JEE Main – 2023

30th JAN 2023 (Evening Shift)

General Instructions

1. The test is of **3 hours** duration and the maximum marks is **300.**
2. The question paper consists of **3 Parts** (Part I: **Physics**, Part II: **Chemistry**, Part III: **Mathematics**). Each Part has **two** sections (Section 1 & Section 2).
3. **Section 1** contains **20 Multiple Choice Questions**. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE CHOICE** is correct.
4. **Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. You will NOT be allowed to attempt the sixth question. If you wish to attempt any other question apart from the five already attempted, then you will have to delete any one response from the five previously answered and then proceed to answer the new one.**
   The answer to each question should be **rounded off to the nearest integer**.
5. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. inside the examination room/hall.

Marking Scheme

1. **Section – 1:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.
2. **Section – 2:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.

SUBJECT I: PHYSICS	MARKS: 100

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. A thin prism P_1 with an angle 6º and made of glass of refractive index 1.54 is combined with another prism P_2 made from glass of refractive index 1.72 to produce dispersion without average deviation. The angle of prism P_2 is :

(1) 4.5º **(2)** 1.3º **(3)** 7.8º **(4)** 6º

2. A point source of 100 W emits light with 5% efficiency. At a distance of 5 m from the source, the intensity produced by the electric field component is :

(1) $\frac{1}{10\pi}\frac{W}{m^2}$ **(2)** $\frac{1}{2\pi}\frac{W}{m^2}$ **(3)** $\frac{1}{20\pi}\frac{W}{m^2}$ **(4)** $\frac{1}{40\pi}\frac{W}{m^2}$

3. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : The nuclear density of nuclides ${}^{10}_{5}B$, ${}^{6}_{3}Li$, ${}^{56}_{26}Fe$, ${}^{20}_{10}Ne$ and ${}^{209}_{83}Bi$ can be arranged as $\rho^N_{Bi} > \rho^N_{Fe} > \rho^N_{Ne} > \rho^N_{Li}$

Reason R : The radius R nucleus is related to its mass number A as $R = R_0 A^{1/3}$, where R_0 is a constant.

In the light of the above statements, choose the correct answer from the options given below

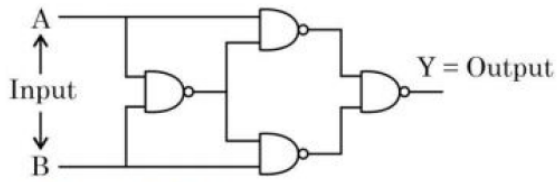
(1) Both A and R are true and R is the correct explanation of A

(2) A is true but R is false

(3) Both A and R are true but R is NOT the correct explanation of A

(4) A is false but R is true

4. The output Y for the inputs A and B of circuit is given by :



Truth table of the shown circuit is :

(1)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(2)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

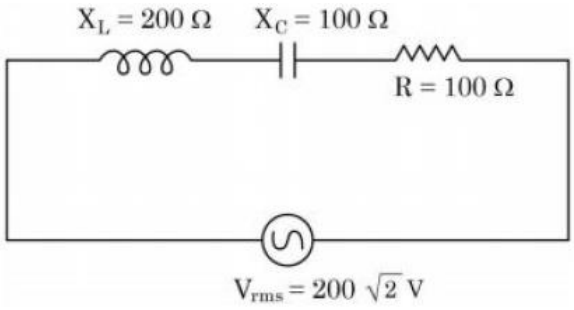
(3)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(4)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

5. In the given circuit, rms value of current $\left(I_{rms}\right)$ through the resistor R is :



(1) 20 A **(2)** 2A **(3)** $\frac{1}{2}A$ **(4)** $2\sqrt{2}A$

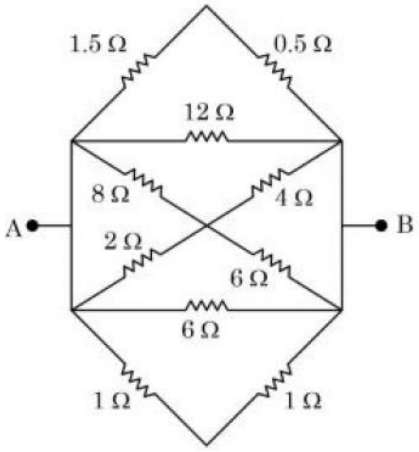
6. An object is allowed to fall from a height R above the earth, where R is the radius of earth. Its velocity when it strikes the earth's surface, ignoring air resistance, will be :

(1) $\sqrt{\frac{gR}{2}}$ **(2)** $2\sqrt{gR}$ **(3)** $\sqrt{2gR}$ **(4)** $\sqrt{gR}$

7. A machine gun of mass 10 kg fires 20 g bullets at the rate of 180 bullets per minute with a speed of $100 ms^{-1}$ each. The recoil velocity of the gun is :

(1) 2.5 *m/s* **(2)** 1.5 *m/s* **(3)** 0.6 *m/s* **(4)** 0.02 *m/s*

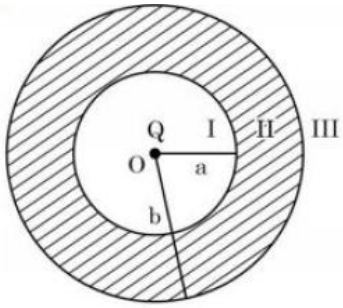
8. The equivalent resistance between A and B is _____.



