JEE Main – 2023

31th JAN 2023 (Morning Shift)

General Instructions

1. The test is of **3 hours** duration and the maximum marks is **300.**

2. The question paper consists of **3 Parts** (Part I: **Physics**, Part II: **Chemistry**, Part III: **Mathematics**). Each Part has **two** sections (Section 1 & Section 2).

3. **Section 1** contains **20 Multiple Choice Questions**. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE CHOICE** is correct.

4. **Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. You will NOT be allowed to attempt the sixth question. If you wish to attempt any other question apart from the five already attempted, then you will have to delete any one response from the five previously answered and then proceed to answer the new one.**
The answer to each question should be **rounded off to the nearest integer**.

5. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. inside the examination room/hall.

Marking Scheme

1. **Section – 1:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.

2. **Section – 2:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.

| SUBJECT I: PHYSICS		MARKS: 100

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. If 1000 droplets of water of surface tension $0.7 N/m$, having same radius 1 mm each, combine to from a single drop. In the process the released surface energy is:

   (1) $8.8 \times 10^{-5} J$ (2) $7.92 \times 10^{-4} J$ (3) $9.68 \times 10^{-4} J$ (4) $7.92 \times 10^{-6} J$

2. A bar magnet with a magnetic moment $5.0 Am^2$ is placed in parallel position relative to a magnetic field of $0.4T$. The amount of required work done in turning the magnet from parallel to antiparallel position relative to the field direction is:

   (1) $2J$ (2) zero   (3) $1J$ (4) $4J$

3. The drift velocity of electrons for conductor connected in an electrical circuit is V_d. The conductor in nor replaced by another conductor with same material and same length but double the area of cross section. The applied voltage remains same. The new drift velocity of electrons will be:

   (1) $2V_d$ (2) $\dfrac{V_d}{4}$ (3) $\dfrac{V_d}{2}$ (4) V_d

4. The pressure of a gas changes linearly with volume from A to B as shown in figure. If no heat is supplied to or extracted from the gas then change in the internal energy of the gas will be:
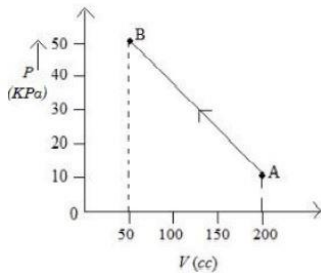
   (1) $4.5J$ (2) $6J$ (3) $-4.5J$ (4) zero

5. Which of the following correctly represents the variation of electric potential (V) of a charged spherical conductor of radius (R) with radial distance (r) from the centre?

   (1)   (2)   (3)   (4)
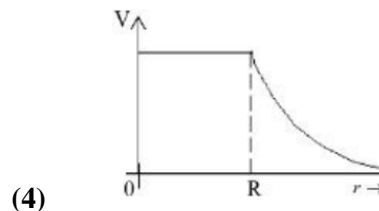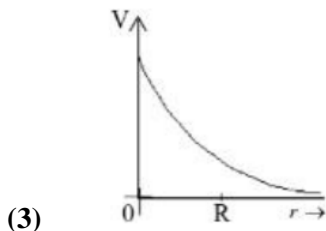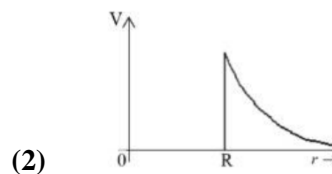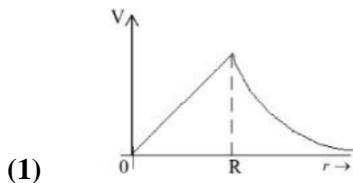
6. If $R, X_L,$ and X_C represent resistance, inductive reactance and capacitive reactance. Then which of the following is dimensionless:

   (1) $RX_L X_C$ (2) $\dfrac{R}{X_L X_C}$ (3) $R\dfrac{X_L}{X_C}$ (4) $\dfrac{R}{\sqrt{X_L X_C}}$

7. Two polaroide A and B are placed in such a way that the pass-axis of polaroids are perpendicular to each other. Now, another polaroid C is placed between A and B bisecting angle between them. If intensity of unpolarized light is I_0 then intensity of transmitted light after passing through polaroid B will be:

(1) zero **(2)** $\dfrac{I_0}{8}$ **(3)** $\dfrac{I_0}{4}$ **(4)** $\dfrac{I_0}{2}$

8. At a certain depth "d" below surface of earth, value of acceleration due to gravity becomes four times that of its value at a height $3R$ above earth surface. Where R is Radius of earth (Take $R = 6400 km$). The depth d is equal to:

(1) 2560 km **(2)** 640 km **(3)** 5260 km **(4)** 4800 km

9. The initial speed of a projectile fired from ground is u. at the highest point during its motion, the speed of projectile is $\dfrac{\sqrt{3}}{2}u$. The time of flight of the projectile is:

(1) $\dfrac{u}{g}$ **(2)** $\dfrac{\sqrt{3}u}{g}$ **(3)** $\dfrac{2u}{g}$ **(4)** $\dfrac{u}{2g}$

10. Spherical insulating ball and spherical metallic ball of same size and mass are dropped from the same height. Choose the correct statement out of the following {Assume negligible air frication}
   (1) Time taken by them to reach the earth's surface will be independent of the properties of their materials
   (2) Metal ball will reach the earth's surface earlier than the insulating ball
   (3) Insulating ball will reach the earth's surface earlier than the metal ball
   (4) Both will reach the earth's surface simultaneously.

