SOLUTIONS

Joint Entrance Exam | IITJEE-2023

1st FEB 2023 | Morning Shift

PHYSICS

SECTION - 1

1.(1) Average K.E. = $\frac{f}{2}k_B T \propto T$

2.(2) Acceleration due to gravity decreases as we go down below the earth's surface.

3.(2) $[b]=[V]=\left[L^3\right]$

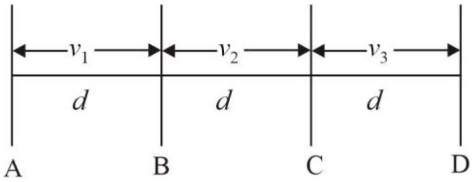
$\because \left[\frac{a}{V^2}\right]=[P] \qquad \therefore [a]=[P]\left[V^2\right]=\left[M^1L^{-1}T^{-2}.L^6\right]=\left[M^1L^5T^{-2}\right]$

$\therefore \left[\frac{b^2}{a}\right]=\left[\frac{L^6}{M^1L^5T^{-2}}\right]=\left[M^{-1}L^1T^2\right]$

[Compressibility] = [1/Bulk Modulus] = $\left[\frac{1}{ML^{-1}T^{-2}}\right]=\left[M^{-1}L^1T^2\right]$

$\therefore \left[\frac{b^2}{a}\right]=$ [Compressibility]

4.(4)



Let say AB = d meter

Then BC = d meter

AD = AB + BC + CD = 3AB

$\therefore \quad AB=BC \Rightarrow 2AB+CD=3AB \qquad \Rightarrow CD=AB$

Hence CD = d meter

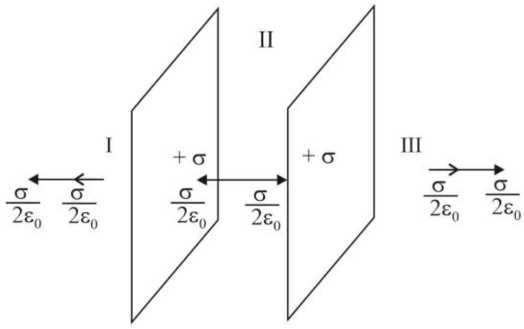
$$\text{Average velocity } = \frac{3d}{\frac{d}{v_1}+\frac{d}{v_2}+\frac{d}{v_3}} = \frac{d}{d}\left(\frac{3}{\frac{1}{v_1}+\frac{1}{v_2}+\frac{1}{v_3}}\right)$$

$$\text{Average velocity } = \frac{3}{\frac{v_2v_3+v_1v_3+v_1v_2}{v_1v_2v_3}} = \frac{3v_1v_2v_3}{v_1v_2+v_2v_3+v_1v_3}$$

5.(3) $\vec{E}_I = -\frac{\sigma}{2\varepsilon_0}\hat{n}-\frac{\sigma}{2\varepsilon_0}\hat{n}=-\frac{\sigma}{\varepsilon_0}\hat{n}$

$\vec{E}_{II} = +\frac{\sigma}{2\varepsilon_0}\hat{n}-\frac{\sigma}{2\varepsilon_0}\hat{n}=\vec{0}$

$\vec{E}_{III} = \frac{\sigma}{2\varepsilon_0}\hat{n}+\frac{\sigma}{2\varepsilon_0}\hat{n}=\frac{\sigma}{\varepsilon_0}\hat{n}$



6.(4) $R = 10^{-3} m$

$$\frac{4\pi}{3}R^3 = (125)\frac{4\pi}{3}r^3 \quad \Rightarrow r = \frac{R}{5} = \frac{10^{-3}}{5}m = 2\times10^{-4}m$$

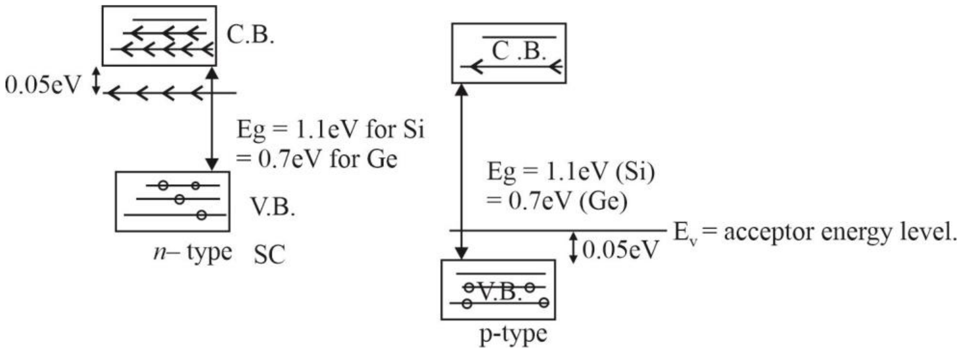
$$SE = SE_f - SE_i = (125)\sigma 4\pi r^2 - \sigma 4\pi R^2$$

$$= 4\pi\sigma\left(125\times4\times10^{-8} - 10^{-6}\right) = 16\pi\sigma\times10^{-6} = 16\times3.14\times0.45\times10^{-6}J = 2.26\times10^{-5}J$$

7.(2) Momentum $= p = \frac{h}{\lambda}$ for both.

$$\therefore \frac{K.E.\ of\ proton}{K.E.\ of\ \alpha - particle} = \frac{p^2/2m_p}{p^2/2m_\alpha} = \frac{m_\alpha}{m_p} = 4:1$$

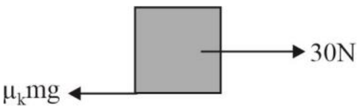
8.(3)



9.(1) FM frequency range starts from 88 MHz and ends at 108 MHz.

