



# SOLUTIONS

**Joint Entrance Exam | IITJEE-2023**

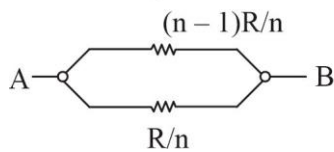
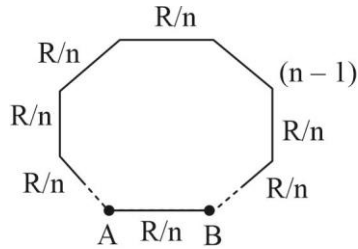
**01<sup>st</sup> FEB 2023 | Evening Shift**

**PHYSICS**

**SECTION - 1**

1.(4) By abruptly changing the direction of magnetic field, the flux will be negative so flux will be changed if sign is considered.

2.(3)



$$Req = \frac{(n-1)(R/n \times R/n)}{(n-1)R/n + R/n}$$

$$Req = \frac{(n-1)R^2}{n^2 R} = \frac{(n-1)R}{n^2}$$

3.(4)  $T = 2\pi\sqrt{\frac{l}{g}}$

$$l = \frac{T^2 g}{4\pi^2}$$

linear relation between  $T^2$  &  $l$  so graph will be a straight line passing through origin.

4.(4)  $\frac{1}{2}$  refer to theory

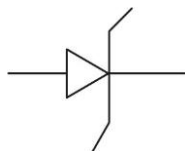
As half the energy is stored in electric field ratio of average electrical energy to the energy is half.

5.(1)  $V_e = \sqrt{2gR}$

$$\therefore \frac{V_A}{V_B} = \frac{\sqrt{2g_A R_A}}{\sqrt{2g_B R_B}} = \frac{1}{2}$$

$$\frac{g_A}{g_B} = \frac{1}{4} \times \frac{R_B}{R_A} = \frac{1}{4} \times \frac{3}{1} = \frac{3}{4}$$

6.(4) Refer to theory



Zener diode is a highly doped  $p$ - $n$  Junction diode which works as voltage regulator in reverse Bias.

7.(4) Area of force and time graph gives impulse.

$$8.(4) \quad |f| = \frac{R}{2} = 20 \text{ cm}$$

Using mirror formula

$$v = \frac{uf}{u-f}$$

$$\text{for } u_1 = 15\text{cm}, v_1 = \frac{(-15)(-20)}{-15 - (-20)} = +60\text{cm}$$

$$\text{for } u_2 = 25\text{cm}, v_2 = \frac{(-25)(-20)}{-25 - (-20)} = -100\text{cm}$$

$$\text{distance between images} = |v_1| + |v_2| = 160 \text{ cm}$$

$$9.(2) \quad B \text{ at centre} = \frac{\mu_0 I \times \theta}{4\pi R}$$

for semicircle  $\theta = \pi$

$$B = \frac{\mu_0 I}{4R} = \frac{4\pi \times 10^{-7} \times 3}{4 \times \frac{\pi}{10}} = 3\mu T$$

10.(4) As charge always resides on surface of conductor so for same potential and radius, charge will be same for both sphere so Assertion is false but reason is correct as capacitance of sphere is  $4\pi \epsilon_0 R$

11.(2) To have minimum error in measuring voltage it is advised to use high resistance voltmeter so assertion is wrong where as reason is correct.

$$12.(2) \quad \eta = 1 - \frac{T_2}{T_1} = \frac{1}{3} \quad \dots(1)$$

$$\eta' = 1 - \frac{(T_2 + x)}{T_1} = \frac{1}{6} \quad \dots(2)$$

$$\text{and } T_1 = 99 + 273 = 372k$$

$$1 - \frac{T_2}{372} = \frac{1}{3}$$

$$T_2 = 248k$$

On solving  $x = 62k$

$$13.(4) \quad \frac{1}{2}mv_1^2 = h(2f_0) - hf_0 \quad \dots(1)$$

$$\frac{1}{2}mv_2^2 = h(5f_0) - hf_0 \quad \dots(2)$$

$$\text{solving } \frac{v_1^2}{v_2^2} = \frac{1}{4}$$

$$\therefore \frac{v_1}{v_2} = \frac{1}{2}$$

14.(2) Velocity at highest point is not zero.

15.(1)  $h = [M^1 L^2 T^{-1}] = [\text{energy}] \times [\text{Time}]$

$$G = [M^{-1} L^3 T^{-2}] = \frac{[\text{force}] \times [\text{distance}]^2}{[\text{Mass}]^2}$$

$$C = [L T^{-1}]$$

$$[h^x G^y C^z] = [M^1 L^0 T^0]$$

$$[M^1 L^2 T^{-1}]^x [M^{-1} L^3 T^{-2}]^y [L T^{-1}]^z = M^1 L^0 T^0$$

$$[M^{x-y} L^{2x+3y+z} T^{-x-2y-z}] = M^1 L^0 T^0$$

$$M^{x-y} L^{2x+3y+z} T^{-x-2y-z} = M^1 L^0 T^0$$

$$x - y = 1 \quad \dots(1)$$

$$2x + 3y + z = 0 \quad \dots(2)$$

$$-x - 2y - z = 0 \quad \dots(3)$$

$$(2) + (3)$$

$$x + y = 0$$

$$x = -y$$

$$x + x = 1$$

$$x = \frac{1}{2}$$

$$y = \frac{-1}{2}$$

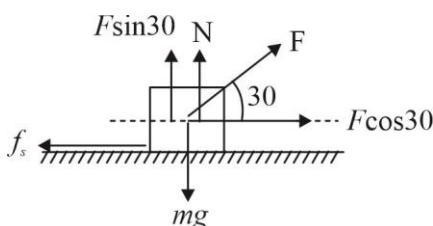
$$z = \frac{1}{2}$$

$$[M] = \left[ h^{\frac{1}{2}} G^{-\frac{1}{2}} C^{\frac{1}{2}} \right]$$

16.(3) For ideal gas  $T = 0\text{K}$  or  $-273^\circ\text{C}$ ,  $P = 0$

$Pv = nRT$  as molecules have no thermal energy they will not exert any force or pressure on walls due to collision at absolute zero temperature

17.(4) FBD of block



$\therefore$  vertical forces are balanced

$$N + F \sin 30 = mg$$

$$N = mg - \frac{F}{2} \quad \dots(1)$$

Horizontal force

$$F \cos 30 = F_{s \max} = \mu_s N$$

$$\therefore \frac{F\sqrt{3}}{2} = 0.25 \left( mg - \frac{F}{2} \right)$$

$\therefore$  solving for F

$$F = 25.2N$$

18.(3) modulation Index =  $\frac{A_m}{A_C}$

$$\text{Index } \alpha = \frac{1}{A_C}$$

$$\frac{I_1}{I_2} = \frac{A_{C_2}}{A_{C_1}} = \frac{2y}{y} = 2:1$$

19.(4) Using Hooke's law and assuming wire to be massless

$$\therefore \frac{F}{A} = \gamma \frac{\Delta L}{L}$$

$$\therefore \Delta L = \frac{FL}{yA} = \frac{M \frac{g}{4} \times 6}{2 \times 10^{11} \times (1 \times 10^{-4})}$$