11. If a source of electromagnetic radiation having power $15 kW$ produces 10^{16} photons per second, the radiation belongs to a part of spectrum is:

(Take Planck constant $h = 6 \times 10^{-34} Js$)

   (1) Radio waves **(2)** Gamma rays **(3)** Ultraviolet rays **(4)** Micro waves

12. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R
   Assertion A: The beam of electrons show wave nature and exhibit interference and diffraction.
   Reason R: Davisson germer experimentally verified the wave nature of electrons.
   In the light of the above statements, choose the most appropriate answer from the options given below:
   (1) A is not correct but R is correct
   (2) A is correct but R is not correct
   (3) Both A and R are correct but R is not the correct explanation of A
   (4) Both A and R are correct and R is the correct explanation of A

13. The amplitude of $15\sin(1000\pi t)$ is modulated by $10\sin(4\pi t)$ single. The amplitude modulated signal contains frequency (ics) of.
   (A) $500 Hz$ **(B)** $2 Hz$ **(C)** $250 Hz$ **(D)** $498 Hz$
   (E) $502 Hz$
   Choose the correct answer from the options given below:
   (1) A and B only **(2)** B only **(3)** A only **(4)** A, D and E only

14. A rod with circular cross – section area $2cm^2$ and length $40cm$ is wound uniformly with 400 turns of an insulated wire. If a current of $0.4A$ flows in the wire windings, the total magnetic flux produced inside windings is $4\pi \times 10^{-6} Wb$. The relative permeability of the rod is:

(Given: Permeability of vacuum $\mu_0 = 4\pi \times 10^{-7} NA^{-2}$)

(1) 12.5   **(2)** $\dfrac{5}{16}$ **(3)** 125   **(4)** $\dfrac{32}{5}$

15. A free neutron decays into a proton but a free proton does not decay into neutron. This is because
(1) Proton is a charged particle
(2) neutron is a composite particle made of a proton and an electron
(3) neutron has larger rest mass than proton
(4) neutron is an uncharged particle

16. The effect of increase in temperature on the number of electrons in conduction band (n_e) and resistance of a semiconductor will be is:

(1) n_e increases, resistance decrease   **(2)** n_e decreases, resistance increases

(3) Both n_e and resistance increase   **(4)** Both n_e and resistance decrease

17. The correct relation between $\gamma = \dfrac{c_p}{c_v}$ and temperature T is:

(1) $\gamma \alpha T$ **(2)** $\gamma \alpha \dfrac{1}{\sqrt{T}}$ **(3)** $\gamma \alpha \dfrac{1}{T}$ **(4)** $\gamma \alpha T^0$

18. The maximum potential energy of a block executing simple harmonic motion is $25J$. A is amplitude of oscillation. At $\dfrac{A}{2}$, the kinetic energy of the block is:

(1) $9.75J$ **(2)** $18.75J$ **(3)** $12.5J$ **(4)** $37.5J$

19. 100 balls each of mass m moving with speed v simultaneously strike a wall normally and reflected back with same speed, in time t s. the total force exerted by the balls on the wall is:

(1) $\dfrac{mv}{100t}$ **(2)** $\dfrac{200mv}{t}$ **(3)** $200mvt$ **(4)** $\dfrac{100mv}{t}$

20. As shown in figure, a $70\,kg$ garden roller is pushed with a force of $\vec{F} = 200N$ at an angle of $30°$ with horizontal. The normal on the roller is:

(Given $g = 10\,ms^{-2}$)
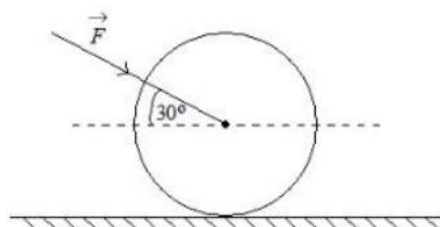
**(1)** $600N$ **(2)** $800\sqrt{2}N$ **(3)** $800N$ **(4)** $200\sqrt{3}N$

SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be **rounded off to the nearest integer**.

21. In a medium the speed of light wave decrease to 0.2 times to its speed in free space the ratio of relative permittivity to the refractive index of the medium is $x:1$. The value of x is_____:

(Given speed of light in free space $= 3\times10^8\,m\,s^{-1}$ and for the given medium $\mu_r = 1$)

22. The speed of a swimmer is $4\,km\,h^{-1}$ in still water. If the swimmer makes his strokes normal to the flow of river of width 1 km, he reaches a point 750 m down the stream on the opposite bank.

The speed of the river water is_____ $km\,h^{-1}$

23. A lift of mass $M = 500\,kg$ is descending with speed of $2\,ms^{-1}$. Its supporting cable begins to slip thus allowing it to fall with a constant acceleration of $2\,ms^{-2}$. The kinetic energy of the lift at the end of fall through to a distance of 6 m will be _____kJ.