10.(4) $S = \frac{1}{2}at^2$

$$\Rightarrow 50 = \frac{1}{2}a(10)^2 \Rightarrow a = 1m/s^2$$



$$f_{net} = ma$$

$$\Rightarrow \quad 30 - \mu_k mg = ma \quad \Rightarrow \quad 30 - \mu_k 50 = 5$$

$$\therefore \quad \mu_k = 0.50$$

11.(1) $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{70}{7\times10^{-3}}}m/s = 100m/s$

12.(1) $B.E. = \Delta m.c^2 = \left[2m_p + 2m_n - m_{He}\right].c^2 = \left[(2)(1.0073+1.0087) - 4.0015\right]\times931.5MeV$

$$= 0.0305\times931.5MeV = 28.4MeV$$

13.(4) $TV^{\gamma-1} =$ constant

$$\Rightarrow \frac{T_f}{T_i} = \left(\frac{V_i}{V_f}\right)^{\gamma-1} = \left(\frac{1}{2}\right)^{1/2} = \frac{1}{\sqrt{2}} \quad \therefore T_f = \frac{T}{\sqrt{2}}$$

$$W = \frac{nR\Delta T}{\gamma-1} = 2R\left(T - \frac{T}{\sqrt{2}}\right) = RT\left(2-\sqrt{2}\right)$$

14.(1) $V_y = \sqrt{2gh} = \sqrt{200} = 10\sqrt{2}m/s$

$V_x = 5m/s$

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{25+200}m/s = 15m/s$

15.(4) Klystron value is used in ovens to generate microwaves.

16.(3) $B = \frac{\mu_0 i}{4\pi r} + \frac{\mu_0 i}{4r} = \frac{\mu_0 i}{2r}\left(\frac{1}{2} + \frac{1}{2\pi}\right)$

17.(2) AC generator works on the principle of electromagnetic induction.

Transformer works on mutual induction.

Resonance occurs when L and C both are present.

Quality factor defines sharpness of Resonance.

18.(4) Using Malus' law $I = I_0 \cos^2\theta$

$\therefore$ After n polarisation

$I = I_0 \cos^2 45°.\cos^2 45°$..... upto n terms

$I = I_0\left(\cos^2 45°\right)^n \quad \Rightarrow I = \frac{I_0}{2^n} = \frac{I_0}{64} \quad \Rightarrow 2^n = 64 \quad \therefore n = 6$

19.(1) $M_p = \frac{M}{9}$

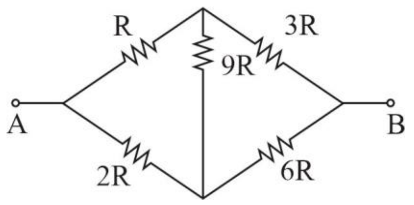
$R_p = \frac{R}{2}$

$v_e = \sqrt{\frac{2GM}{R}}$

Escape velocity on planet $= \frac{v_e}{3}\sqrt{x} = \sqrt{\frac{2GM_p}{R_p}} = \sqrt{\frac{4GM}{9R}}$

$\Rightarrow \frac{v_e}{3}\sqrt{x} = \frac{\sqrt{2}}{3}\sqrt{\frac{2GM}{R}} \quad \Rightarrow \frac{v_e}{3}\sqrt{x} = \frac{\sqrt{2}}{3}v_e \quad \therefore x = 2$

20.(2) Wheat stone network is balanced as $\frac{R}{2R} = \frac{3R}{6R}$



$\therefore$ 9R is removed

$\therefore \quad R_{AB} = (R+3R)||(2R+6R) = \frac{(4R)(8R)}{12R} = \frac{8}{3}R$

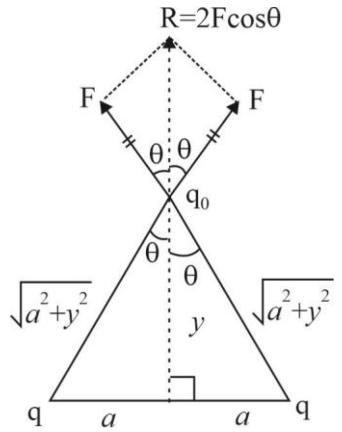
SECTION – 2

21.(2) $F = \dfrac{kqq_0}{a^2 + y^2}$

Resultant force R $= 2F\cos\theta = \dfrac{2kqq_0}{a^2+y^2}\dfrac{y}{\sqrt{a^2+y^2}}$

$$R = 2kqq_0 \frac{y}{\left(a^2+y^2\right)^{3/2}}$$

For R to be maximum, $\dfrac{dR}{dy} = 0$



$$\Rightarrow \frac{2kqq_0}{\left(a^2+y^2\right)^3}\left[\left(a^2+y^2\right)^{3/2} - y\frac{3}{2}\left(a^2+y^2\right)^{1/2} 2y\right] = 0$$

$$\Rightarrow \frac{2kqq_0}{\left(a^2+y^2\right)^{5/2}}\left(a^2+y^2-3y^2\right) = 0 \Rightarrow \quad 2y^2 = a^2$$

$\therefore \quad y = \pm\dfrac{a}{\sqrt{2}} \quad \therefore\ x = 2$

22.(828) $13.6eV - 3.4eV = 10.2eV$ for 1st excited state

$13.6eV - 1.51eV = 12.09eV$ for 2nd excited state

$13.6eV - 0.85eV = 12.75eV$ for 3rd excited state

$\because\ 13.6eV - \dfrac{13.6eV}{n^2} \le 12.75eV$

$\Rightarrow 1 - \dfrac{1}{n^2} \le \dfrac{12.75}{13.60} \quad \Rightarrow n^2 \le 16$

$\therefore\ n \le 4 \quad \therefore\ n = 4$

$L = \dfrac{nh}{2\pi} = \dfrac{4h}{2\pi} = \dfrac{2}{\pi}(4.14)\times 10^{-15} eVs$