On solving  $\Delta L = 10^{-4} = 0.1 \text{ mm}$

20.(2)  $|\Delta E| = 13.6 \times z^2 \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right) = 13.6 \times (4)^2 \left( \frac{1}{2^2} - \frac{1}{4^2} \right)$

on solving  $|\Delta E| = 40.8 \text{ eV}$

## SECTION – 2

21.(4) Optical path difference =  $(\mu - 1)t$

$$(\Delta x)_{\text{optical}} = (1.2 - 1) \times 10 \times 10^{-6} = 2 \times 10^{-6} \text{ m}$$

$$\therefore \text{net paths difference } (\Delta x) = \frac{dy}{D} - 2 \times 10^{-6}$$

for zero order maxima  $\Delta x = 0$

$$\therefore \frac{dy}{D} - 2 \times 10^{-6} = 0$$

$$\frac{d}{D} \times x \beta_0 = 2 \times 10^{-6}$$

$$\therefore \frac{d}{D} \times x \times \frac{\lambda D}{d} = 2 \times 10^{-6}$$

$$x = \frac{2 \times 10^{-6}}{500 \times 10^{-9}} = \frac{2}{5} \times 10 = 4$$

22.(6) Binding energy of A = no of nucleons  $\times$  B.E per nucleon

$$B.E_A = 34 \times 1.2 = 40.8 eV$$

$$B.E_B = 26 \times 1.8 = 46.8 eV$$

$$\therefore \text{Difference } \Delta BE = 46.8 - 40.8 = 6 eV$$

23.(200)

$$\text{for 1st case } 0 = u^2 - 2a(S)$$

$$\therefore 0 = (20)^2 - 2a(500) \quad \dots(1)$$

$$\text{for IInd case } (\sqrt{x})^2 = (20)^2 - 2a(250) \quad \dots(2)$$

solving eq 1 and 2

$$\therefore x = 200$$

24.(132)

$$\int dw = \int_{y=2}^{y=5} \vec{F} \cdot d\vec{y}$$

$$\Rightarrow \int_2^5 (5 + 3y^2) dy = [5y]_2^5 + [y^3]_2^5$$

on solving  $W = 132J$

25.(2) By continuity equation

$$A \frac{dh}{dt} = av \text{ where } v \text{ in the velocity of efflux}$$

$$\therefore (750) \times \frac{dh}{dt} = 500 \times 10^{-2} \times 30$$

$$\therefore (750) \times x \times 10^{-3} \times 10^2 = 500 \times 10^{-2} \times 30$$

$$\Rightarrow x = 2$$

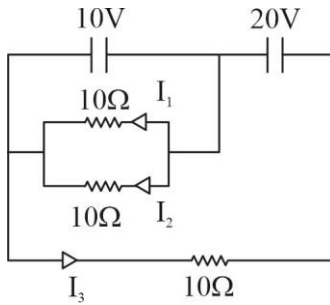
26.(3) Using parallel axis theorem

$$\therefore I = I_{cm} + M(R)^2$$

$$I = \frac{MR^2}{2} + MR^2 = \frac{3}{2} MR^2$$

$$\therefore x = 3$$

27.(2)



Using Ohm's law and Kirchoff's voltage law

$$I_1 = \frac{10}{10} = 1 \text{ Amp}, \quad I_2 = \frac{10}{10} = 1 \text{ Amp}$$

$$I_3 = \frac{20-10}{10} = 1 \text{ Amp}$$

$$\therefore \left| \frac{I_1 + I_3}{I_2} \right| = \frac{1+1}{1} = 2$$

28.(288)

As the electric field is only in the x-axis, hence flux will only be associated with faces parallel to the y-z plane.

$$\text{Net flux } \phi_{net} = \phi_1 + \phi_2$$

$$= \vec{E}_1 \cdot (a^2 \hat{i}) + \vec{E}_2 \cdot (a^2 \hat{i}) = 0 \cdot (a^2 \hat{i}) + E_0 a^2$$

$$\phi_{net} = E_0 a^3$$

Using Gauss's law

$$\phi_{net} = \frac{q_{enclosed}}{\epsilon_0}$$

$$\epsilon_0 E_0 a^3 = q_{enclosed} = Q \times 10^{-14}$$

Putting the values and solving  $Q = 288$

29.(44)

Induced EMF

$$E_{induced} = NBA\omega \sin(\omega t)$$

$$= 600 \times 0.4 \times (70 \times 10^{-4}) \times \frac{2\pi \times 500 \sin(30)}{60} = 44 \text{ V}$$

30.(67)

$$\frac{1}{2} k A^2 = \frac{1}{2} k \left( \frac{A}{2} \right)^2 + 0.25$$

$$\frac{3}{8} k A^2 = 0.25$$

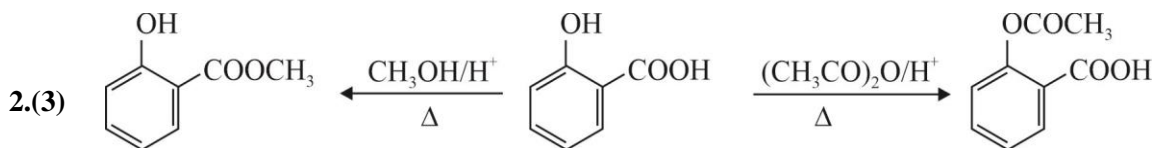
$$\frac{3}{8} k \frac{1}{100} = 0.25$$

$$k = \frac{200}{3} = 67$$

## CHEMISTRY

## SECTION - 1

1.(3) In statement correct option should be mentioned.



Hence Y is  $\text{CH}_3\text{OH}/\text{H}^+, \Delta$

X is  $(\text{CH}_3\text{CO})_2\text{O}/\text{H}^+$

3.(1)  $\text{Cu}^{+2}$  is more stable in water than  $\text{Cu}^+$  due to more hydration energy

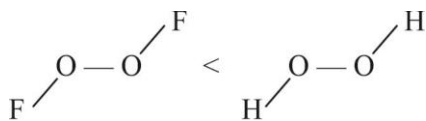
$$\text{H.E.} \propto \frac{e}{r}$$

Hence,  $(\text{H.E.})_{\text{Cu}^{+2}} > (\text{H.E.})_{\text{Cu}^+}$

4.(1)  $\text{K}_2\text{S}_2\text{O}_8 + 2\text{D}_2\text{O}(\text{l}) \longrightarrow 2\text{KDSO}_4(\text{aq}) + \text{D}_2\text{O}_2(\text{l})$

This method is used for the laboratory preparation of  $\text{D}_2\text{O}_2$ .

5.(4)



As electronegativity of  $\text{F} > \text{O}$ ,

So, % s-character is less in  $\text{O}-\text{F}$  bond as compared to  $\text{O}-\text{H}$  bond,

So  $\text{O}-\text{F}$  bond is longer than  $\text{O}-\text{H}$  bond and similarly  $\text{O}-\text{O}$  bond is longer in  $\text{H}_2\text{O}_2$ .

Hence, X  $\rightarrow$  longer and Y  $\rightarrow$  Shorter

6.(3)  $\text{K}^+, \text{Cl}^-, \text{Ca}^{2+}, \text{Sc}^{3+}$  (No. of electrons = 18)

Isoelectronic species have the same number of electrons.

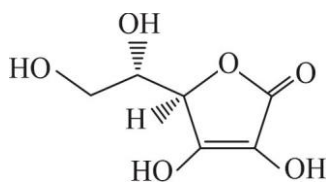
7.(1) Proper condition not given, incomplete question.

8.(4) Order of bond enthalpy (Kj/mol) is

$$\begin{array}{cccc} \text{C}-\text{C} & > & \text{Si}-\text{Si} & > & \text{Ge}-\text{Ge} & > & \text{Sn}-\text{Sn} \\ 348 & & 297 & & 260 & & 240 \end{array}$$

9.(1) Industrial production of urea

10.(3) Most stable structure of vitamin C



11.(2)  $\text{KOH}$  absorbs  $\text{CO}_2$  gas when excess  $\text{CO}_2$  is absorbed by  $\text{KOH}$ ,  $\text{KHCO}_3$  is formed which remains soluble in the solution



12.(3) Oxygen is not present in Nessler's reagent ( $K_2HgI_4$ )

13.(2)

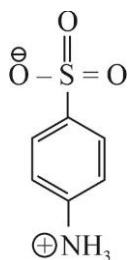
$$\log \frac{x}{m} = \frac{1}{n} \log P + \log k \rightarrow \text{Freundlich adsorption isotherm equations}$$

$$y = 3x + 2.505$$

On comparing both equation.

$$\frac{1}{n} = \text{slope} = 3, \log k = \text{intercept} = 2.505.$$

14.(1) Sulphanilic acid does not give Esterification test for Carboxyl group.



