VMC Vidyamandir Classes SINCE 1986
IIT JEE | MEDICAL | FOUNDATION

SOLUTIONS

Joint Entrance Exam | IITJEE-2023

25^{th} JAN 2023 | Evening Shift

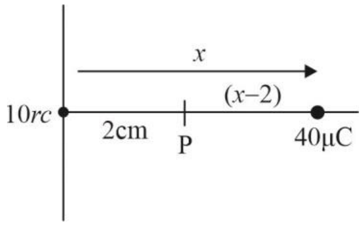
PHYSICS

SECTION - 1

1.(1) $R_{new} = 5^2 \times R_0 = 25 \times 5 = 125\Omega$

2.(1) $\frac{K10\mu C}{4} = \frac{K40\mu C}{(x-2)^2}$

Solving $x = 6cm$



3.(2) $C_v = \frac{5}{2}R + 2 \times \frac{1}{2}R$ ($\because$ Vibrational mode = 2 deg of freedom)

$= \frac{7}{2}R$

4.(4) Theory based

5.(4) Theory based

6.(1) Assuming $\vec{v}$ is $\perp$ to length of wire

$E = Blv = 16$ volt

7.(2) $\tau = K\theta = 4 \times 10^{-5} \times 5 \times 10^{-2} = 2 \times 10^{-6} N - m$

$NIAB = \tau = 2 \times 10^{-6} N - m$

$N = 200 \;\; I = 10 \times 10^{-3} A \;\; B = 10^{-2} T$

$A = 10^{-4} m^2$

$A = 1 cm^2$

8.(4) $V = \frac{dx}{dt} = 8t$

$V = 8 \times 5 = 40 m/s$

9.(2) Let $x = A\sin\omega t$

at $t = t'$ $\quad x = \frac{A}{2}$

$\frac{A}{2} = A\sin\omega t'$

$t' = \frac{\pi}{6\omega} = 2\sec$

$\frac{\pi}{\omega} = 12\sec$

$\frac{T}{4} = \frac{\pi}{2\omega} = 6\sec$ (time taken from 0 to A)

$\frac{T}{4} = t' + t''$ time taken from $\frac{A}{2}$ to A is t''

$t'' = 4\sec$

10.(4) Theory based

11.(3) Theory based

12.(1) Theory based

13.(1) Theory based

14.(3) Theory based

15.(4) Theory based

16.(1) $\frac{t_Q - 0}{100 - 0} = \frac{t_p - 30}{180 - 30}$

17.(3) $E_{ev} = \frac{hc}{\lambda e} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{1241 \times 10^{-10} \times 1.6 \times 10^{-19}} = 0.010 \times 1000 = 10 eV$

18.(4) Theory based

19.(2) $\frac{g \sin 45 + \mu g \cos 45}{g \sin 45 - \mu g \cos 45} = 2$

Solving we get $\mu = \frac{1}{3}$

20.(3) $U_i = \frac{-GmMe}{\text{Re}}$

$U_f = \frac{-GmMe}{3\text{Re}} \qquad (\because h = 2\text{Re})$

$\Delta U = U_f - U_i$

$\Delta U = \frac{2}{3} \frac{GMem}{\text{Re}} \qquad \left(\because\ g = \frac{GMe}{\text{Re}^2}\right)$

$\Delta U = \frac{2}{3} Mg\,\text{Re}$

SECTION – 2

21.(6) $C' = \frac{A\varepsilon_0}{(d - t) + \frac{t}{k}} \qquad C = \frac{\varepsilon_0}{d} = 5\mu F \qquad \text{(given)}$

$C' = \frac{A\varepsilon_0}{\left(d - \frac{d}{2}\right) + \frac{d}{2 \times \frac{3}{2}}} = \frac{A\varepsilon_0}{\frac{d}{2} + \frac{d}{3}} = \frac{6A\varepsilon_0}{5d} = 6\mu F$

22.(5) $I_S = M_S R^2 \left(\frac{7}{5}\right)$

$I_D = M_D R^2 \left(\frac{5}{4}\right)$

$\frac{I_D}{I_S} = \frac{\frac{5}{4} M_D R^2}{\frac{7}{5} M_S R^2} = \frac{25}{28} \times \frac{M_D}{M_S} = \frac{5}{7}$

23.(400) $f' = f_0\left(\frac{V}{V - V_S}\right)$

$= 320\left(\frac{330}{330-66}\right) = 400\mu z$

24.(1) Let original drop radius be R final drop radius be r

$\frac{4}{3}\pi R^3 = 1000\frac{4}{3}\pi r^3$

$R = 10r$

$\frac{U_f}{U_i} = \frac{T \times 1000 \times 4\pi r^2}{T \times 4\pi R^2} = 1000\frac{r^2}{R^2} = 10$

$x = 1$

25.(4) $M_1 = 1kg \qquad M_2 = 3kg$

$U_i = 2 \qquad U_2 = 0$

$V_1 = -2m/s$

$V_1 = \left(\frac{M_1 - M_2}{M_1 + M_2}\right)U_1 + \left(\frac{2M_2}{M_1 + M_2}\right)U_2$

$-2 = \frac{(1-3)}{1+3}U_1 \qquad 4m/s = U_1$

26.(1) $E_{eq} = \frac{\left(\frac{E_1}{r_1} - \frac{E_2}{r_2}\right)}{\frac{1}{r_1} + \frac{1}{r_2}} = 6V$