$= \dfrac{828}{\pi}\times 10^{-17} eVs$

$\therefore\ x = 828$

23.(25) $\phi l_1 = E_1$

$\phi l_2 = E_2$

$$\frac{E_2}{E_1} = \frac{l_2}{l_1} = \frac{100cm}{60cm}$$

$$\Rightarrow \frac{E}{1.5V} = \frac{5}{3}$$

$$\therefore \; E = 2.5V = \frac{25}{10}V$$

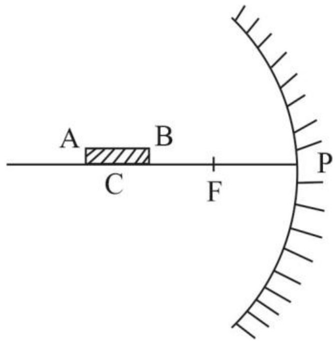
$\therefore \; x = 25$

24.(32) $f = -20cm$

$l = 10cm$

$u_A = -45cm$

$u_B = -35cm$



$$v = \frac{fu}{u-f}$$

$$v_A = \frac{(-20)(-45)}{+20-45}cm = -36cm$$

$$v_B = \frac{(-20)(-35)}{+20-35}cm = -\frac{140}{3}cm$$

length of image $= \left(\frac{140}{3} - 36\right)cm = \frac{32}{3}cm \;\; \therefore x = 32$

25.(40) $\vec{S} = \vec{r}_f - \vec{r}_i = 3\hat{i} - 5\hat{j} + 5\hat{k}$

$\vec{F} = 5\hat{i} + 2\hat{j} + 7\hat{k}$

$W = \vec{F}.\vec{S} = 15 - 10 + 35 = 40J$

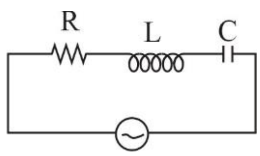
26.(1) $B = \dfrac{P}{-\dfrac{\Delta V}{V}}$

$$\frac{B_{water}}{B_{liquid}} = \frac{0.03\%}{0.01\%} = \frac{3}{1} = \frac{3}{x}$$

$\therefore \; x = 1$

27.(40) $R = 100\Omega$

$X_L = 79.6\Omega$



To maximize the average power supply, current should be maximum.

i.e., resonance occurs.

i.e., $X_C = X_L = 79.6\Omega = \dfrac{1}{\omega C}$

$\therefore\ C = \dfrac{1}{100\pi \times 79.6} F$

$\therefore\ C = 40\mu F$

28.(2) Kinetic energy = $\dfrac{5}{4}$ potential energy

Total energy = $\dfrac{1}{2}kA^2$

$\Rightarrow\ KE. + P.E = \dfrac{1}{2}kA^2 \quad \Rightarrow\ \dfrac{5}{4}PE + P.E. = \dfrac{1}{2}kA^2$

$\Rightarrow\ \dfrac{9}{4}PE = \dfrac{1}{2}kA^2 \quad \Rightarrow\ \dfrac{9}{4}\dfrac{1}{2}kx^2 = \dfrac{1}{2}kA^2 \quad \Rightarrow\ \dfrac{9}{4}x^2 = A^2$

$\therefore\ x = \dfrac{2}{3}A = \dfrac{2}{3}(3cm) = 2cm$

29.(2) $v = \sqrt{\dfrac{2gh}{1+\dfrac{k^2}{R^2}}} = \sqrt{\dfrac{(20)(0.6\sin 30^\circ)}{1+\dfrac{1}{2}}} m/s$

$\therefore\ v = \sqrt{\dfrac{20\times 0.3\times 2}{3}} m/s \ = 2m/s$

30.(144) $q = 2\mu C$

$B = 4mT = 4\times 10^{-3}T$

$V = 100\, volt$

$R = 0.03m$

$R = \dfrac{mv}{qB} = \dfrac{\sqrt{2mK}}{qB} = \dfrac{\sqrt{2mqV}}{qB}$

$m = \dfrac{(qRB)^2}{2qV} = \dfrac{\left(2\times 10^{-6}\times 0.03\times 4\times 10^{-3}\right)^2}{2\times 2\times 10^{-6}\times 100} kg$

$m = 144\times 10^{-18} kg$

CHEMISTRY

SECTION - 1

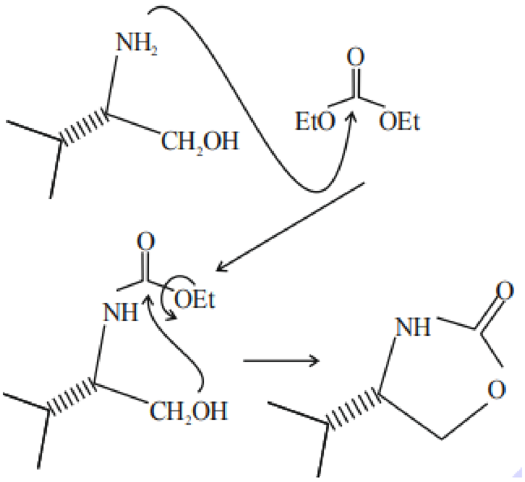
1.(3) Dehydration of alcohol is directly proportional to the stability of carbocation.

2.(2)

List I		List II	
Test		**Functional group/Class of compound**	
(A)	Molisch's Test	**(II)**	Carbohydrate
(B)	Biuret Test	**(I)**	Peptide
(C)	Carbylamine Test	**(III)**	Primary amine
(D)	Schiff s Test	**(IV)**	Aldehyde

3.(2) No pollution occurs by combustion of hydrogen and very low density of hydrogen.

4.(4) Initially lone pair electron of $-NH_2$ attack on electrophilic carbon, after then lone pair electron of oxygen attacks leading to formation of cyclic compound.



5.(3) Resonating structure are hypothetical and resonance hybrid is real structure which is weighted average of all the resonating structures.

6.(1) Fact

7.(1) Fact

8.(1) $2C(s)+O_2(g)\rightarrow 2CO(g)$

$\Delta_r S^\circ$ is +ve, $\Delta_r G^\circ = \Delta_r H^\circ - T\Delta_r S^\circ$; thus slope is negative

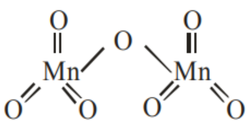
As temperature increases $\Delta_r G^\circ$ becomes more negative thus it has lower tendency to get decomposed.

9.(2) By Haworth structure of mannose.

10.(1) By using catalytic convertors.

11.(1) Adsorption $\propto$ vanderwaal attraction forces $Z_c = \frac{3}{8}$ for all real gases

12.(2)



13.(1) Applying electrical neutrality principle in metal deficiency defect.

