{2}{5} \Rightarrow \frac{\frac{\lfloor n}{\lfloor r-1 \lfloor n-r+1}}{\frac{\lfloor n}{\lfloor r \lfloor n-r}} \times \frac{1}{2} \Rightarrow \frac{r}{n-r+1} = \frac{4}{5}$

$5r = 4n - 4r + 4$

$9r = 4n + 4$ ...(iv)

$\frac{{}^nC_r 2^r}{{}^nC_{r+1}(2)^{r+1}} = \frac{5}{8}$

$\frac{\frac{\lfloor n}{\lfloor r \lfloor n-r}}{\frac{\lfloor n}{\lfloor r+1 \lfloor n-r-1}} \times \frac{1}{2} = \frac{5}{8}$

$\frac{r+1}{n-r} = \frac{5}{4}$

$4r + 4 = 5n - 5r$

$9r = 5n - 4$ ...(v)

From equation (iv) and (v)

$n = 8$

$r = 4$

Hence ${}^8C_4 . 2^4 = 1120$

29.(10)

$f(x+y)=f(x)+f(y)\forall x,y\in N$

$f(1+1)=2f(1)=2\times\frac{1}{5}$

$f(3)=f(2)+f(1)=2\times\frac{1}{5}+\frac{1}{5}=3\times\frac{1}{5}$

Now, $f(x)=\left(x\times\frac{1}{5}\right)$

$\sum_{n=1}^{m}\frac{n/5}{n(n+1)(n+2)}=\frac{1}{12}$

$\frac{1}{5}\sum_{n=1}^{m}\left[\frac{1}{n+1}-\frac{1}{n+2}\right]=\frac{1}{12}$

$=\frac{1}{5}\left[\frac{1}{2}-\frac{1}{m+2}\right]=\frac{1}{12}$

$\frac{1}{5}\left[\frac{m+2-2}{2(m+2)}\right]=\frac{1}{12}$

$12m=10(m+2)$

$6m=5m+10$

$m=10$

30.(1) Coefficient of x^9 in $\left(\alpha x^3+\frac{1}{\beta x}\right)^{11}={}^{11}C_r\left(\alpha x^3\right)^{11-r}\left(\frac{1}{\beta x}\right)^r$

$={}^{11}C_r\alpha^{11-r}.x^{33-3r}.\beta^{-r}.x^{-r}$

Hence, $33-4r=9$

$-4r=-24$

$r=6$

Coefficient of x^9 is ${}^{11}C_6\alpha^5\beta^{-6}$ …(i)

Coefficient of x^{-9} in $\left(\alpha x-\frac{1}{\beta x^3}\right)^{11}$

${}^{11}C_r(\alpha x)^{11-r}(-1)^r(\beta)^{-r}\left(x^{-3r}\right)$

$(-1)^r\,{}^{11}C_r\alpha^{11-r}x^{11-r-3r}\beta^{-r}$

$11-4r=-9\quad\Rightarrow\quad r=5$

Coefficient of x^{-9} is = $-{}^{11}C_5\alpha^6\beta^{-5}$

${}^{11}C_6\alpha^5\beta^{-6}=-{}^{11}C_5\alpha^6\beta^{-5}\Rightarrow\alpha\beta=-1$

$(\alpha\beta)^2=1$