(1) $\frac{1}{2}\Omega$ **(2)** $\frac{3}{2}\Omega$ **(3)** $\frac{2}{3}\Omega$ **(4)** $\frac{1}{3}\Omega$

9. As shown in the figure, a point charge Q is placed at the centre of conducting spherical shell of inner radius *a* and outer *b*. The electric field due to charge Q in three different regions I, II and III is given by:

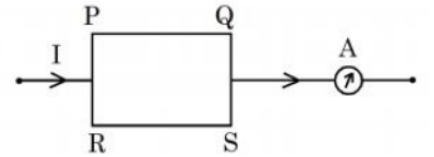
I. $r < a$ **II.** $a < r < b$ **III.** $r > b$



(1) $E_I = 0, E_{II} = 0, E_{III} \neq 0$ **(2)** $E_I = 0, E_{II} = 0, E_{III} = 0$

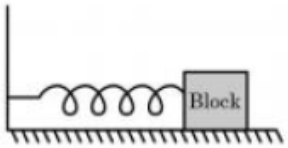
(3) $E_I \neq 0, E_{II} = 0, E_{III} = 0$ **(4)** $E_I \neq 0, E_{II} = 0, E_{III} \neq 0$

10. A current carrying rectangular loop PQRS is made of uniform wire. The length PR = QS = 5 cm and PQ = RS = 100 cm. If ammeter current reading changes from I to 2I, the ratio of magnetic forces per unit length on the wire PQ due to wire RS in the two cases respectively $\left(f_{PQ}^{I} : f_{PQ}^{2I}\right)$ is :



(1) 1 : 3 **(2)** 1 : 2 **(3)** 1 : 4 **(4)** 1 : 5

11. For a simple harmonic motion in a mass spring shown, the surface is frictionless. When the mass of the block is 1 *kg*, the angular frequency is ω_1. When the mass block is 2 *kg* the angular frequency is ω_2. The ratio $\frac{\omega_1}{\omega_2}$ is :



(1) $\frac{1}{\sqrt{2}}$ **(2)** 2 **(3)** $\sqrt{2}$ **(4)** $\frac{1}{2}$

12. An electron accelerated through a potential difference V_1 has a de-Broglie wavelength of λ. When the potential is changed to V_2, its de-Broglie wavelength increases by 50%. The value of $\left(\frac{V_1}{V_2}\right)$ is equal to:

(1) $\frac{9}{4}$ **(2)** 3 **(3)** 4 **(4)** $\frac{3}{2}$

13. Other is labelled as Reason R

Assertion A : Efficiency of a reversible heat engine will be highest at $-273°C$ temperature of cold reservoir.

Reason R : The efficiency of Carnot's engine depends not only one temperature of cold reservoir but it depends on the temperature of hot reservoir too and is given as $n = \left(1 - \frac{T_2}{T_1}\right)$.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both A and R are true and R is the correct explanation of A

(2) A is true but R is false

(3) Both A and R are true but R is NOT the correct explanation of A

(4) A is false but R is true

14. A vehicle travels 4 *km* with speed of 3 *km*/*h* and another 4 *km* with speed of 5 *km*/*h*, then its average speed is :

(1) 3.50 *km*/*h* **(2)** 4.25 *km*/*h* **(3)** 4.00 *km*/*h* **(4)** 3.75 *km*/*h*

15. A flask contains hydrogen and oxygen in the ratio of 2 : 1 by mass at temperature $27°C$. The ratio of average kinetic energy per molecule of hydrogen and oxygen respectively is :

(1) 1 : 1 **(2)** 4 : 1 **(3)** 1 : 4 **(4)** 2 : 1

16. Match List I and List II :

	List I		List II
(A)	Torque	**I.**	$kg\,m^{-1}s^{-2}$
(B)	Energy density	**II.**	$kg\,m^{-1}$
(C)	Pressure gradient	**III.**	$kg\,m^{-2}s^{-2}$
(D)	Impulse	**IV.**	$kg\,m^{2}s^{-2}$

Choose the correct answer from the options given below :

Options :

(1) A-IV, B-III, C-I, D-II **(2)** A-IV, B-I, C-III, D-II

(3) A-IV, B-I, C-II, D-III **(4)** A-I, B-IV, C-III, D-II

17. Match List I and List II :

	List I		List II
(A)	Attenuation	**I.**	Combination of a receiver and transmitter.
(B)	Transducer	**II.**	Process of retrieval of information from the carrier wave at receiver
(C)	Demodulation	**III.**	Converts one form of energy into another
(D)	Repeater	**IV.**	Loss of strength of a signal while propogating through a medium