24. In the figure given below, a block of mass $M = 490g$ placed on a frictionless table is connected with two springs have same spring constant $\left(K = 2N\,m^{-1}\right)$. If the block is horizontally displaced through 'X' m then the number of complete oscillations it will make in 14π seconds will be_____.
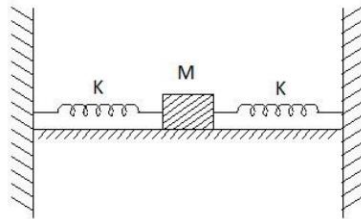
25. A thin rod having a length of 1 m and are of cross – section $3\times10^{-6}\,m^2$ is suspended vertically from one end. The rod is cooled from $210°C$ to $160°C$. After colling, a mass M is attached at the lower end of the rod such that the length of rod again becomes 1 m. young's modulus and coefficient of linear expansion of the rod are $2\times10^{11}\,Nm^2$ and $2\times10^{-5}\,K^{-1}$, respectively. The value of M is_____ kg.

(Take $g = 10\,ms^{-2}$)

26. An inductor of 0.5 mH, a capacitor of $20\,\mu F$ and resistance of $20\,\Omega$ are connected in series with a $220Vac$ source. If the current in the phase with the emf, the amplitude of current of the circuit is $\sqrt{x}$ A. The value of x is:

27. Expression for an electric field is given by $\vec{E} = 4000x^2\hat{i}\,\dfrac{V}{m}$. The electric flux through the cube of side 20 cm when placed in electric field (as shown in the figure) is _____$V\,cm$.
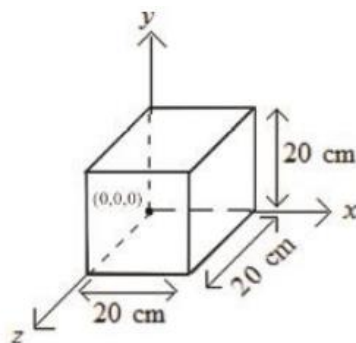
28. For hydrogen atom, λ_1 and λ_2 Are the wavelengths corresponding to the transitions 1 and 2 respectively as shown in figure. The ratio of λ_1 and λ_2 is $\dfrac{x}{32}$. The value of x is_____.
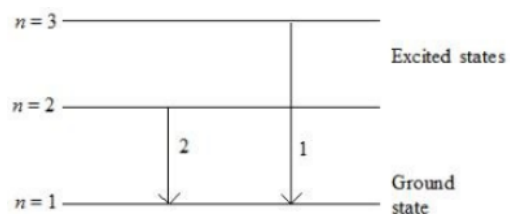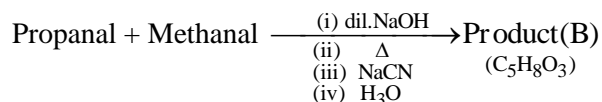

29. A solid sphere of mass 1 *kg* rolls without slipping on a plane surface. Its kinetic energy is $7 \times 10^{-3} J$, the speed of the centre of mass of the sphere is _____ $cm\ s^{-1}$.

30. Two identical cells, when connected either in parallel or in series gives same current in an external resistance 5Ω. The internal resistance of each cell will be____ Ω.

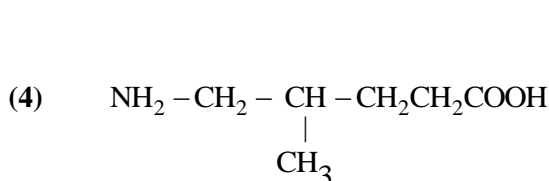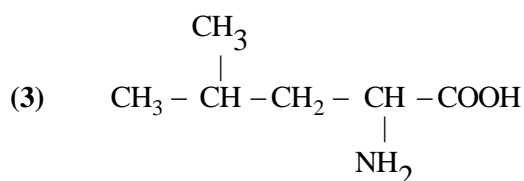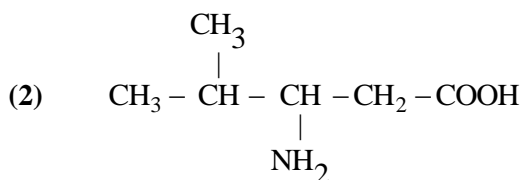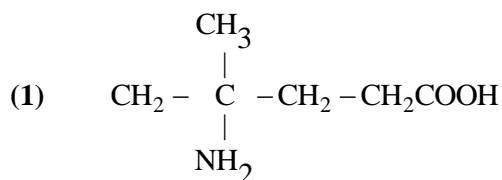| SUBJECT II: CHEMISTRY		MARKS: 100

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. Consider the following reaction

$$\text{Propanal + Methanal} \xrightarrow[\substack{\text{(ii)} \quad \Delta \\ \text{(iii) NaCN} \\ \text{(iv) } H_3O}]{\text{(i) dil.NaOH}} \text{Product(B)} \quad (C_5H_8O_3)$$
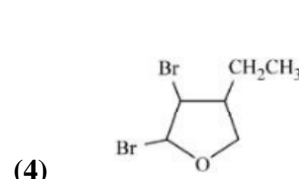
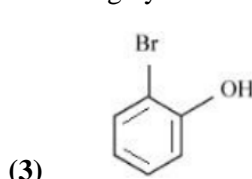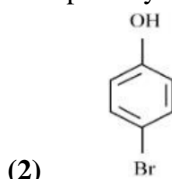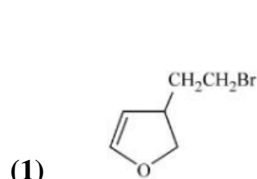The correct statement for product B is. It is:
- **(1)** Optically active and adds one mole of bromine
- **(2)** Optically active alcohol and is neutral
- **(3)** Racemic mixture and is neutral
- **(4)** Racemic mixture and gives a gas will saturated $NaHCO_3$ solution

