JEE Main – 2023

29th JAN 2023 (Evening Shift)

General Instructions

1. The test is of **3 hours** duration and the maximum marks is **300.**
2. The question paper consists of **3 Parts** (Part I: **Physics**, Part II: **Chemistry**, Part III: **Mathematics**). Each Part has **two** sections (Section 1 & Section 2).
3. **Section 1** contains **20 Multiple Choice Questions**. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE CHOICE** is correct.
4. **Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. You will NOT be allowed to attempt the sixth question. If you wish to attempt any other question apart from the five already attempted, then you will have to delete any one response from the five previously answered and then proceed to answer the new one.**
   The answer to each question should be **rounded off to the nearest integer**.
5. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. inside the examination room/hall.

Marking Scheme

1. **Section – 1:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.
2. **Section – 2:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.

SUBJECT I: PHYSICS	MARKS: 100

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. An object moves at a constant speed along a circular path in a horizontal plane with centre at the origin. When the object its velocity is $-4\hat{j}\,m/s$. The object's velocity (v) and acceleration (A) at $x=-2m$ will be:

(1) $v = 4\hat{i}m/s,\ a=8\hat{j}\,m/s^2$ **(2)** $v=-4\hat{j}\,m/s, a=8\hat{i}\,m/s^2$

(3) $v=4\hat{i}m/s,\ a=-8\hat{j}m/s^2$ **(4)** $v=4\hat{j}m/s, a=8\hat{i}m/s^2$

2. The ratio of de-Broglie wavelength of an α particle and a proton accelerated from rest by the same potential is $\frac{1}{\sqrt{m}}$, the value of m is:

(1) 2 **(2)** 4 **(3)** 16 **(4)** 8

3. The equation of a circle is given by $x^2+y^2=a^2$, where a is the radius. If the equation is modified to change the origin other than (0, 0), then find out the correct dimensions of A and B in a new equation: $(x-At)^2+\left(y-\frac{t}{B}\right)^2=a^2$. The dimension of t $[T^{-1}]$.

(1) $A=[LT], B=[L^{-1}T^{-1}]$ **(2)** $A=[L^{-1}T^{-1}], B=[LT]$

(3) $A=[L^{-1}T], B=LT^{-1}]$ **(4)** $A=[L^{-1}T^{-1}], B=[LT^{-1}]$

4. Identify the correct statements from the following:

(A) Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket is negative

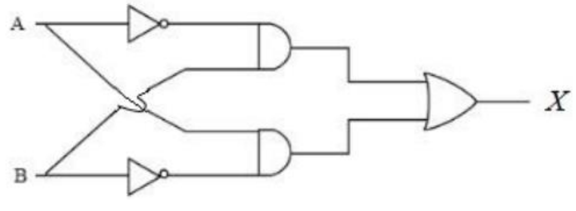
(B) Work done by gravitational force in lifting a bucket out of a well by a rope tied to the bucket is negative

(C) Work done by friction on a body sliding down an inclined plane is positive

(D) Work done by an applied force on a body moving on a rough horizontal plane

(1) B and D only **(2)** B and E only **(3)** A and C only **(4)** B, D and E only

5. For the given logic gates combination, the correct truth table will be:



(1)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

(2)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

(3)

A	B	X
0	0	1
0	1	0
1	0	1
1	1	0

(4)

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

6. A fully loaded boeing aircraft has a mass of $5.4\times10^5 kg$. Its total wing area is $500\,m^2$. It is in level flight with a speed of $1080\,km/h$. If the density of air ρ is $1.2\,kgm^{-3}$, the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface in percentage will be: $(g=10m/s^2)$

(1) 6 **(2)** 10 **(3)** 8 **(4)** 16

7. Heat energy of 184 kJ is given to i.e. of mass 600 g at $-12°C$. Specific heat of ice is $2222.3J\,kg^{-1}°C^{-1}$ and latent heat of ice in $336\,kJ/kg^{-1}$.

(A) Final temperature of system will be 0°C.
(B) Final temperature of the system will be greater than 0°C.
(C) The final system will have a mixture of ice and water in the ratio of 5:1.
(D) The final system will have a mixture of ice and water in the ratio of 1:5.