Sulphanilic acid contains N,S and C which gives  $NaSCN$  with  $FeCl_3$  solution it gives blood red colouration because of  $Fe(SCN)_3$  Complex

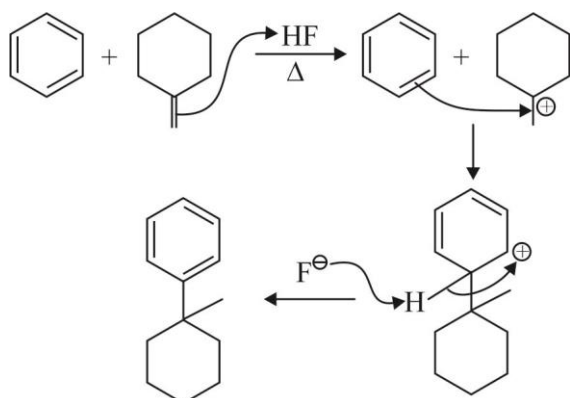
15.(2) Gypsum is used for making Fireproof wall boards

Gypsum is unstable at high temperatures

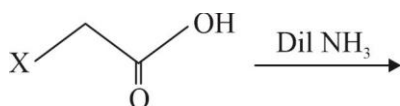
16.(4)  $[Co(NH_3)_5NO_2]^{2+}$  shows linkage isomerism as  $-NO_2$  is an ambidentate ligand.

$[Co(NH_3)_5NO_2]^{2+}$  and  $[Co(NH_3)_5(ONO)]^{2+}$  two isomers.

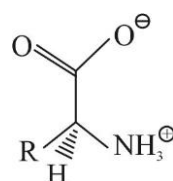
17.(1)

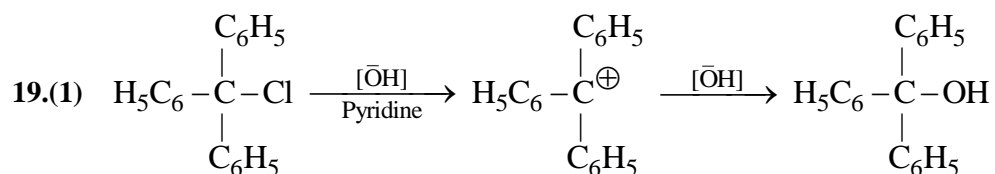


18.(3)



Amino acids exist in Zwitter ion form in Aqueous medium as





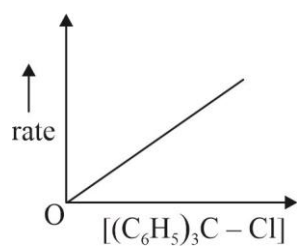
Tertiary Carbocation

Follow  $\text{S}_{\text{N}}^1$  pathway

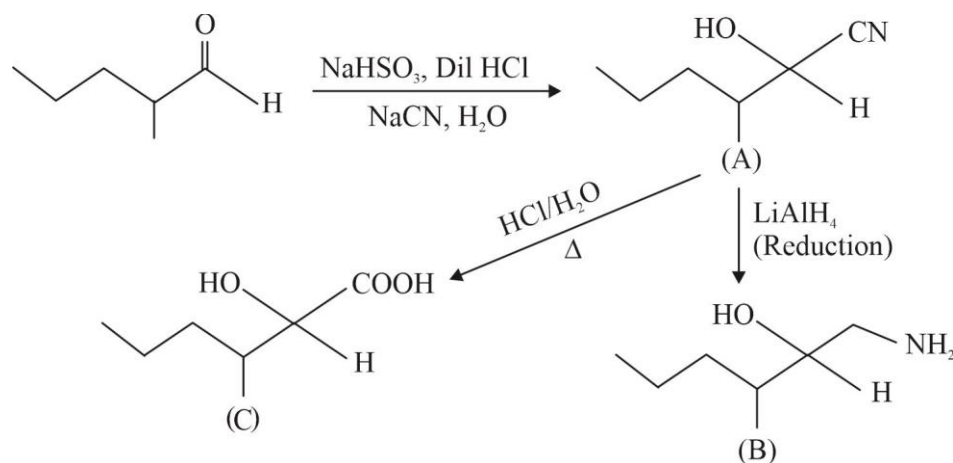
for  $\text{S}_{\text{N}}^1$  reaction

Rate =  $K[\text{substrate}]$

$$= K [(\text{C}_6\text{H}_5)_3\text{C} - \text{Cl}]$$



20.(1)



## SECTION – 2

21.(6)  $[\text{Mn}(\text{H}_2\text{O})_6]^{2+}$

$$\text{Mn}^{2+} = [\text{Ar}]3d^5$$

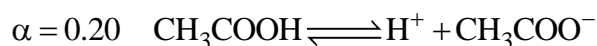
$$\text{Spin only magnetic moment } (\mu_{\text{spin only}}) = \sqrt{n(n+2)} \text{ B.M} = \sqrt{5(5+2)} \text{ B.M} = \sqrt{35} \text{ B.M} = 5.91 \text{ B.M}$$

22.(3) Copper matte is made up of a combination of copper sulfide ( $\text{Cu}_2\text{S}$ ) and iron sulfide ( $\text{FeS}$ )

$$23.(4) \quad \text{Density } (d) = \frac{Zm}{N_a a^3} \quad \begin{array}{l} Z = 2(\text{Bcc}) \\ Z = 4(\text{Fcc}) \end{array}$$

$$\frac{(d)_{\text{Fcc}}}{(d)_{\text{Bcc}}} = \frac{\frac{4 \times m}{N_a a_{\text{Fcc}}^3}}{\frac{2 \times m}{N_a a_{\text{Bcc}}^3}} = \frac{4 \times m}{N_a \times 2^3} \times \frac{N_a \times (2.5)^3}{m \times 2} = \frac{15.625}{4} = 3.9$$

24.(372)



for dissociation

$$i = 1 + 0.20$$

$$i = 1 + \alpha$$

$$\text{molality (m)} = \frac{5 \times 1000}{60 \times 500} = \frac{1}{6}$$

depression in freezing point:

$$\Delta T_F = i k_F m = 1.2 \times 1.86 \times \frac{1}{6} = 0.372 = 372 \times 10^{-3} \text{ } ^\circ\text{C}$$

25.(139)

10% (V/V) means

Volume of solute ( $\text{Br}_2$ ) = 10 ml

Volume of solution = 100 ml

Volume of solvent ( $\text{CCl}_4$ ) = 90 ml

mass of  $\text{Br}_2 = d \times V \Rightarrow 3.2 \times 10 \Rightarrow 32 \text{ g}$

$$\text{Number of moles} = \frac{32}{160} = 0.2 \text{ moles}$$

$$\text{Molality} = \frac{\text{moles of solute}}{\text{mass of solvent (kg)}}$$

$$\Rightarrow \frac{0.2 \text{ moles}}{1.6 \text{ g cm}^{-3} \times 90 \text{ ml}}$$

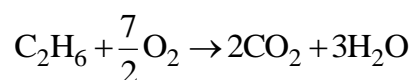
$$\Rightarrow \frac{0.2}{1.6 \times 90} \text{ mol/g} \Rightarrow 10^3 \times \frac{0.2}{1.6 \times 90} \text{ mol/g} \Rightarrow \frac{2 \times 10^2}{16 \times 9}$$

$$= 1.39 \text{ mol/kg} \Rightarrow 139 \times 10^{-2} \text{ mol/kg}$$

26.(1006)

