

JEE Main – 2023

25th JAN 2023 (Morning Shift)

General Instructions

- 1. The test is of **3 hours** duration and the maximum marks is **300**.
- The question paper consists of 3 Parts (Part I: Physics, Part II: Chemistry, Part III: Mathematics). Each Part has two sections (Section 1 & Section 2).
- **3.** Section 1 contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.
- 4. Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. You will NOT be allowed to attempt the sixth question. If you wish to attempt any other question apart from the five already attempted, then you will have to delete any one response from the five previously answered and then proceed to answer the new one.

The answer to each question should be **rounded off to the nearest integer**.

5. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. inside the examination room/hall.

Marking Scheme

- **1. Section 1:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.
- 2. Section 2: +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.

SUBJECT I: PHYSICS

MARKS: 100

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. Electron beam used in an electron microscope, when accelerated by a voltage of 20kV, has a de-Broglie wavelength of λ_0 . If the voltage is increased to 40kV, then the de-Broglie wavelength associated with the electron beam would be:

(1)
$$3\lambda_0$$
 (2) $\frac{\lambda_0}{\sqrt{2}}$ (3) $9\lambda_0$ (4) $\frac{\lambda_0}{2}$

2. Given below are two stateemnts: one is labelled as Assertion A and the other is labelled as Reason R. Assertion A: Photodiodes are used in forward bias usually for measuring the light intensity. Reason R: For a p-n junction diode, at applied voltage V the current in the forward bias is more than the current in the reverse bias for $|V_z| > \pm V \ge |V_0|$ where V_0 is the threshold voltage and V_z is the breakdown voltage.

In the light of the above statements, choose the correct answer from the options given below.

- (1) Both A and R are true and R is correct explanation A
- (2) Both A and R are true but R is NOT the correct explanation A
- (3) A is false but R is true
- (4) A is true but R is false
- 3. Assume that the earth is a solid sphere of uniform density and a tunnel is dug along its diameter throughout the earth. It is found that when a particle is released in this tunnel, it executes a simple harmonic motion. The mass of the particle is 100g. The time period of the motion of the particle will be (approximately):

(Take $g = 10 ms^{-2}$, radius of earth = 640 km) (1) 12 hours (2) 24 hours (3) 1 hour 40 minutes (4) 1 hour 24 minutes

4. In Young's double slits experiment, the position of 5^{th} bright fringe from the central maximum is 5*cm*. the distance between slits and screen is 1m and wavelength of used monochromatic light is 600nm. The separation between the slits is:

| (1) | 60 µm | (2) | 12 µm |
|-----|-------|-----|-------|
| (3) | 48 µm | (4) | 36 µm |

5. A bowl filled with very hot soup cools from 98° C to 86° C in 2 minutes when the room temperature is 22° C. How long it will take to cool from 75° C to 69° C?

| (1) | 0.5 minute | (2) | 2 minute |
|-----|------------|-----|----------|
| (3) | 1.4 minute | (4) | 1 minute |

6. A uniform metallic wire carries a current 2*A*, when 3.4*V* battery is connected across it. The mass of uniform metallic wire is $8.92 \times 10^{-3} kg$, density is $8.92 \times 10^{3} kg / m^{3}$ and resistivity is $1.7 \times 10^{-8} \Omega - m$. The length of wire is:

| (1) | l = 100 m | (2) | l = 10m |
|-----|-----------|-----|---------|
| (1) | i = 100 m | | i - 10m |

(3) l = 5m (4) l = 6.8m

- 7. A car is moving with a constant speed of 20m/s in a circular horizontal track of radius 40m. a bon is suspended from the roof of the car by a massless string. The angle made by the string with the vertical will be: (Take $g = 10m/s^2$)
 - (1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$
- 8. A car travels a distance of 'x' with speed v_1 and then same distance 'x' with speed v_2 in the same direction. The average speed of the car is:
 - (1) $\frac{v_1 + v_2}{2}$ (2) $\frac{v_1 v_2}{2(v_1 + v_2)}$ (3) $\frac{2v_1 v_2}{v_1 + v_2}$ (4) $\frac{2x}{v_1 + v_2}$