$r_{eq} = 2\Omega \left(\frac{1}{r_{eq}} = \frac{1}{3} + \frac{1}{6}\right)$

$R = 4\Omega$

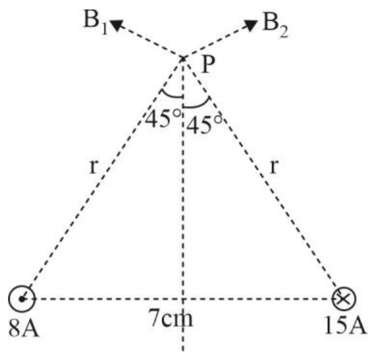
$I = \frac{E_{eq}}{r_{eq} + R} = 1A$

27.(68) $\frac{r}{\sqrt{2}} = \frac{7}{2} \qquad r = \frac{7}{\sqrt{2}}cm$

$f_{met} = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2\pi r}\sqrt{8^2 + 15^2}$

$= \frac{\mu_0}{2\pi r} \times 17 = \frac{\mu_0\sqrt{2}}{2\pi \times 7} \times 17 \times 10^2 = \frac{2 \times 10^{-7} \times \sqrt{2}}{7} \times 17 \times 10^{+2}$

$= 2 \times 17 \times 10^{-5} \times 0.2 = 68 \times 10^{-6}T = 68$



28.(8) $\cos\phi = \frac{R}{z} = \frac{80}{\sqrt{80^2 + (60)^2}} = 0.8$

$x = 8$

29.(2) Let mass number of parent be A

$A_1 + A_2 = A \quad \ldots(i)$

$\frac{A_1}{A_2} = \frac{2}{3}$ (as momentum is conserved)

Solving

$A_1 = \frac{2}{5}A \qquad A_2 = \frac{3}{5}A$

$\frac{r_1}{r_2} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{3}} = \left(\frac{2}{3}\right)^{\frac{1}{3}}$

30.(30) As image of plane mirror is 5 cm behind it. So image of lens is at

$20cm - 5cm = 15cm = V$

$U = ?\ V = +15cm\ \ f = +10cm$ (Using $\frac{1}{V} - \frac{1}{U} = \frac{1}{f}$)

$U = 30cm$

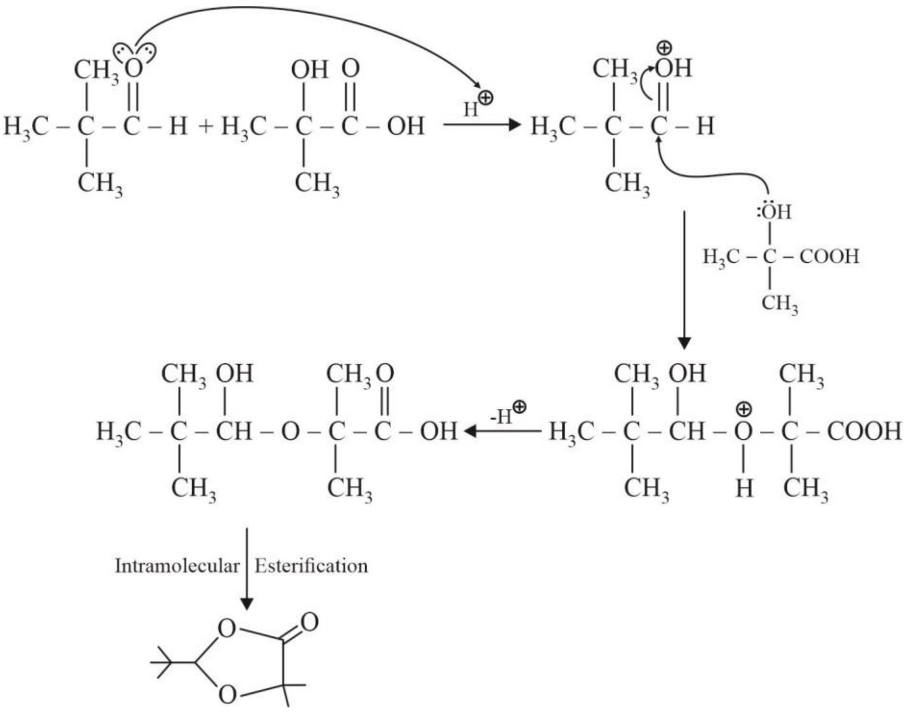
CHEMISTRY

SECTION - 1

1.(3) Ammonium salts produce haze in atmosphere

Polychlorinated biphenyls act as cleaning solvents

2.(4)



3.(2) Butylated hydroxy anisole (BHA) is added to butter because it acts as preservative by lowering the rate of self-oxidation of butter.

4.(2) Metallic character increases down the group and decreases along a period.

$\therefore \text{Si} < \text{Be} < \text{Mg} < \text{K}$

5.(1) In chemisty dipole moment is from '+' to '–'

Statement 1 : False

Statement 2 : True

6.(3) Cobalt catalyst $\rightarrow$Methanol production

Syn gas $\rightarrow$ Coal gasification

Nickel catalyst $\rightarrow$ Water gas production

Brine solution $\rightarrow$ $(H_2 + Cl_2)$ production

7.(2) $A \longrightarrow AgCl$

$B \longrightarrow [Ag(NH_3)_2]Cl$

8.(1) $H_3C-\overset{*}{C}H(Br)-\overset{*}{C}H(D)-CH_3$

9.(1) Na is the weakest reducing agent since its oxidation potential is least among given options.