2. The correct increasing order of the ionic radii is:
- **(1)** $S^{2-} < Cl^- < Ca^{2+} < K^+$
- **(2)** $Cl^- < Ca^{2+} < K^+ < S^{2-}$
- **(3)** $K^{2+} < S^{2-} < Ca^{2+} < Cl^-$
- **(4)** $Ca^{2+} < K^+ < Cl^- < S^{2-}$

3. H_2O_2 acts as a reducing agent in.
- **(1)** $2NaOCl + H_2O_2 \rightarrow 2NaCl + H_2O + O_2$
- **(2)** $Na_2S + 4H_2O_2 \rightarrow NaSO_4 + 4H_2O$
- **(3)** $Mn^{2+} + 2H_2O_2 \rightarrow MnO_2 + 2H_2O$
- **(4)** $2Fe^{2+} + 2H^+ + H_2O_2 \rightarrow 2Fe^{3+} + 2H_2O$

4. A protein 'X' with molecular weight of 70,000 u, on hydrolysis gives amino acids. One of these amino acid is:

(1)

$$CH_2 - \underset{\underset{NH_2}{|}}{\overset{\overset{CH_3}{|}}{C}} - CH_2 - CH_2COOH$$

(2)

$$CH_3 - CH - \underset{\underset{NH_2}{|}}{CH} - CH_2 - COOH \quad (CH_3 \text{ above first CH})$$

(3)

$$CH_3 - CH - CH_2 - \underset{\underset{NH_2}{|}}{CH} - COOH \quad (CH_3 \text{ above first CH})$$

(4)

$$NH_2 - CH_2 - \underset{\underset{CH_3}{|}}{CH} - CH_2CH_2COOH$$
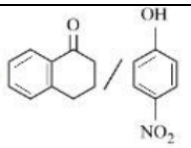
5. Identify X, Y and Z in the following reaction. (Equation not balanced):

$$ClO + NO_2 \rightarrow \underline{X} \xrightarrow{H_2O} \underline{Y} + \underline{Z}$$
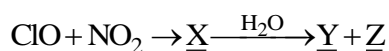
- **(1)** $X = ClNO_2, Y = HCl, Z = HNO_3$
- **(2)** $X = ClONO_2, Y = HOCl, Z = HNO_3$
- **(3)** $X = ClONO_2, Y = HOCl, Z = NO_2$
- **(4)** $X = ClNO_3, Y = Cl_2, Z = NO_2$

6. An organic compound 'A' with empirical formula C_6H_6O gives sooty flame on burning. Its reaction with bromine solution in low polarity solvent results in high yield of B. B is:

(1) CH₂CH₂Br substituted furan   **(2)** 4-bromophenol (OH top, Br bottom)   **(3)** 2-bromophenol (Br and OH)   **(4)** dibromo ethyl tetrahydrofuran

7. Match items of column I and II.

	Column I (Mixture of compounds)		**Column II (Separation Technique)**
A.	H_2O / CH_2Cl_2	**i.**	Crystallization
B.		**ii.**	Differential solvent extraction
C.	Kerosene/ Naphthalene	**iii.**	Column chromatography
D.	$C_6H_{12}O_6$ / NaCl	**iv.**	Fractional distillation

(1) A-(ii), B-(iii), C-(iv), D(i)   **(2)** A-(ii), B-(iv), C-(i), D-(iii)

(3) A-(iii), B-(iv), C-(ii), D-(i)   **(4)** A-(i), B-(iii), C-(ii), D-(iv)

8. Match List I with List II.

	List I		**List II**
A.	XeF_4	**i.**	See -saw
B.	SF_4	**ii.**	Square planar
C.	NH_4^+	**iii.**	Bent T- shaped
D.	BrF_3	**iv.**	Tetrahedral

(1) A-IV, B-III, C-II, D-I   **(2)** A-II, B-I, C-III, D-IV

(3) A-II, B-I, C-IV, D-III   **(4)** A-IV, B-I, C-II, D-III

9. Which one of the following statements is correct for electrolysis of brine solution?
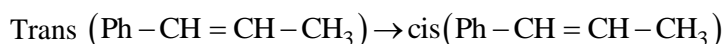
(1) H_2 is formed at anode   **(2)** Cl_2 is formed at cathode

(3) OH^- is formed at cathode   **(4)** O_2 is formed at cathode

10. Choose the correct set of reagents for the following conversion.

Trans $\left(Ph - CH = CH - CH_3\right) \rightarrow cis\left(Ph - CH = CH - CH_3\right)$

(1) $Br_2, alc \bullet KOH, NaNH_2, Na\left(LiqNH_3\right)$

(2) $Br_2, aq \bullet KOH, NaNH_2, Na\left(LiqNH_3\right)$

(3) $Br_2, aq \bullet KOH, NaNH_2, H_2 Lindlar Catalyst$

(4) $Br_2, alc \bullet KOH, NaNH_2, H_2 Lindlar Catalyst$

11. Adding surfactants in non – polar solvent, the micelles structure will look like:
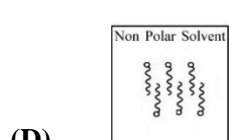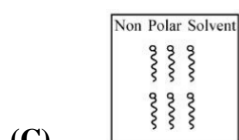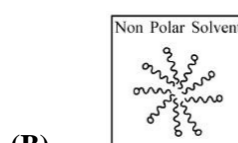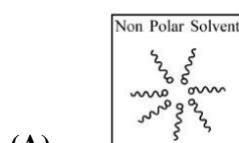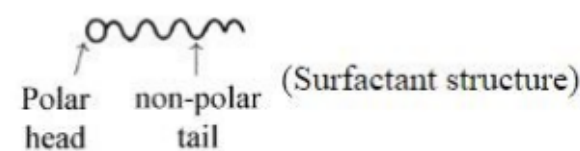
(1) b   **(2)** a   **(3)** d   **(4)** c