9. A solenoid of 1200 turns is wound uniformly in a sample layer on a glass tube 2m long and 0.2m in diameter. The magnetic intensity at the center of the solenoid when a current of 2A flows through it is:

- (1) $2.4 \times 10^{-3} Am^{-1}$ (2) $2.4 \times 10^{3} Am^{-1}$
- (3) $1Am^{-1}$ (4) $1.2 \times 10^3 Am^{-1}$
- **10.** Match List I with List II.

| | List I | List II | | |
|-----------|-------------------------|--|--|--|
| | (Current configuration) | (Magnitude of Magnetic Field at point O) | | |
| A. | \rightarrow | I. | $B_0 = \frac{\mu_0 I}{4\pi r} [\pi + 2]$ | |
| В. | | II. | $B_0 = \frac{\mu_0}{4} \frac{I}{r}$ | |
| C. | | III. | $B_0 = \frac{\mu_0 I}{2\pi r} [\pi - 1]$ | |
| D. | | IV. | $B_0 = \frac{\mu_0 I}{4\pi r} [\pi + 1]$ | |

Choose the correct answer from the options given below:

- (1) A I, B III, C IV, D II (2) A II, B I, C IV, D III(3) A - III, B - IV, C - I, D - II (4) A - III, B - I, C - IV, D - III
- A message signal of frequency 5 kHz is used to modulate a carrier signal of frequency 2 MHz. The

bandwidth for amplitude modulation is:

| (1) | 20 kHz | (2) | 10 kHz |
|-----|--------|-----|---------|
| (3) | 5 kHz | (4) | 2.5 kHz |

12. In an LC oscillator, if values of inductance and capacitance become twice and eight times, respectively, then the resonant frequency of oscillator becomes x times its initial resonant frequency ω_0 . The value of x is:

(1)
$$1/16$$
 (2) 4 (3) $1/4$ (4) 16

11.

- 13. The root mean square velocity of molecules of gas is:
 - (1) Inversely proportional to square root of temperature
 - Proportional to square root of temperature (\sqrt{T}) (2)
 - Proportional to temperature (T) (3)
 - Proportional to square of temperature (T^2) (4)
- 14. Match List I with List II.

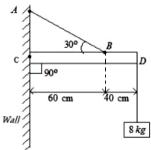
| List I | | | List II | | |
|--------|-----------------|------|--------------------|--|--|
| А. | Surface tension | I. | $Kg m^{-1}s^{-1}$ | | |
| В. | Pressure | II. | Kg ms ⁻ | | |
| C. | Viscosity | III. | $Kg m^{-1}s^{-2}$ | | |
| D. | Impulse | IV. | $Kg s^{-2}$ | | |

Choose the correct answer from the options given below:

| (1) | A - IV, B - III, C - II, D - I | (2) | A – II, B – I, C – III, D – IV |
|-----|--------------------------------|-----|--------------------------------|
| (3) | A - IV, B - III, C - I, D - II | (4) | A - III, B - IV, C - I, D - II |
| (1) | (2) | (3) | (4) |

15. An object of mass 8kg is hanging from one end of a uniform rod CD of mass 2kg and length 1m pivoted at its end C on a vertical wall as shown in figure. It is supported by a cable AB such that the system is in equilibrium. The tension in the cable is: (Take $g = 10m/s^2$)

(1) 30 N (2) 90 N (3) 300 N (4) 240 N



- The ratio of the density of oxygen nucleus $\binom{16}{8}O$ and helium nucleus $\binom{4}{2}He$ is: 16. (2) 8:1 4:1(1) 1:1(3) (4) 2:1
- 17. A Carnot engine with efficiency 50% takes heat from a source at 600 K. In order to increase the efficiency to 70%, keeping the temperature of sink same, the new temperature of the source will be: 300 K 1000 K 900 K 360 K (1) (2)(3)(4)

18. An electromagnetic wave is transporting energy in the negative z direction. At a certain point and certain time the direction of electric field of the wave is along positive y direction. What will be the direction of the magnetic field of the wave at that point and instant?

