VMC Vidyamandir Classes SINCE 1986

IIT JEE | MEDICAL | FOUNDATION

SOLUTIONS

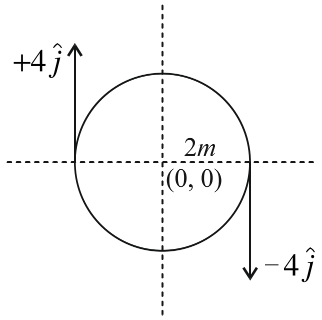
Joint Entrance Exam | IITJEE-2023

29th JAN 2023 | Evening Shift

PHYSICS

SECTION - 1

1.(4)



$x = 2m$, $\quad U_y = -4\hat{j}$

$x = -2m$, $\quad v = ?$, $\quad a = ?$

$$a = \frac{v^2}{R} = \frac{(4)^2}{2} \Rightarrow 8m/s^2$$

$v = +4\hat{j}$ $\quad$, $\quad a = 8\hat{i}\, m/s^2$

2.(4) $\lambda = \frac{h}{\sqrt{2mqv}}$

$$\lambda\alpha = \frac{1}{\sqrt{m.q}}$$

$$\frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{m_p.q_p}{m_\alpha \cdot q_\alpha}} = \sqrt{\frac{1}{8}}$$

$m = 8$

3.(1) $(x - At)^2 + \left(y - \frac{B}{t}\right)^2 = \text{Constant}$

$\therefore$ dim' of t is T^{-1}

dim' of x, y is L

$[A] = ?$ $\quad$ $[B] = ?$

$[At] = L$

$$[A] = \frac{L}{T^{-1}} = [LT]$$

$$y = \frac{t}{B} = L$$

$\frac{T^{-1}}{B} = L$ $\quad$ $B = L^{-1}T^{-1}$

4.(2) Work done by gravity = -ve (B)

Work done by Air Resist. = -ve (E)

5.(1) $y = \overline{A} \cdot B + A \cdot \overline{B}$

$= XOR$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

6.(2) $m = 5.4 \times 10^5 \, kg$

$A = 500 m^2$

$\rho = 1.2 kgm^{-3} = 16000.56 J$

$v = 1080 \, km/hr = 300 \, m/s$

$$\Delta P = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$v_2 - v_1 = \frac{2 \times \Delta P}{(v_1 + v_2) \cdot \rho}$$

$$\frac{(v_2 - v_1)}{v_{avg}} = \frac{\Delta P}{\rho \cdot v_{avg}} \Rightarrow$$

$$F = \Delta P.A = 5.4 \times 10^5 \times 10 = 5.4 \times 10^6$$

$$\Delta P = \frac{5.4 \times 10^6}{500} = \frac{v_2 - v_1}{v_{avg}} = \frac{\left(\frac{5.4 \times 10^6}{500}\right)}{1.2 \times 300^2} = \frac{10 \cdot 8 \times 10^3}{10 \cdot 2 \times 9 \times 10^4}$$

$$= 10^{-1} \times 100 = 10$$

7.(3) $m = \frac{600}{1000} = 0.6 kg$

$Q = MC\Delta T$

$= 0.6 \times 222 L \cdot 3 \times 12$

$= 16000.56 J$

Total heat = 184000 J

$Q = m.L$

$(18400 - 16000.56) = m(336000)$

$m = 0.5 \, kg$

$m = 500 \, g \rightarrow$ Water

Ice $600 - 500 = 100 \, gm$

Ice : Water = 1 : 5

8.(4) $N = 4$

$B = 32T$

$B = \dfrac{\mu_0 \cdot N \cdot I}{2 \cdot R} = 32$

$\left(\dfrac{\mu_0 I}{2R}\right) \times 4 = 32$

$\dfrac{\mu_0 I}{2R_i} = 8T$

$N = 4 \rightarrow N = 1$

$4 \times 2\pi R_i \rightarrow 2\pi R_i$

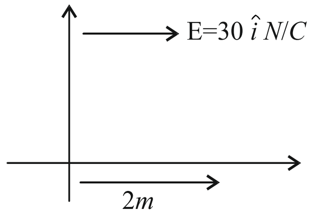
$R \rightarrow 4R_i$

$B' = \dfrac{\mu_0 I}{2 \cdot R} = \dfrac{\mu_0 \cdot I}{2 \cdot (4R_i)} = \dfrac{8}{4}$

$B' = 2T$

9.(3) $q = 2 \times 10^{-2} C$ $\quad\quad E = 30\hat{i}\ N/C$

$(1, 2, 0) \rightarrow (0, 0, 0)$

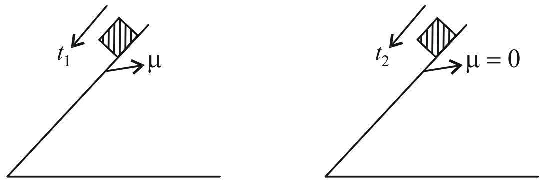


$W_c = -qEx$

$= 2 \times 10^{-2} \times 30 \times 1 = -60 \times 10^{-2} = -600 \times 10^{-3}$

$W_C = -600\ mJ$

10.(4) If $t_1 = nt_2, \quad \mu = ?$



$t_1 = nt_n$

$\sqrt{\dfrac{2S}{a_1}} = n \cdot \sqrt{\dfrac{2S}{a_2}}$

$a_2 = n^2 a_1$

$g \sin\theta = n^2[g\sin\theta - \mu g\cos\theta]$

$1 = n^2(1 - n)$

$\mu = 1 - \dfrac{1}{n^2}$

11.(1) $RP = \frac{1}{d\theta} = \frac{2n\sin\theta}{\lambda}$

$\therefore \quad RP \propto n$ i.e. it increases with increase in refraction index

12.(3) EMW does not carry any charge (electron) with it, so it is not deflected by electric and magnetic field.

$\frac{1}{2}\varepsilon E_0^2 = \frac{1}{2}\frac{B_0^2}{\mu_0} \Rightarrow \quad \frac{E_0}{B_0} = \sqrt{\frac{1}{\varepsilon_0\mu_0}}$

13.(2) **Subs-A**

$N_a = 16$

$t_{1/2} = 1 = \frac{\ln 2}{\lambda_1} \Rightarrow \lambda_1 = \ln 2$

$w_a = 320\ gm$

$N_A = \frac{320}{16} \times 6.6 \times 10^{23}$

$N_t = N_0 e^{-\lambda t} = 20 \times 6.6 \times 10^{23} e^{-\ln 2 \times 2}$

Subs-B

$N_B = 32$

$t_{1/2} = \frac{1}{2} = \frac{\ln 2}{\lambda_2} \Rightarrow \lambda_2 = 2\ln 2$

$w_b = 320\ gm$

$N_B = \frac{320}{32} \times 6.6 \times 10^{23}$

$N_{t_B} = 10 \times 6.6 \times 10^{23} \times e^{-2\ln 2 \times 2}$

$N_{t_A} + N_{t_B} = 33 \times 10^{23} + 4.125 \times 10^{23} = 3.38 \times 10^{24}$ atoms

14.(1) $a = \frac{F}{20},\ t = 20\sec,\ u = 0$

$v = at = F$

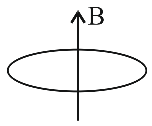
$F \times 10 = 50 \Rightarrow F = 5N$

15.(1) $A = 26 \times 10^{-4} m^2$

$R = 10\Omega$

$B = 40T$

$t = 1\sec$



$$\varepsilon = \frac{\Delta\phi}{\Delta t} = \frac{B.A}{\Delta t} = \frac{40\times25\times10^{-4}}{1} = \frac{1}{10}v$$

$$i = \frac{\varepsilon}{R} = 0.01A$$

$$e = (\text{change in } PE) = MB\cos\theta = MB$$

$$\Rightarrow\ w = \frac{1}{100}\times25\times10^{-4} = 10^{-3}J$$

16.(4) $K = \dfrac{\Delta V}{\Delta L}$ (Potential Gradient)

Sensitivity $\propto \dfrac{1}{K}$

17.(3) $T^2 \propto R^3$

$$\frac{(24)^2}{R^3} = \frac{T^2}{\left(\frac{R}{4}\right)^3} \Rightarrow T^2 = \frac{24\times24}{4\times4\times4} \Rightarrow T = \frac{6}{2} = 3h$$