$3A^{2+}$ are replaced by $2A^{3+}$, thus one vacant site per pair of A^{3+} is created.

14.(3) Double salt contain two or more types of salts. $CuSo_4, 4NH_3.H_2O$ and $Fe(CN)_2.4KCN$ are complex compounds.

15.(4) **A.** Beryllium oxide is amphoteric in nature.

B. Beryllium carbonate is kept in the atmosphere of CO_2 because it is thermally less stable.

C. Beryllium sulphate is readily soluble in water due to high degree of hydration.

D. Beryllium shows anomalous behaviour due to small size, high ionization energy and high value of ϕ (polarising power).

16.(4) Chlorine oxides, Cl_2O, ClO_2, Cl_2O_6 and Cl_2O_7 are highly reactive oxidizing agents and tend to explode.

17.(4) Formation of Prussian blue complex takes place.

18.(4) Statements A and B are correct and C and D are incorrect.

19.(4) $\bar{C}N$ is a strong field ligand so maximum splitting in d orbitals take place.

20.(2) In alcoholic KOH, elimination reaction takes place.

SECTION – 2

21.(2) As per the language of given question, the best possible isomeric structure is $Ph-CH=CH-O-CH_3$ (cis and trans). So, the answer is 2.

22.(3) $\underset{\downarrow}{KCl} + \underset{\downarrow}{AgNO_3} \rightarrow AgCl + KNO_3$

V = 25ml V = 20ml

M = 1M

At equivalence point, mmole of KCl = mmole of $AgNO_3$ = 20 mmole

Volume of solution = 25 ml

Mass of solution = 25 gm

Mass of solvent = 25 – mass of solute

$$= 25 - \left[20 \times 10^{-3} \times 74.5\right] = 23.51 gm$$

Molality of KCl = $\dfrac{\text{mole of KCl}}{\text{mass of solvent in kg}}$

$$= \frac{20 \times 10^{-3}}{23.51 \times 10^{-3}} = 0.85$$

$$\text{i of KCl} = 2(100\% \text{ ionisation})$$

$$\Delta T_f = i \times K_f \times m = 2 \times 2 \times 0.85 = 3.4 \simeq 3$$

23.(2) (A) $V_e = 1000 m/s; h = 6 \times 10^{-34} Js;$

$m_e = 9 \times 10^{-31} kg$

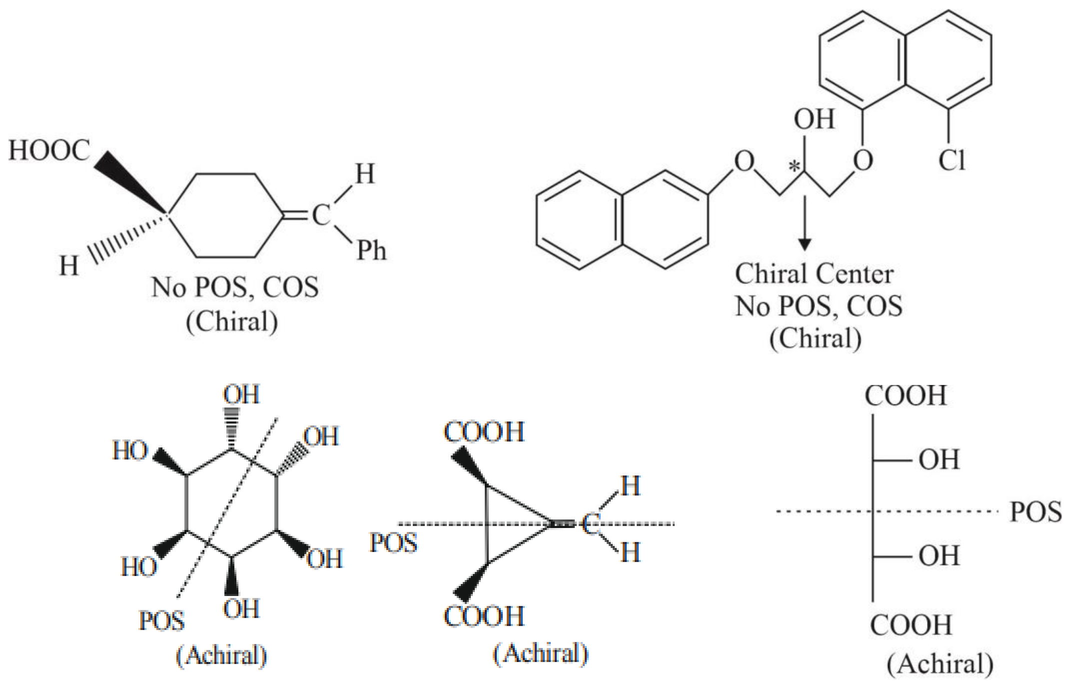
$$\lambda = \frac{h}{mv} = \frac{6 \times 10^{-34}}{9 \times 10^{-31} \times 1000} = 666.67 \times 10^{-9} m = 666.67 nm$$

(B) The characteristic of electrons emitted is independent of the material of the electrodes of the cathode ray tube.

(C) The cathode rays start from cathode and move towards anode.

(D) The nature of the emitted electrons is independent on the nature of the gas present in cathode ray tube.