10.(2) (i) 500 gm of 0.25 molal aqueous solution

(ii) 250 mL of 0.25 molal aqueous solution

$$m = \frac{\text{moles of solute}}{\text{mass of solvent (in kg)}}$$

$$0.25 = \frac{n_1 \times 1000}{500 - n_1 \times 62} \quad \ldots\text{(i)} \qquad \left(n_1 = \frac{w_1}{62}\right)$$

$$0.25 = \frac{n_2 \times 1000}{250 - n_2 \times 62} \quad \ldots\text{(ii)} \qquad \left(n_2 = \frac{w_2}{62}\right)$$

Solving both equations for $\frac{w_1}{w_2}$ ratio, we get

$$\frac{w_1}{w_2} = \frac{2}{1}$$

11.(2) When $K_2Cr_2O_7$ acts as oxidizing agent. Oxidation state of Cr changes from +6 to +3

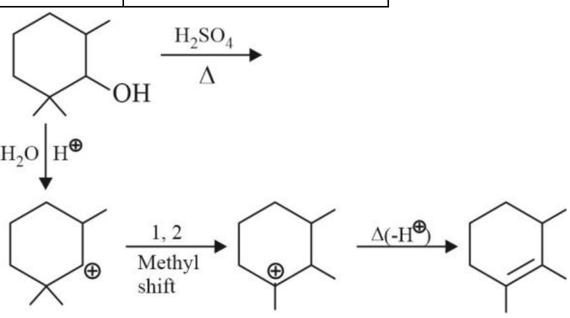
12.(1) In froth floatation method ; the purpose of rotating paddle is to avoid settling of ore.

13.(3) Alkali metals impart color to oxidizing flame.

14.(4)

Polymer	Use
Glyptal	Paints and Lacquers
Neoprene	Gaskets
Acrilan	Synthetic wool
LDP	Flexible pipes

15.(1)

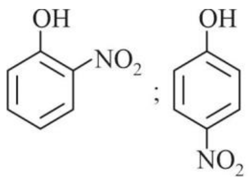


16.(2) (A) $CH_3CH_2CH_2NH_2$; $CH_3CH_2NH-CH_3$ are functional isomers

(B) $CH_3CH_2CH_2CH_2-\underset{\underset{O}{\|}}{C}-CH_3$; $CH_3-CH_2-CH_2-\underset{\underset{O}{\|}}{C}-CH_2-CH_3$ are metamers

(C) $CH_3-\underset{\underset{O}{\|}}{C}-NH_2$; $CH_3-\underset{\underset{OH}{|}}{C}=NH$ are tautomers

(D)



are positional isomer

17.(4) CO : Neutral ; sparingly soluble in water

CO_2 : Acidic ; dissolve in small quantity in water to produce H_2CO_3

18.(4) Order of basic strength in aqueous medium

$Et_2NH > Et_3N > EtNH_2 > PhNH_2$

Order of pK_b

$Et_2NH < Et_3N < EtNH_2 < PhNH_2$

(3.00) (3.25) (3.29) (9.38)

19.(1) Order of crystal field splitting (Δ) energy

α strength of ligand

α no. of ligands

$C > B > A > D$

Wavelength of light absorbed $\propto \dfrac{1}{\text{splitting energy}}$

$\underset{\substack{(310)\\ \text{nm}}}{C} < \underset{\substack{(475)\\ \text{nm}}}{B} < \underset{\substack{(535)\\ \text{nm}}}{A} < \underset{\substack{(600)\\ \text{nm}}}{D}$

20.(4) If $[H^+]$ is $\uparrow$ by 1000 times

pH should decrease by 3 units.

SECTION – 2

21.(5) $[Co(NH_3)_4Cl_2]Cl \longrightarrow [Co(NH_3)_4Cl_2]^+ + Cl^-$ (1)

$[Ni(H_2O)_6]Cl_2 \longrightarrow [Ni(H_2O)_6]^{+2} + 2Cl^-$ (2)

$[Pd(NH_3)_4]Cl_2 \longrightarrow [Pd(NH_3)_4]^{+2} + 2Cl^-$ (2)

$\therefore$ 5 moles of AgCl

22.(2) Water vapours are absorbed by anhydrous calcium chloride

ΔH is defined per mole

23.(5) d_{xy}, d_{yz}, d_{xz} orbitals have electron density diagonally.

24.(2) $k = 4.6 \times 10^{-3} s^{-1}$

For first order reaction

$A_t = A_0 e^{-kt}$ or $2.303 \log\left(\dfrac{A_0}{A_t}\right) = kt$

For 10% completion

$$t_{1/10} = \frac{2.303}{k} \log\left(\frac{A_0}{0.9A_0}\right) \quad \ldots(i)$$

For 90% completion

$$t_{9/10} = \frac{2.303}{k}\log\left(\frac{A_0}{0.1A_0}\right) \quad \ldots\text{(ii)}$$

$$\frac{t_{1/10}}{t_{9/10}} = \frac{\log\left(\frac{10}{9}\right)}{\log(10)}$$

$$t_{9/10} = 25 \times t_{1/10}$$

$$\Rightarrow \alpha = 1 - e^{-kt}$$

25.(3) $Pt(s) \mid H_2(g)(1bar) \mid H^+(aq.)(1M) \parallel M^{+3}(aq.), M^+(aq.) \mid Pt(s)$

$$A: H_2 \longrightarrow 2H^+ + 2e^-$$
$$C: M^{+3} + 2e^- \longrightarrow M^+$$

$$H_2(g) + M^{+3}(aq.) \longrightarrow 2H^+(aq.) + M^+(aq.)$$

$$E_{cell} = E^0_{cell} - \frac{0.059}{n}\log Q \qquad (E^0_{cell} = 0.2\,V)$$

$$0.1115 = 0.2 - \frac{0.059}{2}\log\frac{[H^+]^2[M^+]}{[P_{H_2}][M^{+3}]}$$

$$0.1115 = 0.2 - \frac{0.059}{2}\log 10^a \qquad \left(\because \frac{[M^+]}{[M^{+3}]} = 10^a\right)$$

$$\therefore\ a = 3$$

26.(2) Surface tension is due to uneven forces acting on the molecules present on the surface.