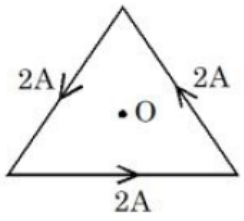
Choose the correct answer from the options given below :

Options :

(1) A-IV, B-III, C-I, D-II **(2)** A-IV, B-III, C-II, D-I

(3) A-II, B-III, C-IV, D-I **(4)** A-I, B-II, C-III, D-IV

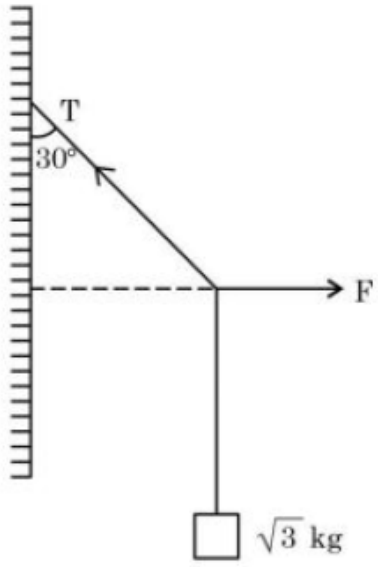
18. As shown in the figure, a current of 2A flowing in an equilateral triangle of side $4\sqrt{3}\,cm$. The magnetic field at the centroid O of the triangle is :



(Neglect the effect of earth's magnetic field)

(1) $\sqrt{3}\times10^{-4}T$ **(2)** $4\sqrt{3}\times10^{-4}T$ **(3)** $3\sqrt{3}\times10^{-5}T$ **(4)** $4\sqrt{3}\times10^{-5}T$

19. A block of $\sqrt{3}kg$ is attached to a string whose other end is attached to the wall. An unknown force F is applied so that the string makes an angle of $30°$ with the wall. The tension T is : (Given $g = 10ms^{-2}$)



(1) 15 N **(2)** 25 N **(3)** 10 N **(4)** 20 N

20. A force is applied to a steel wire 'A', rigidly clamped at one end. As a result elongation in the wire is 0.2 mm. If same force is applied to another steel wire 'B' of double the length and diameter 2.4 times that of the wire 'A', the elongation in the wire 'B' will be (wire having uniform circular cross sections)

(1) $3.0\times10^{-2}mm$ **(2)** $6.9\times10^{-2}mm$

(3) $2.77\times10^{-2}mm$ **(4)** $6.06\times10^{-2}mm$

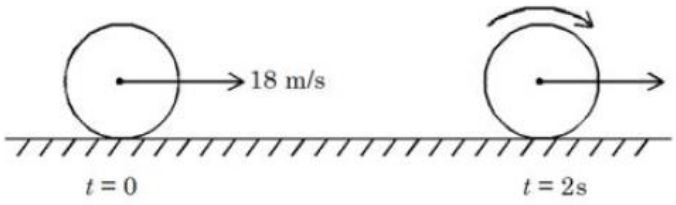
SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be **rounded off to the nearest integer**.

21. A body of mass 2 *kg* is initially at rest. It starts moving unidirectionally under the influence of a source of constant power *P*. Its displacement in 4*s* is $\frac{1}{3}\alpha^2\sqrt{P}m$. The value of α will be _____.

22. The velocity of a particular executing SHM varies with displacement (*x*) as $4v^2 = 50 - x^2$. The time period of oscillations is $\frac{x}{7}s$. The value of *x* is _____. $\left(\text{Take } \pi = \frac{22}{7}\right)$

23. A uniform disc of mass 0.5 *kg* and radius *r* is projected with velocity 18 *m*/*s* at *t* = 0*s* on a rough horizontal surface. It starts off with a purely sliding motion at *t* = 0*s*. After 2*s* it acquires a purely rolling motion (see figure). The total kinetic energy of the disc after 2s will be _____ J (Given, coefficient of friction is 0.3 and $g = 10 m/s^2$).



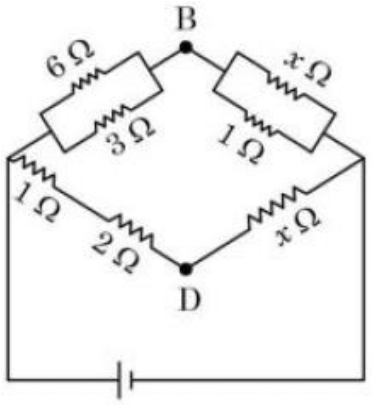
24. A stone tied to 180 cm long string at its end is making 28 revolutions in horizontal circle in every minute. The magnitude of acceleration of stone is $\frac{1936}{x}ms^{-2}$. The value of *x* _____. $\left(\text{Take } \pi = \frac{22}{7}\right)$

25. In a Young's double slit experiment, the intensities at two points, for the path differences $\frac{\lambda}{4}$ and $\frac{\lambda}{3}$ (λ being the wavelength of light used) are I_1 and I_2 respectively. If I_0 denotes intensity produced by each one of the individual slits, then $\frac{I_1 + I_2}{I_0} =$ _____.