24.(2)



25.(3) $MnO_4^- + 8H^+ + 5e^- \rightleftharpoons Mn^{2+} + 4H_2O$

$$E = E^0 - \frac{0.059}{5}\log\frac{\left[Mn^{2+}\right]}{\left[MnO_4^-\right]\left[H^+\right]^8}$$

$$1.282 = 1.54 - \frac{0.059}{5}\log\frac{10^{-3}}{10^{-1}\times\left[H^+\right]^8}$$

By solving this $\left[H^+\right] = 10^{-3}$ Hence, $pH = 3$

26.(12) $HBrO_3$ (Bromic acid)

Ox. State of Br = + 5

$HBrO_4$ (per bromic acid)

Ox. State of Br = + 7

Sum of Ox. State = 12

27.(15) $[A]_t = [A]_0 e^{-kt}$

For A : Let $[A]_t$ be y and $[A]_0$ be x; $k = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{15\,min}$

$$y = xe^{-kt} = xe^{-\left(\frac{\ln 2}{15}\right)t}$$

For B : $[B]_t = [B]_0 e^{-kt}$

Let $[B]_t = y; [B]_0 = 4x; k = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{5\,\text{min}}$

$$y = 4xe^{-\left(\frac{\ln 2}{5}\right)t}$$

$$\Rightarrow\ xe^{-\left(\frac{\ln 2}{15}\right)t} = 4xe^{-\left(\frac{\ln 2}{5}\right)t}$$

$$e^{t\left(\frac{\ln 2}{5}-\frac{\ln 2}{15}\right)} = 4$$

$$t \times \left(\frac{\ln 2}{5} - \frac{\ln 2}{15}\right) = \ln 4$$

$$t \times \ln 2\left[\frac{1}{5} - \frac{1}{15}\right] = 2\ln 2$$

$$t = 15\,\text{min}$$

28.(12) (i) $x(g) \rightleftharpoons y(g) + z(g)$ $\quad k_{p_1} = 3$

	x(g)	y(g)	z(g)
Initial moles	n	–	–
At equilibrium	$n - \alpha n$	αn	αn

$P_{total} = p_1$

$$k_{p_1} = \frac{\left(\frac{\alpha}{1\times\alpha}\times p_1\right)\left(\frac{\alpha}{1+\alpha}\times p_1\right)}{\frac{1-\alpha}{1+\alpha}\times p_1}$$

$$3 = \frac{\alpha^2}{(1+\alpha)} \times \frac{p_1}{(1-\alpha)}$$

$$3 = \frac{\alpha^2 p_1}{1-\alpha^2}$$

(ii) $A(g) \rightleftharpoons 2B(g)$ $\quad k_{p_2} = 1$

	A(g)	2B(g)
Initial moles	n	–
At equilibrium	$x - \alpha n$	$2\alpha n$

$P_{total} = p_2$

$$k_{p_2} = \frac{\left(\frac{2\alpha}{1-\alpha^2}\times p_2\right)^2}{\frac{1-\alpha}{1+\alpha}\times p_2}$$

$$1 = \frac{4\alpha^2 \times p_2}{1-\alpha^2}$$

$$\frac{k_{p_1}}{k_{p_2}} = \frac{p_1}{4p_2}$$

$\frac{3}{1} = \frac{p_1}{4p_2}$ $\therefore\ p_1 : p_2 = 12:1$

$x = 12$

29.(499) $2H_2O(g) \to 2H_2(g) + O_2(g)$ $+(242\times2)\,kJ\ mol^{-1}$

$H_2(g) + O_2(g) \to 2OH$ $+78\ kJ\ mol^{-1}$

$H_2(g) \to 2H$ $+436kJ\ mol^{-1}$

$2H_2O \to 2H + 2OH$ $+998\ kJ\ mol^{-1}$

$H_2O \to H + OH$ $998 \times \frac{1}{2} = +499kJ\ mol^{-1}$

30.(364) $m = \frac{1000\times M}{1000\times d - M\times M.W\ of\ solute} = \frac{1000\times3}{1000\times1-(3\times58.5)} = 3.64 = 364\times10^{-2}$

MATHEMATICS

SECTION - 1

1.(1) $f(x) = 2x + \tan^{-1} x \text{ and } g(x) = \ln\left(\sqrt{1+x^2} + x\right)$ and $x \in [0, 3]$

Clearly $f(x) \text{ and } g(x)$ both are increasing functions in [0, 3]

$\therefore\ f(3) > g(3)$

2.(4) Equation of the pair of angle bisector for the homogenous equation $ax^2 + 2hxy + by^2 = 0$ is given as

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

Here $a = 2, h = \frac{1}{2} \text{ and } b = -3$

Equation will become $\frac{x^2 - y^2}{2 - (3)} = \frac{xy}{1/2}$

$$x^2 - y^2 = 10xy$$

$$x^2 - y^2 - 10xy = 0$$

3.(3) $\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$

$$(\lambda + 2)\begin{vmatrix} 1 & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$$(\lambda + 2)\left[1\left(\lambda^2 - 1\right) - 1(\lambda - 1) + (1 - \lambda)\right] = 0$$

$$(\lambda + 2)\left(\lambda^2 - 2\lambda + 1\right) = 0$$

$$(\lambda + 2)(\lambda - 1)^2 = 0 \Rightarrow \lambda = -2, \lambda = 1$$

at $\lambda = 1$ system has infinite solution, for inconsistent $\lambda = -2$

so $\sum\left(|-2|^2 + |-2|\right) = 6$

4.(4) $\sum_{r=1}^{26} \frac{1}{(2r-1)!((51)! - (2r-1))!} = \sum_{r=1}^{26} {}^{51}C_{(2r-1)} \frac{1}{51!} = \frac{51}{51!}\left\{{}^{51}C_1 + {}^{51}C_3 + \ldots + {}^{51}C_{51}\right\} = \frac{1}{51!}\left(2^{50}\right)$

5.(4) The solution of D.E.: $y(\sec x) = x\tan x - \ln(\sec x) + c$

Given $y(0) = 1 \Rightarrow c = 1$ $\quad\quad \therefore\ y(\sec x) = x\tan x - \ln(\sec x) + 1$

At $x = \frac{\pi}{6}, y = \frac{\pi}{12} + \frac{\sqrt{3}}{2}\ln\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$

6.(4) $\lim_{n\to\infty}\left(\frac{1}{1+n}+....+\frac{1}{n+n}\right)=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n+r}$ = $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\left(\frac{1}{1+\frac{r}{n}}\right)$

$\int\frac{1}{1+x}dx\left[\ell n(1+x\right]_0^1=\ell n2$

7.(2) Equation of line PM $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z-3}{-1}=\lambda$

any point on line $=(\lambda+2,\,2\lambda-1,-\lambda+3)$

for point M $(\lambda+2)+2(2\lambda-1)-(3-\lambda)=0 \quad \Rightarrow \lambda=\frac{1}{2}$

point $M\left(\frac{1}{2}+2,\,2\times\frac{1}{2}-1,\frac{-1}{2}+3\right)=\left(\frac{5}{2},0,\frac{5}{2}\right)$