The molecules on the surface are responsible for vapour pressure if the system is a closed system.

27.(12) Let the mass of hydrocarbon be 100 gm

$\therefore$ mass of carbon = 85.8 gm

mass of hydrogen = 14.2 gm

molar ratio of $C:H = \left(\frac{85.8}{12}\right) \times \left(\frac{1}{14.2}\right) = 1:2$

Molecular weight of hydrocarbon is given 84 g/mol

$\therefore\ C_6H_{12}$

28.(4) (A) $\pi_1 = 0.5 \times R \times T$

$\pi_2 = 2 \times 0.25RT$

$\pi_1 = \pi_2$

(B) $\pi_1 = 5 \times 0.1RT$

$\pi_2 = 5 \times 0.1RT$

$\pi_1 = \pi_2$

(C) $\pi_1 = 5 \times 0.05RT$

$\pi_2 = 2 \times 0.25RT$

$\pi_1 \neq \pi_2$

(D) $\pi_1 = 2 \times 0.15RT$

$\pi_2 = 3 \times 0.1RT$

$\pi_1 = \pi_2$

(E) $\pi_1 = 5 \times 0.02RT$

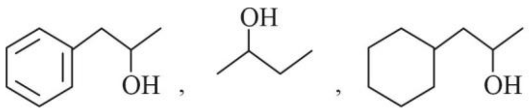
$\pi_2 = 2 \times 0.05RT$

$\pi_1 = \pi_2$

$\therefore$ (A), (B), (D), (E) are correct

29.(3) Alcohols give red coloration with ceric ammonium nitrate

2° methylated alcohols give positive iodoform test



are correct options

30.(847) $C_2H_4(g) + 3O_2(g) \longrightarrow 2CO_2(g) + 2H_2O(\ell)$

xL - 2xL -

$CH_4(g) + 2O_2(g) \longrightarrow CO_2(g) + 2H_2O(\ell)$] Considering T, P constant

yL - yL -

$x + y = 16.8$

$2x + y = 28$

$\Rightarrow x = 11.2$

$y = 5.6$

Heat evolved $= (n_{C_2H_4} \times 1400 + n_{CH_4} \times 900)$kJ

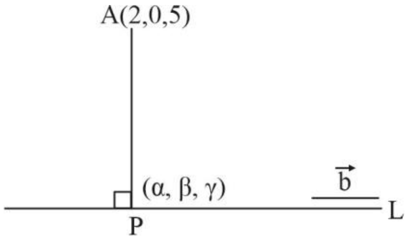
$$= \left(\frac{1 \times 11.2}{0.0821 \times 298} \times 1400\right) + \left(\frac{1 \times 5.6}{0.0821 \times 298} \times 900\right)$$

= 640.894 + 206.001= 846.895 = 847

MATHEMATICS

SECTION - 1

1.(2) Let $L = \frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1} = \lambda$



Let foot of perpendicular is

$P(2\lambda - 1, 5\lambda + 1, -\lambda - 1)$

$\overrightarrow{PA} = (3 - 2\lambda)\hat{i} - (5\lambda + 1)\hat{j} + (6 + \lambda)\hat{k}$

Direction ratio of line $\Rightarrow\ \vec{b} = 2\hat{i} + 5\hat{j} - \hat{k}$

Now, $\Rightarrow \overrightarrow{PA} \cdot \vec{b} = 0$

$\Rightarrow\ 2(3 - 2\lambda) - 5(5\lambda + 1) - (6 + \lambda) = 0$

$\Rightarrow\ \lambda = \frac{-1}{6}$

$P(2\lambda - 1, 5\lambda + 1, -\lambda - 1) \equiv P(\alpha, \beta, \gamma)$

$\Rightarrow\ \alpha = 2\left(-\frac{1}{6}\right) - 1 = -\frac{4}{3} \Rightarrow \boxed{\alpha = -\frac{4}{3}}$

$\Rightarrow\ \beta = 5\left(-\frac{1}{6}\right) + 1 = \frac{1}{6} \Rightarrow \boxed{\beta = \frac{1}{6}}$

$\Rightarrow\ \gamma = -\lambda - 1 = \frac{1}{6} - 1 \Rightarrow \boxed{\gamma = -\frac{5}{6}}$

$\frac{\beta}{\gamma} = \frac{\frac{1}{6}}{\frac{-5}{6}} = \frac{-1}{5}$

2.(3) $\vec{a} \times \vec{b} = (\hat{i} - \hat{j})$

Taking cross product with $\vec{a}$

$\Rightarrow\ \vec{a} \times (\vec{a} \times \vec{b}) = \vec{a} \times (\hat{i} - \hat{j})$