26. A faulty thermometer reads $5°C$ in melting ice and $95°C$ in stream. The correct temperature on absolute scale will be _____ K when the faulty thermometer reads $41°C$.

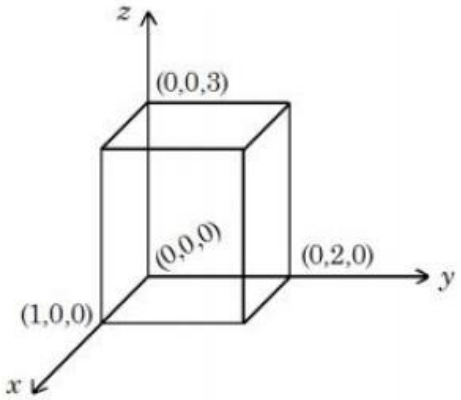
27. In an ac generator, a rectangular coil of 100 turns each having area $14 \times 10^{-2} m^2$ is rotated at 360 rev/min about an axis perpendicular to a uniform magnetic field of magnitude 3.0 T. The maximum value of the emf produced will be _____ V. $\left(\text{Take } \pi = \frac{22}{7}\right)$

28. If the potential difference between B and D is zero, the value of x is $\frac{1}{n}\Omega$. The value of n is _____.



29. A radioactive nucleus decays by two different process. The half life of the first process is 5 minutes and that of the second process is 30 s. The effective half-life of the nucleus is calculated to be $\frac{\alpha}{11}s$. The value of α is _____.

30. As shown in figure, a cuboid lies in a region with electric field $E = 2x^2\hat{i} - 4y\hat{j} + 6\hat{k}\frac{N}{C}$. The magnitude of charge within the cuboid is $n \in_0 C$. The value of n is _____. (If dimension of cuboid is $1\times 2\times 3m^3$)

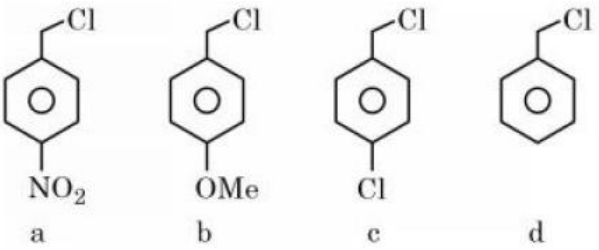


SUBJECT II: CHEMISTRY	MARKS: 100

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. Decreasing order towards S_N1 reaction for the following compounds is :



(1) b > d > c > a **(2)** a > b > c > d **(3)** d > b > c > a **(4)** a > c > d > b

2. Chlorides of which metal are soluble in organic solvents :

(1) Mg **(2)** Ca **(3)** K **(4)** Be

3. Which of the following reaction is correct ?

(1) $2LiNO_3 \longrightarrow 2Li + 2NO_2 + O_3$

(2) $4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 2N_2O_4 + O_2$

(3) $4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 4NO_2 + O_2$

(4) $4LiNO_3 \xrightarrow{\Delta} 2NaNO_2 + O_2$

4. Bond dissociation energy of "E-H" bond of the "H_2E" hydrides of group 16 elements (given below), follows order.

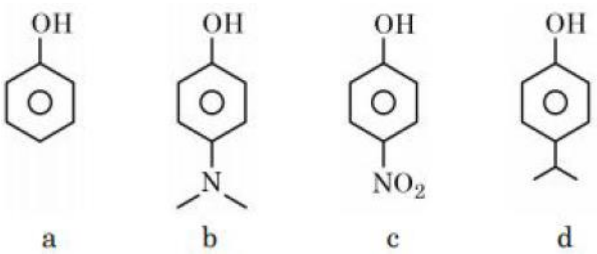
A. O B. S C. Se D. Te

(1) A > B > C > D **(2)** D > C > B > A **(3)** A > B > D > C **(4)** B > A > C > D

5. Formulae for Nessler's reagent is :

(1) $KHgI_3$ **(2)** HgI_2 **(3)** KHg_2I_2 **(4)** K_2HgI_4

6. The correct order of pK_a values for the following compounds is :



(1) b > a > d > a **(2)** b > d > a > c **(3)** a > b > c > d **(4)** c > a > d > b

7. The wave function (ψ) of 2s is given by :

$$\psi_{2s}\frac{1}{2\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{1/2}\left(2-\frac{r}{a_0}\right)e^{-r/2a_0}$$

At $r = r_0$, radial node is formed. Thus, r_0 in terms of a_0

(1) $r_0 = 4a_0$ **(2)** $r_0 = 2a_0$ **(3)** $r_0 = \frac{a_0}{2}$ **(4)** $r_0 = a_0$

8. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Antihistamines do not effect the secretion of acid in stomach.