- (1) Positive direction of x(2) Negative direction of x
- Negative direction of y(4) Positive direction of z(3)
- A parallel plate capacitor has plate area $40 cm^2$ and plates separation 2mm. The space between the 19. plates is filled with a dielectric medium of a thickness 1mm and dielectric constant 5. The capacitance of the system is:

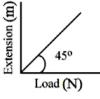
(1)
$$\frac{3}{10}\varepsilon_0 F$$
 (2) $10\varepsilon_0 F$ (3) $24\varepsilon_0 F$ (4) $\frac{10}{3}\varepsilon_0 F$

- T is the time period of simple pendulum on the earth's surface. Its time period become x T when taken 20. to a height R (equal to earth's radius) above the earth's surface. Then, the value of x will be: $\frac{1}{2}$
 - $\frac{1}{4}$ (1) 2 (4) (2) (3) 4

SECTION-2

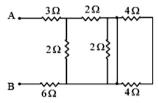
Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be rounded off to the nearest integer.

- 21. A ray of light is incident from air on a glass plate having thickness $\sqrt{3} cm$ and refractive index $\sqrt{2}$. The angle of incidence of a ray is equal to the critical angle for glass-air interface. The lateral displacement of the ray when it passes through the plate is _____×10⁻² cm. (Given sin 15° = 0.26)
- 22. As shown in the figure, in an experiment to determine young's modulus of a wire, the extension-load curve is plotted. The curve is a straight line passing through the origin and makes an angle of 45° with the load axis. The length of wire is 62.8 cm and its diameter is 4mm. The Young's modulus is found to be $x \times 10^4 Nm^{-2}$. The value of x is



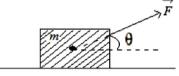
Ω

23. In the given circuit, the equivalent resistance between the terminal A and B is _____



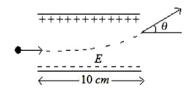
24. An object of mass 'm'initially at rest on a smooth horizontal plane starts moving under the action of force F = 2N. In the process of its linear motion, the angle θ (as shown in figure) between the direction of force and horizontal varies as $\theta = kx$, where k is a constant and x is the distance covered by the object

from its initial position. The expression of kinetic energy of the object will be $E = \frac{n}{k} \sin \theta$. The value of *n* is ______.



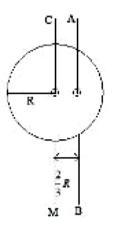
Smooth horizontal surface

25. A uniform electric field of 10 N/C is created between two parallel charged plates (as shown in figure). An electron enters the field symmetrically between the plates with a kinetic energy 0.5eV. the length of each plate is 10*cm*. The angle (θ) of deviation of the path of electron as it comes out of the field is _____ (in degree).



- 26. An LCR series circuit of capacitance 62.5nF and resistance of 50Ω , is connected to an A.C. source of frequency 2.0 kHz. For maximum value of amplitude of current in circuit, the value of inductance is _____ mH. (Take $\pi^2 = 10$)
- 27. The distance between two consecutive points with phase difference of 60° in a wave of frequency 500 Hz is 6.0*m*. the velocity with which wave is traveling is _____ km/s.

- 28. If $\vec{P} = 3\hat{i} + \sqrt{3}\hat{j} + 2\hat{k}$ and $\vec{Q} = 4\hat{i} + \sqrt{3}\hat{j} + 2.5\hat{k}$ then, The unit vector in the direction of $\vec{P} \times \vec{Q}$ is $\frac{1}{x}(\sqrt{3}i + \hat{j} 2\sqrt{3}\hat{k})$. The value of x is:
- 29. The wavelength of the radiation emitted is λ_0 when an electron jumps from the second excited state to the first excited state of hydrogen atom. If the electron jumps from the third excited state to the second orbit of the hydrogen atom, the wavelength of the radiation emitted will be $\frac{20}{x}\lambda_0$. the value of x is ______.
- 30. I_{CM} is the moment of inertia of a circular disc about an axis (CM) passing through its center and perpendicular to the plane of disc. I_{AB} is it's moment of inertia about an axis Ab perpendicular to plane and parallel to axis CM at a distance $\frac{2}{3}R$ from center. Where R is the radius of the disc. The ratio of I_{AB} and I_{CM} is x:9. The value of x is ______.