$\Rightarrow (\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$

$\Rightarrow \vec{a} - 3\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$

$\Rightarrow\ 2\vec{a} - 6\vec{b} = 2\hat{i} + 2\hat{j} + 4\hat{k}$

$\Rightarrow\ \vec{a} - 6\vec{b} = 3\hat{i} + 3\hat{j} + 3\hat{k}$

3.(1) Let $A:(3,-4,2)$ $C:(-2,-1,3)$

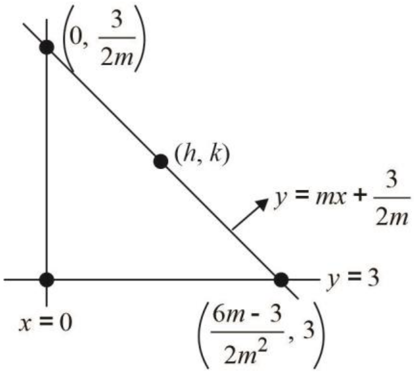
$B:(1,2,-1)$ $D:(5,-2\alpha,4)$

A, B, C, D are coplanar points, then

$$\Rightarrow \begin{vmatrix} 1-3 & 2+4 & -1-2 \\ -2-3 & -1+4 & 3-2 \\ 5-3 & -2\alpha+4 & 4-2 \end{vmatrix} = 0 \quad \Rightarrow \alpha = \frac{73}{17}$$

4.(2) $y^2 = 6x$ & $y^2 = 4ax$

$$\Rightarrow 4a = 6 \Rightarrow a = \frac{3}{2}$$



$$y = mx + \frac{3}{2m}; (m \neq 0)$$

$h = \dfrac{6m-3}{4m^2},\ k = \dfrac{6m+3}{4m}$. Now eliminating m, we get

$$\Rightarrow 3h = 2(-2k^2 + 9k - 9) \quad \Rightarrow 4y^2 - 18y + 3x + 18 = 0$$

5.(3) $\dfrac{x+1}{1} = \dfrac{y}{\frac{1}{2}} = \dfrac{z}{\frac{-1}{12}}$ & $\dfrac{x}{1} = \dfrac{y+2}{1} = \dfrac{z-1}{\frac{1}{6}}$ $\Rightarrow$ Shortest distance $= \dfrac{(\vec{b}-\vec{a})\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}$

$$S.D = \left(-\hat{i} + 2\hat{j} - \hat{k}\right) \cdot \frac{(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}$$

$$\left(\vec{p}\times\vec{q} \equiv \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \frac{1}{2} & \frac{-1}{12} \\ 1 & 1 & \frac{1}{6} \end{vmatrix} = \frac{1}{6}\hat{i} - \frac{1}{4}\hat{j} + \frac{1}{2}\hat{k} \text{ or } 2\hat{i} - 3\hat{j} + 6\hat{k} \right)$$

$$S.D. = \frac{\left(-\hat{i} + 2\hat{j} - \hat{k}\right)\cdot\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)}{\sqrt{2^2 + 3^2 + 6^2}} = \left|\frac{-14}{7}\right| = 2$$

6.(3) $f(x)=2x^n+\lambda$

$f(4)=133$

$f(5)=255$

$133=2\times 4^n+\lambda$ ...(i)

$255=2\times 5^n+\lambda$ ...(ii)

(ii) – (i)

$122=2(5^n-4^n)$

$\Rightarrow\ 5^n-4^n=61$

$\therefore\ n=3$

Now, $f(3)-f(2)=2(3^3-2^3)=38$

Number of divisors is 1, 2, 19, 38 ; & their sum is 60

7.(4) $I=16\int_1^2\frac{dx}{x^3(x^2+2)^2}=16\int_1^2\frac{dx}{x^3x^4\left(1+\frac{2}{x^2}\right)^2}$

Let $1+\frac{2}{x^2}=t\Rightarrow\frac{-4}{x^3}dx=dt$

$I=-4\int_3^{\frac{3}{2}}\frac{dt}{\left(\frac{2}{t-1}\right)^2t^2}$

$I=-4\int_3^{\frac{3}{2}}\left(\frac{t-1}{2}\right)^2\frac{dt}{t^2}$

$I=-1\left[t-2\ell n|t|-\frac{1}{t}\right]_3^{\frac{3}{2}}$

$I=-1\left[\left(\frac{3}{2}-2\ell n\frac{3}{2}-\frac{2}{3}\right)-\left(3-2\ell n3-\frac{1}{3}\right)\right]$

$I=-1\left[2\ell n2-\frac{11}{6}\right]$

$I=\frac{11}{6}-\ell n4$

8.(4) $\frac{dy}{dt}+\alpha y=\gamma e^{-\beta t}$

I.F. $=e^{\int\alpha\,dt}=e^{\alpha t}$

Solution $\Rightarrow y.e^{\alpha t} = \int \gamma e^{-\beta t}.e^{\alpha t} dt$

$$\Rightarrow ye^{\alpha t} = \gamma \frac{e^{(\alpha-\beta)t}}{(\alpha-\beta)} + c \qquad \Rightarrow y = \frac{\gamma}{e^{\beta t}(\alpha-\beta)} + \frac{c}{e^{\alpha t}}$$

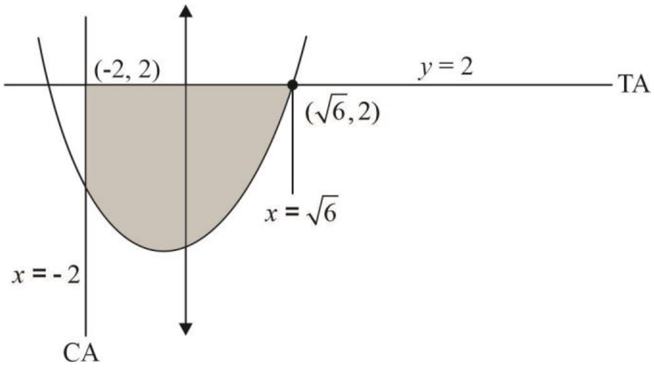
$$\lim_{t\to\infty} y(t) = \frac{\gamma}{\infty} + \frac{c}{\infty} = 0$$