Given 0.3 g ethane ( $\text{C}_2\text{H}_6$ )

change in temperature ( $\Delta T$ ) = 0.5



$$\Delta n_g = 2 - \left(1 + \frac{7}{2}\right)$$

$$\Rightarrow 2 - \frac{9}{2}$$

$$\Rightarrow -\frac{5}{2} \Rightarrow -2.5$$

$$q_v = c \times q \times M / m$$

$$\Rightarrow 20 \text{KJ K}^{-1} \times 0.5 \text{K} \times \frac{30 \text{g mol}^{-1}}{0.3 \text{g}}$$

$$\Rightarrow 20 \times 0.5 \times \frac{30}{0.3} \text{KJ mol}^{-1}$$

$$\Rightarrow 1000 \text{KJ mol}^{-1}$$

$$q_p = q_v + \Delta n g R T$$

$$\Rightarrow 1000 \text{KJ mol}^{-1} + 2.5 \times 8.314 \text{ JK}^{-1} \text{ mol}^{-1} \times 298 \text{K}$$

$$\Rightarrow 1000 \text{KJ mol}^{-1} + 6193.93 \text{ J mol}^{-1} \Rightarrow 1000 \text{KJ mol}^{-1} + 6193.93 \times 10^{-3} \text{KJ mol}^{-1}$$

$$\Rightarrow 1000 \text{KJ mol}^{-1} + 6.1939 \text{KJ mol}^{-1} \Rightarrow 1006 \text{KJ mol}^{-1}$$

27.(13039)

$$\kappa = \kappa_{\text{Ag}^+} + \kappa_{\text{Br}^-} + \kappa_{\text{NO}_3^-}$$

$$\kappa_{\text{sp}} \text{ of AgBr} = [\text{Ag}^+] [\text{Br}^-]$$

$$4.9 \times 10^{-13} = [s + 10^{-5}] [s]$$

$$\frac{4.9 \times 10^{-13}}{10^{-5}} = [s]$$

$$4.9 \times 10^{-8} = [s]$$

$$[\text{Ag}^+] = 4.9 \times 10^{-8} + 10^{-5} \approx 10^{-5} \text{ mole / litre} = 10^{-2} \text{ mole m}^{-3}$$

$$[\text{Br}^-] = 4.9 \times 10^{-8} \text{ mole / litre} = 4.9 \times 10^{-5} \text{ mole m}^{-3}$$

$$[\text{NO}_3^-] = 10^{-5} \text{ mole / litre} = 10^{-2} \text{ mole m}^{-3}$$

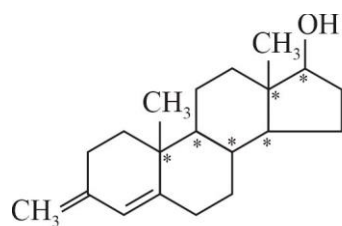
$$\kappa_{\text{Ag}^+} = 10^{-2} \times 6 \times 10^{-3} = 6 \times 10^{-5} = 6000 \times 10^{-8}$$

$$\kappa_{\text{Br}^-} = 4.9 \times 8 \times 10^{-3} \times 10^{-5} = 39.2 \times 10^{-8} = 39.2 \times 10^{-8}$$

$$\kappa_{\text{NO}_3^-} = 7 \times 10^{-3} \times 10^{-2} = 7 \times 10^{-5} = 7000 \times 10^{-8}$$

$$\kappa = 6000 \times 10^{-8} + 39.2 \times 10^{-8} + 7000 \times 10^{-8} = 10^{-8} \times 13039.2 \text{ sm}^{-1}$$

28.(6) Asymmetric carbon atoms has four different groups attached.



Testosterone

\* = Chiral/ Asymmetric  
carbon atoms  
= 6

29.(75)  $A \longrightarrow B$  (zero order reaction)

$$t_{1/2} \text{ (half life)} = 50 \text{ min}$$

$$\text{For zero order } [A]_0 - [A]_t = Kt \quad \dots(1)$$

$$\therefore [A]_0 = \text{initial conc.}$$

$$[A]_t = \text{conc. at time 't'}$$

$$t_{1/2} = \frac{[A]_0}{2k} \quad \dots(2)$$

Also, Time required to reduce  $\frac{1}{4}$  th of  $[A]_0$

$$[A]_0 - \frac{[A]_0}{4} = k t_{1/4}$$

$$t_{1/4} = \frac{3[A]_0}{4K} \quad \dots(3)$$

dividing equation (3) by (2), get

$$\frac{t_{1/4}}{t_{1/2}} = \frac{\frac{2[A]_0}{4k}}{\frac{[A]_0}{2k}} = \frac{3}{2}$$

$$t_{1/4} = \frac{3}{2} \times t_{1/2} = \frac{3}{2} \times 50 = 75 \text{ min}$$

30.(3) Chloroliazepoxide, veronal, valium are Tranquilizers.

**MATHEMATICS**

**SECTION - 1**

1.(3) Let equation of required plane be

$$(2x+3y-z-2)+\lambda(x+2y+3z-6)=0$$

$$\Rightarrow (2+\lambda)x+(3+2\lambda)y+(-1+3\lambda)z-2-6\lambda=0 \quad \dots(1)$$

since (1) is perpendicular to  $2x+y-z+1=0$

$$2(2+\lambda)+1(3+2\lambda)-1(-1+3\lambda)=0$$

$$8+\lambda=0 \Rightarrow \lambda=-8$$

$\therefore$  required plane is  $-6x-13y-25z+46=0$

$$d = \frac{|(-6)(-7)-13(1)-25(1)+46|}{\sqrt{(-6)^2+(-13)^2+(-25)^2}} = \frac{50}{\sqrt{830}}$$

$$d^2 = \frac{2500}{830} = \frac{250}{83}$$

2.(4) (A)

$p$	$q$	$p \wedge q$	$p \vee (p \wedge q)$	$\rightarrow$ (not a tautology)
$T$	$T$	$T$	$T$	$T$
$T$	$F$	$F$	$T$	$T$
$F$	$T$	$F$	$F$	$F$
$F$	$F$	$F$	$F$	$F$

(B)

$p$	$q$	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$p \rightarrow (p \wedge (p \rightarrow q))$	$\rightarrow$ (not a tautology)
$T$	$T$	$T$	$T$	$T$	$T$
$T$	$F$	$F$	$F$	$F$	$F$
$F$	$T$	$T$	$F$	$T$	$T$
$F$	$F$	$T$	$F$	$T$	$T$

(C)

$p$	$q$	$(p \wedge (p \rightarrow q))$	$\sim q$	$(p \wedge (p \rightarrow q)) \rightarrow \sim q$	(not a tautology)
$T$	$T$	$T$	$F$	$F$	$F$
$T$	$F$	$F$	$T$	$T$	$T$
$F$	$T$	$F$	$F$	$T$	$T$
$F$	$F$	$F$	$T$	$T$	$T$

(D)

$p$	$q$	$p \wedge q$	$\sim p \rightarrow q$	$(p \wedge q) \rightarrow (\sim p \rightarrow q)$	(tautology)
$T$	$T$	$T$	$T$	$T$	$T$
$T$	$F$	$F$	$T$	$T$	$T$
$F$	$T$	$F$	$T$	$T$	$T$
$F$	$F$	$F$	$F$	$T$	$T$

**2nd Method**

$$p \wedge q \rightarrow (\sim p \rightarrow q)$$

$$\text{Apply } P \rightarrow Q = \sim P \vee Q$$

$$(p \wedge q) \rightarrow (p \vee q)$$

$$= \sim (p \wedge q) \vee (p \vee q) \text{ (Tautology)}$$

$$\{(P \cap Q)' \cup (P \cup Q) = \cup\}$$

3.(2) Let  $I = \int_{-\pi/4}^{\pi/4} \frac{x + \frac{\pi}{4}}{2 - \cos 2x} dx$

$$I = \int_{-\pi/4}^{\pi/4} \frac{-x + \frac{\pi}{4}}{2 - \cos 2x} dx \quad \left[ \text{Using } \int_a^b f(x) dx = \int_a^b f(a+b-x) \right]$$

$$2I = 2 \times \frac{\pi}{4} \int_{-\pi/4}^{\pi/4} \frac{dx}{2 - \cos 2x}$$

$$I = \frac{\pi}{4} \int_{-\pi/4}^{\pi/4} \frac{dx}{2 - (2\cos^2 x - 1)} = \frac{\pi}{4} \int_{-\pi/4}^{\pi/4} \frac{dx}{3 - 2\cos^2 x}$$

$$= \frac{\pi}{4} \int_{-\pi/4}^{\pi/4} \frac{\sec^2 x dx}{3\sec^2 x - 2} = \frac{\pi}{4} \int_{-\pi/4}^{\pi/4} \frac{\sec^2 x dx}{3(1 + \tan^2 x) - 2}$$

$$= \frac{\pi}{4} \int_{-\pi/4}^{\pi/4} \frac{\sec^2 x dx}{3\tan^2 x + 1} = \frac{\pi}{4\sqrt{3}} \tan^{-1}(\sqrt{3} \tan x) \Big|_{-\pi/4}^{\pi/4}$$

$$= \frac{\pi}{4\sqrt{3}} [\tan^{-1} \sqrt{3} - \tan^{-1}(-\sqrt{3})]$$

$$= \frac{\pi}{4\sqrt{3}} \left[ \frac{\pi}{3} - \left( -\frac{\pi}{3} \right) \right] = \frac{\pi}{4\sqrt{3}} \times \frac{2\pi}{3} = \frac{2\pi^2}{12\sqrt{3}} = \frac{\pi^2}{6\sqrt{3}}$$