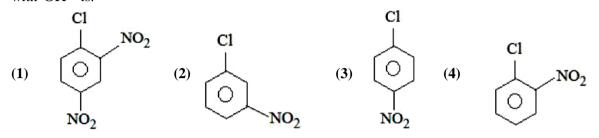
SUBJECT II: CHEMISTRY

MARKS: 100

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. The compound which will have the lowest rate towards nucleophilic aromatic substitution on treatment with OH⁻ is:



- 2. The correct order in aqueous medium of basic strength in case of methyl substituted amines is:
 - (1) $Me_2NH > MeNH_2 > Me_3N > NH_3$ (2) $NH_3 > Me_3N > MeNH_2 > Me_2NH$

(3)
$$Me_2NH > Me_3N > MeNH_2 > NH_3$$
 (4) $Me_3N > Me_2Nh > MeNH_2 > NH_3$

3. Match List I with List II.

| | List I | List II | | |
|----|----------|------------------------------|-------------|--|
| | Elements | Colour imparted to the flame | | |
| А. | K | I. Brick Red | | |
| В. | Са | II. | Violet | |
| C. | Sr | III. | Apple Green | |
| D. | Ва | IV. | Crimson Red | |

Choose the correct answer from the options given below:

(1) A - II, B - I, C - III, D - IV

(2) A - II, B - IV, C - I, D - III

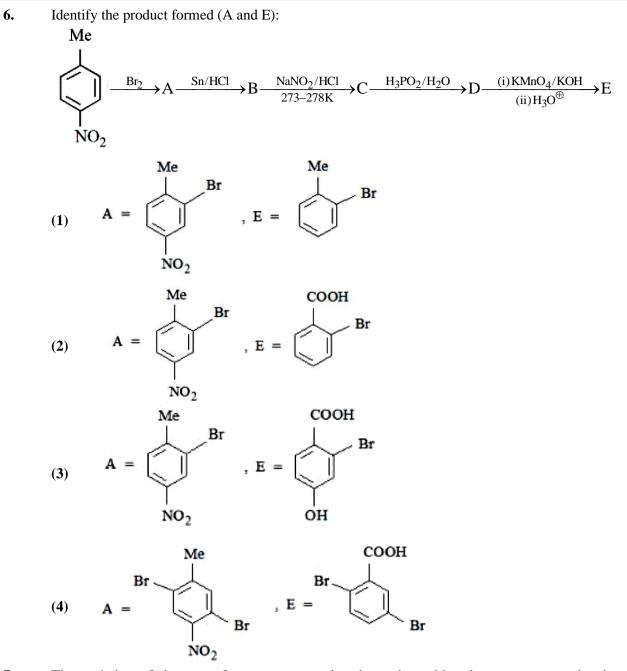
- $(3) \qquad A-IV, B-III, C-II, D-I$
- $(4) \qquad A-II, B-I, C-IV, D-III$
- 4. Which one of the following reactions does not occur during extraction of copper?

(1)
$$2\text{FeS} + 3\text{O}_2 \longrightarrow 2\text{FeO} + 2\text{SO}_2$$
 (2) $\text{CaO} + \text{SiO}_2 \longrightarrow \text{CaSiO}_3$

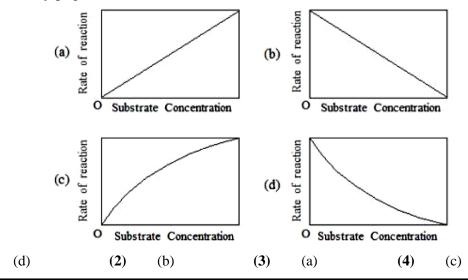
(3)
$$\operatorname{FeO} + \operatorname{SiO}_2 \longrightarrow \operatorname{FeSiO}_3$$
 (4) $2\operatorname{Cu}_2 S + 3\operatorname{O}_2 \longrightarrow 2\operatorname{Cu}_2 O + 2\operatorname{SO}_2$

5. '25 volume' hydrogen peroxide means

- (1) 1 L marketed solution contains 75 g of H_2O_2
- (2) 1 L marketed solution contains 25 g of H_2O_2
- (3) 1 L marketed solution contains 250 g of H_2O_2
- (4) 100 mL marketed solution contains 25 g of H_2O_2

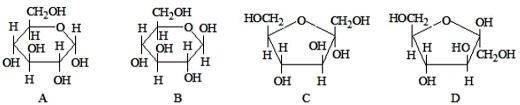


7. The variation of the rate of an enzyme catalyzed reaction with substrate concentration is correctly represented by graph.