9.(2) $16(x^2+4x)-(y^2-4y)+44=0$

$$16(x+2)^2-64-(y-2)^2+4+44=0$$

$$16(x+2)^2-(y-2)^2=16$$

$$\frac{(x+2)^2}{1}-\frac{(y-2)^2}{16}=1$$



$$A = \int_{-2}^{\sqrt{6}} (2-(x^2-4))\,dx = \int_{-2}^{\sqrt{6}} (6-x^2)\,dx = \left(6x - \frac{x^3}{3}\right)_{-2}^{\sqrt{6}}$$

$$A = \left(6\sqrt{6} - \frac{6\sqrt{6}}{3}\right) - \left(-12 + \frac{8}{3}\right)$$

$$A = \frac{12\sqrt{6}}{3} + \frac{28}{3}$$

$$A = 4\sqrt{6} + \frac{28}{3}$$

10.(4) If $\Delta = \vee, \nabla = \vee$

p	q	$(p\to q)$	$(p\vee q)$	$(p\to q)\vee(p\wedge q)$
T	T	T	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

Hence it is taulogogy.

11.(2) $\lim_{x\to\frac{\pi^+}{2}} e^{\frac{\cot 6x}{\cot 4x}} = \lim_{x\to\frac{\pi^+}{2}} e^{\frac{\sin 4x.\cos 6x}{\sin 6x.\cos 4x}} = e^{2/3}$

$\Rightarrow \lim_{x\to\frac{\pi^-}{2}} \left(1+|\cos x|\right)^{\frac{\lambda}{|\cos x|}} = e^{\lambda} \qquad \Rightarrow f\left(\frac{\pi}{2}\right)=\mu$

For continuous function $\Rightarrow e^{2/3}=e^{\lambda}=\mu$

$\lambda=\frac{2}{3}, \mu=e^{2/3}$

$9\lambda+6\log_e\mu+\mu^6-e^{6\lambda}=10$

12.(4) $\sum_{k=0}^{6} {}^{51-k}C_3$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + ... + {}^{45}C_3 = {}^{45}C_3 + {}^{46}C_3 + ... + {}^{51}C_3$

$= {}^{45}C_4 + {}^{45}C_3 + {}^{46}C_3 + ... + {}^{51}C_3 - {}^{45}C_4$

$({}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r)$

$= {}^{52}C_4 - {}^{45}C_4$

13.(3) $f:\{1,2,3,4\}\to\{a\in\mathbb{Z}:|a|\le 8\}$

$f(n)+\frac{1}{n}f(n+1)=1, \forall n\in\{1,2,3\}$

$f(n+1)$ must be divisible by n

$f(4)\Rightarrow -6,-3,0,3,6$

$f(3)\Rightarrow -8,-6,-4,-2,0,2,4,6,8$

$f(2)\Rightarrow -8,........,8$

$f(1)\Rightarrow -8,........,8$

$\frac{f(4)}{3}$ must be odd since $f(3)$ should be even therefore 2 solution possible

$f(4)$	$f(3)$	$f(2)$	$f(1)$
-3	2	0	1
3	0	1	0

14.(1) $\log_{\sqrt{m}}((\sin x-\cos x)+m-2)\in\left[\log_{\sqrt{m}}(m-4), \log\frac{m}{\sqrt{m}}\right]$

$\therefore \log_{\sqrt{m}}(m-4)=0 \ \& \ \log\frac{m}{\sqrt{m}}=2$

$\therefore m=5$

15.(1) Given, $A^T = A,\ B^T = -B,\ C^T = -C$

Let $M = A^{13}B^{26} - B^{26}A^{13}$

Then, $M^T = (A^{13}B^{26} - B^{26}A^{13})^T$

$= (A^{13}B^{26})^T - (B^{26}A^{13})^T = (B^T)^{26}(A^T)^{13} - (A^T)^{13}(B^T)^{26} = B^{26}A^{13} - A^{13}B^{26} = -M$

Hence, M is skew symmetric

Let $N = A^{26}C^{13} - C^{13}A^{26}$

Then, $N^T = (A^{26}C^{13})^T - (C^{13}A^{26})^T = -(C)^{13}(A)^{26} + A^{26}C^{13} = N$

Hence, N is symmetric $\quad\therefore\quad$ Only S2 is true.

16.(1) $f(x) = 2x^3 + (2p-7)x^2 + 3(2p-9)x - 6$

$f'(x) = 6x^2 + 2(2p-7)x + 3(2p-9)$

$f'(0) < 0 \qquad \therefore\ 3(2p-9) < 0$

$p < \dfrac{9}{2}$

$p \in \left(-\infty, \dfrac{9}{2}\right)$

17.(2) $n(s) = 36$

Given : $N-2,\ \sqrt{3N},\ N+2$ are in G.P.