4.(2) Total sample space =  $n(S) = 6^2 = 36$

$$A = \{(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 5), (4, 6), (5, 6)\}$$

$$B = \{(2, 1), (2, 3), (2, 5), (4, 1), (4, 3), (4, 5), (6, 1), (6, 3), (6, 5)\}$$

$$C = \{(1, 2), (1, 4), (1, 6), (3, 2), (3, 4), (3, 6), (5, 2), (5, 4), (5, 6)\}$$

$$\text{Now } n(A \cup B \cap C) = \{(1, 2), (1, 4), (1, 6), (3, 4), (3, 6), (5, 6)\} = 6$$

5.(1)  $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$

$$\Rightarrow (\vec{r} \times \vec{a}) - (\vec{c} \times \vec{a}) = \vec{0}$$

$$\Rightarrow (\vec{r} - \vec{c}) \times \vec{a} = \vec{0}$$

$$\Rightarrow (\vec{r} - \vec{c}) = \lambda \vec{a}$$

$$\vec{r} = \vec{c} + \lambda \vec{a} \quad \dots(1)$$

Given,  $\vec{r} \cdot \vec{b} = 0$

from (1)  $\vec{r} \cdot \vec{b} = \vec{c} \cdot \vec{b} + \lambda \vec{a} \cdot \vec{b}$

$$\Rightarrow (1-3) + \lambda(2+5) = 0$$

$$\Rightarrow 7\lambda = 2 \Rightarrow \lambda = \frac{2}{7}$$

$$\vec{r} = (\hat{i} + 2\hat{j} - 3\hat{k}) + \frac{2}{7}(2\hat{i} + 7\hat{j} + 5\hat{k})$$

$$\Rightarrow \frac{(7\hat{i} + 14\hat{j} - 21\hat{k}) + 4\hat{i} - 14\hat{j} + 10\hat{k}}{7}$$

$$\vec{r} = \frac{11\hat{i} - 11\hat{k}}{7}$$

$$|\vec{r}| = \sqrt{\frac{121}{49} + \frac{121}{49}} = \sqrt{2 \times \frac{121}{49}} \Rightarrow \frac{11}{7} \sqrt{2}$$

6.(2)  $R_1 : (A \cap B') \cup (B \cap A') = \phi$

$$\Rightarrow A = B$$

$$R_2 : (A \cup B') = (B \cup A')$$

$$\Rightarrow A = B$$

Hence,  $R_1$  and  $R_2$  are equivalence.

7.(3) 
$$A = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} = A = \begin{pmatrix} \cos \frac{\pi}{3} & \sin \frac{\pi}{3} \\ -\sin \frac{\pi}{3} & \cos \frac{\pi}{3} \end{pmatrix}$$

Let  $A = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$

then  $A^n = \begin{pmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{pmatrix} \quad n \in N$

$$A^{30} = \begin{pmatrix} \cos 10\pi & \sin 10\pi \\ -\sin 10\pi & \cos 10\pi \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$A^{25} = \begin{pmatrix} \cos \frac{25\pi}{3} & \sin \frac{25\pi}{3} \\ -\sin \frac{25\pi}{3} & \cos \frac{25\pi}{3} \end{pmatrix}$$

Note:  $a = \cos\left(\frac{25\pi}{3}\right)$



$$\Rightarrow \cos\left(8\pi + \frac{\pi}{3}\right) = \cos \frac{\pi}{3}$$

$$b = \sin\left(\frac{25\pi}{3}\right) = \sin\left(8\pi + \frac{\pi}{3}\right) = \sin \frac{\pi}{3}$$

$$A^{25} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} = A$$

$$A^{30} = I \quad \dots(1)$$

$$A^{25} = A \quad \dots(2)$$

Add (1) and (2)

$$A^{30} + A^{25} = A + I \Rightarrow A^{30} + A^{25} - A = I$$

$$8.(4) \quad \frac{dy}{dx} = \frac{1-xy^2}{2x^2y} = \frac{1}{2x^2y} - \frac{xy^2}{2x^2y}$$

$$\frac{dy}{dx} = \frac{1}{2x^2y} - \frac{y}{2x}$$

$$\frac{dy}{dx} + \frac{y}{2x} = \frac{1}{2x^2y}$$

$$y \frac{dy}{dx} + \frac{y^2}{2x} = \frac{1}{2x^2}$$

$$\text{Let } \boxed{y^2 = z} \quad 2y \frac{dy}{dx} = \frac{dz}{dx}$$

$$\frac{1}{2} \frac{dz}{dx} + \frac{z}{2x} = \frac{1}{2x^2}$$

$$I.F = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

$$z \cdot x = \int \frac{1}{x^2} \cdot x dx$$

$$z \cdot x = \ln x + \ln c$$

$$xy^2 = \ln x + \ln c$$

$$\text{Given at } x = 2, y = \sqrt{\ln 2}$$

$$2 \ln 2 = \ln 2 + \ln C$$

$$\ln 4 - \ln 2 = \ln C$$

$$\ln 2 = \ln C$$

$$C = 2$$

$$xy^2 = \ln x + \ln 2 = \ln(2x) = 2x = e^{xy^2}$$

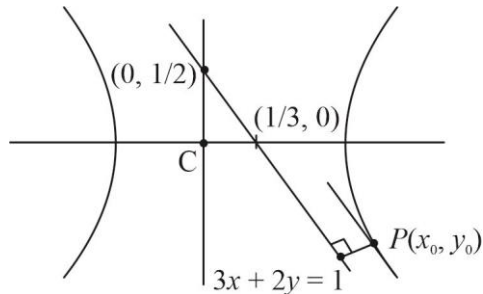
By comparing

$$\alpha = 2; \beta = 1; \gamma = 2$$

Then  $\alpha + \beta - \gamma$

$$3 - 2 = 1$$

9.(2)



Shortest distance lies along common normal, differentiating  $3x^2 - 4y^2 = 36$  w.r.t.  $x$ , and put

$$x = x_0; \quad y = y_0$$

$$\frac{dy}{dx} = \frac{3x_0}{4y_0} = -\frac{3}{2}$$

$$\Rightarrow x_0 = -2y_0 \dots(1)$$

$(x_0, y_0)$  also lies on the curve  $3x^2 - 4y^2 = 36$

$$= 3x_0^2 - 4y_0^2 = 36 \quad \dots(2)$$

$$= 3 \times 4y_0^2 - 4y_0^2 = 36$$

$$= 8y_0^2 = 36$$

$$= y_0^2 = \frac{9}{2} \Rightarrow y_0 = \pm \frac{3}{\sqrt{2}}$$

put  $y_0^2 = \frac{9}{2}$  in equation (2)

$$3x_0^2 - 4y_0^2 = 36$$

$$\Rightarrow x_0 = \mp 3\sqrt{2}$$

Req. point lies in 4th quadrant:

$$x_0 = 3\sqrt{2}, \quad \& \quad y_0 = -\frac{3}{\sqrt{2}}$$

$$\text{Now } \sqrt{2}(y_0 - x_0) = \sqrt{2}\left(-\frac{3}{\sqrt{2}} - 3\sqrt{2}\right) = -3 - 6 = -9$$