18.(1) $\mu = \text{Amp modulation} = \dfrac{A_{max} - A_{min}}{A_{max} + A_{min}}$

$$\Rightarrow\ \mu = \frac{14-6}{14+6} = \frac{8}{20} = 0.4$$

19.(3) $I_{rms(a)} = \dfrac{V_{rms}}{R} = \dfrac{226}{40} = 5.5A$

$$w = 2\pi\times50 = 100\pi = 314$$

$$R = 40\Omega,\ X_L = wL = 314\times50\times10^{-23},$$

$$X_L = 15.7\Omega$$

$$X_C = \frac{1}{314\times0.5\times10^{-6}} = X_C = \frac{10^6}{157} = 6370\Omega$$

$$i_{rms} = \frac{V}{Z} = \frac{V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$\because Z > R\ \Rightarrow\ i_{rms(b)} < i_{rms}(a)$$

20.(4) $V_{rms} = \sqrt{\dfrac{3RT}{M}} = \sqrt{\dfrac{\alpha+5}{\alpha}}\times\sqrt{\dfrac{8RT}{\pi M}}$

$$\Rightarrow\ 3 = \left(\frac{\alpha+5}{\alpha}\right)\times\frac{8}{22}\times7$$

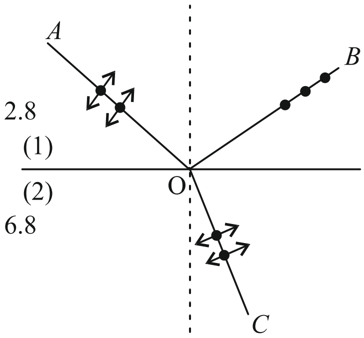
$$\Rightarrow 66\alpha = 56\alpha + 280$$

$$\Rightarrow\ \alpha = 28$$

SECTION – 2

21.(7) Reflected Ray c Polarized

Incident angle $\to$ Brewster angle



$C = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$, $v = \frac{1}{\sqrt{\varepsilon_0 \varepsilon_r \mu_0 \mu_r}}$ | $\mu = \frac{c}{v}$

$\mu = \sqrt{\varepsilon_r \mu_r} = \sqrt{\varepsilon_r}$ | $\mu_r = 1$

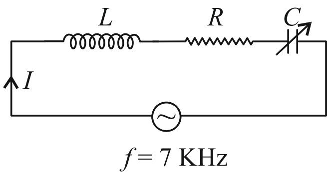
$\mu_1 = \sqrt{2.8}$; $\mu_2 = \sqrt{6.8}$

$$\tan i_p = {}^1\mu_2 = \frac{\mu_2}{\mu_1} = \sqrt{\frac{6.8}{2.8}} = \sqrt{\frac{17}{7}}$$

$$\sqrt{1 + \frac{10}{\theta}} = \sqrt{\frac{17}{7}}$$

$\theta = 7$

22.(3872)



$f = 7\,KHz$

$L = 2\mu H$

Current will be maximum at resonance

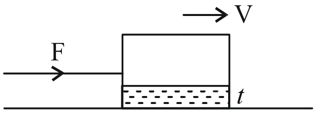
So, $f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$

$f^2 = \frac{1}{4\pi^2} \times \frac{1}{LC}$; $C = \frac{1}{4\pi^2 L \times f^2}$

$$= \frac{1}{4 \times \frac{22}{7} \times \frac{22}{7} \times 2 \times 10^{-6} \times 7 \times 7 \times 10^6} = \frac{1}{3872}$$

$x = 3872$

23.(25)



A= 0.20 m^2
t = 0.25 mm
F = 0.1 N
V = constant
$\eta = 50 \times 10^{-3}$ PJ

$$|F| = \eta A \frac{dv}{dx}$$

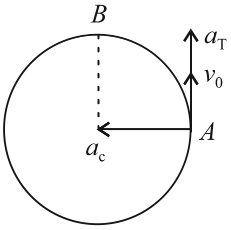
$$\frac{dv}{dx} = \frac{v-0}{t} = \frac{v}{t}$$

$$F = \eta A \times \frac{v}{t}$$

$$v = \frac{F \times t}{\eta A} = \frac{0.9 \times 0.25 \times 10^{-3}}{5 \times 10^{-3} \times 0.2 \times 100} = \frac{5}{2} \times 10^{-2} = 2.5 \times 10^{-2}$$

$$V = 25 \times 10^{-3} m/s$$

24.(40)



$R = 600\ m$
$v_0 = 54$ km/hr = 15 m/s

$$|a_c| = |a_T|$$

$$t_{AB} = ?$$

$$\omega_0 = \frac{V_c}{R} = \frac{15}{600} = \frac{1}{40}$$

$$\omega^2 R = R\alpha = \frac{d\omega}{dt}$$

$$\int_0^t dt = \int_{\omega_0}^{\omega} \frac{d\omega}{\omega^2}$$

$$t = \left[\frac{1}{\omega}\right]_{\omega_0}^{\omega} = -\left[\frac{1}{\omega} - \frac{1}{\omega_0}\right]$$

$$t = 40 - \frac{1}{\omega}$$

$$\frac{1}{\omega} = 40 - t$$

$$\omega = \frac{1}{40 - t}$$

$$\frac{d\theta}{dt}=\frac{1}{40-t}$$

$$\int_0^{\pi/2} d\theta=\int_0^t \frac{dt}{40-t}$$

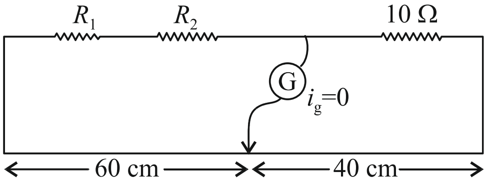
$$\frac{\pi}{2}=\frac{[\ln(40-t)]_0^t}{-1}$$

$$-\frac{\pi}{2}=\ln\frac{40-t}{40}$$

$$\frac{40-t}{40}=e^{-\pi/2}$$

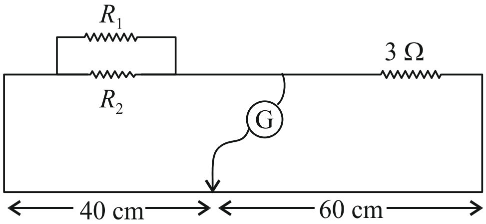
$$t=40(1-e^{-\pi/2})$$

25.(30)



$$\frac{R_1+R_2}{10}=\frac{60}{40}=\frac{3}{2}$$

$$R_1+R_2=15\Omega$$



$$\frac{R_1R_2}{3(R_1+R_2)}=\frac{40}{60}=\frac{2}{3}$$

$$R_1R_2=30\ \Omega$$

26.(40) $m=250\ g=\frac{1}{4}\ kg$; $F=-25x, V_{\max}=4\,m/s$

$A\rightarrow$ Amplitude = ? ; $F=-25x=-kx$; $k=25\ N/m$

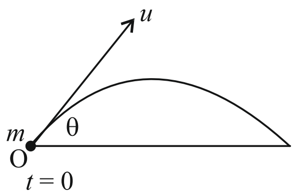
$$V_{\max}=\omega A$$

$$4=\sqrt{\frac{k}{m}}\times A=\sqrt{\frac{25}{1}\times 4}\times A=10A$$

$$A=\frac{4}{10}m$$

$$A=40\ cm$$

27.(800)



$m = 100\ g,\ u = 20\ m/s,\ \theta = 45^{\circ}$

$T = \dfrac{2u\sin\theta}{g} = \dfrac{2\times 20}{10}\times\dfrac{1}{\sqrt{2}}$

$T = 2\sqrt{2}$

At $t = 2s$

$u_x = u\cos\theta = 20\times\dfrac{1}{\sqrt{2}} = 10\sqrt{2}$

$u_y = u\sin\theta = 20\times\dfrac{1}{\sqrt{2}} = 10\sqrt{2}$

$v_x = 10\sqrt{2}$ (constant)