$3N = (N-2)(N+2)$

$3N = N^2 - 4$

$(N-4)(N+1) = 0 \Rightarrow \boxed{N=4}$ or $N = -1$ rejected

(Sum = 4) $\equiv \{(1,3), (3,1), (2,2)\}$

$n(A) = 3$

$P(A) = \dfrac{3}{36} = \dfrac{1}{12} = \dfrac{4}{48} \Rightarrow k = 4$

18.(4) $AA^T = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{-3}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$B^2 = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2i \\ 0 & 1 \end{bmatrix}$

$B^3 = \begin{bmatrix} 1 & -3i \\ 0 & 1 \end{bmatrix}$

⋮

$B^{2023} = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

$$M = A^T BA$$

$$M^2 = M.M = A^T BAA^T BA = A^T B^2 A$$

$$M^3 = M^2.M = A^T B^2 AA^T BA = A^T B^3 A$$

$$\vdots$$

$$M^{2023} = ...A^T B^{2023} A$$

$$AM^{2023}A^T = \underbrace{AA^T}\, B^{2023}\, \underbrace{AA^T} = B^{2023} = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$$

Inverse of $(AM^{2023}A^T)$ is $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

19.(3) $(z-2i)(\bar{z}+2i) = 4(z+i)(\bar{z}-i)$

$$z\bar{z} + 4 + 2i(z-\bar{z}) = 4(z\bar{z} + 1 + i(\bar{z}-z))$$

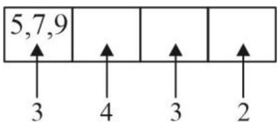
$$3z\bar{z} - 6i(z-\bar{z}) = 0$$

$$x^2 + y^2 - 2i(2iy) = 0$$

$$x^2 + y^2 + 4y = 0$$

20.(3) Numbers between 5000 & 10000

Using digits, 1,3,5,7,9

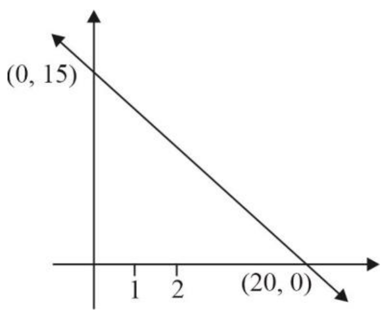


Total numbers $= 3 \times 4 \times 3 \times 2 = 72$

SECTION – 2

21.(31) $y = \dfrac{(60-3x)}{4}$

$$x = 1,\ y = \frac{57}{4} = 14.25$$



$$(1, 1)(1, 2)......(1, 14) \Rightarrow 14\, pts$$

If $x=2,\ y=\frac{27}{2}=13.5$

$(2,2)(2,4)....(2,12)\ \Rightarrow\ 6\,pts$

If $x=3,\ y=\frac{51}{4}=12.75$

$(3,3)(3,6).....(3,12)\Rightarrow\ 4\,pts$

If $x=4,\ y=12$

$(4,4)(4,8)\Rightarrow 2\,pts$

If $x=5,\ y=\frac{45}{4}=11.25$

$(5,5)(5,10)\Rightarrow 2\,pts$

If $x=6,\ y=\frac{21}{2}=10.5$

$(6,6)\Rightarrow 1\,pt$

If $x=7,\ y=\frac{39}{4}=9.75$

$(7,7)\Rightarrow 1\,pt$

If $x=8,\ y=9$

$(8,8)\Rightarrow 1\ pt$

If $x=9,\ y=\frac{33}{4}=8.25\Rightarrow$ no. pt

Total = 31*pts*.

22.(7) $(2023)^{2023}$

$=(2030-7)^{2023}$

$=(35K-7)^{2023}$

$= {}^{2023}C_0(35K)^{2023}(-7)^0+{}^{2023}C_1(35K)^{2022}(-7)+....+{}^{2023}C_{2023}(-7)^{2023}$

$=35N-7^{2023}$

Now, $-7^{2023}=-7\times 7^{2022}=-7(7^2)^{1011}$

$=-7(50-1)^{1011}$

$=-7({}^{1011}C_\alpha 50^{2011}-{}^{2011}C_1(50)^{1010}+......{}^{1011}C_{1011})$

$=-7(5\lambda-1)$

$=-35\lambda+7$

$\therefore$ when $(2023)^{2023}$ is divided by 35 remainder is 7

23.(18) $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k})$ $\qquad \vec{r} = \vec{a} + \lambda\vec{p}$

$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} - \hat{j})$ $\qquad \vec{r} = \vec{b} + \mu\vec{q}$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -1 & 0 \end{vmatrix} = \hat{i} + 2\hat{j} - 3\hat{k}$$

$$d = \left| \frac{(\vec{b} - \vec{a}) \cdot (\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|} \right|$$

$$d = \left| \frac{(-3\hat{j} - \hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})}{\sqrt{14}} \right| = \left| \frac{-6+3}{\sqrt{14}} \right| = \frac{3}{\sqrt{14}}$$

$$\alpha = \frac{3}{\sqrt{14}}$$

Now, $28\alpha^2 = 28 \times \frac{9}{14} = 18$

24.(20) $\int_{\frac{1}{3}}^{3} |\ln x|\, dx = \int_{\frac{1}{3}}^{1} (-\ln x)\, dx + \int_{1}^{3} (\ln x)\, dx$

$$= -[x\ln x - x]_{1/3}^{1} + [x\ln x - x]_{1}^{3}$$

$$= -\left[-1 - \left(\frac{1}{3}\ln\frac{1}{3} - \frac{1}{3}\right)\right] + [3\ln 3 - 3 - (-1)]$$

$$= \left[-\frac{2}{3} - \frac{1}{3}\ln\frac{1}{3}\right] + [3\ln 3 - 2] \quad = -\frac{4}{3} + \frac{8}{3}\ln 3$$

$$= \frac{4}{3}(2\ln 3 - 1) \quad = \frac{4}{3}\left(\ln\frac{9}{e}\right)$$

$\therefore\ m = 4, n = 3$

Now, $m^2 + n^2 - 5 = 16 + 9 - 5 = 20$

25.(6860) (I) 7 Red apple (RA), 5 white apple (WA), 8 oranges (O)

5 fruits to be selected (Note:- Fruits taken different)