(1)

- 8. A cubic solid is made up of two elements X and Y. Atoms of X are present on every alternate corner and one at the center of cube. Y is at $\frac{1}{3}$ rd of the total faces. The empirical formula of the compound is:
 - (1) $XY_{2.5}$ (2) $X_{2.5}Y$ (3) $X_{1.5}Y_2$ (4) $X_2Y_{1.5}$
- 9. Match items of Row I with those of Row II. Row I :



Row II :

(i) $\alpha - D - (-) - Fructofuranose,$ (ii) (iii) $\alpha - D - (-)Glucopyranose,$ (iv) Correct match is: (1) A - iv, B - iii, C - i, D - ii(3) A - iii, B - iv, C - ii, D - i

 β -D-(-)-Fructofuranose, β -D-(-)-Glucopyranose

- (2) A iii, B iv, C i, D ii(4) A - i, B - ii, C - iii, D - iv
- **10.** Match the List-I with List-II.

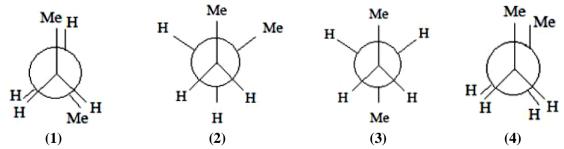
| | List I | List II | | |
|----|--------------------------------------|----------------|--|--|
| | Cations | Group reagents | | |
| А. | $A \longrightarrow Pb^{2+}, Cu^{2+}$ | i. | H_2S gas is presence of dilute HCl | |
| В. | $B \longrightarrow Al^{3+}, Fe^{3+}$ | ii. | $(NH_4)_2CO_3$ in presence of NH_4OH | |
| C. | $C \longrightarrow Co^{2+}, Ni^{2+}$ | iii. | NH ₄ OH in presence of NH ₄ Cl | |
| D. | $D \longrightarrow Ba^{2+}, Ca^{2+}$ | iv. | H_2S in presence of NH_4OH | |

Choose the correct answer from the options given below:

(1)
$$A - iii, B - i, C - iv, D - ii$$

(2) $A - iv, B - ii, C - iii, D - ii$
(3) $A - i, B - iii, C - iv, D - ii$
(4) $A - i, B - iii, C - ii, D - iv$

11. Which of the following conformations will be the most stable?



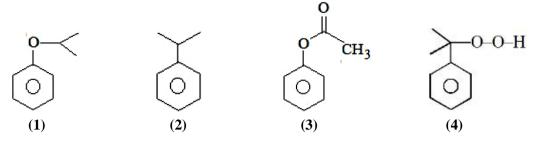
12. Some reactions of NO₂ relevant to photochemical smog formation are: Identify A, B, X and Y

- (1) $X = [O], Y = NO, A = O_2, B = O_3$ (2) $X = \frac{1}{2}O_2, Y = NO_2, A = O_3, B = O_2$ NO₂ sunlight X + YA B B
- (3) $X = NO, Y = [O], A = O_2, B = N_2O_3$
- (4) $X = N_2O, Y = [O], A = O_3, B = NO$