Reason R : Antiallergic and antacid drugs work on different receptors.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both A and R are true and R is the correct explanation of A

(2) A is false but R is true

(3) A is true but R is false

(4) Both A and R are true but R is not the correct explanation of A

9. $KMnO_4$ oxidises I^- in acidic and neutral/faintly alkaline solution, respectively, to :

(1) IO_3^- and I_2 **(2)** I_2 and IO_3^- **(3)** IO_3^- and IO_3^- **(4)** I_2 and I_2

10. The $Cl-Co-Cl$ bond angle values in a $fac-[Co(NH_3)_3Cl_3]$ complex is(are) :

(1) $90°$ **(2)** $180°$ **(3)** $90°$ and $120°$ **(4)** $90°$ and $180°$

11. Match List I and List II :

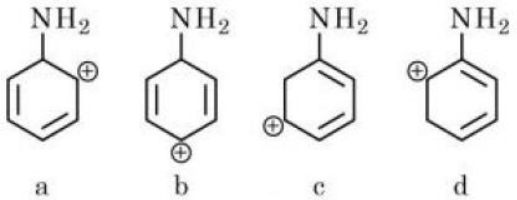
	List I (Mixture)		List II (Separation Technique)
(A)	$CHCl_3 + C_6H_5NH_2$	**I.**	Steam distillation
(B)	$C_6H_{14} + C_5H_{12}$	**II.**	Differential extraction
(C)	$C_6H_5NH_2 + H_2O$	**III.**	Distillation
(D)	Organic compound in H_2O	**IV.**	Fractional distillation

Options :

(1) A-IV, B-I, C-III, D-II **(2)** A-III, B-I, C-IV, D-II

(3) A-III, B-I, C-III, D-IV **(4)** A-III, B-IV, C-I, D-II

12. The most stable carbocation for the following is :



(1) d **(2)** b **(3)** c **(4)** a

13. Maximum number of electrons that can be accommodated in shell with n = 4 are :

(1) 72 **(2)** 16 **(3)** 50 **(4)** 32

14. Match List I and List II :

	List I (Complexes)		List II (Hybridisation)
(A)	$[Ni(CO)_4]$	**I.**	sp^3
(B)	$[Cu(NH_3)_4]^{2+}$	**II.**	dsp^2
(C)	$[Fe(NH_3)_6]^{2+}$	**III.**	sp^3d^2
(D)	$[Fe(H_2O)_6]^{2+}$	**IV.**	d^2sp^3

Options :

(1) A-II, B-I, C-IV, D-III **(2)** A-I, B-II, C-III, D-IV

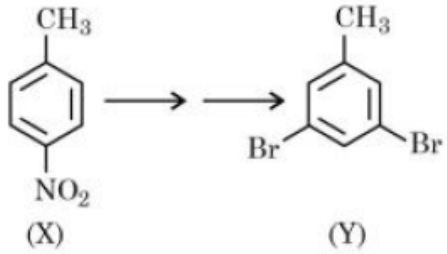
(3) A-II, B-I, C-III, D-IV **(4)** A-I, B-II, C-IV, D-III

15. The water quality of a pond was analysed and its BOD was found to be 4. The pond has :

(1) Very clean water **(2)** Slightly polluted water

(3) Highly polluted water **(4)** Water has high amount of fluoride compounds

16.



(X) (Y)

In the above conversion of compound (X) to product (Y), the sequence of reagents to be used will be :

(1) (i) Fe, H^+ (ii) $Br_2(aq)$ (iii) HNO_2 (iv) H_3PO_2

(2) (i) Br_2, Fe (ii) Fe, H^+ (iii) $LiAlH_4$

(3) (i) Fe, H^+ (ii) $Br_2(aq)$ (iii) HNO_2 (iv) CuBr

(4) (i) $Br_2(aq)$ (ii) $LiAlH_4$ (iii) H_3O^+

17. Boric acid is solid, whereas BF_3 is gas at room temperature because of :

(1) Strong ionic bond in Boric acid

(2) Strong covalent bond in BF_3

(3) Strong hydrogen bond in Boric acid

(4) Strong van der Waal's interaction in Boric acid

18. 1 L, 0.02 M solution of $[Co(NH_3)_5SO_4]Br$ is mixed with 1 L, 0.02 M solution of $[Co(NH_3)_5Br]SO_4$. The resulting solution is divided into two equal parts (X) and treated with excess of $AgNO_3$ solution and $BaCl_2$ solution respectively as shown below :

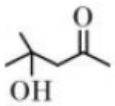
1 L solution (X) + $AgNO_3$ solution (excess) $\rightarrow$ Y

1 L solution (X) + $BaCl_2$ solution (excess) $\rightarrow$ Z

(1) 0.02, 0.01 **(2)** 0.01, 0.02

(3) 0.02, 0.02 **(4)** 0.01, 0.01

19. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A :  can be easily reduced using Zn-Hg/HCl to 

Reason R : Zn-Hg/HCl is used to reduce carbonyl group to $-CH_2-$ group.