10.(2) Put  $x = \tan \theta \Rightarrow$  given  $0 < x < 1$  &  $\theta = \tan^{-1} x \Rightarrow 0 < \theta < \frac{\pi}{4} \Rightarrow \theta \in \left(0, \frac{\pi}{4}\right)$

$$2 \tan^{-1} \left( \frac{1 - \tan \theta}{1 + \tan \theta} \right) = \cos^{-1} \left( \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right)$$

$$\Rightarrow 2 \tan^{-1} \left( \tan \left( \frac{\pi}{4} - \theta \right) \right) = \cos^{-1}(\cos 2\theta)$$

$$2 \left( \frac{\pi}{4} - \theta \right) = 2\theta$$

$$\Rightarrow \frac{\pi}{2} - 2\theta = 2\theta$$

$$\Rightarrow 4\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{8}$$

$$\tan \frac{\pi}{8} = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}} = \sqrt{2}-1$$

$$\alpha x + y + z = 1$$

11.(1)  $x + \alpha y + z = 1$

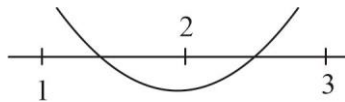
$$x + y + \alpha z = \beta$$

for infinite solution

$$\alpha = 1, \& \beta = 1$$

**Note:** For  $\alpha = 1 \& \beta = 1$  given three planes overlap to each other and provide infinitely many solution.

12.(1)



$$f(1)f(2) < 0$$

$$\Rightarrow (k-6)(k-8) < 0$$

$$k \in (6, 8) \quad \dots(1)$$

and  $f(2)f(3) < 0$

$$\Rightarrow (k-8)(k-6) < 0$$

$$6 < k < 8$$

$$= k \in (6, 8) \quad \dots(2)$$

(i)  $\cap$  (ii)  $k \in (6, 8)$   $k = 7$  is only possible integer

13.(3)  $f(x) = x^x, x > 0$

$$\ln y = x \ln x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = x \times \frac{1}{x} + \ln x \cdot 1$$

$$\frac{dy}{dx} = y(1 + \ln x)$$

$$\Rightarrow y'(x) = x^x(1 + \ln x)$$

$$\Rightarrow y'(2) = 4(1 + \ln 2)$$

$$2y'(2) = 8(1 + \ln 2) \quad \dots(1)$$

$$y'(x) = y(1 + \ln x)$$

$$y''(x) = y \times \frac{1}{x} + (1 + \ln x) y'$$

$$\Rightarrow \frac{x^{-x}}{x} + (1 + \ln x)(x^x)(1 + \ln x)$$

$$y''(2) = 2 + 4(1 + \ln 2)^2$$

$$2y'(2) = 8(1 + \ln 2)$$

$$= 2 + 4(1 + \ln 2)[1 + \ln 2 - 2]$$

$$\Rightarrow 2 + 4(1 + \ln 2)(\ln 2 - 1)$$

$$\Rightarrow 2 + 4(\ln 2)^2 - 4$$

$$\Rightarrow 2 + 4(\ln 2)^2 - 4$$

$$4(\ln 2)^2 - 2$$

14.(4) Given  $x_1 = 9$

$$\text{mean } \bar{X} = \frac{x_1 + x_2 + \dots + x_7}{7}$$

$$\Rightarrow \frac{a + a + d + \dots + a + 6d}{7}$$

$$\Rightarrow \frac{7a + d(1 + 2 + \dots + 6)}{7}$$

$$\Rightarrow \frac{7a + d \times \frac{6}{2}(7)}{7}$$

$$\frac{7a + 21d}{7}$$

$$\Rightarrow a + 3d = a_4 = \bar{x} \quad \dots(1)$$

$$\sigma^2 = 16 = \sum \frac{(x_i - \bar{x})^2}{N} \quad \dots(2)$$

(i) Put  $i = 1, x_1 - \bar{x}$

$$\Rightarrow a - (a + 3d) = -3d$$

(ii) Put  $i = 2, x_2 - \bar{x} = (a + d) - (a + 3d) \Rightarrow -2d$

(iii) Put  $i = 3, x_3 - \bar{x} = (a + 2d) - (a + 3d) \Rightarrow -d$

(iv) Put  $i = 4, x_4 - \bar{x} = (a + 3d) - (a + 3d) = 0$

(v) Put  $i = 5, x_5 - \bar{x} = (a + 4d) - (a + 3d) \Rightarrow d$

(vi) Put  $i = 6, x_6 - \bar{x} = (a + 5d) - (a + 3d) \Rightarrow 2d$

(vii) Put  $i = 7, x_7 - \bar{x} = (a + 6d) - (a + 3d) \Rightarrow 3d$

Put in equation (2)

$$\frac{2[d^2 + 4d^2 + 9d^2]}{7} = 16$$

$$14d^2 = 8 \times 7 \Rightarrow d = 2 \text{ and } a = 9$$

$$\bar{x} = 9 + 3 \times 2 \Rightarrow 15$$

$$x_6 = a + 5d \Rightarrow 9 + 5 \times 2 = 19$$

$$15 + 19 = 34$$

15.(2)  $f(x) + f\left(\frac{1}{1-x}\right) = 1+x$

put  $x = 2$ ,  $f(2) + f(-1) = 3$

put  $x = -1$ ,  $f(-1) + f\left(\frac{1}{2}\right) = 0$

subtract  $\overline{\quad\quad\quad}$   
 $f(2) - f\left(\frac{1}{2}\right) = 3 \quad \dots(1)$

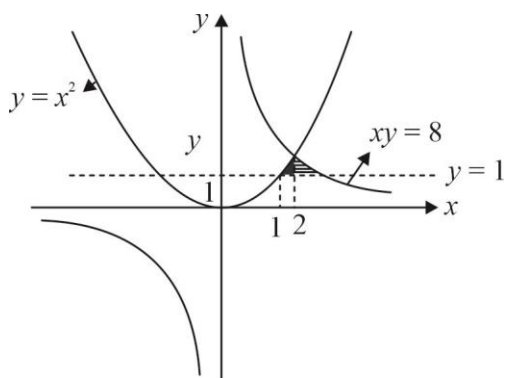
put  $x = \frac{1}{2}$ ,  $f\left(\frac{1}{2}\right) + f(2) = \frac{3}{2} \quad \dots(2)$

add (1) and (2),  $2f(2) = 3 + \frac{3}{2}$

$$= 2f(2) = \frac{9}{2} = f(2) = \frac{9}{4}$$

16.(4)

$$\left. \begin{array}{l} xy = 8 \\ y = x^2 \end{array} \right\} x = 2, y = 4$$



$$\text{Required Area} = \int_1^4 \left( \frac{8}{y} - \sqrt{y} \right) dy = 8 \ln y - \frac{y^{3/2}}{\frac{3}{2}} \Bigg|_1^4 = \left( 8 \ln 4 - \frac{2}{3} \times 4^{3/2} \right) - \left( 0 - \frac{2}{3} \right)$$

$$= 8 \ln 4 - \frac{2}{3} \times 8 + \frac{2}{3} = 16 \ln 2 - \frac{14}{3}$$

17.(4) Projection of  $\vec{a}$  on  $\vec{b}$  is  $\vec{a} \cdot \hat{b}$

$$= (5\hat{i} - \hat{j} - 3\hat{k}) \cdot \frac{(\hat{i} + 3\hat{j} + 5\hat{k})}{\sqrt{35}}$$

$$= \frac{5 - 3 - 15}{\sqrt{35}} = \frac{-13}{\sqrt{35}}$$

Since projection is -ve,

$\therefore$  projection of  $\vec{a}$  on  $\vec{b}$  is in direction opposite to that of  $\vec{b}$ .

As no option matches, therefore it should be Bonus.