$v_y = uy + ayt$

$10\sqrt{2} - 20,\ v = v_x\hat{i} + v_y\hat{j}$

$x = 4xt = 20\sqrt{2}$

$y = u_y t = \dfrac{1}{2}a_y t^2$

$= 20\sqrt{2} - \dfrac{1}{2}\times 10\times 2^2 = 20\sqrt{2} - 20$

$r = x\hat{i} + y\hat{j}$

$\vec{i} = \vec{r}\times\vec{p}$

$= \vec{r}\times m\vec{v}$

$= m[\vec{r}\times\vec{v}]$

$= m[(x\hat{i} + y\hat{j})\times(v_x\hat{i} + v_y\hat{j})]$

$= m[0 + xv_y\hat{k} - yv_x\hat{k} + 0]$

$= 0.1[20\sqrt{2}(10\sqrt{2} - 20) - (20\sqrt{2} - 20)10\sqrt{2}]\hat{k}$

$= 0.1\times(-200\sqrt{2})\hat{k}$

$= 20\sqrt{2}(-\hat{k})$

$|L| = 20\sqrt{2} = \sqrt{800}$

$k = 800$

28.(41) 20 div. main scale = 1 *cm*

1 div main scale $= \frac{1}{20} cm = 0.05\ cm = 0.5\ mm$

50 div VS = 49 div MS

1 div VS $= \frac{49}{50}$ div MS

$$= \frac{49}{50} \times 0.5mm = 0.49\ mm$$

$$LC = 1MS - 1\ VS$$

$= 0.5 - 0.49 =$ 001 mm $= 10^{-2} mm = 10 \times 10^{3} mm$

Real thickness = 5.25 mm

App thickness = 5 mm

$$\mu_{max} = \frac{5.25 + 0.01}{5.00 - 0.01} = 1.0541$$

$$\mu_{min} = \frac{5.25 - 0.01}{5.00 + 0.01} = 1.0459$$

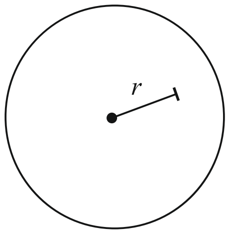
Uncertainty $= \frac{\mu_{max} - \mu_{min}}{2} = 41 \times 10^{-4}$

29.(12) $V = 2ar^2 + b$

$$\rho = -\lambda a \varepsilon$$

$$\frac{dV}{dr} = 4ar$$

$$E = -\frac{dv}{dr} = -4ar$$

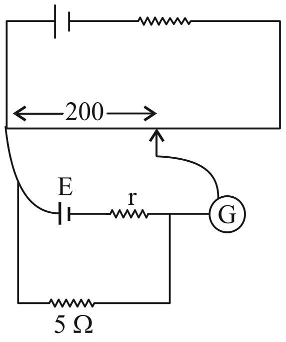


$$E = \frac{\rho r}{3\varepsilon}$$

$$-4ar = \frac{-(\lambda a \varepsilon) r}{3\varepsilon}$$

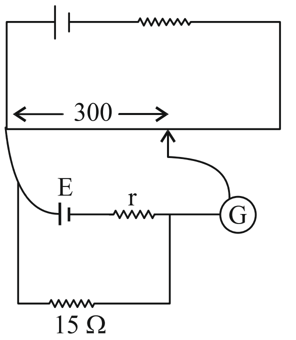
$$\lambda = 12$$

30.(5)



$$r = \left(\frac{E}{V} - 1\right) \times R = \left(\frac{l_1}{l_2} - 1\right) \times R$$

$$r = \left(\frac{l_1}{200} - 1\right) \times 5 \qquad \ldots\ldots\ldots.(1)$$



$$r = \left(\frac{l_1}{300} - 1\right) \times 15 \qquad \ldots\ldots\ldots.. (2)$$

$l_1 \to$ null point length for only EMF will remain same

$$\left(\frac{l_1}{300} - 1\right) \times 15 = \left(\frac{l_1}{200} - 1\right) \times 5$$

$$\frac{3l_1}{300} - \frac{l_1}{200} = 3 - 1 = 2$$

$$\frac{6l_1 - 3l_1}{600} = 2$$

$$l_1 = 400$$

From equation (1)

$$r\left(\frac{400}{200} - 1\right) \times 5 = 5\Omega$$

CHEMISTRY

SECTION - 1

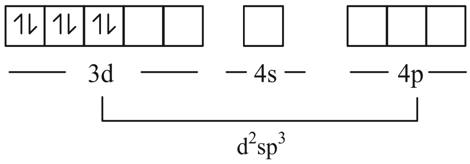
1.(3) $[Co(C_2O_4)_3]^{3-}$

$x+3(-2)=-3$

$x^2=-3+6=+3$

Co^{+3}

$\Rightarrow Co^{+3}=d^6$

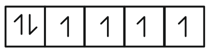


Unpaired $e^- = 0$

$M.M.=0$

$[CoF_6]^{3-} \Rightarrow Co^{+3}$

$d^6 \Rightarrow$ no pairing



Unpaired $e^- = 4$

$MM=\sqrt{4(4+2)}$ BM

$=4.9$ BM

$[FeF_6]^{3-} \Rightarrow x+6(-1)=-3$

$x=+3$

$Fe^{+3} \Rightarrow d^5$

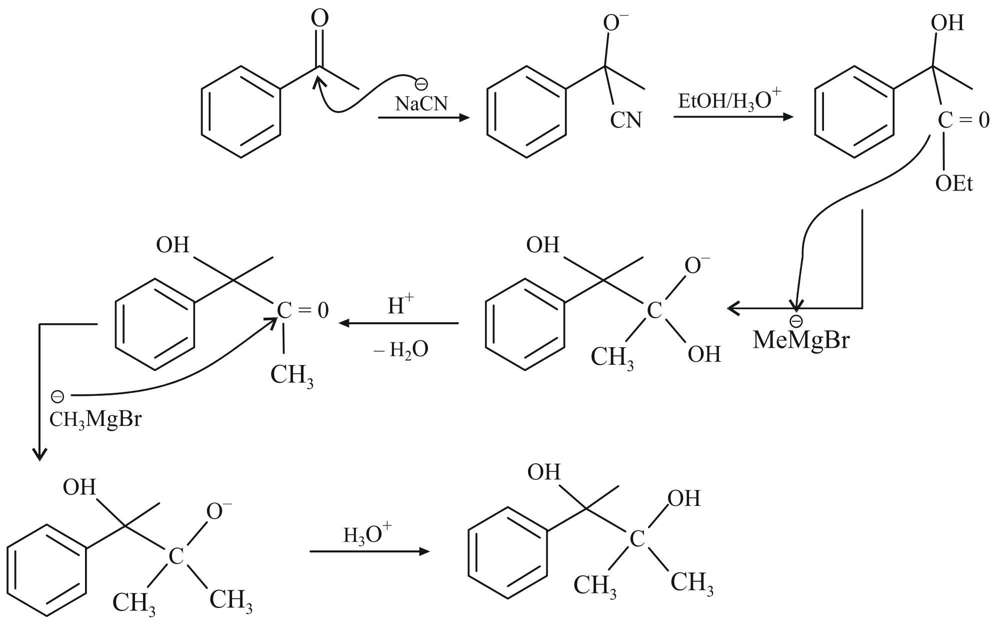
Unpaired $e^- = 5$

M.M. = 5.9 BM

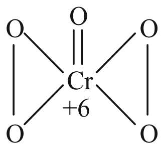
Order:

$[FeF_6]^{3-} > [CoF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$

2.(2)



3.(3) CrO_5 :



: Blue

4.(1) $CH_4 + H_2O \xrightarrow[Ni]{1270K} \underbrace{CO + 3H_2}_{\text{Syn gas}}$

$\text{Vegitable oil} \rightarrow \text{Alkene} \xrightarrow{Ni/H_2} \text{Ghee}$

Group 14 elements such as Si forms electron precise hydrides

5.(4) **(i)** $MnO_4^- \Rightarrow Mn^{+7}$

(ii) $R_4^{+8}O_4,\ O_5^{+8}O_4$

(iii) Sc exists as Sc^{+3}

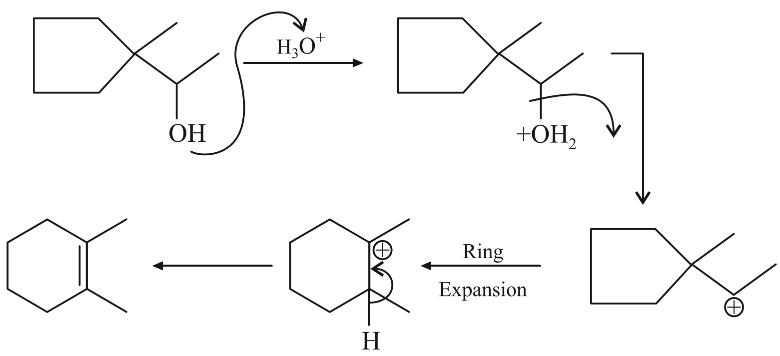
(iv) $K_2Cr_2O_7 \longrightarrow Cr^{+3}$

i, ii, & iv are true

6.(1) X = Starch

A = Iodine

7.(1)