(1) B and D only **(2)** A and E only **(3)** A and D only **(4)** A and C only

8. The electric current in a circular coil of four turns produces a magnetic induction 32 RT at its centre. The coil is unwound and is rewound into a circular oil of single turn, the magnetic induction at the centre of the coil by the same current will be:

(1) 8T **(2)** 4T **(3)** 16T **(4)** 2T

9. A point charge $2\times10^{-2}C$ is moved from P to S in a uniform electric field of $30NC^{-1}$ directed along positive x-axis. If coordinates of P and S are (1, 2, 0)*m* and (0, 0, 0)*m* respectively, the work done by electric field will be:

(1) $-1200mJ$ **(2)** $1200mJ$ **(3)** $-600mJ$ **(4)** $600\,mJ$

10. The time taken by an object to down 45° rough inclined plane is *n* times as it takes to slide down a perfectly smooth $45°$ incline plane. The coefficient of kinetic friction between the object and the in line plane is:

(1) $1+\frac{1}{n^2}$ **(2)** $\sqrt{\frac{1}{1-n^2}}$ **(3)** $\sqrt{1-\frac{1}{n^2}}$ **(4)** $1-\frac{1}{n^2}$

11. A scientist is observing a bacteria through a compound microscope. For better analysis and to improve its resolving power he should. (Select the best option)

(1) Increase the refractive index of the medium between the object and objective lens
(2) Decrease the diameter of the objective lens
(3) Decrease the focal length of the eye piece
(4) Increase the wave length of the light

12. Given below are two statements:

Statement I : Electromagnetic waves are not defected by electric and magnetic field.

Statement II : The amplitude of electric field and the magnetic field in electromagnetic waves are released to each other as $E_0=\sqrt{\frac{\mu_0}{\varepsilon_0}}B_0$.

In the light of the above statement, choose the correct answer from options given below:

(1) Both Statement I and Statement II are true
(2) Statement I is false but statement II is true
(3) Statement I true but statement II is false
(4) Both Statement I and Statement II are false

13. Substance A has atomic mass number 16 and half life of 1 day. Another substance B has atomic mass number 32 and half life of $\frac{1}{2}$ day. If both A and B simultaneously start undergo radio activity at the same time with initial mass 320 g each, how many total atoms of A and B combined would be left after 2 days.

(1) 1.69×10^{24} **(2)** 3.38×10^{24} **(3)** 6.76×10^{23} **(4)** 6.76×10^{24}

14. A force acts for 20*s* on a body of mass 20 kg, starting from rest, after which the fore ceases and then body describes 50*m* in the next 10*s*. The value of force will be:

(1) 5N **(2)** 10 N **(3)** 40 N **(4)** 20 N

15. A square loop of area $25cm^2$ has a resistance of 10Ω. The loop is placed in uniform magnetic field of magnitude 40.0T. The plane of loop is perpendicular to the magnetic filed. The work done in pulling the loop out of the magnetic field slowly and uniformly in 1.0 sec, will be:

(1) $1.0 \times 10^{-3} J$ **(2)** $5 \times 10^{-3} J$ **(3)** $2.5 \times 10^{-3} J$ **(4)** $1.0 \times 10^{-4} J$

16. With the help potentiometer, we can determine the value of emf of given cell. The sensitivity of the potentiometer is:

(A) Directly proportional to the length of the potentiometer wire.
(B) Directly proportional to the potential gradient of the wire.
(C) Inversely proportional to the potential gradient of the wire.
(D) Inversely proportional to the length of the potentiometer wire.

Choose the correct option for the above statements

(1) A only **(2)** C only **(3)** A and C only **(4)** B and D only

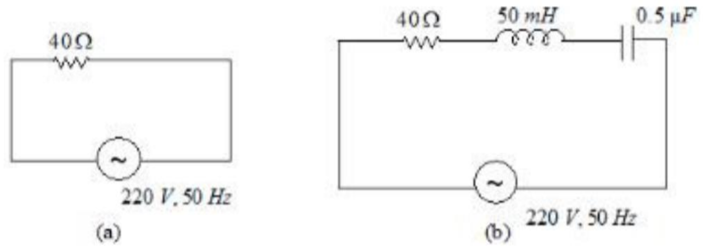
17. The time period of a satellite of earth is 24 hours. If the separation between the earth and the satellite is decreased to only fourth of the previous value, then its new time period will become.

(1) 6 hours **(2)** 12 hours **(3)** 3 hours **(4)** 4 hours

18. The modulation index for an A.M. wave having maximum and minimum peak-to-peak voltages of 14 mV and 6 mV respectively is:

(1) 0.4 **(2)** 0.2 **(3)** 1.4 **(4)** 0.6

19. For the given figures, choose the correct options:



(1) The rms current in circuit (b) can be larger than that in (a)
(2) At resonance, current in (b) is less than that in (a)
(3) The rms current in circuit (b) an never be larger than that in (a)
(4) The rms current in figure (a) is always equal to that in figure (b)

20. At 300 K, the rms speed of oxygen molecules is $\sqrt{\frac{\alpha+5}{\alpha}}$ times to that of its average speed in the gas. Then, the value of α will be:

(used $\pi = \frac{22}{7}$)

(1) 27 **(2)** 32 **(3)** 24 **(4)** 28

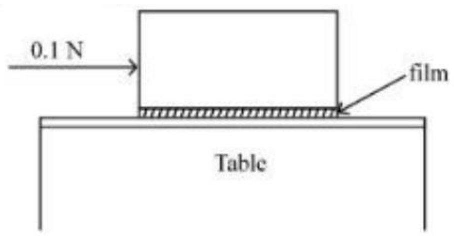
SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be **rounded off to the nearest integer**.