| | Vidyamandir Classes: Innovating For Your Success | | | | | | |
|-----|--|--|--|--|--|--|--|
| 13. | The radius of the 2^{nd} orbit of Li^{2+} is x. The expected radius of the 3^{rd} orbit of Be^{3+} is: | | | | | | |
| | (1) $\frac{4}{9}x$ (2) $\frac{9}{4}x$ (3) $\frac{27}{16}x$ (4) $\frac{16}{27}x$ | | | | | | |
| 14. | Inert gases have positive electron gain enthalpy. Its correct order is:(1)He < Kr < Xe < Ne(2)He < Xe < Kr < Ne(3)He < Ne < Kr < Xe(4)Xe < Kr < Ne < He | | | | | | |
| 15. | Which of the following statements is incorrect for antibiotics? (1) An antibiotic is a synthetic substance produced as a structural analogue of naturally occurring (2) An antibiotic should promote the growth or survival of microorganisms (3) An antibiotic must be a product of metabolism (4) An antibiotic should be effective in low concentrations | | | | | | |
| 16. | $\begin{array}{c} & & & & & & \\ & & & & \\ P & & & & & \\ \hline Q & & & & \\ \hline P & & & & & \\ \hline Q & & & & \\ \hline P & & & & & \\ \hline Q & & & & \\ \hline R & & & \\ \hline \end{array}$ The correct sequence of reagents for the preparation of Q and R is: (1) (i) $\operatorname{CrO}_2\operatorname{Cl}_2, \operatorname{H}_3\operatorname{O}^+$; (ii) $\operatorname{Cr}_2\operatorname{O}_3$, 770 K, 20 atm; (iii) NaOH; (iv) $\operatorname{H}_3\operatorname{O}^+$ (2) (i) $\operatorname{Cr}_2\operatorname{O}_3$, 770 K, 20 atm; (ii) $\operatorname{CrO}_2\operatorname{Cl}_2, \operatorname{H}_3\operatorname{O}^+$; (iii) NaOH; (iv) $\operatorname{H}_3\operatorname{O}^+$ (3) (i) KMnO ₄ , OH ⁻ ; (ii) Mo ₂ O ₃ , Δ ; (iii) NaOH; (iv) $\operatorname{H}_3\operatorname{O}^+$ (4) (i) Mo ₂ O ₃ , Δ ; (ii) $\operatorname{CrO}_2\operatorname{Cl}_2, \operatorname{H}_3\operatorname{O}^+$; (iii) NaOH; (iv) $\operatorname{H}_3\operatorname{O}^+$ | | | | | | |
| 17. | Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R: Assertion A: Acetal/Ketal is stable in basic medium. Reason R: The high leaving tendency of alkoxide ion gives the stability to acetal/ketal in basic medium. In the light of the above statements, choose the correct answer from the options given below: (1) Both A and R are true and R is the correct explanation of A (2) A is false but R is true (3) Both A and R are true but R is NOT the correct explanation of A (4) A is true but R is false | | | | | | |
| 18. | Compound A reacts with NH ₄ Cl and forms a compound B. Compound B reacts with H ₂ O and excess of CO ₂ to form compound C which on passing through or reaction with saturated NaCl solution forms sodium hydrogen carbonate. Compound A, B and C, are respectively. (1) $CaCl_2, NH_4^{\oplus}, (NH_4)_2CO_3$ (2) $CaCl_2, NH_3, NH_4HCO_3$ (3) $Ca(OH)_2, NH_3, NH_4HCO_3$ (4) $Ca(OH)_2, NH_4^{\oplus}, (NH_4)_2CO_3$ | | | | | | |
| 19. | Reaction of thionyl chloride with white phosphorus forms a compound [A], which on hydrolysis gives [B], a dibasic acid. [A] and [B] are respectively. (1) PCl_3 and H_3PO_3 (2) P_4O_6 and H_3PO_3 | | | | | | |

 PCl_5 and $\mathrm{H_3PO}_4$ $\mathrm{POCl}_3\,\mathrm{and}\,\mathrm{H}_3\mathrm{PO}_4$ (3) (4)

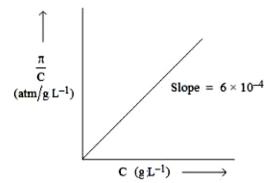
20. In the cumene to phenol preparation in presence of air, the intermediate is:



SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be rounded off to the nearest integer.

- In sulphur estimation, 0.471g of an organic compound gave 1.4439g of barium sulphate. The percentage of sulphur in the compound is ______. (Nearest Integer) (Given: Atomic mass Ba: 137u, S:32u, O:16u)
- 22. The number of paramagnetic species from the following is ______. $[Ni(CN)_4]^{2-}, [Ni(CO)_4], [NiCl_4]^{2-}, [Fe(CN)_6]^{4-}, [Cu(NH_3)_4]^{2+}, [Fe(CN)_6]^{3-} and [Fe(H_2O)_6]^{2+}$
- 23. The total number of lone pairs of electrons on oxygen atoms of ozone is ______.
- How many of the following metal ions have similar value of spin only magnetic moment in gaseous state
 ? (Given: Atomic number: V, 23; Cr, 24; Fe, 26; Ni, 28)
 V³⁺, Cr³⁺, Fe²⁺, Ni³⁺
- 25. A litre of buffer solution contains 0.1 mole of each of NH_3 and NH_4Cl . On the addition of 0.02 mole of HCl by dissolving gaseous HCl, the pH of the solution is found to be _____×10⁻³ (Nearest integer)[Given: $pK_b(NH_3) = 4.745$, log2 = 0.301, log3 = 0.477, T = 298 K]
- 26. The osmotic pressure of solutions of PVC in cyclohexanone at 300 K are plotted on the graph. The molar mass of PVC is _____ g mol⁻¹ (Nearest integer) [Given: R = 0.083 L atm $K^{-1}mol^{-1}$]