For Image $Q(\alpha,\beta,\gamma)$ $\frac{\alpha+2}{2}=\frac{5}{2},\frac{\beta-1}{2}=0,\ \frac{\gamma+3}{2}=\frac{5}{2}$

$Q:(3,1,2)$

$d=\left|\frac{3(3)+2(1)+2+29}{\sqrt{3^2+2^2+1^2}}\right|$

$d=\frac{42}{\sqrt{14}}=3\sqrt{14}$

8.(2) $\sim(q\vee((\sim q)\wedge p))$

$=\sim q\wedge\sim((\sim q)\wedge p)=\sim q\wedge(q\vee\sim p)=(\sim q\wedge q)\vee(\sim q\wedge\sim p)=(\sim q\wedge\sim p)$

9.(4) $\sqrt{(x-2)^2+y^2}=2\sqrt{(x-3)^2+y^2}$

$=x^2+y^2-4x+4=4x^2+4y^2-24x+36=3x^2+3y^2-20x+32=0$

$=x^2+y^2-\frac{20}{3}x+\frac{32}{3}=0=(\alpha,\beta)=\left(\frac{10}{3},0\right)$

$\gamma=\sqrt{\frac{100}{9}-\frac{32}{3}}=\sqrt{\frac{4}{9}}=\frac{2}{3}$

$3(\alpha,\beta,\gamma)=3\left(\frac{10}{3}+\frac{2}{3}\right)=12$

10.(4) As $3(a-a)+\sqrt{7}=\sqrt{7}$ which belongs to relation so relation is reflexive

Check for symmetric: Take $a=\frac{\sqrt{7}}{3},b=0$

Now (a, b) $\in R$ but $(b,a)\notin R$

As $(b-a)+\sqrt{7}=0$ which is rational so relation is not symmetric.

Check for transitivity: Take $(a,b)\ as\left(\frac{\sqrt{7}}{3},1\right)$

and $(b,c)\ as\left(1,\frac{2\sqrt{7}}{3}\right)$

So now $(a,b)\in R\ and\ (b,c)\in R$ but $(a,c)\notin R$ which means relation is not transitive.

11.(4) $\frac{dy}{dx}+\frac{x+a}{y-2}=0$

$\frac{dy}{dx}=\frac{x+a}{2-y}$

$(2-y)dy=(x+a)dx\quad 2y\frac{-y}{2}=\frac{x^2}{2}+ax+c$

$a+c=-\frac{1}{2}\ as\ y(1)=0$

$x^2+y^2+2ax-4y-1-2a=0$

$\pi r^2=4\pi$

$r^2=4$

$4=\sqrt{a^2+4+1+2a}$

$(a+1)^2=0$

$P,Q=\left(0,2\pm\sqrt{3}\right)$

Equation of normal at P, Q are $y-2=\sqrt{3}(x-1)$

$y-2=-\sqrt{3}(x-1)$

$R=\left(1-\frac{2}{\sqrt{3}},0\right)$

$S=\left(1+\frac{2}{\sqrt{3}},0\right)$

$RS=\frac{4}{\sqrt{3}}=4\frac{\sqrt{3}}{3}$

12.(2) Shortest distance between two lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{a_2}=\frac{z-z_1}{a_3}$ & $\frac{x-x_2}{b_1}=\frac{y-y_2}{b_2}=\frac{z-z_2}{b_3}$ is given

as $\frac{\begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}}{\sqrt{(a_1b_3-a_3b_2)^2+(a_1b_3-a_3b_1)^2+(a_1b_2-a_2b_1)^2}}$

$$\frac{\begin{vmatrix} 8 & 7 & 3 \\ 1 & 2 & -3 \\ 1 & 4 & -5 \end{vmatrix}}{\sqrt{(2)^2+(2)^2+(2)^2}} = \frac{|8(-10+12)-7(-5+3)+3(4-2)|}{\sqrt{4+4+4}}$$

$$= \frac{16+14+6}{\sqrt{12}} = \frac{36}{\sqrt{12}} = \frac{36}{2\sqrt{3}} = \frac{18}{\sqrt{3}} = 6\sqrt{3}$$

13.(2) $\frac{1+3+5+a+b}{5} = 5$

$a+b=16$ … (1)

$$\sigma^2 = \frac{\sum x_1^2}{5} - \left(\frac{\sum x}{5}\right)^2$$

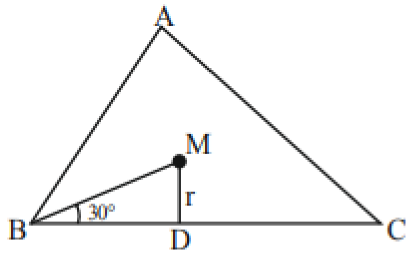
$$8 = \frac{1^2+3^2+5^2+a^2+b^2}{5} - 25$$

$a^2+b^2=130$ ….(2)

by (1), (2)

$a=7, b=9$ or $a=9, b=7$

14.(4) If $\cos 2A + \cos 2B + \cos 2C$ is minimum then A = B = C = 60°



So ΔABC we have $\tan 30° = \frac{MD}{BD} = \frac{r}{a/2} = \frac{6}{a}$

$1/\sqrt{3} = \frac{1}{a} = a = 6\sqrt{3}$

Perimeter of $\Delta ABC = 18\sqrt{3}$

Area of $\Delta ABC = \frac{\sqrt{3}}{4}a^2 = 27\sqrt{3}$

15.(3) $np + npq = 5, (np)(npq) = 6$

$np(1+q) = 5, n^2p^2q = 6$

$n^2p^2(1+q)^2 = 25, n^2p^2q = 6$

$\frac{6}{q}(1+q)^2 = 25$

$6q^2 + 12q + 6 = 25q$

$6q^2 - 13q + 6 = 0$

$6q^2 - 9q - 4q + 6 = 0$

$(3q-2)(2q-3) = 0$

$q = \frac{3}{2}, \frac{2}{3}, q = \frac{2}{3}$ is accepted

$p = \frac{1}{3} \Rightarrow n.\frac{1}{3} + n.\frac{1}{3}.\frac{2}{3} = 5$

$\frac{3n+2n}{9} = 5$

$n = 9$

So $6(n+p-q) = 6\left(9 + \frac{1}{3} - \frac{2}{3}\right) = 52$

16.(1) $T_r = \frac{(r^2+r+1)-(r^2-r+1)}{2(r^4+r^2+1)}$

$\Rightarrow T_r = \frac{1}{2}\left[\frac{1}{r^2-r+1} - \frac{1}{r^2+r+1}\right]$