21. Unpolarised light is incident on the boundary between two dielectric media, whose dielectric constants are 2.8 (medium –1) and 6.8 (medium – 2), respectively. To satisfy the condition, so that the reflected and refracted rays are perpendicular to each other, the angle of incidence should be $\tan^{-1}\left(1+\frac{10}{\theta}\right)^{\frac{1}{2}}$ the value of θ is________.
(Given for dielectric media, $u_r = 1$)

22. An inductor of inductance $2\mu H$ is connected in series with a resistance, a variable capacitor and an AC source of frequency 7 kHz. The value of capacitance for which maximum current is drawn into the circuit is $\frac{1}{x}F$, where the value of *x* is________.
(Take $\pi\frac{22}{7}$)

23. A metal block of base area $0.20\,m^2$ is placed on a table, as shown in figure. A liquid film of thickness 0.22 *mm* is inserted between the block and the table. The block is pushed by a horizontal force of 0.1 N and moves with a constant speed. If the viscosity of the liquid is $5.0\times10^{-3}P1$, the speed of block is_______$\times10^{-3}m/s$

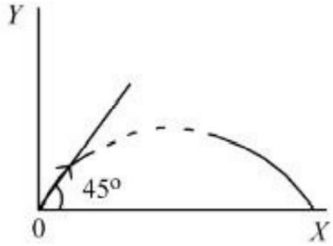


24. A car is moving on a circular path of radius 600 *m* such that the magnitudes of the tangential acceleration and centripetal acceleration are equal. The time taken by the car to complete first quarter of revolution, if it is moving with an initial speed of 54 *km/hr* $t(1-e^{-\pi/2})s$. The value of *t* is __________.

25. When two resistances R_1 and R_2 connected in series and introduction into the left gap of a mater bridge and a resistance of 10Ω is introduced into the right gap, a null point is found at 60 cm from left side. When R_1 and R_2 are connected in parallel and introduced into the left gap, a resistance of 3Ω is introduced into the right-gap to get null point at 40 cm from left end. The product of R_1R_2 is_____Ω^2.

26. A particle of mass 250 g executes a simple harmonic motion under a periodic force $F=(-25x)N$. The particle attains a maximum speed of 4 m/s during its oscillation. The amplitude of the motion is _______ cm.

27. A particle of mass 100 *g* is projected at time $t = 0$ with a speed $20\,ms^{-1}$ at an angle 45° to the horizontal as given in the figure. The magnitude of the angular momentum of the particle about the starting point at time $t = 2s$ is found to be $\sqrt{K}kg\,m^2/s$. The value of K is________.
(Take g = 10 ms^{-2})



28. In an experiment of measuring the refractive index of a glass slab using travelling microscope in physics lab, a student measures real thickness of the glass slab as 5.25 mm and apparent thickness of the glass slab as 5.00 mm. Travelling microscope has 20 divisions in one cm on main scale and 50 divisions on vernier scale is equal to 49 divisions on main scale. The estimated uncertainty in the measurement of refractive index of the slab is $\frac{x}{10}\times10^{-3}$, where *x* is __________.

29. For a charged spherical ball, electrostatic potential inside the ball varies with *r* as $V = 2ar^2 + b$.
Here, *a* and *b* are constant r is the distance from the centre. The volume charge density inside the ball is $-\lambda a\varepsilon$. The value of λ is _____________.
$\varepsilon =$ permittivity of the medium.

30. A null point is found at 200 cm is potentiometer when cell in secondary circuit is shunted by 5Ω. When a resistance of 15Ω is used for shunting, null point moves to 300 cm. The internal resistance of the cell is _________Ω.

SUBJECT II: CHEMISTRY	MARKS: 100

SECTION-1

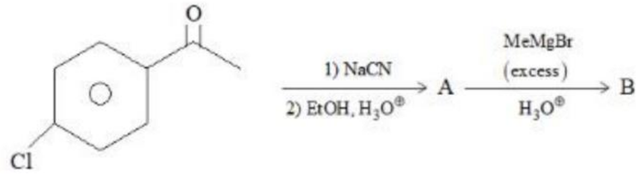
This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

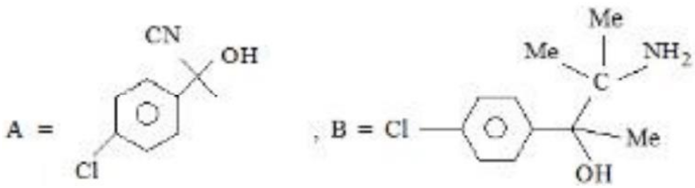
1. Correct order of spin only magnetic moment of the following complex ions is:
(Given At. No. Fe : 26, Co : 267)