Possible selections : (2O, 1RA, 2WA) or (2O, 2RA, 1WA) or (3O, 1RA, 1WA)

$\Rightarrow\ {}^8C_2\,{}^7C_1\,{}^5C_2 + {}^8C_2\,{}^7C_2\,{}^5C_1 + {}^8C_3\,{}^7C_1\,{}^5C_1$

$\Rightarrow 1960 + 2940 + 1960$

$\Rightarrow 6860$

(II) Fruits taken identical

Possible selections = 1 + 1 + 1 = 3

26.(3) $a, b, \frac{1}{18} \rightarrow GP$

$\frac{a}{18} = b^2 \qquad \ldots(i)$

$\frac{1}{a}, 10, \frac{1}{b} \rightarrow AP$

$\frac{1}{a} + \frac{1}{b} = 20$

$\Rightarrow a + b = 20ab,$ from equation (i) ; we get

$\Rightarrow 18b^2 + b = 360b^3 \quad \Rightarrow 360b^2 - 18b - 1 = 0 \quad \{\because b \neq 0\}$

$\Rightarrow b = \frac{18 \pm \sqrt{324 + 1440}}{720} \qquad \Rightarrow b = \frac{18 + \sqrt{1764}}{720} \quad \{\because b > 0\}$

$\Rightarrow b = \frac{1}{12} \qquad \Rightarrow a = 18 \times \frac{1}{144} = \frac{1}{8}$

Now, $16a + 12b = 16 \times \frac{1}{8} + 12 \times \frac{1}{12} = 3$

27.(3) $m_{PQ} \cdot m_{QR} = -1$

$\Rightarrow \frac{10-2}{9+3} \times \frac{10-4}{9-\alpha} = -1 \Rightarrow \alpha = 13$

$m_{OP} \cdot m_{QS} = -1 \Rightarrow m_{QS} = -\frac{4}{7}$

Equation of QS

$y - 10 = -\frac{4}{7}(x - 9)$

$\Rightarrow 4x + 7y = 106 \qquad \ldots(i)$

$m_{OR} \cdot m_{RS} = -1 \Rightarrow m_{RS} = -8$

Equation of RS

$y - 4 = -8(x - 13)$

$\Rightarrow 8x + y = 108 \qquad \ldots(ii)$

Solving equation (i) & (ii)

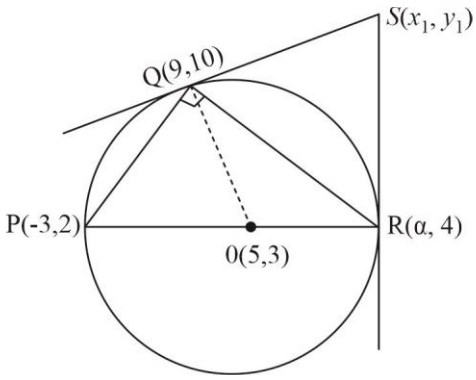
$x_1 = \frac{25}{2} \; y_1 = 8$

$S(x_1, y_1)$ lies on $2x - ky = 1$

$25 - 8k = 1$

$\Rightarrow 8k = 24$

$\Rightarrow k = 3$



28.(25) $\cos 2\theta \cdot \cos\frac{\theta}{2} = \cos 3\theta \cdot \cos\frac{9\theta}{2}$

$\Rightarrow\ 2\cos 2\theta \cdot \cos\frac{\theta}{2} = 2\cos\frac{9\theta}{2}\cdot\cos 3\theta\ \Rightarrow\ \cos\frac{5\theta}{2}+\cos\frac{3\theta}{2}=\cos\frac{15\theta}{2}+\cos\frac{3\theta}{2}$

$\Rightarrow\ \cos\frac{15\theta}{2}=\cos\frac{5\theta}{2}\quad\Rightarrow\ \frac{15\theta}{2}=2k\pi\pm\frac{5\theta}{2}$

$5\theta = 2k\pi \text{ or } 10\theta = 2k\pi$

$\theta=\frac{2k\pi}{5}\qquad \theta=\frac{k\pi}{5}$

$\therefore\ \theta=\left\{-\pi, \frac{-4\pi}{5}, \frac{-3\pi}{5}, \frac{-2\pi}{5}, \frac{-\pi}{5}, 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \pi\right\}$

$m=5,\ n=5$

$\therefore\ m.n=25$

29.(9) $P(S)=\frac{1}{4}\qquad P(N)=\frac{3}{4}$

$$P\left(\frac{\text{Person is smoker}}{\text{Person diagnosed with cancer}}\right)=\frac{\frac{1}{4}\cdot 27P}{\frac{1}{4}\cdot 27P+\frac{3}{4}P}=\frac{27}{30}=\frac{9}{10}$$

$\therefore\ K=9$

30.(45) $x^2+60^{\frac{1}{4}}x+a=0$

Let the roots be $\alpha \,\&\, \beta$

$\alpha+\beta=-60^{\frac{1}{4}}\ \&\ \alpha\beta=a$

Given $\alpha^4+\beta^4=-30\quad\Rightarrow\ \left(\alpha^2+\beta^2\right)^2-2\alpha^2\beta^2=-30$

$\left[(\alpha+\beta)^2-2\alpha\beta\right]^2-2a^2=-30$

$\Rightarrow\ \left\{60^{\frac{1}{2}}-2a\right\}^2-2a^2=-30\Rightarrow\ 60+4a^2-4a\times 60^{\frac{1}{2}}-2a^2=-30$

$\Rightarrow\ 2a^2-4\cdot 60^{\frac{1}{2}}a+90=0$

Product $=\frac{90}{2}=45$