12. The correct order of melting points of dichlorobenzenes is:

(1)

(2)

(3)

(4)

13. When Cu^{2+} ion is treated with KI, a white precipitate, X appears in solution. The solution is titrated with sodium thiosulphate, the compound Y is formed, X and Y respectively are:
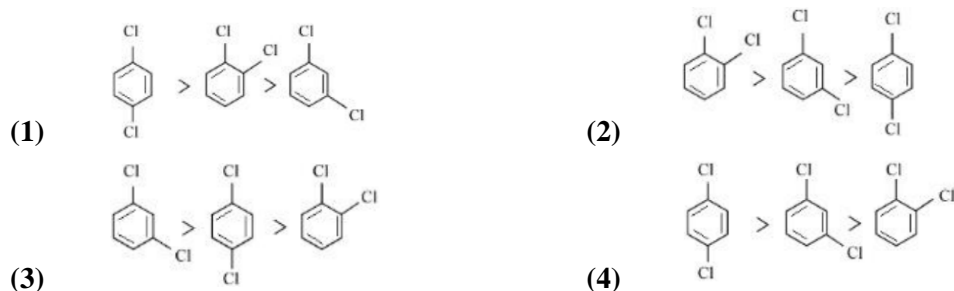
(1) $X = CuI_2 \ Y = Na_2S_4O_6$

(2) $X = CU_2I_2 \ Y = Na_2S_4O_5$

(3) $X = CuI_2 \ Y = Na_2S_2O_3$

(4) $X = Cu_2I_2 \ Y = Na_2S_4O_6$

14. Which transition in the hydrogen spectrum would have the same wavelength as the Balmer type transition from $n = 4$ to $n = 2$ of He^+ spectrum.

(1) $n = 1$ to $n = 3$ **(2)** $n = 2$ to $n = 1$ **(3)** $n = 1$ to $n = 2$ **(4)** $n = 3$ to $n = 2$

15. The correct order of basicity of oxides of vanadium is:

(1) $V_2O_3 > V_2O_4 > V_2O_5$

(2) $V_2O_4 > V_2O_3 > V_2O_5$

(3) $V_2O_3 > V_2O_5 > V_2O_4$

(4) $V_2O_5 > V_2O_4 > V_2O_3$

16. Which of the following artificial sweeteners has the highest sweetness value in comparison to cane sugar?

(1) Sucralose **(2)** Saccharin **(3)** Alitame **(4)** Aspartame

17. Cobalt chloride when dissolved in water forms pink colored complex $\underline{X}$ which has octahedral geometry. This solution on treating with conc HCl forms deep blue complex, $\underline{Y}$ which has a $\underline{Z}$ geometry. X, Y and Z, respectively are:

(1) $X = \left[Co(H_2O)_6 \right]^{3+}, Y = \left[CoCl_6 \right]^{3-}, Z = $ Octahedral

(2) $X = \left[Co(H_2O)_6 \right]^{2+}, Y = \left[CoCl_2 \right]^{2-}, Z = $ Tetrahedral

(3) $X = \left[Co(H_2O)_6 \right]^{2+}, Y = \left[CoCl_6 \right]^{3-}, Z = $ Octahedral

(4) $X = \left[Co(H_2O)_4 Cl_2 \right]^{+}, Y = \left[CoCl_4 \right]^{2-}, Z = $ Tetrahedral

18. The methods NOT involved in concentration of ore are:

(A) Liquation **(B)** Leaching **(C)** Electrolysis **(D)** Hydraulic washing
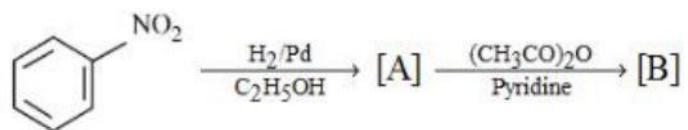**(E)** Froth floatation

Choose the correct answer from the options given below:

(1) C, D and E only

(2) B, D and C only

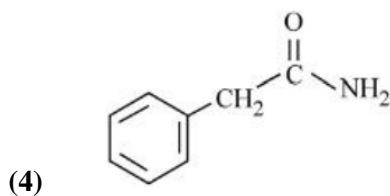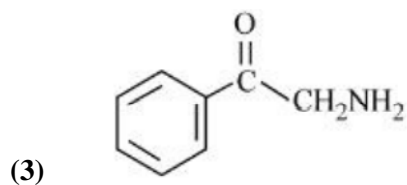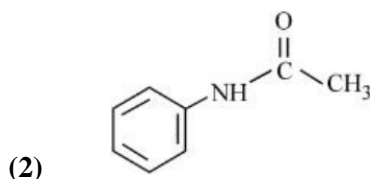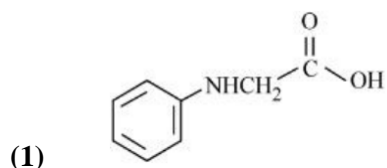**(3)** A and C only

(4) B, D and E only

19. $Nd^{2+} = $ _____

(1) $4f^4 6s^2$ **(2)** $4f^2 6s^2$ **(3)** $4f^3$ **(4)** $4f^4$

20.