8.(3) Electronic configuration:

$O_2^{2-} = \sigma_{1s^2}\ \sigma^*_{1s^2}\ \sigma_{2s^2}\ \sigma^*_{2s^2}\ \sigma^2_{2z^2}\ \pi^2_{2px} = \pi^2_{2py}\ \pi^2_{2px} = \pi^2_{2py}$

B.O.$=\dfrac{(\text{Bonding } e^-)-(\text{ABMO})e^-}{2}$

$=\dfrac{6-4}{2}=1$

$NO^+ : \pi^2_{2px} = \pi^2_{2py}\ \sigma^2_{2py}\ \pi^{*1}_{2px}$

$B.O. = \dfrac{6-0}{2} = 3$

$CO : \sigma_{1s^2}\sigma^*_{1s^2}\ \sigma_{2s^2}\ \sigma^*_{2s^2}\ \pi^2_{2px}\ \pi^2_{2py}\ \pi^2_{2pz}$

B.O. $= \dfrac{6-0}{2} = 3$

9.(2) Calamine : $ZnCO_3$

Sphalerite : ZnS

Siderite : $FeCO_3$

Malachite : $CuCO_3 \cdot Cu(OH)_2$

10.(2) **(A)** $\Delta U = q + w = q - P\Delta V$

(B) $\Delta G = \Delta H - T\Delta S$

(C) $\Delta S = \dfrac{q_{rev3}}{T}$

(D) $\Delta H = \Delta U + \Delta n_g RT$

B & C are correct, A & D are incorrect

11.(4) Statement 1 says DECREASE in 1st IE from Al to Ga. But IE of Ga>Al.
Hence statement 1 is wrong.

12.(3) Depression and Hypertension

Equanil is used in case of Hypertension and depression

13.(1) $\underset{6}{X}\ \ \underset{5}{Y}$

14.(3) Ethylamine

$CH_3CH_2CONH_2 \xrightarrow{Br_2/KOH} CH_3CH_2NH_2$

15.(2) A-III, B-I, C-II, D-IV

Van't Hoff factor$=\dfrac{\text{Normal mass}}{\text{Abnormal mass}}$

K_f – Cryoscopic constant

Solution with $\longrightarrow$ Isotonic solution

Same composition of $\longrightarrow$ Azeotrope

16.(3) A-III, B-I, C-IV, D-II

Osmosis – Solvent molecules pass through semi permeable towards solution side

Reverse osmosis- Solvent pass through semipermeable towards solution side.

Electroosmosis- Dispersion medium moves in electric field

Electro phoresies – Movement of charged colloidal particles under influence of applied electric potential towards oppositely charged electrodes.

17.(4) **2-Bromopentane**

$$\underset{\substack{|\\ \text{Br}}}{\text{C}-\text{C}-\text{C}-\text{C}-\text{C}} \rightarrow \text{C}-\text{C}-\text{C}-\text{C}=\text{C} \rightarrow \underset{\substack{|\\ \text{cis+trans}}}{\text{C}-\text{C}-\text{C}=\text{C}-\text{C}}$$

18.(1) A-IV, B-III, C-I, D-II

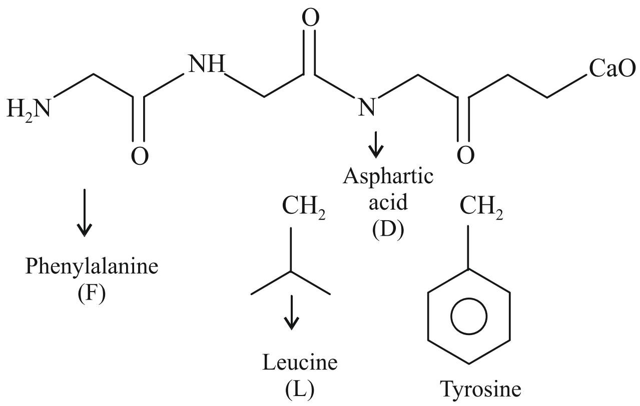
Elastomeric – Neoprene

Fibre – Polyster

Thermosetting – Urea resin

Thermoplastic - Polystyrene

19.(2)



20.(2) $CaH_6 + \left(a + \frac{b}{4}\right) O_2 \rightarrow aCO_2 + \frac{b}{2} H_2O$

Equivalent of $H_2O = 3$

$\frac{b}{2} = 3$

$b = 6$

$a + \frac{b}{4} = 9.5$

$a = 8$

SECTION – 2

21.(4) **(1)** $N_2(g)+3H_2(g) \rightleftharpoons 2NH_3(g),\ R_1=4\times10^5$

(2) $N_2(g)+O_2(g) \rightleftharpoons 2NO(g), k_2=1.6\times10^{12}$

(3) $H_2(g)+\frac{1}{2}O_2(g) \rightleftharpoons H_2O(g)\ ;\ k_3=1.0\times10^{-13}$

Reverse Equation (1)

(4) $2NH_3 \rightleftharpoons N_2(g)+3H_2(g),\ k_1^{'}=\frac{1}{k_1}=\frac{1}{4\times10^5}$

Multiply equation (3) by 3

(5) $3H_2(g)+\frac{3}{2}O_2(g) \rightleftharpoons 3H_2O(g)k_3^{'}=(k_3)^3$

Now, add equation (2), (4) & (5)

$$2NH_3(g)+\frac{5}{2}O_2(g) \rightleftharpoons 2NO(g)+3H_2O(g)\,k''$$

$$k''=k_2\times k_1^{'}\times k_3^{'}$$

$$=1.6\times10^{12}\times\frac{1}{4\times10^5}\times(10^{-13})^3$$

$$=0.4\times10^7\times10^{-39}$$

$$=0.4\times10^{-32}=4\times10^{-33}$$

22.(36) N atoms of HCP forms 2N Oh voids & N Td voids.

1 mole atoms of HCP $\longrightarrow$ 3 mole total voids

0.2 mole $\longrightarrow$ $0.02\times3\times6.02\times10^{23}$

$$=0.3612\times10^{23}=36.12\times10^{21}$$

23.(2) $A \xrightarrow{b} B$

$b=4.6\times10^{-5}\,L\ mol^{-1}\,sec^{-1}$

Rate $=b[A]^n$

$$\frac{mol}{L\times sec}=\frac{L}{mol\times sec}\left(\frac{mol}{L}\right)^n$$

$$\left(\frac{mol}{L}\right)^2=\left(\frac{mol}{L}\right)^n \Rightarrow n=2$$

24.(3) $4LiNO_3 \xrightarrow{\Delta} 2Li_2O+4NO_2+O_2$

25.(15) Strength of HCl = 73 g/L

Molarity of HCl = $\left(\frac{73}{36.5}\right)$M

$$2Na + 2H_2O \rightarrow 2NaOH + H_2$$

Moles of Na $= \frac{0.69}{23} = 0.03$

Moles of NaOH = moles of Na = 0.03

Now, $NaOH + HCl \rightarrow NaCl + H_2O$

For complete neutralization,

Moles of HCl = Moles of NaOH

Molarity $\times$ Volume = 0.03

$$\left(\frac{73}{36.5}\right) \times V(L) = 0.03$$

$$V(L) = \frac{0.03 \times 36.5}{73}$$

$$V(ml) = \frac{0.03 \times 36.5}{73} \times 10^3 = 15 \text{ ml}$$

26.(17) $E_r^0 = \frac{0.059}{n} \log k_{eq}$

$$E_r^0 = \frac{0.059}{2} \times \log(10^{20})$$

$$E_r^0 = \frac{0.059}{2} \times 20 = 0.59$$

Let standard reduction potential of Sn / Sn^{2+} be 'x'

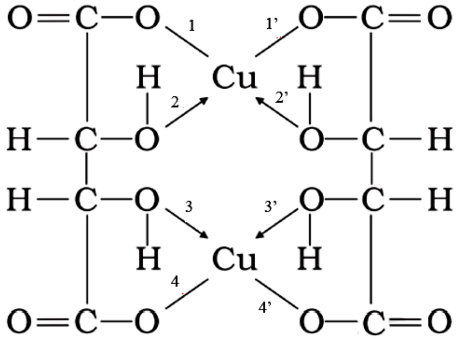
$$x - (-0.76) = 0.59$$

$$x = 0.59 - 0.76 = -0.17$$

But we have to find the oxidation potential of Sn / Sn^{2+}

So, answer should be '–x' i.e., 0.17

27.(4)