In the light of the above statements, choose the correct answer from the options given below:

(1) A is false but R is true

(2) A is true but R is false

(3) Both A and R are true but R is not the correct explanation of A

(4) Both A and R are true and R is the correct explanation of A

20. Given below are two statements :

Statement I : During Electrolytic refining, the pure metal is made to act as anode and its impure metallic form is used as cathode.

Statement II : During the Hall-Heroult electrolysis process, purified Al_2O_3 is mixed with Na_3AlF_6 to lower the melting point of the mixture.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both Statement I and Statement I are incorrect

(2) Statement I is correct but Statement I is incorrect

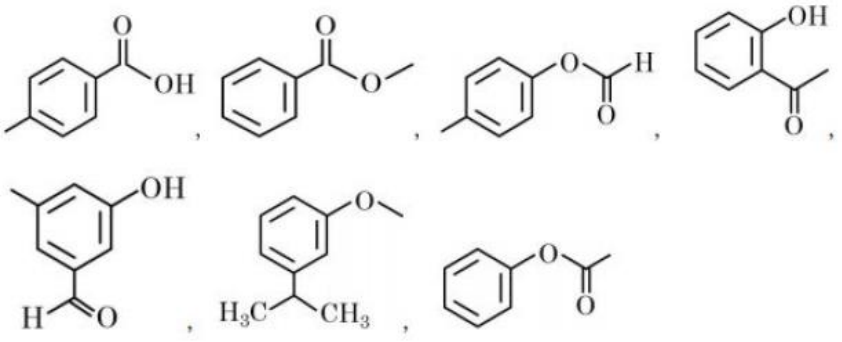
(3) Both Statement I and Statement II ae correct

(4) Statement I is incorrect but Statement II is correct

SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be **rounded off to the nearest integer**.

21. Number of compounds from the following which will not dissolve in cold $NaHCO_3$ and NaOH solutions but will dissolve in hot NaOH solution is _____.



22. 1 mole of ideal gas is allowed to expand reversibly and adiabatically from a temperature of $27°C$. The work done is $3kJ\,mol^{-1}$. The final temperature of the gas is _____ K (Nearest integer). (Given $C_V = 20\,J\,mol^{-1}K^{-1}$)

23. The electrode potential of the following half cell at 298 K $X|X^{2+}(0.001M)||Y^{2+}(0.01M)|Y$ is_____ $\times 10^{-2}V$ (Nearest integer).

Given : $E^{\circ}_{X^{2+}|X} = -2.36\,V$

$E^{\circ}_{Y^{2+}|Y} = +0.36\,V$

$\frac{2.303RT}{F} = 0.06\,V$

24. Iron oxide FeO, crystallises in a cubic lattice with a unit cell edge length of $5.0\,Å$. If density of the FeO in the crystal is $4.0\,g\,cm^{-3}$, then the number of FeO units present per unit cell is _____. (Nearest integer) (Given : Molar mass of Fe and O is 56 and $16\,g\,mol^{-1}$ respectively)

25. The strength of 50 volume solution of hydrogen peroxide is _____ g/L. (Nearest integer)

Given : Molar mass of H_2O_2 is $34\,g\,mol^{-1}$

Molar volume of gas at STP = 22.7 L.

26. A short peptide on complete hydrolysis produces 3 moles of glycine (G), two moles of leucine (L) and two moles of valine (V) per mole of peptide. The number of peptide linkages in it are _____.

27. Lead storage battery contains 38% by weight solution of H_2SO_4. The van't Hoff factor is 2.67 at this concentration. The temperature is Kelvin at which the solution in the battery will freeze is _____. (Nearest integer). (Given $K_f = 1.8\,K\,kg\,mol^{-1}$)

28. An organic compound undergoes first order decomposition. If the time taken for the 60% decomposition is 540 s, then the time required for 90% decomposition will be is _____ s. (Nearest integer). (Given : ln 10 = 2.3 ; log 2 = 0.3)

29. The graph of $\log\frac{x}{m}$ vs log p for an adsorption process is a straight line inclined at an angle of $45°$ with intercept equal to 0.6020. The mass of gas absorbed per unit mass of absorbent at the pressure of 0.4 atm is _____ $\times 10^{-1}$ (Nearest integer). Given : log 2 = 0.3010

30. Consider the following equation :

The number of factors which will increase the yield of SO_3 at equilibrium from the following is _____.

A. Increasing temperature

B. Increasing pressure

C. Adding more SO_2

D. Adding more O_2

E. Addition of catalyst

SUBJECT III: MATHEMATICS	MARKS: 100

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. If the functions $f(x)=\frac{x^3}{3}+2bx+\frac{ax^2}{2}$ and $g(x)=\frac{x^3}{3}+ax+bx^2$, $a\neq 2b$ have a common extreme point, then $a+2b+7$ is equal to :

(1) 4 **(2)** 3 **(3)** $\frac{3}{2}$ **(4)** 6

2. The solution of the differential equation $\frac{dy}{dx}=-\left(\frac{x^2+3y^2}{3x^2+y^2}\right)$, $y(1)=0$ is :

(1) $log_e|x+y|-\frac{2xy}{(x+y)^2}=0$ **(2)** $log_e|x+y|+\frac{2xy}{(x+y)^2}=0$