Consider the above reaction and identify the product B.

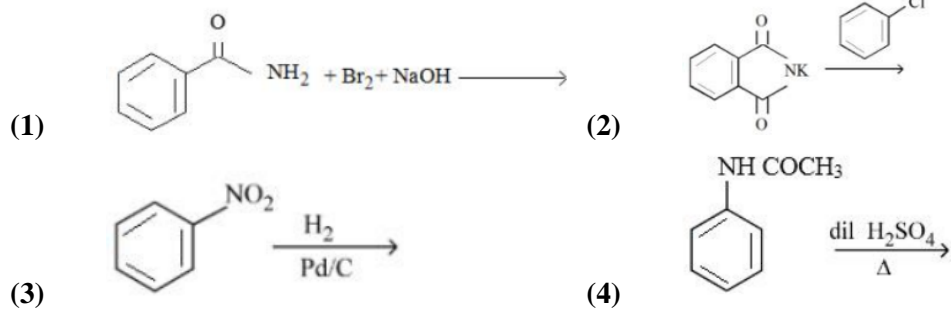**(1)**

(2)

(3)

(4)

SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be **rounded off to the nearest integer**.

21. The logarithm of equilibrium constant for the reaction $Pd^{2+} + 4Cl^- \rightleftharpoons PdCl_4^{2-}$ is_____ (Nearest integer).

Given: $\dfrac{2.303RT}{F} = 0.06V$

$Pd_{(aq)}^{2+} + 2e^- \rightleftharpoons Pd(s) \quad E^\theta = 0.83V$

$PdCl_4^{2-}(aq) + 2e^- \rightleftharpoons Pd(s) + 4Cl^-(aq) \quad E^\theta = 6.65V$

22. The total pressure of a mixture of non-reacting gases $X(0.6g)$ and $Y(0.45g)$ in a vessel is 740 mm of Hg. The partial pressure of the gas X is_____ mm of Hg.

(Given: molar mass $X = 20$ and $Y = 45 g\,mol^{-1}$)

23. At 27°C, a solution containing 2.5 g of solute in 250.0 mL of solution exerts an osmotic pressure of 400Pa. The molar mass of solute is _____ g mol^{-1}.

(Nearest integer)

(Given: R=0.083L bar $K^{-1}mol^{-1}$)

24. The oxidation state of phosphorus in hypophosphoric acid is+_____.

25. $A \rightarrow B$

The rate constants of the above reaction at 200K and 300K are $0.03\,min^{-1}$ and

$0.05\,min^{-1}$ respectively. The activation energy for the reaction is _____ J

(Nearest integer)

(Given: ln 10 = 2.3

$R = 8.3 JK^{-1}mol^{-1}$

$\log 5 = 0.70$

$\log 3 = 0.48$

$\log 2 = 0.30$)

26. On complete combustion, 0.492 g of an organic compound gave 0.792 g if CO_2. The % of carbon in the organic compound is_____ (Nearest integer).

27. Zinc reacts with hydrochloric acid to give hydrogen and zinc chloride. The volume of hydrogen gas produced at STP from the reaction of 11.5 g of zinc with excess HCl is _____L (nearest integer).

(Given: Molar mass of Zn is 65.4g mol^{-1} and molar volume of H_2 at STP = 22.7L)

28. For reaction: $SO_2(g) + \dfrac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$

$K_p = 2 \times 10^{12}$ at 27°C and 1 atm pressure. The K_c for the same reaction is _____$\times 10^{13}$. (Nearest integer).

(Given $R = 0.082L\,atm\,K^{-1}mol^{-1}$)

29. How many of the transformations given below would result in aromatic amines?

(1)

$NH_2 + Br_2 + NaOH \longrightarrow$

(2)

NK

Cl

(3)

NO_2

$\xrightarrow[\text{Pd/C}]{H_2}$

(4)

$NH\,COCH_3$

$\xrightarrow[\Delta]{\text{dil } H_2SO_4}$

30. The enthalpy change for the conversion of $\frac{1}{2}Cl_2(g)$ to $Cl^-(aq)$ is $(-)$ _____ kJ mol^{-1} (nearest integer):

Given: $\Delta_{dis}H^{\theta}_{Cl_{2(g)}} = 240\text{kJ mol}^{-1}, \Delta_{eg}H^{\theta}_{Cl_{(g)}} = -350\text{kJ mol}^{-1}$,

$\Delta_{hyd}H^{\theta}_{Cl^-_{(g)}} = -380\text{kJ mol}^{-1}$

| SUBJECT III: MATHEMATICS | MARKS: 100 |

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. Let $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$, and $\vec{b}$ and $\vec{c}$ be two nonzero vectors such that
$\left|\vec{a} + \vec{b} + \vec{c}\right| = \left|\vec{a} + \vec{b} - \vec{c}\right|$ and $\vec{b} \cdot \vec{c} = 0$. Consider the following two statements:

   (A) $\left|\vec{a} + \lambda\vec{c}\right| \geq \left|\vec{a}\right|$ for all $\lambda \in R$

   (B) $\vec{a}$ and $\vec{c}$ are always parallel.