Copper tartarate complex

In this above complex two tartrate ligands are present

So denticity is 4

28.(270)

$$r_n = \left(a \times \frac{n^2}{x}\right) Å$$

$$a = 0.6$$

Now, radius of 3rd Bohr orbit of He^+ in pm

$$r_n = \left(a \frac{n^2}{x}\right) Å \; ; \; r_3 = 0.6 \times \frac{(3)^2}{2} \times 10^{-10}$$

$$= \frac{9 \times 0.3 \times 10^{-10} \times 10^{-2}}{10^{-2}} = \frac{2.7 \times 10^{-12}}{10^{-2}} = 270 \text{ pm}$$

29.(4) Acidic oxides $\rightarrow$ N_2O_3, NO_2, SO_2, Cl_2O_7

30.(200)

$$n_{CO_2} = \frac{4.4}{44} = 0.1 \text{ mole} = n_C$$

$$m_C = 0.1 \times 12 = 1.2 \text{ g}$$

In 0.01 mol of organic compound $\rightarrow$ 0.1 mol of C

In 1 mol $\Rightarrow$ 10 mol of C

$$10 \times 12 = \frac{60}{100} \times M$$

$$M = 200 \text{ gmol}^{-1}$$

MATHEMATICS

SECTION - 1

1.(1) $B \rightarrow (\sim A \vee B)$, $(\sim A \vee B)$ is equivalent of $A \rightarrow B$

So, $B \rightarrow (\sim A \vee B)$ is equivalent to $B \rightarrow (A \rightarrow B)$

2.(2) Let vector $\vec{c}$ be $xi + yi + z\hat{k}$

$$\vec{a} \times \vec{b} = 15i - 20j - 25k = 5(3i - 4j - 5k)$$

Given,

$$\vec{c} \cdot (\vec{a} \times \vec{b}) + 25 = 0$$

$$3x - 4y - 5z = -5 \qquad \text{- (I)}$$

$$\vec{c} \cdot (i + j + k) = 4$$

$$x + y + z = 4 \qquad \text{-(2)}$$

Projection of $\vec{c}$ on $\vec{a}$ in $\dfrac{\vec{c} \cdot \vec{a}}{|a|} = 1$

$$4x + 3y = 5 \qquad \text{-(3)}$$

Solving (1), (2) & (3)

$$x = 2, y = -1, z = 3$$

Now projection of $\vec{c}$ on $\vec{b}$ is $\dfrac{\vec{c} \cdot \vec{b}}{|\vec{b}|} = \dfrac{25}{5\sqrt{2}} = \dfrac{5}{\sqrt{2}}$

3.(2) Let $I = \displaystyle\int_{1/2}^{2} \frac{\tan^{-1} x}{x} dx$ -(1)

Replaying x by $\dfrac{1}{x}$

$$I = \int_{2}^{1/2} x\left(\tan^{-1}\frac{1}{x}\right)\left(-\frac{1}{x^2}\right) dx$$

$$I = \int_{1/2}^{2} \frac{1}{x} \tan^{-1/x} dx \qquad \text{-(2)}$$

(1) + (2)

$$2I = \int_{1/2}^{2} \frac{1}{x}\left(\tan^{-1} x + \tan^{-1}\frac{1}{x}\right) dx$$

$$2I = \int_{1/2}^{2} \frac{\pi}{2}\frac{dx}{x} = \frac{\pi}{2} |\ln x|_{1/2}^{2}$$

$$I = \frac{\pi}{2} \ln 2$$

4.(2) $f''(x) = g''(x) + 6x$

Integrating both side

$$f'(x) = g'(x) + 3x^2 + c$$

Now $f'(1) = g'(1) + 3 + c = g$ (Given)

$3 + 3 + c = 9$ $\left[\therefore g'(1) = \frac{12}{4} = 3\right]$

$c = 3$ -(1)

hence $f'(x) = g'(x) + 3x^2 + 3$ →from (1)

→ (2)

Integrating (2) both side

$$f(x) = g(x) + x^3 + 3x + c$$

Given

$f(2) = g(2) + 8 + 6 + c = 12$ $\left[\therefore\ g(2) = 12/3 = 4\right]$

$$4 + 8 + 6 + c = 12$$

$$c = -6$$

So, $f(x) - g(x) = x^3 + 3x - 6$ - (3)

From (2) $f'(x) - g'(x) = 3x^2 + 3$ - (4)

$f(x) - g(x)$ is increasing function

$$[f(x) - g(x)]_{\text{minimum}} = -10$$

$$[f(x) - g(x)]_{\text{maximum}} = 8$$

So option 2 is in correct

5.(4) $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+3}{1} = \lambda_1$

$\therefore$ $x = \lambda_1 + 1$

$y = 2\lambda_1 + 2$

$z = \lambda_1 - 3$

$\frac{x-a}{2} = \frac{y+2}{3} = \frac{z-3}{1} = \lambda_2$

$x = 2\lambda_2 + a;\ y = 3\lambda_2 - 2;\ z = \lambda_2 + 3$

By comparing and solving $\lambda_1 = 22,\ \lambda_2 = 16$

$\therefore$ $a = -9, x = 23, y = 46, z = 19$

Now distance of point $P(23, 46, 19)$ then plane $z + 9 = 0$ is $\left|\frac{19+9}{1}\right| = 28$

6.(2) Number of letters start with

a---------- = 4! = 24
H--------- = 4! = 24
O---------- = 4! = 24
Ta--------- = 3! = 6
TH-------- = 3! = 6
TOG--------- = 2! =2
TOH--------- = 2! = 2
TOUGH = 1

Serial number of the word 89

7.(4) Number of 3- digit number divisible by 3 is $999 = 102 + (n-1)\times 3$

$n = 300$ .... (1)

Number of 4- digit number divisible by 4 is $996 = 100 + (n-1)\times 4$

$n = 225$ .... (2)

Number of number divisible by 12 is $996 = 108 + (n-1)\times 3$

$n = 75$ .... (3)

There are 20 number divisible by 48 from 1 to 1000 but we need 3- digit number 18 numbers are divisible by 48

So, total number of required number $= 300 + 225 - 75 - 18 = 432$

8.(3) Sum of add coefficient $\frac{2^{99}}{2} = 2^{98} = K$ -(1)

Middle term is (101) th term $= {}^{200}C_{100}\frac{(2)^{100}}{2^{50}}$

$a = {}^{200}C_{100}2^{50}$ -(2)

$$\frac{{}^{200}C_{99}K}{a} = \frac{200!\times 2^{98}}{100!\,99!\times {}^{200}C_{100}\times 2^{50}} \rightarrow \text{From (1) and (2)}$$

$$= \frac{100}{101}\times 2^{48} = \frac{25}{101}\times 2^{50}$$

Comparing with $\frac{2^{\ell}m}{n}, \ell = 50, M = 101$

9.(3) $\cos^2 2x - 2\sin^4 x - 2\cos^2 x = \lambda$

$$\left(1-2\sin^2 x\right)^2 - 2\left(\sin^4 x\right) - 2\left(1-\sin^2 x\right) = \lambda$$

$$1 + 4\sin^4 x - 4\sin^2 x - 2\sin^4 x - 2 + 2\sin^2 x = \lambda$$

$$2\left[\sin^4 x - \sin^2 x - \frac{1}{2}\right] = \lambda$$

$$2\left[\left(\sin^2 x - \frac{1}{2}\right)^2 - \frac{3}{4}\right] = \lambda \quad \text{........ (1)}$$

$\therefore$

$$-1 \le \sin x \le 1$$

$$0 \le \sin^2 x \le 1$$

$$-\frac{1}{2} \le \sin^2 x - \frac{1}{2} \le \frac{1}{2}$$

$$0 \le \left(\sin^2 x - \frac{1}{2}\right)^2 \le \frac{1}{4}$$

$$-\frac{3}{4} \le \left(\sin^2 x - \frac{1}{2}\right)^2 - \frac{3}{4} \le -\frac{1}{2}$$

$$-\frac{3}{4} \le 2\left(\sin^2 x - \frac{1}{2}\right)^2 - \frac{3}{4} \le -1$$

$$-\frac{3}{4} \le \lambda \le -\frac{1}{2}$$

10.(4) $y^2 = 3x \rightarrow \text{tangent} \qquad x+2y=1 \qquad \text{Slope} = -\frac{1}{2}$