18.(3) 
$$\sum_{n=1}^{\infty} \frac{2n^2 + 3n + 4}{(2n)!} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{2n(2n-1) + 8n + 8}{(2n)!}$$

$$\frac{1}{2} \sum_{n=1}^{\infty} \left[ \frac{2n(2n-1)}{(2n)!} + \frac{8n}{(2n)!} + \frac{8}{(2n)!} \right]$$

$$\frac{1}{2} \sum_{n=1}^{\infty} \left[ \frac{1}{(2n-2)!} + \frac{4}{(2n-1)!} + \frac{8}{(2n)!} \right]$$

$$\frac{1}{2} \left( 1 \left[ 1 + \frac{1}{2!} + \frac{1}{4!} + \dots \right] + 4 \left[ \frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \dots \right] + 8 \left[ \frac{1}{2!} + \frac{1}{4!} + \dots \right] \right) \dots (1)$$

we know  $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

$$e^1 = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots$$

$$e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots$$

$$\frac{e + e^{-1}}{2} = 1 + \frac{1}{2!} + \frac{1}{4!} + \dots$$

$$\frac{e - e^{-1}}{2} = \frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \dots$$

putting in (1)

$$\frac{1}{2} \left( 1 \left( \frac{e + e^{-1}}{2} \right) + 4 \left( \frac{e - e^{-1}}{2} \right) + 8 \left( \frac{e + e^{-1}}{2} - 1 \right) \right)$$

$$\frac{13e}{4} + \frac{5}{4e} - 4$$

19.(1) 
$$f(x) = \begin{cases} -(x^2 - 5x + 6) - 3x + 2 & 2 \leq x \leq 3 \\ x^2 - 5x + 6 - 3x + 2 & -1 \leq x < 2 \end{cases}$$

$$f(x) = \begin{cases} -x^2 + 2x - 4 & 2 \leq x \leq 3 \\ x^2 - 8x + 8 & -1 \leq x < 2 \end{cases}$$

$$f(x) = -x^2 + 2x - 4 \quad \text{in } x \in [2, 3]$$

$$f'(x) = -2x + 2 < 0 \quad \text{for } x \in [2, 3]$$

$$f(x) = x^2 - 8x + 8 \quad \text{in } x \in [-1, 2)$$

$$f'(x) = 2x - 8 < 0 \quad \text{for } x \in [-1, 2)$$

$$\therefore f(-1) = 17 \text{ absolute maximum value}$$

$$f(3) = -7 \text{ absolute minimum value}$$

$$\text{sum} = 17 + (-7) = 10$$

$$20.(2) \quad \left| \frac{1+ai}{b+i} \right| = 1 \Rightarrow |1+ai|^2 = |b+i|^2$$

$$(1+ai)(1-ai) = (b+i)(b-i)$$

$$1+a^2 = b^2+1 \Rightarrow a = \pm b$$

$$\text{Since } ab < 0, \therefore a = -b \dots(1)$$

Using

$$|z-1| = |2z|$$

$$\Rightarrow |z-1|^2 = (2|z|)^2$$

$$\Rightarrow (z-1)(\bar{z}-1) = 4z\bar{z}$$

$$\Rightarrow z\bar{z} - z - \bar{z} + 1 = 4z\bar{z}$$

$$3z\bar{z} + z + \bar{z} - 1 = 0$$

$$3|z|^2 + z + \bar{z} - 1 = 0 \dots(2)$$

Since  $z = a+ib$  satisfies (2)

$$3(a^2+b^2) + (a+ib) + (a-ib) - 1 = 0$$

$$3(a^2+b^2) + 2a - 1 = 0 \dots(3)$$

Solving (1) and (3)

$$\text{we get } 3(a^2+b^2) + 2a - 1 = 0$$

$$3(a^2+b^2) + 2a - 1 = 0$$

$$6a^2 + 2a - 1 = 0$$

$$a = \frac{-2 \pm \sqrt{28}}{12} = \frac{-2 \pm 2\sqrt{7}}{12}$$

Case-1

$$a = \frac{-1 + \sqrt{7}}{6}$$

$$[a] = 0$$

$$b = -a = \frac{1 - \sqrt{7}}{6}$$

$$\therefore \frac{1+[a]}{4b} = \frac{1+0}{4\left(\frac{1-\sqrt{7}}{6}\right)} = \frac{3}{2(1-\sqrt{7})}$$

Case-2

$$a = \frac{-1 - \sqrt{7}}{6}$$

$$[a] = -1$$

$$b = -a = \frac{1 + \sqrt{7}}{6}$$

$$\therefore \frac{1+[a]}{4b} = \frac{1-1}{4\left(\frac{1+\sqrt{7}}{6}\right)} = 0$$

As no option matches, therefore it should be Bonus.

SECTION – 2

21.(105)

$$x \geq 1, y \geq 3, z \geq 4$$

$$x-1 \geq 0, y-3 \geq 0, z-4 \geq 0$$

$$\text{Let } x-1 = X$$

$$y-3 = Y$$

$$z-4 = Z$$

$$\therefore X \geq 0, Y \geq 0, Z \geq 0$$

$$x + y + z = 21$$

$$(X+1) + (Y+3) + (Z+4) = 21$$

$$X + Y + Z = 13$$

$$\text{Not of integral solution} = {}^{13+3-1}C_{3-1} = {}^{15}C_2 = 105$$

22.(16)  $y^2 - 4y = 8x + 4$

$$(y-2)^2 = 8(x+1)$$

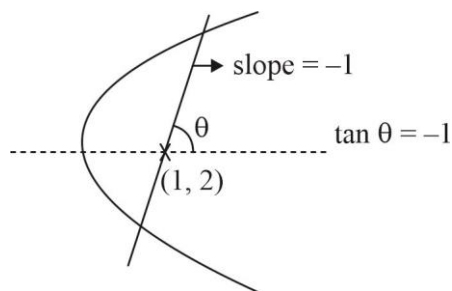
$$a = 2, \text{ vertex } (-1, 2), \text{ focus } (1, 2)$$

Since x-int of focal chord is 3

$\therefore$  It passes through (3, 0)

$$\text{equation of focal chord is } y-0 = -1(x-3)$$

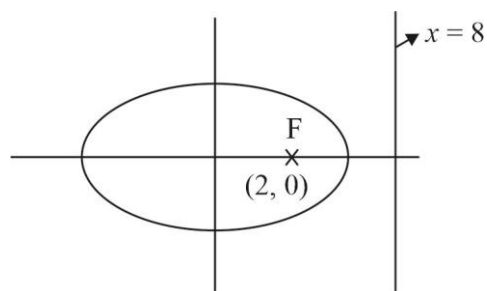
$$x + y = 3$$



$$\text{Length of focal chord} = 4a \operatorname{cosec}^2 \theta$$

$$= 4 \times 2 \times [1 + (-1)^2] = 16$$

23.(39)





$$\left. \begin{aligned} ae &= 2 \\ \frac{a}{e} &= 8 \end{aligned} \right\} a = 4, e = 1/2$$

$$a^2 e^2 = a^2 - b^2$$

$$\Rightarrow 4 = 16 - b^2$$

$$b^2 = 12$$

$$b = 2\sqrt{3}$$

$$E: \frac{x^2}{16} + \frac{y^2}{12} = 1$$

Let  $P(4\cos\theta, 2\sqrt{3}\sin\theta)$  be a point in 1st quadrant equation of tangent is

$$\frac{x\cos\theta}{4} + \frac{y\sin\theta}{2\sqrt{3}} = 1$$

It passes through  $(0, 4\sqrt{3})$

$$2\sin\theta = 1 \Rightarrow \sin\theta = \frac{1}{2}$$

$$\therefore \cos\theta = \frac{\sqrt{3}}{2}$$

$$P(2\sqrt{3}, \sqrt{3})$$

for Q put  $y = 0$

$$x = 4\sec\theta = \frac{8}{\sqrt{3}}$$

$$Q\left(\frac{8}{\sqrt{3}}, 0\right)$$

$$PQ = \sqrt{\left(2\sqrt{3} - \frac{9}{\sqrt{3}}\right)^2 + (\sqrt{3} - 0)^2} = \sqrt{\frac{4}{3} + 3} = \sqrt{\frac{13}{3}}$$

$$(3PQ)^2 = 9 \times \frac{13}{3} = 39$$

$$24.(4) \sqrt{2^{\log_2(10-3^x)}} = (10-3^x)^{1/2}$$

$$\sqrt[5]{2^{(x-2)\log_2 3}} = (3^{(x-2)})^{1/5} = 3^{\frac{x-2}{5}}$$

$$T_6 = {}^m C_5 (10-3^x)^{\frac{m-5}{2}} \left(3^{\frac{x-2}{5}}\right)^5 = {}^m C_5 (10-3^x)^{\frac{m-5}{2}} \cdot 3^{x-2} = 21 \quad \dots(1)$$