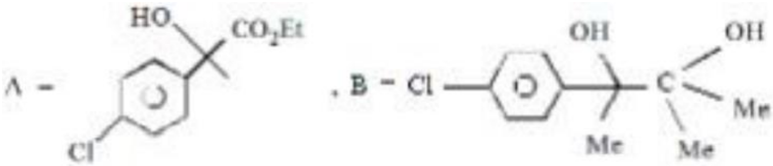
(1) $[Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-} > [FeF_6]^{3-}$ **(2)** $[FeF_6]^{3-} > [Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-}$

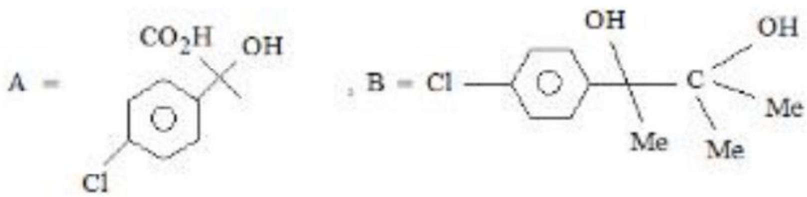
(3) $[FeF_6]^{3-} > [CoF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$ **(4)** $[CoF_6]^{3-} > [FeF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$

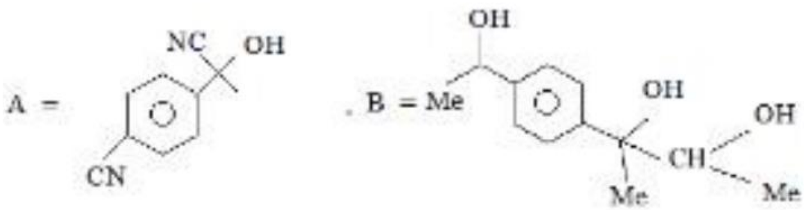
2. Find out the major products from the following reaction sequence.



(1) 

(2) 

(3) 

(4) 

3. A solutions of C_rO_5 in amyl alcohol has a ______________ colour.

(1) Yellow **(2)** Green **(3)** Blue **(4)** Orange-Red

4. Given below are two statements:
Statement I : Nickle is being used as the catalyst for producing syn gas and edible fats.
Statement II : Silicon forms both electron rich and electron deficient hydrides.
In the light of the above statements, choose the most appropriate answer from the options given below :
(1) Statement I is correct but statement II is incorrect
(2) Both the statements I and II are correct
(3) Both the statements I and II are incorrect
(4) Statement I is incorrect but statement II is correct

5. The set of correct statements is:

(i) Manganese exhibits + 7 oxidation state in its oxide.

(ii) Ruthenium and Osmium exhibit + 8 oxidation in their oxides.

(iii) Sc shows + 4 oxidation state which is oxidizing in nature.

(iv) Cr shows oxidizing nature in +6 oxidation state.

(1) (i) and (iii) **(2)** (ii), (iii) and (iv)

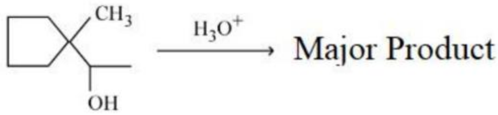
(3) (ii) and (iii) **(4)** (i), (ii) and (iv)

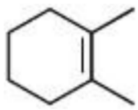
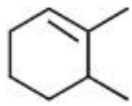
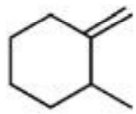
6. An indicator 'X' is used for studying the effect of variation in concentration of iodide on the rate of reaction of iodide ion with H_2O_2 at room temp. The indicator 'X' forms blue coloured complex with compound 'A' present in the solution. The indictor 'X' and compound 'A' respectively are:

(1) Starch and iodine **(2)** Starch and H_2O_2

(3) Methyl orange and H_2O_2 **(4)** Methyl orange and iodine

7. Find out the major product for the following reaction.



(1)  **(2)**  **(3)**  **(4)** 

8. According to MO theory the bond orders for O_2^{2-}, CO and NO^+ respectively, are:

(1) 2, 3 and 3 **(2)** 1, 2 and 3 **(3)** 1, 3 and 3 **(4)** 1, 3 and 2

9. The major component of which of the following ore is sulphide based mineral?

(1) Calamine **(2)** Sphalerite **(3)** Siderite **(4)** Malachite

10. Which of the following relations are correct?

(A) $\Delta U = q + p\Delta V$ (B) $\Delta G = \Delta H - T\Delta S$

(C) $\Delta S = \frac{q_{rev}}{T}$ (D) $\Delta H = \Delta U - \Delta nRT$

Choose the most appropriate answer from the options given below:

(1) C and D only **(2)** B and C only **(3)** A and B only **(4)** B and D only

11. Given below are two statements:

Statement I : The decrease in first ionization enthalpy from B to Al is much larger than the from Al to Ga

Statement II : The d orbitals in Ga are completely filled

In the light of the above statement, choose the most appropriate answer from the options given below

(1) Both the statements I and II are correct

(2) Statement I is correct but statement II is incorrect

(3) Both the statements I and II are incorrect

(4) Statement I is incorrect but statement II is correct

12. A doctor prescribed the drug Equanil to a patient. The patient was likely to have symptoms of which disease?

(1) Hyperacidity

(2) Anxiety and stress

(3) Depression and hypertension

(4) Stomach ulcers

13. The concentration of dissolved Oxygen in water for growth of fish should be more than <u>X</u> ppm and Biochemical Oxygen Demand in clean water should be less than <u>Y</u> ppm. X and Y in ppm are, respectively.