Point of contact

$$\left(\frac{a}{m^2}, \frac{2a}{m}\right) = \left(\frac{\frac{3}{4}}{\frac{1}{4}}, \frac{2\times\frac{3}{4}}{-\frac{1}{2}}\right) = (3, -3)$$

Point of contact on ellipse $\qquad \rightarrow$ Slope of tangent = –1

$$\left(-\frac{a^2m}{c}, \frac{b^2}{c}\right) \qquad C = \pm\sqrt{4+1} = \pm 5$$

$$\left(-\frac{4\times(-1)}{\pm\sqrt{5}}, \frac{1}{\pm\sqrt{5}}\right) = \left(\frac{4}{\pm\sqrt{5}}, \frac{1}{\pm\sqrt{5}}\right)$$

Area of ΔPQR

$$= \frac{1}{2}\begin{vmatrix} 3 & -3 & 1 \\ \frac{4}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 1 \\ -\frac{4}{\sqrt{5}} & -\frac{1}{\sqrt{5}} & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 3 & -3 & 1 \\ 0 & 0 & 2 \\ -\frac{4}{\sqrt{5}} & -\frac{1}{\sqrt{5}} & 1 \end{vmatrix} = \frac{1}{2}\times 2\left(-\frac{3}{\sqrt{5}} - \frac{12}{\sqrt{5}}\right) = \frac{15}{\sqrt{5}} = \frac{\sqrt{5}\times\sqrt{5}\times 3}{\sqrt{5}} = 3\sqrt{5}$$

11.(3) $\int_1^2 \frac{t^4+1}{t^6+1}\,dt = \int_1^2 \frac{t^4+1+2t^2-2t^2}{t^6+1}\,dt = \int_1^2 \frac{(t^2+1)^2}{t^6+1}\,dt - \int_1^2 \frac{2t^2}{t^6+1}\,dt$

$$= \int_1^2 \frac{(t^2+1)^2}{(t^2+1)(t^4+1-t^2)}\,dt - 2\int_1^2 \frac{t^2}{(t^3)^2+1}\,dt$$

$$= \int_1^2 \frac{t^2+1}{(t^4+1-t^2)}\,dt - \frac{2}{3}\left(\tan^{-1}t^3\right)^2 = \int_1^2 \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}-1}\,dt - \frac{2}{3}(\tan^{-1}8 - \tan^{-1}1)$$

$$t - \frac{1}{t} = z\ ;\ \left(1+\frac{1}{t^2}\right)dt = dz = \int_0^{3/2} \frac{dz}{z^2+1} - \frac{2}{3}\left[\tan^{-1}8 - \tan^{-1}1\right]$$

$$= \left[\tan^{-1}z\right]_0^{3/2} - \frac{2}{3}\left[\tan^{-1}8 - \tan^{-1}1\right] = \tan^{-1}\frac{3}{2} - \frac{2}{3}\tan^{-1}8 + \frac{2}{3}\times\frac{\pi}{4}$$

$$= \tan^{-1}\frac{3}{2} - \tan^{-1}8 + \frac{1}{3}\tan^{-1}8 + \frac{\pi}{6}$$

$$\tan^{-1}\left(\frac{\frac{3}{2}-8}{1+12}\right) + \frac{1}{3}\tan^{-1}8 + \frac{\pi}{6}$$

$$\tan^{-1}\left(-\frac{1}{2}\right) + \frac{1}{3}\tan^{-1}8 + \frac{\pi}{6}$$

$$\frac{\pi}{2} - \tan^{-1}\frac{1}{2} + \frac{1}{3}\tan^{-1}8 + \frac{\pi}{6} - \frac{\pi}{2}$$

$$\cot^{-1}\frac{1}{2} + \frac{1}{3}\tan^{-1}8 - \frac{\pi}{3} \Rightarrow \tan^{-1}2 + \frac{1}{3}\tan^{-1}8 - \frac{\pi}{3}$$

12.(1) $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$

$\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$

$\frac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{|(\vec{b}_1\times\vec{b}_2)|}$

$a_1=i-8\hat{j}+4\hat{k}$ $\qquad \vec{b}_1=2\hat{i}-7\hat{j}+5\hat{k}$

$a_2=i+2\hat{j}+6\hat{k}$ $\qquad \vec{b}_2=2\hat{i}+\hat{j}-3\hat{k}$

$\vec{a}_2-\vec{a}_1=10\hat{j}+2\hat{k}$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -7 & 5\\ 2 & 1 & -3\end{vmatrix}=16\hat{i}+16\hat{j}+16\hat{k}$$

$d=\frac{(10+2)16}{16\sqrt{3}}=4\sqrt{3}$ units

13.(3)

$2k+3$ Possible outcomes 5, 7, 9, 11, 13, 15,….	$2k+6$ 8, 10, 12, 14, 16	$2k+9$ 11, 13, 15, ………	$2k+12$ 19, 16, ….

Sequence $2k+9$ is repeating in $2k+3$ and sequence $2k+12$ is repeating in $2k+6$ only $2k+3$ and $2k+6$ are unique sequence

Let $p(\omega_1)=\alpha$

$p(\omega_2)=\frac{\alpha}{2}$

$p(\omega_3)=\frac{\alpha}{4}$

……………

……………

……………

$p(\omega)+p(\omega_2)........=1$

$\frac{\alpha}{1-\frac{1}{2}}=1 \Rightarrow \alpha=\frac{1}{2}$ and $p(\omega_r)=\alpha^r=\left(\frac{1}{2}\right)^r$

$p(\omega_5)+p(\omega_7)+p(\omega_9)=\left(\frac{1}{2}\right)^5+\left(\frac{1}{2}\right)^7+\left(\frac{1}{2}\right)^9.......=\frac{1}{3}\times\frac{1}{8}$

$p(\omega_8)+p(\omega_{10})+p(\omega_{12})=\left(\frac{1}{2}\right)^8+\left(\frac{1}{2}\right)^{10}+\left(\frac{1}{2}\right)^{12}......=\frac{1}{3}\times\frac{1}{64}$

So $p(B)=\frac{1}{3}\left(\frac{1}{8}+\frac{1}{64}\right)=\frac{3}{64}$

14.(2) $A=\begin{bmatrix} e^t & e^{-t}(\sin t-2\cos t) & e^{-t}(-2\sin t-\cos t) \\ e^t & e^{-t}(2\sin t-\cos t) & e^{-t}(\sin t-2\cos t) \\ e^t & e^{-t}(\cos t) & e^{-t}(\sin t) \end{bmatrix}$ $\quad t\in R$

$$|A|\Rightarrow e^{-t}\begin{vmatrix} 1 & \sin t-2\cos t & -2\sin t-\cos t \\ 1 & 2\sin t+\cos t & \sin t-2\cos t \\ 1 & \cos t & \sin t \end{vmatrix}$$

$$|A|\Rightarrow e^{-t}\begin{vmatrix} 1 & \sin t-2\cos t & -2\sin t-\cos t \\ 0 & \sin t+3\cos t & 3\sin t-\cos t \\ 0 & 2\sin t & -2\cos t \end{vmatrix}$$

$$=e^{-t}\left[(\sin t+3\cos t)(-2\cos t)-2\sin t(3\sin t-\cos t)\right]$$

$$=e^{-t}\left[-2\sin t\cos t-6\cos^2 t-6\sin^2 t+2\sin t\cos t\right]=-6e^{-1}\neq 0 \qquad t\in R$$

15.(1) $\vec{a}=\hat{i}+2\hat{k}$

$\vec{b}=\hat{i}+\hat{j}+\hat{k}$

$\vec{c}=7\hat{i}-3\hat{j}+4\hat{k}$

$\vec{r}\times\vec{b}+\vec{b}\times\vec{c}=0$

$\vec{r}\cdot\vec{a}=0$

$\vec{r}\cdot\vec{c}=?$

$\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}$

$\vec{r}\times\vec{b}+\vec{b}\times\vec{c}=\vec{0}$

$\vec{a}\times(\vec{r}\times\vec{b})+\vec{a}\times(\vec{b}\times\vec{c})=\vec{0}$

$(\vec{a}\cdot\vec{b})\vec{r}-(\vec{a}\cdot\vec{r})\vec{b}+(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}=0$