27. Consider the cell

Pt(s) | H₂(g)(latm) | H⁺(aq,[H⁺]=1) || Fe³⁺(aq), Fe²⁺(aq) | Pt(s) Given $E_{Fe^{3+}/Fe^{2+}}^{\circ} = 0.771 V$ and $E_{H^{+}/\frac{1}{2}H_{2}}^{\circ} = 0.V, T = 298 K$

If the potential of the cell is 0.712 V, the ratio of concentration of Fe^{2+} to Fe^{3+} is ______ . (Nearest integer)

- 28. For the first order reaction $A \longrightarrow B$, the half life is 30 min. The time taken for 75% completion of the reaction is _____ min. (Nearest integer) (Given: $\log 2 = 0.3010$, $\log 3 = 0.4771$, $\log 5 = 0.6989$)
- 29. An athlete is given 100g of glucose $(C_6H_{12}O_6)$ for energy. This is equivalent to 1800kJ of energy. The 50% of this energy gained is utilized by the athlete for sports activities at the event. In order to avoid

storage of energy, the weight of extra water he would need to perspire is ______ g (Nearest integer) Assume that there is not other way of consuming stored energy.

Given: The enthalpy of evaporation of water is 45 kJ mol^{-1}

Molar mass of C, H & O are 12, 1 and 16 g mol^{-1} .

30. The density of a monobasic strong acid (Molar mass 24.2 g/mol) is 1.21 kg/L. the volume of its solution required for the complete neutralization of 25 mL of 0.24 M NaOH is $___$ ×10⁻² mL (Nearest integer)

SUBJECT III: MATHEMATICS

MARKS: 100

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. Let
$$S_1$$
 and S_2 be respectively the sets of all $a \in R - \{0\}$ for which the system of linear equations.

ax+2ay-3az=1(2a+1)x+(2a+3)y+(a+1)z=2

(3a+5)x+(a+5)y+(a+2)z=3

has unique solution and infinitely many solutions. Then:

(1) $S_1 = \Phi \text{ and } S_2 = R - \{0\}$ (2) $S_1 = R - \{0\} \text{ and } S_2 = \Phi$

(3)
$$S_1$$
 is an infinite set and $n(S_2) = 2$ (4) $n(S_1) = 2$ and S_2 is an infinite set

2. The value of
$$\lim_{n \to \infty} \frac{1 + 2 - 3 + 4 + 5 - 6 + \dots + (3n - 2) + (3n - 1) - 3n}{\sqrt{2n^4 + 4n + 3} - \sqrt{n^4 + 5n + 4}}$$
 is:

- (1) $3(\sqrt{2}+1)$ (2) $\frac{3}{2\sqrt{2}}$ (3) $\frac{3}{2}(\sqrt{2}+1)$ (4) $\frac{\sqrt{2}+1}{2}$
- **3.** The mean and variance of the marks obtained by the students in a test are 10 and 4 respectively. Later, the marks of one of the students is increased from 8 to 12. If the new mean of the marks is 10.2, then their new variance is equal to:

4. Let $z_1 = 2 + 3i$ and $z_2 = 3 + 4i$. The set $S = \left\{ z \in C : |z - z_1|^2 - |z - z_2|^2 = |z_1 - z_2|^2 \right\}$ represents a:

- (1) Straight line with the sum of its intercepts on the coordinate axes equals -18
- (2) Hyperbola with eccentricity 2
- (3) Hyperbola with the length of the transverse axis 7
- (4) Straight line with the sum of its intercepts on the coordinate axes equals 14