(1) X-6 ; Y-5 **(2)** X-6 ; Y - 12 **(3)** X-4 ; Y-8 **(4)** X-4 ; Y-15

14. Reaction of propenamide with $Br_2 / KOH(aq)$ produces:

(1) Propylamine **(2)** Propanenitrile **(3)** Ethylamine **(4)** Ethylnitrile

15. Match List I and List II

	List I		**List II**
A	Van't Hoff factor, i	I	Cryoscopic constant
B	k_f	II	Isotonic solutions
C	Solutions with same osmotic pressure	III	$\frac{\text{Normal molar mass}}{\text{Abnormal molar mass}}$
D	Azeotropes	IV	Solutions with same composition of vapour above it

Choose the correct answer from the options given below :

	A	B	C	D		A	B	C	D
(1)	I	III	II	IV	**(2)**	III	I	II	IV
(3)	III	II	I	IV	**(4)**	III	I	IV	II

16. Match List I and List II

	List I		**List II**
A	Osmosis	I	Solvent molecules pass through semi permeable membrane towards solvent side
B	Reverse osmosis	II	Movement of charged colloidal particles under the influence of applied electric potential towards oppositely charged electrodes.
C	Electro osmosis	III	Solvent molecules pass through semi permeable membrance towards solution side
D	Electroporesis	IV	Despersion medium moves in an electric field

Choose the correct answer from the options given below:

	A	B	C	D		A	B	C	D
(1)	I	III	IV	II	**(2)**	I	III	II	IV
(3)	III	I	IV	II	**(4)**	III	I	II	IV

17. The one giving maximum number of isomeric alkenes on dehydrohalogenation reaction is (excluding rearrangement)

(1) 2-Bromo-3, 3-dymethypentane **(2)** 1-Bromo-2-methylbutane

(3) 2-Bromopropane **(4)** 2-Bromopentane

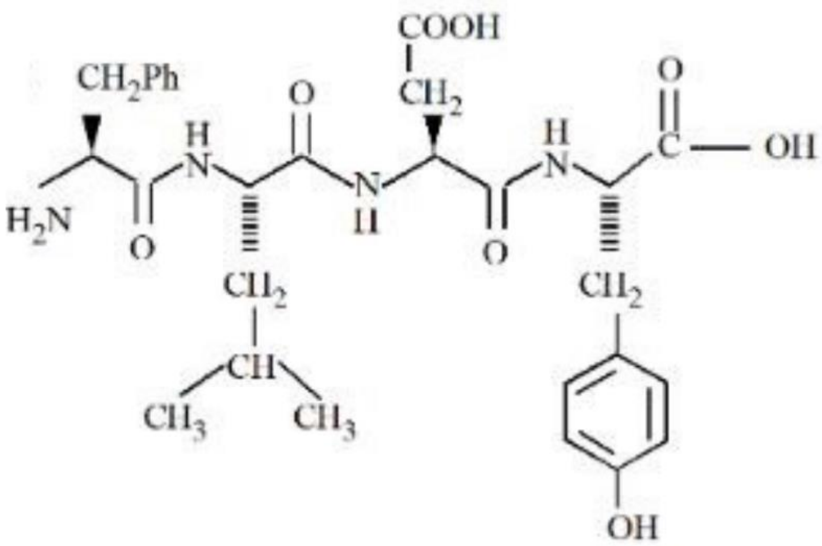
18. Match List I and List II

	List I		**List II**
A	Elastomeric polymer	I	Urea formaldehyde resin
B	Fibre Polymer	II	Polystyrene
C	Thermosetting Polymer	III	Polyster
D	Thermoplatic polymer	IV	Neoprene

Choose the correct answer from the options given below:

	A	B	C	D		A	B	C	D
(1)	IV	III	I	II	**(2)**	IV	I	III	II
(3)	II	I	IV	III	**(4)**	II	III	I	IV

19. Following tetrapeptide can be represented as:



(F, L D, Y, I, Q, P are one letter codes for amino acids)

(1) PLDY **(2)** FLDY **(3)** YQLF **(4)** FIQY

20. When a hydrocarbon A undergoes combustion in the presence of air, it requires 9.5 equivalents of oxygen and produces 3 equivalents of water. What is the molecular formula of A?