$T_1 = \frac{1}{2}\left[\frac{1}{1} - \frac{1}{3}\right]$

$T_2 = \frac{1}{2}\left[\frac{1}{3} - \frac{1}{7}\right]$

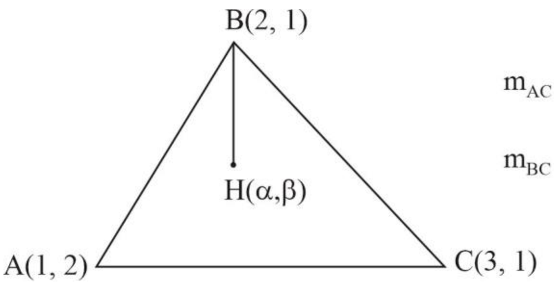
$T_3 = \frac{1}{2}\left[\frac{1}{2} - \frac{1}{13}\right]$

$\vdots$

$T_{10} = \frac{1}{2}\left[\frac{1}{91} - \frac{1}{111}\right]$

$\Rightarrow \sum_{r=1}^{10} T_r = \frac{1}{2}\left[1 - \frac{1}{11}\right] = \frac{55}{11}$

17.(4)



$m_{AC} = -\frac{1}{2}$

$m_{BC} = -2$

Here $(m)_{BH} \times (m)_{AC} = -1$

$\left(\frac{\beta-3}{\alpha-2}\right)\left(\frac{1}{-2}\right) = -1$

$\beta - 3 = 2\alpha - 4$

$\beta = 2\alpha - 1$

$m_{AH} \times m_{BC} = -1$

$\Rightarrow \quad \left(\frac{\beta-2}{\alpha-1}\right)(-2) = -1 \;\Rightarrow 2\beta - 4 = \alpha - 1$

$\Rightarrow \quad 2(2\alpha-1) = \alpha + 3 \;\Rightarrow 3\alpha = 5$

$\alpha = \frac{5}{3}, \beta = \frac{7}{5} \Rightarrow H\left(\frac{5}{3}, \frac{7}{3}\right)$

$\alpha + 4\beta = \frac{5}{3} + \frac{28}{3} = \frac{33}{3} = 11$

$\beta + 4\alpha = \frac{7}{3} + \frac{20}{3} = \frac{27}{3} = 9$

$x^2 - 20x + 99 = 0$

18.(1) $C_1 \to C_1 + C_2 + C_3$

$$f(x) = \begin{vmatrix} 2+\sin 2x & \cos^2 x & \sin 2x \\ 2+\sin 2x & 1+\cos^2 x & \sin 2x \\ 2+\sin 2x & \cos^2 x & 1+\sin 2x \end{vmatrix}$$

$$f(x) = (2+\sin 2x)\begin{vmatrix} 1 & \cos^2 x & \sin 2x \\ 1 & 1+\cos^2 x & \sin 2x \\ 1 & \cos^2 x & 1+\sin 2x \end{vmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$f(x) = (2+\sin 2x)\begin{vmatrix} 1 & \cos^2 x & \sin 2x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = (2+\sin 2x)(1) = 2 + \sin 2x$$

$= \sin 2x \in \left[\frac{\sqrt{3}}{2}, 1\right]$

Hence $2 + \sin 2x \in \left[2 + \frac{\sqrt{3}}{2}, 3\right]$

19.(3) $\cos^{-1}(2x) - 2\cos^{-1}\sqrt{1-x^2} = \pi$

$\cos^{-1}(2x) - \cos^{-1}\left(2\left(1-x^2\right)-1\right) = \pi$

$\cos^{-1}(2x) - \cos^{-1}\left(1-2x^2\right) = \pi$

$-\cos^{-1}\left(1-2x^2\right)=\pi-\cos^{-1}(2x)$

Taking cos both sides we get $\cos\left(-\cos^{-1}\left(1-2x^2\right)\right)=\cos\left(\pi-\cos^{-1}(2x)\right)$

$1-2x^2=-2x$

$2x^2-2x-1=0$

On solving, $x=\frac{1-\sqrt{3}}{2},\frac{1+\sqrt{3}}{2}$

As $x=[-1/2,1/2]$, $x=\frac{1+\sqrt{3}}{2}$ = rejected

So $x=\frac{1-\sqrt{3}}{2}\Rightarrow x^2-1=-\sqrt{3}/2\;\; =2\sin^{-1}\left(x^2-1\right)=2\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)=\frac{-2\pi}{3}$

20.(4) Let $\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}=t$

$t+\frac{1}{t}=10$

$\Rightarrow \quad t=5+2\sqrt{6},5-2\sqrt{6} \quad \Rightarrow \quad \left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}=5+2\sqrt{6},5-2\sqrt{6}$

$\Rightarrow \quad x^2-4=2,-2 \;\; or \;\; x^2=6,2 \quad \Rightarrow \quad x=\pm\sqrt{2},\pm\sqrt{6}$