(3) $log_e|x+y|+\frac{xy}{(x+y)^2}=0$ **(4)** $log_e|x+y|-\frac{xy}{(x+y)^2}=0$

3. A vector $\vec{v}$ in the first octant is inclined to the *x*-axis at $60°$, to the *y*-axis at $45°$ and to the *z*-axis at an acute angle. If a plane passing through the points $\left(\sqrt{2},-1,1\right)$ and (*a*, *b*, *c*), is normal to $\vec{v}$, then :

(1) $\sqrt{2}a-b+c=1$ **(2)** $\sqrt{2}a+b+c=1$

(3) $a+b+\sqrt{2}c=1$ **(4)** $a+\sqrt{2}b+c=1$

4. Let $\vec{a}$ and $\vec{c}$ be two vectors, let $|\vec{a}|=1, |\vec{b}|=4$ and $\vec{a}\cdot\vec{b}=2$. If $\vec{c}=\left(2\vec{a}\times\vec{b}\right)-3\vec{b}$, then the value of $\vec{b}\cdot\vec{c}$ is :

(1) –48 **(2)** –84 **(3)** –60 **(4)** –24

5. If *P* is a 3×3 real matrix such that $P^T=aP+(a-1)I$, where $a>1$, then :

(1) $|Adj\,P|=\frac{1}{2}$ **(2)** *P* is a singular matrix

(3) $|Adj\,P|>1$ **(4)** $|Adj\,P|=1$

6. Let *f*, *g* and *h* be the real valued functions defined on *IR* as :

$$f(x)=\begin{cases}\frac{x}{|x|}, & x\neq 0\\ 1, & x=0\end{cases},\quad g(x)=\begin{cases}\frac{sin(x+1)}{(x+1)}, & x\neq -1\\ 1, & x=-1\end{cases}$$

and $h(x)=2[x]-f(x)$, where [*x*] is the greatest integer $\leq x$.

Then the value of $\lim\limits_{x\to 1} g\left(h(x-1)\right)$ is :

(1) 0 **(2)** 1 **(3)** –1 **(4)** *sin* (1)

7. The parabolas : $ax^2+2bx+cy=0$ and $dx^2+2ex+fy=0$ intersect on the line $y = 1$. If a, b, c, d, e, f are positive real numbers and a, b, c are in G.P., then :

(1) d, e, f are in G.P. **(2)** d, e, f are in A.P.

(3) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P. **(4)** $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

8. Let $a_1=1, a_2, a_3, a_4, \ldots.$ be consecutive natural numbers.

Then $tan^{-1}\left(\frac{1}{1+a_1a_2}\right)+tan^{-1}\left(\frac{1}{1+a_2a_3}\right)+\ldots.+tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$ is equal to :

(1) $\frac{\pi}{4}-tan^{-1}(2022)$ **(2)** $cot^{-1}(2022)-\frac{\pi}{4}$

(3) $\frac{\pi}{4}-cot^{-1}(2022)$ **(4)** $tan^{-1}(2022)-\frac{\pi}{4}$

9. $\lim\limits_{n\to\infty}\frac{3}{n}\left\{4+\left(2+\frac{1}{n}\right)^2+\left(2+\frac{2}{n}\right)^2+\ldots+\left(3-\frac{1}{n}\right)^2\right\}$ is equal to :

(1) 12 **(2)** $\frac{19}{3}$ **(3)** 0 **(4)** 19

10. Let $a, b, c>1, a^3, b^3$ and c^3 be in A.P., and $log_a b, log_e a$ and $log_c b$ be in G.P. If the sum of first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a-8b+c}{10}$ is -444, then abc is equal to :

(1) $\frac{343}{8}$ **(2)** 343 **(3)** $\frac{125}{8}$ **(4)** 216

11. The number of ways of selecting two numbers a and b, $a\in\{2, 4, 6, \ldots., 100\}$ and $b\in\{1, 3, 5, \ldots., 99\}$ such that 2 is the remainder when $a + b$ is divided by 23 is :

(1) 186 **(2)** 54 **(3)** 108 **(4)** 268

12. The range of the function $f(x)=\sqrt{3-x}+\sqrt{2+x}$ is :

(1) $\left[\sqrt{2}, \sqrt{7}\right]$ **(2)** $\left[\sqrt{5}, \sqrt{13}\right]$ **(3)** $\left[2\sqrt{2}, \sqrt{11}\right]$ **(4)** $\left[\sqrt{5}, \sqrt{10}\right]$

13. Let A be a point on the x-axis. Common tangents are drawn from A to the curves $x^2+y^2=8$ and $y^2=16x$. If one of these tangents touches the two curves at Q and R, then $(QR)^2$ is equal to :