   (1)   only (A) is correct   **(2)**   only (B) is correct
   (3)   neither (A) nor (B) is correct   **(4)**   both (A) and (B) are correct

2. Let $\alpha \in (0,1)$ and $\beta = \log_e(1-\alpha)$. Let $P_n(x) = x + \dfrac{x^2}{2} + \dfrac{x^2}{3} + ... + \dfrac{x^n}{n}, x \in (0,1)$.

   Then the integral $\displaystyle\int_0^{\alpha} \dfrac{t^{50}}{1-t} dt$ is equal to:

   (1) $\beta + P_{50}(\alpha)$ **(2)** $-\left(\beta + P_{50}(\alpha)\right)$**(3)** $\beta - P_{50}(\alpha)$ **(4)** $P_{50}(\alpha) - \beta$

3. For all $z \in C$ on the curve $C_1 : |z| = 4$, let the locus of the point $z + \dfrac{1}{z}$ be the curve C_2. then:

   (1)   The curves C_1 and C_2 intersect at 2 points

   (2)   The curves C_1 and C_2 intersect at 4 points

   (3)   The curve C_2 lies inside C_1

   (4)   The curve C_1 lies inside C_2

4. If $\sin^{-1}\dfrac{\alpha}{17} + \cos^{-1}\dfrac{4}{5} - \tan^{-1}\dfrac{77}{36} = 0, 0 < \alpha < 13$, then $\sin^{-1}(\sin\alpha) + \cos^{-1}(\cos\alpha)$ is equal to:

   (1)   16   **(2)**   0   **(3)** $16 - 5\pi$ **(4)** π

5. $(S1) = (p \Rightarrow q) V(p \wedge (\sim q))$ is a tautology

   $(S2)((\sim p) \Rightarrow (\sim q)) \wedge ((\sim p) \vee q)$ is a contradiction.

   Then.

   (1)   both $(S1)$ and $(S2)$ are correct   **(2)**   both $(S1)$ and $(S2)$ are wrong

   (3)   Only $(S2)$ is correct   **(4)**   only $(S1)$ is correct

6. A bag contains 6 balls. Two balls are drawn from it at random and both are found to be black. The probability that the bag contains at least 6 black balls is:

   (1) $\dfrac{5}{6}$ **(2)** $\dfrac{3}{7}$ **(3)** $\dfrac{5}{7}$ **(4)** $\dfrac{2}{7}$

7. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{pmatrix}$. Then the sum of the diagonal elements of the matrix $(A+I)^{11}$ is equal to:

   (1)   4097   **(2)**   6144   **(3)**   4094   **(4)**   2050

8. A wire of length $20\,m$ is to the cut into two pieces. A piece of length l_1 is bent to make a square of area A_1 and the other piece of length l_2 is made into a circle of area A_2. If $2A_1 + 3A_2$ is minimum then $(\pi l_1) : l_2$ is equal to:

(1) 6:1   (2) 3:1   (3) 4:1   (4) 1:6

9. Let the shortest distance between the lines $L : \dfrac{x-5}{-2} = \dfrac{y-\lambda}{0}, \dfrac{z+\lambda}{1}, \lambda \geq 0$ and $L_1 : x+1 = y-1 = 4-z$ be $2\sqrt{6}$. If (α, β, γ) lies on L, then which of the following is NOT possible?

(1) $2\alpha - \gamma = 9$ (2) $\alpha - 2\gamma = 19$ (3) $\alpha + 2\gamma = 24$ (4) $2\alpha + \gamma = 7$

10. If the sum and product of four positive consecutive terms of a G.P., are 126 and 1296, respectively, then the sum of common ratios of all such *GPs* is:

(1) $\dfrac{9}{2}$ (2) 12   (3) 3   (4) 7

11. Let a circle C_1 be obtained on rolling the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ upwards 4 units on the tangent T to it at the point (3,2). Let C_2 be the image of C_1 in T. let A and B be the center of circles C_1 and C_2 respectively, and M and N be respectively the feet of perpendiculars drawn From A and B on the x-axis. Then the area of trapezium $AMNB$ is:

(1) $2\left(2+\sqrt{2}\right)$ (2) $3+2\sqrt{2}$ (3) $2\left(1+\sqrt{2}\right)$ (4) $4\left(1+\sqrt{2}\right)$

12. If the domain of the function $f(x) = \dfrac{[x]}{1+x^2}$, where $[x]$ is greatest integer $\leq x$, is $[2,6)$, then the range is:

(1) $\left(\dfrac{5}{37}, \dfrac{2}{5}\right] - \left\{\dfrac{9}{29}, \dfrac{27}{109}, \dfrac{18}{89}, \dfrac{9}{53}\right\}$ (2) $\left(\dfrac{5}{26}, \dfrac{2}{5}\right] - \left\{\dfrac{9}{29}, \dfrac{27}{109}, \dfrac{18}{89}, \dfrac{9}{53}\right\}$

(3) $\left(\dfrac{5}{37}, \dfrac{2}{5}\right]$ (4) $\left(\dfrac{5}{26}, \dfrac{2}{5}\right]$

13. If the maximum distance of normal to the ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{b^2} = 1, b < 2,$ from the origin is 1, then the eccentricity of the ellipse is:

(1) $\dfrac{1}{2}$ (2) $\dfrac{\sqrt{3}}{2}$

(3) $\dfrac{\sqrt{3}}{4}$ (4) $\dfrac{1}{\sqrt{2}}$

14. Let $y = f(x) = \sin^3\left(\dfrac{\pi}{3}\left(\cos\left(\dfrac{\pi}{3\sqrt{2}}\left(-4x^3 + 5x^2 + 1\right)^{\frac{3}{2}}\right)\right)\right)$. Then, at $x = 1$.