${}^m C_1, {}^m C_2, {}^m C_3$  are 1st, 3rd, 5th terms of A.P.

$$a = {}^m C_1$$

$$a + 2d = {}^m C_2$$

$$a + 4d = {}^m C_3$$

$$\therefore {}^m C_3 - {}^m C_2 = {}^m C_2 - {}^m C_1$$

$${}^m C_3 + {}^m C_1 = 2 {}^m C_2$$

$$\frac{m(m-1)(m-2)}{6} + m = \frac{2m(m-1)}{2}$$

$$\frac{(m-1)(m-2)}{6} + 1 = m-1$$

$$m^2 - 3m + 2 = 6(m-2)$$

$$m^2 = 9m + 14 = 0 \Rightarrow m = 7, 2 \text{ (rejected)}$$

$$\therefore m = 7, \text{ put } m = 7 \text{ in (1)}$$

$${}^7 C_5 (10 - 3^x)^1 3^{x-2} = 21$$

$$(10 - 3^x) \cdot \frac{3^x}{9} = 1$$

$$(10 - t)t = 9$$

$$t^2 - 10t + 9 = 0$$

$$t = 1, 9$$

$$3^x = 1, 9 \Rightarrow x = 0, 2$$

$$\text{Required sum} = 0^2 + 2^2 = 4$$

25.(6)  $\alpha x + \beta y + \gamma z = 1$  is not an equation of plane.

Correct equation of plane should be  $\alpha x + \beta y + \gamma z = 1$ .

There is printing mistake in question. Instead of  $\gamma z$ , it should be  $\gamma z$ .

$$26.(1) T_{r+1} = {}^{22} C_r (x^{2/3})^{22-r} \cdot \left( \frac{\alpha}{x^3} \right)^r$$

$$\text{Exponent of } x = \frac{2}{3}(22-r) - 3r = 0$$

$$r = 4$$

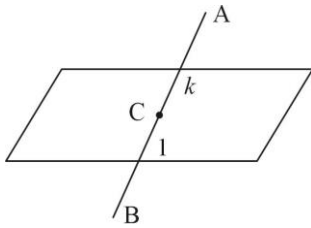
$$\therefore T_5 = {}^{22} C_4 \times \alpha^4$$

$$7315 = \frac{22 \cdot 21 \cdot 20 \cdot 19}{4!} \alpha^4$$

$$1 = \alpha^4$$

$$\therefore |\alpha| = 1$$

27.(10) Equation of AB  $\frac{x+3}{5} = \frac{y+6}{10} = \frac{z-1}{-4}$



Let Any pt of on AB be  $(5\lambda-3, 10\lambda-6, -4\lambda+1)$  ... (1)

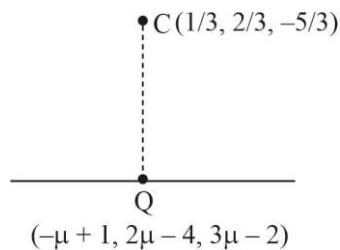
$$9(5\lambda-3) + (10\lambda-6) + 2(-4\lambda+1) = 0$$

$$42\lambda - 28 = 0$$

$$\lambda = \frac{2}{3}$$

put  $\lambda = \frac{2}{3}$  in (1),  $C\left(\frac{1}{3}, \frac{2}{3}, \frac{-5}{3}\right)$

Now any pt. on line 2  $\frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3}$



bc  $Q(-\mu+1, 2\mu-4, 3\mu-2)$

$$\vec{CQ} \cdot \vec{b} = 0$$

$$\vec{b} = -\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\left(-\mu+1-\frac{1}{3}\right) \cdot (-1) + \left(2\mu-4-\frac{2}{3}\right) \cdot (2) + \left(3\mu-2+\frac{5}{3}\right) \cdot 3 = 0$$

$$\mu - \frac{2}{3} + 4\mu - \frac{28}{3} + 9\mu - 1 = 0$$

$$14\mu = 11 \Rightarrow \mu = \frac{11}{14}$$

d.r's of foot of  $\perp \left(-\mu + \frac{2}{3}, 2\mu - \frac{14}{3}, 3\mu - \frac{1}{3}\right)$

$$\left(\frac{-11}{14} + \frac{2}{3}, \frac{22}{14} - \frac{14}{3}, \frac{3 \times 11}{14} - \frac{1}{3}\right)$$

$$\left(\frac{-5}{42}, \frac{-130}{42}, \frac{85}{42}\right) \text{ or } (-1, -26, 17)$$

$$a = -1, b = -26, c = 17$$

$$|a+b+c| = 10$$

28.(81)

-----4 fixed

Case-1 '4' → 6 times    Number = 1

Case-2 '4' → 4 times

'5' → 1 times

'9' → 1 times

$$\text{Numbers} = \frac{5!}{3!} = 20$$

Case 3 '4' → 2 times

'5' → 2 times

'9' → 2 times

$$\text{Number} = \frac{5!}{2!2!} = 30$$

Case 4 '4' → 1 time

'5' → 1 time

'9' → 4 time

$$\text{Numbers} = \frac{5!}{4!} = 5$$

Case 5 '4' → 3 times

'9' → 3 times

$$\text{Numbers} = \frac{5!}{3!2!} = 10$$

Case 6 '4' → 1 time

'5' → 4 times

'9' → 1 time

$$\text{Numbers} = \frac{5!}{4!} = 5$$

Case 7 '4' → 3 times

'5' → 3 times

$$\text{Numbers} = \frac{5!}{3!2!} = 10$$

Total number 81

$$\begin{aligned}
 29.(13) \quad I &= \int_0^{\pi} \frac{5^{\cos x} (1 + \cos x \cos 3x + \cos^2 x + \cos^3 x \cos 3x)}{1 + 5^{\cos x}} dx \\
 &= \int_0^{\pi/2} \left( \frac{5^{\cos x} (1 + \cos x \cos 3x + \cos^2 x + \cos^3 x \cos 3x)}{1 + 5^{\cos x}} + \frac{5^{\cos(\pi-x)} (1 + \cos(\pi-x) \cos(3(\pi-x)) + \cos^2(\pi-x) + \cos^3(\pi-x) \cos 3(\pi-x))}{1 + 5^{\cos(\pi-x)}} \right) dx \\
 &= \int_0^{\pi/2} (1 + \cos x \cos 3x + \cos^2 x + \cos^3 x \cos 3x) dx \\
 &= \int_0^{\pi/2} (1 + \cos x (4 \cos^3 x - 3 \cos x) + \cos^2 x + \cos^3 x (4 \cos^3 x - 3 \cos x)) dx \\
 &= \int_0^{\pi/2} (4 \cos^6 x + \cos^4 x - 2 \cos^2 x + 1) dx \\
 &= 4 \times \left( \frac{5 \times 3 \times 1}{6 \times 4 \times 2} \times \frac{\pi}{2} \right) + \left( \frac{3 \times 1}{4 \times 2} \times \frac{\pi}{2} \right) - 2 \left( \frac{\pi}{4} \right) + \frac{\pi}{2} = \frac{13\pi}{16} \\
 \therefore k &= 13
 \end{aligned}$$

30.(321)

$$3, 7, 11, 15, \dots, 399, \quad d_1 = 4$$

$$2, 5, 8, 11, \dots, 359, \quad d_2 = 3$$

$$2, 7, 12, 17, \dots, 197 \quad d_3 = 5$$

Common terms of 3 A.P's will be in A.P.

whose common difference is L.C.M of {4, 3, 5} = 60

First common term is 47 (by observation)

common terms are 47, 107, 167

sum = 321