SECTION – 2

21.(754) $a_1+a_2+a_3+a_4=50$

$\Rightarrow 32+6d=50 \quad \Rightarrow d=3$

and, $a_{n-3}+a_{n-2}+a_{n-1}+a_n=170$

$\Rightarrow 32+(4n-10).3=170 \quad \Rightarrow n=14$

$a_7=26,\ a_8=29 \quad \Rightarrow a_7.a_8=754$

22.(14) $f(x)=x^2+g'(1)x+g''(2)$

$f'(x)=2x+g'(1)$

$f'(x)=2$

$f(x)=f(1)x^2+x\left[2x+g'(1)\right]+2$

$g'(x)=2f(1)x+4x+g'(1)$

$g''(x)=2f(1)+4$

$g''(x)=0$

$2f(1)+4=0$

$f(1)=-2 \quad -2=1+g'(1)=g'(1)=-3$

So, $\quad f'(x)=2x-3$

$f(x)=x^2-3x+c$

$c=0$

$f(x)=x^2-3x$

$g(x)=-3x+2$

$f(4)-g(4)=14$

23.(3501) $[\vec{u}\vec{v}\vec{w}]=\vec{u}.(\vec{v}\times\vec{w})$

$\min.\left(|u||\vec{v}\times\vec{w}|\cos\theta\right)=-\alpha\sqrt{3401}$

$\Rightarrow \quad \cos\theta=-1$

$|u|=\alpha$ (Given) $|\vec{v}\times\vec{w}|=\sqrt{3401}$

$$\vec{v}\times\vec{w}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \alpha & 2 & -3\\ 2\alpha & 1 & -1\end{vmatrix}$$

$$\vec{v}\times\vec{w}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \alpha & 2 & -3\\ 2\alpha & 1 & -1\end{vmatrix}$$

$\vec{v}\times\vec{w}=\hat{i}-5\alpha\hat{j}-3\alpha\hat{k}$

$|\vec{v}\times\vec{w}|=\sqrt{1+25\alpha^2+9\alpha^2}=\sqrt{3401}$

$34\alpha^2=3400$

$\alpha^2=100$

$\alpha=10$ (as $\alpha>0$)

so, $\vec{u}=\lambda\left(\hat{i}-5\alpha\,\hat{j}-3\alpha\hat{k}\right)$

$\alpha^2=\lambda^2\left(1+25\alpha^2+9\alpha^2\right)$

$100=\lambda^2(1+34\times100)$

$\lambda^2=\frac{100}{3401}=\frac{m}{n}$

24.(63) $\int\left(x^{20}+x^{13}+x^6\right)\left(2x^{21}+3x^{14}+6x^7\right)^{1/7}dx$

$2x^{21}+3x^{14}+6x^7=t$

$42\left(x^{20}+x^{13}+x^6\right)dx=dt$

$$\frac{1}{42}\int_0^{11}t^{\frac{1}{7}}dt=\left(\frac{t^{8/7}}{8/7}\times\frac{1}{42}\right)_0^{11}=\frac{1}{48}\left(t^{8/7}\right)_0^{11}=\frac{1}{48}(11)^{8/7}$$

$l=48, m=8, n=7$

$l+m+n=63.$

25.(18) $A=\int_{-1}^{0}\left(x^2-3x\right)dx+\int_{0}^{2}\left(3x-x^2\right)dx$

$\Rightarrow \quad A=\frac{x^3}{3}-\frac{3x^2}{2}\Big|_{-1}^{0}+\frac{3x^2}{2}-\frac{x^3}{3}\Big|_{0}^{2} \quad \Rightarrow \quad A=\frac{11}{6}+\frac{10}{3}=\frac{31}{6}$

$\therefore \quad 12A=62$

26.(1) $\frac{dy}{dx}+y=k$

$y.e^x=e^x+c$

$f(0)=e^{-2}$

$\Rightarrow c=e^{-2}-k$

$\therefore y=k+\left(e^{-2}-k\right)e^{-x}$

Now $k=\int_0^2\left(k+\left(e^{-2}-k\right)e^{-x}\right)dx$

$\Rightarrow \quad k=e^{-2}-1$

$\therefore \quad y=\left(e^{-2}-1\right)+e^{-x}$

$f(2)=2e^{-2}-1,\ f(0)=e^{-2}$

$2f(0)-f(2)=1$

27.(11) $A(2,6,2)\ B(-4,0,\lambda),C(2,3,-1)\ D(4,5,0)$

Area = $\frac{1}{2}\left|\overrightarrow{BD}\times\overrightarrow{AC}\right|=18$

$\overrightarrow{AC}\times\overrightarrow{BD}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & -3 & -3\\ 8 & 5 & -\lambda\end{vmatrix}=(3\lambda+15)\hat{i}-\hat{j}(-24)+\hat{k}(-24)$

$=\sqrt{(3\lambda+15)^2+(24)^2+(24)^2}=36=\lambda^2+10\lambda+9=0=\lambda=-1,-9$

$|\lambda|\le 5\Rightarrow\lambda=-1$

$5-6\lambda=5-6(-1)=11$

28.(536) Divisible by $2\rightarrow 450$

Divisible by $3\rightarrow 300$

Divisible by $7\rightarrow 128$

Divisible by $2\&7\rightarrow 64$

Divisibly by $2\&7\rightarrow 43$

Divisibly by $2\&3 \to 150$

Divisibly by $2\&3 \to 150$

Divisibly by $2,3\&7 \to 21$

$\therefore$ Total numbers = 450 + 300 – 150 – 64 – 43 + 21 = 514

29.(29) $(21+2)^{200} + (21-2)^{200}$

$\Rightarrow 2\left[{}^{100}C_0\ 21^{200} + 200C_2 21^{198}.2^2 + ... + {}^{200}C_{198} 21^2.\ 2^{198} + 2^{200}\right]$

$\Rightarrow 2\left[49I_1 + 2^{200}\right] = 49I_1 + 2^{201}$

Now, $2^{201} = (8)^{67} = (1+7)^{67} = 49I_2 + {}^{67}C_0\ {}^{67}C_1.7 =$

$49I_2 + 470 = 49I_2 + 49 \times 9 \times 29$

$\therefore$ Remainder is 29

30.(50400) Vowels: A, A, A, I, I, O

Consonants : S, S, S, S, N, N, T

$\because$ Total number of ways in which vowels come together = $\frac{\lfloor 8}{\lfloor 4 \lfloor 2} \times \frac{\lfloor 6}{\lfloor 3 \lfloor 2} = 50400$