(1) $\sqrt{2}y' - 3\pi^2 y = 0$

(2) $2y' + 3\pi^2 y = 0$

(3) $2y' + \sqrt{3}\pi^2 y = 0$

(4) $y' + 3\pi^2 y = 0$

15. The value of $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{(2+3\sin x)}{\sin x(1+\cos x)}\,dx$ is equal to:

(1) $\dfrac{10}{3} - \sqrt{3} + \log_e \sqrt{3}$ 　　　　　　　 **(2)** $\dfrac{7}{2} - \sqrt{3} - \log_e \sqrt{3}$

(3) $\dfrac{10}{3} - \sqrt{3} - \log_e \sqrt{3}$ 　　　　　　　 **(4)** $-2 + 3\sqrt{3} + \log_e \sqrt{3}$

16. Let R be a relation on $N \times B$ defined by $(a,b)\,R\,(c,d)$ if and only if $ad(b-c) = bc(a-d)$. Then R is:
   (1) Symmetric and transitive but not reflexive
   (2) Symmetric but neither reflexive not transitive
   (3) Reflexive and symmetric but not transitive
   (4) Transitive but neither reflexive not symmetric

17. The number of real roots of the equation $\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}$, is:
   (1) 0 　　　　 **(2)** 1 　　　　 **(3)** 3 　　　　 **(4)** 2

18. For the system of linear equations
$$x + y + z = 6$$
$$\alpha x + \beta y + 7z = 3$$
$$x + 2y + 3x = 14,$$
   Which of the following is not true?
   (1) If $\alpha = \beta = 7$, then the system has no solution
   (2) If $\alpha = \beta$ and $\alpha \neq 7$, then the system has a unique solution
   (3) There is a unique point (α, β) on the line $x + 2y + 18 = 0$ for which the system has infinitely many solutions
   (4) For every point $(\alpha, \beta) \neq (7,7)$ on the line $x - 2y + 7 = 0$, the system has infinitely many solutions

19. Let $y = f(x)$ represent a parabola with focus $\left(-\dfrac{1}{2}, 0\right)$ and directrix $y = -\dfrac{1}{2}$. Then

$$S = \left\{ x \in R : \tan^{-1}\left(\sqrt{f(x)}\right) + \sin^{-1}\left(\sqrt{f(x)+1}\right) = \frac{\pi}{2} \right\} :$$

   (1) is an infinite set 　　　　　　　 **(2)** is an empty set
   (3) contains exactly two elements 　　　 **(4)** contains exactly one element

20. Let a differentiable function f satisfy $f(x) + \displaystyle\int_{3}^{x} \frac{f(t)}{t}\,dt = \sqrt{x+1}, x \geq 3$. Then $12 f(8)$ is equal to:

   (1) 19 　　　　 **(2)** 17 　　　　 **(3)** 34 　　　　 **(4)** 1

SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be **rounded off to the nearest integer**.

21. The remainder on dividing 5^{99} by 11 is_____.

22. Let $a > 0$, be the smallest number such that the expansion of $\left(x^{\frac{2}{3}} + \frac{2}{x^3} \right)^{30}$ has a term $\beta x^{-a}, \beta \in N$.

Then a is equal to_____.

23. Let 5 digit numbers be constructed using the digits 0, 2, 3, 4, 7, 9 with repetition allowed, and are arranged in ascending order with serial numbers. Then the serial number of the number 42923 is_____.

24. Number of 4-digit numbers that are less than or equal to 2800 and either divisible by 3 or by 11, is equal to_____.

25. Let for $x \in R$,

$$f(x) = \frac{x + |x|}{2} \text{ and } g(x) = \begin{cases} x, & x < 0 \\ x^2, & x \geq 0 \end{cases}.$$

Then area bounded by the curve $y = (fog)(x)$ and the lines $y = 0, 2y - x = 15$ is equal to_____.

26. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}| = \sqrt{14}, |\vec{b}| = \sqrt{6}$ and $|\vec{a} \times \vec{b}| = \sqrt{48}$. Then $(\vec{a} \cdot \vec{b})^2$ is equal to_____.

27. Let $a_1, a_2, ..., a_n$ be in A.P. if $a_5 = 2a_7$ and $a_{11} = 18$, then

$$12 \left(\frac{1}{\sqrt{a_{10}} + \sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}} + \sqrt{a_{12}}} + ... + \frac{1}{\sqrt{a_{17}} + \sqrt{a_{18}}} \right) \text{ is equal to}\_\_\_\_\_.$$

28. If the variance of the frequency distribution

x_i	2	3	4	5	6	7	8
Frequency f_i	3	6	16	α	9	5	6

Is 3, then α is equal to_____.

29. Let θ be the angle between the planes $P_1 : \vec{r} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 9$ and $P_2 : \vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 15$. Let L be the line that meets P_2 at the point $(4, -2, 5)$ and makes an angle θ with the normal of P_2. If α is the angle between L and P_2, then $(\tan^2 \theta)(\cot^2 \alpha)$ is equal to_____.

30. Let the line $L: \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1}$ intersect the plane $2x + y + 3z = 16$ at the point P. let the point Q be the foot of perpendicular from the point $R(1, -1, -3)$ on the line L. if α is the area of triangle PQR, then α^2 is equal to_____.