$(\vec{a}\cdot\vec{b})\vec{r}-0+15(\hat{i}+\hat{j}+k)-3(7\hat{i}-3\hat{j}+4\hat{k})=0$

$(\vec{a}\cdot\vec{b})\vec{r}-6\hat{i}+24\hat{j}-3\hat{k}=0$

$3\vec{r}=6\hat{i}-24\hat{j}-3\hat{k}$

$\vec{r}=2\hat{i}-8\hat{j}-\hat{k}$

$\vec{r}\cdot\vec{c}=14+24-4=34$

16.(1) $2a+3b$ is the multiple of 5

$2a+3b=5N$

For reflexive $\quad a=b$

$2a+3a=5a$ is multiple of '5'

For Symmetric

$2a+3b=5N$

Now $2b+3a=3b+2a+a-b(a=b)=5N+a-b=5N$

Always multiple of '5'

Relation is symmetric

For Transitive

Let $\quad 2a+3b=5N$

$$2b+3c=5M$$

$$\Rightarrow \quad 2a+5b+3c=5(M+N)$$

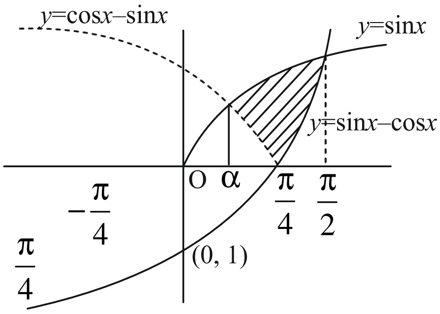
$$2a+3c=5(M+N)-5b=5\lambda$$

So, relation is transitive

17.(2) $A=\left\{(x,\ y):|\cos x-\sin x|\le y\le \sin x,\ 0\le x\le \frac{\pi}{2}\right\}$

$y\ge|\cos x-\sin x|$

$y\le \sin x$



$y=Qx-\sin x: 0\le x\le \frac{\pi}{4}$

$\sin x-Qx \qquad \frac{\pi}{4}\le x\le \frac{\pi}{2}$

Bounded area $=\int_{\alpha}^{\pi/4}(2\sin x-Qx)\ dx+\int_{\pi/4}^{\pi/2}[\sin x-\sin x-Qx]\ dx$

$=[-2Qx-\sin x]_{\alpha}^{\pi/4}+[\sin x]_{\pi/4}^{\pi/2}$

$\left(-\frac{2}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right)-\left(-\frac{4}{\sqrt{5}}-\frac{1}{\sqrt{5}}\right)+1-\frac{1}{\sqrt{2}}=-\frac{3}{\sqrt{2}}+\frac{5}{\sqrt{5}}+1-\frac{1}{\sqrt{2}}$

$1+\sqrt{5}-2\sqrt{2}$

18.(3) $y=y(x)$

$x\log_e x\frac{dy}{dx}+y=x^2\log_e x \ \ (x>1)$

$y(2)=2$

$\frac{dy}{dx}+\frac{1}{x\log_e x}y=x$

$p=\frac{1}{x\log_e x}$

I.F. $=e^{\int\frac{1}{x\log_e x}dx}=e^{\log_e(\log_e x)}=\log_e x$

$y\times\log_e x=\int\log_e x\cdot x\,dx+c$

$y\times\log_e x=\log_e x\frac{x^2}{2}-\frac{1}{4}x^2+c$

$2\times\log_e 2=\log_e 2\cdot 2-1+c$

$c=1$

$$y \times \log_e x = \log_e x \cdot \frac{x^2}{2} - \frac{1}{4}x^2 + 1$$

$$y(e) \cdot 1 = 1 \cdot \frac{e^2}{2} - \frac{e^2}{4} + 1$$

$$y(e) = \frac{e^2}{4} + 1$$

$$y(e) = \frac{e^2 + 4}{4}$$

19.(3) $A(a, -2, 4)$, $B(2, b, -3)$

$$c = \left(\frac{2\times 2 + 1\times a}{3}, \frac{2\times b + 1\times -2}{3}, \frac{2\times(-3) + 1\times 4}{3}\right)$$

$$c = \left[\frac{4+a}{3}, \frac{2b-2}{3}, -\frac{2}{3}\right]$$

Point lies on $2x - y + z = 4$

$$2\left(\frac{4+a}{3}\right) - \left(\frac{2b-2}{3}\right) - \frac{2}{3} = 4$$

$$8 + 2a - 2b = 12$$

$$2a - 2b = 4 \Rightarrow a - b = 2 \qquad ....(1)$$

Now $\quad OC^2 = \sqrt{\left(\frac{4+a}{3}\right)^2 + \left(\frac{2b-2}{3}\right)^2 + \frac{4}{9}} = \sqrt{5}$

$$\Rightarrow a^2 + 4b^2 = 5 \qquad ....(2)$$

By (1) and (2) $(2+b)^2 + 4b^2 = 5$

$$\Rightarrow \quad 5b^2 + 4b - 1 = 0$$

$$\Rightarrow \quad b = \frac{1}{5}, -1$$

When $\quad b = \frac{1}{5}, a = \frac{11}{5}$ and $b = -1, a = 1$

Rejected

So $\quad b = -1, a = 1$

$$p = (2, -1, -3), c = \left(\frac{5}{3}, -\frac{4}{3}, -\frac{2}{3}\right)$$

$$c = \left(\frac{5}{3}, -\frac{4}{3}, -\frac{2}{3}\right)$$

$$cp^2 = \frac{1}{9} + \frac{1}{9} + \frac{49}{9} = \frac{51}{9} = \frac{17}{3}$$

20.(4) Given $f(1) = 1$

$$f(1) + 2f(2) + 3f(3) ..... + xf(x) = x(x+1)f(x)$$

Put $x = 2$

$$f(1) + 2f(2) = 2\times 3f(2)$$

$1=4f(2)$

$f(2)=\frac{1}{4}$

Put $x=3$

$f(1)+2f(2)+3f(3)=3\times 4f(3)$

$1+\frac{1}{2}=9f(3)$

$f(3)=\frac{3}{2\times 9}=\frac{1}{6}$

Similarly

$f(4)=\frac{1}{8}$

.

.

$f(n)=\frac{1}{2n}$

$f(2022)=\frac{1}{4044}$

$f(2028)=\frac{1}{4056}$

$\frac{1}{f(2022)}+\frac{1}{f(2028)}=8100$

SECTION – 2

21.(4) $\frac{dy}{dx}=\frac{x-a}{(x+b)(x-2)}\left[\frac{1}{x-a}-\frac{1}{x+b}-\frac{1}{x-2}\right]=-4$

(1-3) lies on $y=\frac{x-a}{(x+b)(x-2)}$

$3b+a+2=0,\ a=-3b-2$

$\frac{dy}{dx}=\frac{-(1-a)}{1+b}\left[\frac{(1+b)-(1-a)+(1-a)(1+b)}{(1-a)(1+b)}\right]=-4$

$-(b+a+b-a-ab+1)=-4(1+b)^2 \Rightarrow (2b+1-ab)=+(4b^2+8b+4)$

$2b+1-b(-3b-2)=4b^2+8b+4$

$b^2+4b+3=0$

$(b+3)(b+1)=0$

$b=-1,\ b=-3$

$a=1,\ a=9-2=7$

$a+b=0,\ a+b=4$

22.(5) $|A| = 2,\ ad - b^2 = 2$

$$\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix}\begin{bmatrix} a & b \\ b & d \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$$

$2a + b = 1,\ 2b + d = 2 \Rightarrow a = \frac{1-b}{2},\ d = 2(1-b)$

$3a + \frac{3}{2}b = \alpha,\ 3b + \frac{3}{2}d = \beta$

$2a = 1 + \frac{1}{2} \Rightarrow a = \frac{3}{4}$

$S = a + d$

$= \frac{3}{4} + 3 = \frac{15}{4}$

$(1-b)^2 - b^2 = 2$

$1 + b^2 - 2b - b^2 = 2$

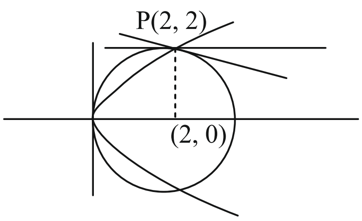
$2b = -1,\ b = -\frac{1}{2}$

$\alpha = \frac{9}{4} - \frac{3}{4} = \frac{6}{4} = \frac{3}{2}$

$\beta = -\frac{3}{2} + \frac{9}{2} = \frac{6}{2}$

$\frac{\beta S}{\alpha^2} = 5$

23.(10)