(1) 72 **(2)** 81 **(3)** 64 **(4)** 76

14. Let $\lambda\in IR, \vec{a}=2\hat{j}-3\hat{k}, \vec{b}=\hat{i}-\lambda\hat{j}+2\hat{k}$.

If $\left(\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}\times\vec{b}\right)\right)\times\left(\vec{a}-\vec{b}\right)=8\hat{i}-40\hat{j}-24\hat{k}$, then $\left|\lambda\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)\right|^2$ is equal to :

(1) 136 **(2)** 132 **(3)** 144 **(4)** 140

15. If a plane passes through the points $(-1,k,0),(2,k,-1),(1,1,2)$ and is parallel to the line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$, then the value of $\frac{k^2+1}{(k-1)(k-2)}$ is :

(1) $\frac{6}{13}$ **(2)** $\frac{5}{17}$ **(3)** $\frac{17}{5}$ **(4)** $\frac{13}{6}$

16. Consider the following statements :

P : I have fever Q : I will not take medicine R : I will take rest

The statement "If I have fever, then I will take medicine and I will take rest" is equivalent to :

(1) $(P\vee\sim Q)\wedge(P\vee\sim R)$ **(2)** $((\sim P)\vee\sim Q)\wedge((\sim P)\vee R)$

(3) $(P\vee Q)\wedge((\sim P)\vee R)$ **(4)** $((\sim P)\vee\sim Q)\wedge((\sim P)\vee\sim R)$

17. Let q be the maximum integral values of p in [0, 10] for which the roots of the equation $x^2-px+\frac{5}{4}p=0$ are rational. Then the area of the region $\left\{(x,y):0\le y\le(x-q)^2,0\le x\le q\right\}$ is :

(1) 25 **(2)**164 **(3)** 243 **(4)** $\frac{125}{3}$

18. Let $x=\left(8\sqrt{3}+13\right)^{13}$ and $y=\left(7\sqrt{3}+9\right)^{9}$. If [$t$] denotes the greatest integer $\le t$, then :

(1) [x] + [y] is even **(2)** [x] is even but [y] is odd

(3) [x] and [y] are both odd **(4)** [x] is odd but [y] is even

19. For $\alpha,\beta\in IR$, suppose the system of linear equations

$$x-y+z=5$$
$$2x+2y+az=8$$
$$3x-y+4z=\beta$$

Has infinitely many solutions. Then α and β are the roots of :

(1) $x^2-10x+16=0$ **(2)** $x^2+18x+56=0$

(3) $x^2-18x+56=0$ **(4)** $x^2+14x+24=0$

20. Let S be the set of all values of a_1 for which the mean deviation about the mean of 100 consecutive positive integers $a_1,a_2,a_3,\ldots,a_{100}$ is 25. Then S is :

(1) {99} **(2)** ϕ **(3)** IN **(4)** {9}

SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be **rounded off to the nearest integer**.

21. The number of seven digits odd numbers, that can be formed using all the seven digits 1, 2, 2, 2, 3, 3, 5 is _____.

22. A bag contains six balls of different colours. Two balls are drawn in succession with replacement. The probability that both the balls are of the same colour is p. Next four balls are drawn in succession with replacement and the probability that exactly three balls of the same colour is q. If $p : q = m : n$, where m and n are coprime, then $m + n$ is equal to _____.

23. Let A be the area of the region $\left\{(x, y): y \geq x^2, y \geq (1-x)^2, y \leq 2x(1-x)\right\}$. Then 540 A is equal to _____.

24. 50^{th} root of a number x is 12 and 50^{th} root of another number y is 18. Then the remainder obtained on dividing $(x + y)$ by 25 is _____.

25. If $\int \sqrt{sec\,2x-1}\,dx = \alpha\, log_e \left| cos\,2x + \beta + \sqrt{cos\,2x\left(1+cos\,\frac{1}{\beta}x\right)}\right| + \text{constant}$, then $\beta-\alpha$ is equal to _____.

26. Let a line L pass through the point P(2, 3, 1) and be parallel to the line

$$x+3y-2z-2=0=x-y+2x$$

If the distance of L from the point (5, 3, 8) is α, then $3\alpha^2$ is equal to _____.

27. If the value of real number $a > 0$ for which $x^2-5ax+1=0$ and $x^2-ax-5=0$ have a common real root is $\frac{3}{\sqrt{2\beta}}$ then β is equal to _____.

28. The 8^{th} common term of the series

$$S_1 = 3+7+11+15+19+\ldots.,$$
$$S_2 = 1+6+11+16+21+\ldots.$$

is _____.

29. Let $P(a_1, b_1)$ and $Q(a_2, b_2)$ be two distinct points on a circle with center $C(\sqrt{2}, \sqrt{3})$. Let O be the origin and OC be perpendicular to both CP and CQ. If the area of the triangle OCP is $\frac{\sqrt{35}}{2}$, then $a_1^2 + a_2^2 + b_1^2 + b_2^2$ is equal to _____.

30. Let A = {1, 2, 3, 5, 8, 9}. Then the number of possible functions $f : A \to A$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in A$ with $m \cdot n \in A$ is equal to _____.