(1) C_9H_9 **(2)** C_8H_6 **(3)** C_6H_6 **(4)** C_9H_6

SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be **rounded off to the nearest integer**.

21. At 298 K

$$N_2(g)+3H_2(g) \rightleftharpoons 2NH_3(g), K_1=4\times10^5$$

$$N_2(g)+O_2(g) \rightleftharpoons 2NO(g), K_2=1.6\times10^{12}$$

$$H_2(g)+\frac{1}{2}O_2(g) \rightleftharpoons H_2O(g), K_3=10^{-13}$$

Based on above equilibria, the equilibrium constant of the reaction,

$$2NH_3(g)+\frac{5}{2}O_2(g) \rightleftharpoons 2NO(g)+3H_2O(g) \text{ is } \_\_\_\_\_\_\_\_\_ \times 10^{-33}$$

22. A metal M forms hexagonal close-packed structure. The total number of voids in 0.02 mol of it is ______ $\times10^{21}$ (Nearest integer).

(Given $N_A=6.02\times10^{23}$)

23. For conversion of compound $A \rightarrow B$, the rate constant of the reaction was found to be $4.6\times10^{-5}\,L\,mol^{-1}s^{-1}$. The order of the reaction is ____________.

24. On heating, $LiNO_3$ gives how many compounds among the following? ________.
$Li_2O, N_2, LiNO_2, NO_2$

25. The volume of HCl, containing $73gL^{-1}$, required to completely neutralize NaOH obtained by reacting 0.69 g of metallic sodium with water, is ________ mL. (Nearest Integer)

(Given : molar Masses of Na, Cl, O, H, are 23, 35, 5, 16 and 1 g mol^{-1})

26. The equilibrium constant for the reaction

$Zn(s)+Sn^{2+}(aq) \rightleftharpoons Zn^{2+}(aq)+Sn(s)$ is 1×10^{20} at 298 K. The magnitude of standard electrode potential of Sn/Sn^{2+} if $E^0_{Zn^{2+}}/Zn=-76V$ is ________ $\times10^{-2}V$ (Nearest Integer)

Given : $\frac{2.303RT}{F}=0.059V$

27. The density of the ligand present in the Fehling's reagent is _________.

28. Assume that the radius of the first Bohr orbit of hydrogen atom is $0.6Å$. The radius of the third Bohr orbit of He^+ is __________ picometer. (Nearest Integer)

29. Total number of acidic oxides among
$N_2O_3, NO_2, N_2O, Cl_2O_7, SO_2, CO, CaO, Na_2O$ and NO is _________.

30. When 0.01 mol of an organic compound containing 60% carbon was burnt completely, 4.4 g of CO_2 was produced. The molar mass of compound is ___________ $g\,mol^{-1}$ (Nearest Integer)

SUBJECT III: MATHEMATICS	MARKS: 100

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. The statement $B \Rightarrow ((\sim A) \vee B)$ is equivalent to :

(1) $B \Rightarrow (A \Rightarrow B)$ **(2)** $A \Rightarrow ((\sim A) \Rightarrow B)$

(3) $A \Rightarrow (A \Leftrightarrow B)$ **(4)** $B \Rightarrow ((\sim A) \Rightarrow B)$

2. Let $\vec{a} = 4\hat{i} + 3\hat{j}$ and $\vec{b} = 3\hat{i} - 4\hat{j} + 5\hat{k}$. If $\vec{c}$ is a vector such that $\vec{c}.\left(\vec{a} \times \vec{b}\right) + 25 = 0$, $\vec{c}.(\hat{i} + \hat{j} + \hat{k}) = 4$, and projection of $\vec{c}$ on $\vec{a}$ is 1, then the projection of $\vec{c}$ on $\vec{b}$ equals :

(1) $\frac{1}{\sqrt{2}}$ **(2)** $\frac{5}{\sqrt{2}}$ **(3)** $\frac{1}{5}$ **(4)** $\frac{3}{\sqrt{2}}$

3. The value of the integral $\int_{1/2}^{2} \frac{\tan^{-1} x}{x} dx$ is equal to:

(1) $\frac{1}{2}\log_e 2$ **(2)** $\frac{\pi}{2}\log_e 2$ **(3)** $\pi \log_e 2$ **(4)** $\frac{\pi}{4}\log_e 2$

4. Let f and g be twice differentiable functions on $\mathbb{R}$ such that

$f''(x) = g''(x) + 6x$

$f'(1) = 4g'(1) - 3 = 9$

$f(2) = 3g(2) = 12$

Then which of the following is **NOT** true?