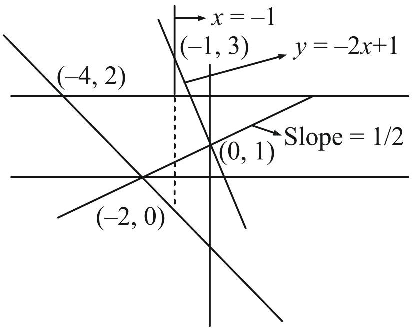
$2y = 2\left(\frac{x+2}{2}\right)$

$2y = x + 2$ ....(1)

$2x + 2y - 4\left(\frac{x+2}{2}\right) = 0$

$x + y - x - 2 = 0$

$y = 2$



Circumradius $=\sqrt{(-1-2)^2+(3-2)^2}=\sqrt{10}$

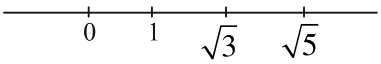
$r^2=10$

24.(9) $x^7+3x^5-13x^3-15x=0$

$x(x^6+3x^4-13x^2-15)=0$

$x^2=t$

$x(t+1)(t+5)(t-3)=0$



$\alpha_1\alpha_2-\alpha_3\alpha_4+\alpha_5\alpha_6$

$=\sqrt{5}i\times\sqrt{5}i-\sqrt{3}\times-\sqrt{3}+i\times-i$

$5+3+1=9$

25.(14) $\alpha=8-14i$

$A=\left\{z\in C:\dfrac{\alpha Z-\bar{\alpha}\bar{Z}}{Z^2-(\bar{Z})^2-112i}=1\right\}$

$B=\{Z\in C:|z+3i|=4\}$

$\alpha Z-\bar{\alpha}\bar{Z}=Z^2-[\bar{Z}]^2-112i$

$\alpha Z-\bar{\alpha}\bar{Z}=(Z+\bar{Z})(Z-\bar{Z})-112i$

$(8-14i)(x+iy)-(8+14i)(x-iy)=2x\times 2iy-112i$

$8x+8iy-14ix+14y-8x+8iy-14ix-14y=4ixy-112i$

$16iy-28ix=4ixy-112i$

$4y-7x=xy-28$

$4y+28-7x-xy=0$

$4(y+7)-x(y+7)=0$

$(x-4)(y+7)=0$

$x=4, y=-7\Rightarrow z=4+iy$ or $Z=x-7i$

$|Z+3i|=4$

$x^2+(y+3)^2=16$

$x=4, \quad 16+(y+3)^2=16$

$y+3=0$

$y=-3$

$Z=4-3i$

$y=-7, \qquad x^2+(-7+3)^2=16$

$\qquad x^2+16=16, \ x=0$

$Z=0-7i$

$=\sum_{Z\in A\cap B}(\operatorname{Re}Z-\operatorname{Im}Z)$

$=4-(-3)+0-(-7)$

$=7+7=14$

26.(9) $C_2=5=4r_1+4r_2$

$r_1+r_2=\frac{5}{4}$

$r_1^2+r_2^2+2r_1r_2=\frac{25}{16} \Rightarrow 2r_1r_2=\frac{12}{16}, \ r_1r_2=\frac{3}{8}$

$2r_1\cdot 2r_2=\frac{3}{2}$ **……………..(1)**

$C_3=\frac{13}{4}=4\left(r_1^2+r_2^2\right)$

$r_1^2+r_2^2=\frac{13}{16}$

$(2r_1)^2+(2r_2)^2=\frac{13}{4}$ ……….(2)

Solving (1) and (2)

$r_1=\frac{1}{2}, \ r_2=\frac{3}{4}, \qquad ab=4\cdot\left(\frac{1}{2}\right)^5=\frac{1}{8}$

$C_k=4\left(\frac{1}{2}\right)^{k-1}+4\cdot\left(\frac{3}{4}\right)^{k-1}, \qquad b_4=4\cdot\left(\frac{3}{4}\right)^3=\frac{27}{16}$

$\sum_{k=1}^{\infty}C_k=4\cdot\frac{1}{1-\frac{1}{2}}+4\cdot\frac{1}{1-\frac{3}{4}}=8+16$

$\left(\sum_{k=1}^{\infty}C_k\right)-(12a_6+8b_4)=24-\left(12\times\frac{1}{8}+8\times\frac{27}{16}\right)=24-15=9$

27.(461)

Let $a_1 = b_1 = 1$

$a_n = a_{n-1} + (n-1)$; $b_n = b_{n-1} + a_{n-1} \quad \forall\ n \geq 2$

$$\sum_{n=2}^{n} a_n - a_{n-1} = (a_n - a_1) = \sum_{n=2}^{n} (n-1) \Rightarrow\ a_n = 1 + \frac{n(n-1)}{2} = 1 + {}^{n-1}C_2$$

$$\sum_{n=2}^{n} b_n - b_{n-1} = \sum_{n=2}^{n} a_{n-1}\ ;\ b_n - b_1 = n - 1\sum_{n=2}^{n} {}^{n-1}C_2\ ;\ b_n = n + \sum {}^{n-1}C_2$$

$$S = \sum_{n=1}^{10} \frac{n}{2^n} + \sum_{n=1}^{10} \frac{{}^{n-1}C_2}{2^n}$$

$$2S = \sum_{n=1}^{n} \frac{n}{2^{n-1}} + \sum_{n=1}^{n} \frac{{}^{n-1}C_2}{2^{n-1}} = \sum_{n=1}^{8} \frac{n}{2^{n-1}} + \frac{9}{2^8} + \frac{10}{2^9} + \left(\frac{1}{24} + \frac{3}{8} + \ldots\ldots - \frac{36}{2^9}\right)$$

$$2S - T = \frac{28}{2^9} + \left(\frac{1}{4} + \frac{3}{8} + \ldots\ldots \frac{36}{2^9}\right)$$

$$2^7(2S - T) = 7 + (2^5 + 3.2^4 + \ldots. + 36.4) = 461$$

28.(3000)

Total 4-digit numbers = 9000

Having G.C.D, 2 with $54 = 3^3 \times 2^1$

Not multiple of $3 \rightarrow 3000$

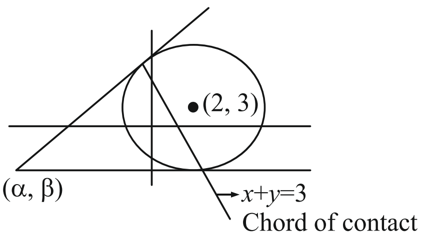
Exactly multiple of $2 = \frac{6000}{2} = 3000$

Or

Even numbers = 4500

Not multiple of 3 = 3000

29.(11)



$$(x-2)^2 + (y-3)^2 = 16$$

$$\alpha x - 4\left(\frac{x+\alpha}{2}\right) + 4 + \beta y - 2 \times 3\left(\frac{y+\beta}{2}\right) + 9 = 16$$

$$(\alpha - 2)x + (\beta - 3)y - 2\alpha - 3\beta - 3 = 0$$

$$\frac{\alpha - 2}{1} = \frac{\beta - 3}{1} = \frac{-2\alpha - 3\beta - 3}{-3}$$

$$3\alpha - 6 = 2\alpha + 3\beta + 3$$

$$\alpha - 3\beta = 9 \Rightarrow \beta = 5$$

$$3\beta - 9 = 2\alpha + 3\beta + 3$$

$$2\alpha = -12,\ \alpha = -6$$

$$= 4\alpha - 7\beta = 4 \times (-6) - 7 \times -5 = 11$$

30.(603)

$x = \{11, 12, 13, ...41\}$

$y = \{61, 62, ...91\}$

$\bar{x} = \dfrac{11 + ...41}{31} = 26$

$\bar{y} = \dfrac{61 + ...91}{31} = 76$

$x \cup y = \{11, 12, ...41, 61, 62, ...91\}$

Variable $\dfrac{\sum (x_i)^2 + (y_i)^2}{62} - \left(\dfrac{\bar{x} + \bar{y}}{2}\right)^2$

$= \dfrac{(11^2 + ...(41)^2) + (61^2 + ...91^2)}{62} - (51)^2 = 3306 - 2601 = 705$

$|\bar{x} + \bar{y} - 705| = |26 + 76 - 705| = |102 - 705| = 603$