(1) $g(-2) - f(-2) = 20$

(2) If $-1 < x < 2$ then, $|f(x) - g(x)| < 8]$

(3) $|f'(x) - g'(x)| < 6 \Rightarrow -1 < x < 1$

(4) There exists $x_0 \in (1, 3/2)$ such that $f(x_0) = g(x_0)$

5. If the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{1}$ and $\frac{x-a}{2} = \frac{y+2}{3} = \frac{z-3}{1}$ intersect at the point, P then the distance of the point P from the plane z = a is:

(1) 10 **(2)** 22 **(3)** 16 **(4)** 28

6. The letters of the word OUGHT are written in all possible ways and these words are arranged as in a directory, in a series. Then the serial number of the word TOUGH is:

(1) 86 **(2)** 89 **(3)** 79 **(4)** 84

7. The number of 3 digit numbers, that are divisible by either 3 or 4 but not divisible by 48, is:

(1) 472 **(2)** 507 **(3)** 400 **(4)** 432

8. Let K be the sum of the coefficients of the odd powers of *x* in the expansion of $(1+x)^{99}$. Let a be the middle term in the expansion of $\left(2 + \frac{1}{\sqrt{2}}\right)^{200}$. If $\frac{^{200}C_{99} K}{a} = \frac{2^l m}{n}$, where *m* and *n* are odd numbers, then the ordered pair (*l*, *n*) is equal to:

(1) (50, 51) **(2)** (51, 101) **(3)** (50, 101) **(4)** (51, 99)

9. The set of all values of λ for which the equation $\cos^2 2x - 2\sin^4 x - 2\cos^2 x = \lambda$ has a real solutions *x*, is:

(1) $[-2,-1]$ **(2)** $\left[-1,-\frac{1}{2}\right]$ **(3)** $\left[-\frac{3}{2},-1\right]$ **(4)** $\left[-2,-\frac{3}{2}\right]$

10. If the tangent at a point P on the parabola $y^2 = 3x$ is parallel to the line $x + 2y = 1$ and the tangents at the points Q and R on the ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ are perpendicular to the line x–y = 2, then the area of the triangle PQR is:

(1) $\frac{3}{2}\sqrt{5}$ **(2)** $3\sqrt{5}$ **(3)** $5\sqrt{3}$ **(4)** $\frac{9}{\sqrt{5}}$

11. The value of the integral $\int_1^2 \left(\frac{t^4+1}{t^6+1}\right) dr$ is:

(1) $\tan^{-1}\frac{1}{2}+\frac{1}{3}\tan^{-1}8-\frac{\pi}{3}$ **(2)** $\tan^{-1}2-\frac{1}{3}\tan^{-1}8+\frac{\pi}{3}$

(3) $\tan^{-1}2+\frac{1}{3}\tan^{-1}8-\frac{\pi}{3}$ **(4)** $\tan^{-1}\frac{1}{2}-\frac{1}{3}\tan^{-1}8+\frac{\pi}{3}$

12. The shortest distance between the lines $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$ and $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$ is:

(1) $4\sqrt{3}$ **(2)** $5\sqrt{3}$ **(3)** $2\sqrt{3}$ **(4)** $3\sqrt{3}$

13. Let $S = \{w_1, w_2 ....\}$ be the sample space associated to a random experiment. Let $P(w_n) = \frac{P(w_{n-1})}{2}, n \ge 2$. Let $A = \{2k + 3l : k, l \in \mathbb{N}\}$ and $B = \{w_n : n \in A\}$. Then P(B) is equal to :

(1) $\frac{3}{32}$ **(2)** $\frac{1}{16}$ **(3)** $\frac{3}{64}$ **(4)** $\frac{1}{32}$

14. The set of all values of $t \in \mathbb{R}$, for which the matrix

$$\begin{bmatrix} e^t & e^{-t}(\sin t - 2\cos t) & e^t(-2\sin t - \cos t) \\ e^t & e^{-t}(2\sin t + \cos t) & e^{-t}(\sin t - 2\cos t) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{bmatrix}$$ in invertible, is:

(1) $\left\{(2k+1)\frac{\pi}{2},\ k \in \mathbb{Z}\right\}$ **(2)** $\mathbb{R}$

(3) $\{k\pi,\ k \in \mathbb{Z}\}$ **(4)** $\left\{k\pi + \frac{\pi}{4}, k \in \mathbb{Z}\right\}$

15. If $\vec{a} = \hat{i} + 2\hat{k}, \vec{b} = \hat{i} + \hat{j} + \hat{k}, \vec{c} = 7\hat{i} - 3\hat{j} + 4\hat{k}, \vec{r} \times \vec{b} + \vec{b} \times \vec{c} = \vec{0}$ and $\vec{r} \cdot \vec{a} = 0$. Then $\vec{r} \cdot \vec{c}$ is equal to:

(1) 34 **(2)** 36 **(3)** 30 **(4)** 32

16. Let R be a relation defined on $\mathbb{N}$ as a R b if $2a + 3b$ is multiple of $5, a, b, \in \mathbb{N}$. Then R is:

(1) An equivalence relation **(2)** Not reflexive

(3) Symmetric but not transitive **(4)** Transitive but not symmetric

17. The area of the region $A=\left\{(x,y):|\cos x-\sin x|\le y\le\sin x, 0\le x\le\frac{\pi}{2}\right\}$ is:

(1) $\frac{3}{\sqrt{5}}-\frac{3}{\sqrt{2}}+1$ **(2)** $\sqrt{5}-2\sqrt{2}+1$

(3) $1-\frac{3}{\sqrt{2}}+\frac{4}{\sqrt{5}}$ **(4)** $\sqrt{5}+2\sqrt{2}-4.5$

18. Let $y=y(x)$ be the solution of the differential equation $x\log_e x\frac{dy}{dx}+y=x^2\log_e x, (x>1)$. If $y(2)=2$, then $y(e)$ is equal to:

(1) $\frac{1+e^2}{4}$ **(2)** $\frac{2+e^2}{2}$ **(3)** $\frac{4+e^2}{4}$ **(4)** $\frac{1+e^2}{2}$

19. The plane $2x-y+z=4$ intersects the line segment joining the points $A(a,-2,4)$ and $B(2,b,-3)$ at the point C in the ratio 2 :1 and the distance of the point C from the origin is $\sqrt{5}$. If $ab<0$ and P is the point $(a-b, 2b-a)$ then CP^2 is equal to:

(1) $\frac{73}{3}$ **(2)** $\frac{16}{3}$ **(3)** $\frac{17}{3}$ **(4)** $\frac{97}{3}$

20. Consider a function $f: IN \to IR$, satisfying

$f(1)+2f(2)+3f(3)+....+xf(x)=x(x+1); x\ge 2$ with $f(1)=1$

Then $\frac{1}{f(2022)}+\frac{1}{f(2028)}$ is equal to:

(1) 8000 **(2)** 8200 **(3)** 8400 **(4)** 8100

SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be **rounded off to the nearest integer**.

21. If the equation of the normal to the curve $y=\dfrac{x-a}{(x+b)(x-2)}$ at the point $(1,-3)$ is $x-4y=13$, then the value of $a+b$ is equal to _______.

22. Let A be a symmetric matrix such that |A| = 2 and $\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix} A - \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$.

If the sum of the diagonal elements of A is s, then $\dfrac{\beta s}{\alpha^2}$ is equal to ______________.

23. A triangle is formed by the tangents at the point (2, 2) on the curves $y^2=2x$ and $x^2+y^2=4x$, and the line $x+y+2=0$. If r is the radius of its circumcircle, then r^2 is equal to _________.

24. Let $\alpha_1, \alpha_2, \ldots\ldots \alpha_7$ be the roots of the equation $x^7+3x^5-13x^3-15x=0$ and $|\alpha_1| \geq |\alpha_2| \geq \ldots\ldots \geq |\alpha_7|$. Then $a_1\alpha_2-\alpha_3\alpha_4+\alpha_5\alpha_6$ is equal to ________

25. Let $\alpha=8-14i, A=\left\{z\in c:\dfrac{\alpha z-\bar{\alpha}\bar{z}}{z^2-(\bar{z})^2-112i}1\right\}$ and $B=\{z\in C:|z+3i|=4\}$.

Then $\sum\limits_{z\in A\cap B}(\operatorname{Re} z-\operatorname{Im} z)$ is equal to ___________.

26. Let $\{a_k\}$ and $\{b_k\}$, $k\in\mathbb{N}$, be two G.P.s with common ratios r_1 and r_2 respectively such that $a_1=b_1=4$ and $r_1<r_2$. Let $c_k=a_k+b_k, k\in\mathbb{N}$. If $c_2=5$ and $c_3=\dfrac{13}{4}$ then $\sum\limits_{k=1}^{\infty}c_k-(12a_6+8b_4)$ is equal to ________.

27. Let $a_1=b_1=1$ and $a_n=a_{n-1}+(n-1), b_n=a_{n-1}, \forall n\geq 2$. If $S=\sum\limits_{n=1}^{10}\dfrac{b_n}{2^n}$ and $T=\sum\limits_{n=1}^{8}\dfrac{n}{2^{n-1}}$, then $2^7(2S-T)$ is equal to ________.

28. The total number of 4-digit numbers whose greatest common divisor with 54 is 2, is ______.

29. A circle with centre (2, 3) and radius 4 intersects the line $x+y=3$ at the points P and Q. If the tangents at P and Q intersect at the point $S(\alpha,\beta)$, then $4\alpha-7\beta$ is equal to ___________.

30. Let X = {11, 12, 13, …., 40, 41} and Y = {61. 62, 63, ….., 90, 91} be the two sets of observations. If $\bar{x}$ and $\bar{y}$ are their respective means and σ^2 is the variance of all the observations in $X\bigcup Y$, then $|\bar{x}+\bar{y}-\sigma^2|$ is equal to ______.