5. Let
$$y = y(x)$$
 be the solution curve of the differential equation $\frac{dy}{dx} = \frac{y}{x} \left(1 + xy^2(1 + \log_e x)\right)$,

$$x > 0, y(1) = 3. \text{ Then } \frac{y^2(x)}{9} \text{ is equal to:}$$
(1) $\frac{x^2}{5-2x^3(2+\log_e x^3)}$
(2) $\frac{x^2}{2x^3(2+\log_e x^3)-3}$
(3) $\frac{x^2}{7-3x^3(2+\log_e x^2)}$
(4) $\frac{x^2}{3x^3(1+\log_e x^2)-2}$
Let $y(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})$. Then $y'-y''$ at $x = -1$ is equal to:
(1) 944
(2) 464
(3) 496
(4) 976
The vector $\vec{a} = -\hat{i} + 2\hat{j} + \hat{k}$ is rotated through a right angle, passing through the y-axis in it

7. The vector $\vec{a} = -\hat{i} + 2\hat{j} + k$ is rotated through a right angle, passing through the y-axis in its way and the resulting vector is \vec{b} . Then the projection of $3\vec{a} + \sqrt{2}\hat{b}$ on $\vec{c} = 5\hat{i} + 4\hat{j} + 3\hat{k}$ is:

(1)
$$3\sqrt{2}$$
 (2) $\sqrt{6}$ (3) 1 (4) $2\sqrt{3}$

6.

| 8. | The points of intersection of the line $ax+by=0$, $(a \neq b)$ and the circle $x^2 + y^2 - 2x = 0$ are $A(\alpha, 0)$ and $B(1, \beta)$. The image of the circle with AB as a diameter in the line $x + y + 2 = 0$ is: | | | | | | |
|-----|--|--|--|--|--|--|--|
| | | | | | | | |
| | (1) $x^{2} + y^{2} + 3x + 5y + 8 = 0$ (2) (3) $x^{2} + y^{2} + 3x + 3y + 4 = 0$ (4) | $x^{2} + y^{2} + 5x + 5y + 12 = 0$ | | | | | |
| 9. | Let \vec{a}, \vec{b} and \vec{c} be three non zero vectors such that | $\vec{b} \cdot \vec{c} = 0$ and $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b} - \vec{c}}{2}$. If \vec{d} be a vector | | | | | |
| | such that $\vec{b} \cdot \vec{d} = \vec{a} \cdot \vec{b}$, then $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$ is equal t | | | | | | |
| | (1) $-\frac{1}{4}$ (2) $\frac{1}{2}$ (3) Let $f:(0,1) \rightarrow IR$ be a function defined by $f(x) =$ | $\frac{3}{4}$ (4) $\frac{1}{4}$ | | | | | |
| 10. | Let $f:(0,1) \rightarrow IR$ be a function defined by $f(x) =$ | $\frac{1}{1-x}$, and $g(x) = (f(-x) - f(x))$. Consider two | | | | | |
| | statements. | 1-e | | | | | |
| | (I) g is an increasing function in (0, 1) | | | | | | |
| | (II) g is one-one in $(0, 1)$ | | | | | | |
| | Then : (1) Only (II) is true (2) | Neither (I) nor (II) is true | | | | | |
| | (3) Only (I) is true (4) | Both (I) and (II) are true | | | | | |
| 11. | The statement $(p \land (\sim q)) \Rightarrow (p \Rightarrow (\sim q))$ is: | | | | | | |
| | (1) A contradiction (2) | Equivalent to $p \lor q$ | | | | | |
| | (3) A tautology (4) | Equivalent to $(\sim p) \lor (\sim q)$ | | | | | |
| 12. | The minimum value of the function $f(x) = \int_{0}^{2} e^{ x-t } dt$ (1) 2 (2) $2(e-1)$ (3) | is: | | | | | |
| | (1) 2 (2) $2(e-1)$ (3) | 2e-1 (4) $e(e-1)$ | | | | | |
| 13. | Consider the lines L_1 and L_2 given by: | | | | | | |
| | $L_1: \frac{x-1}{2} = \frac{y-3}{1} = \frac{z-2}{2}$ | | | | | | |
| | $L_2: \frac{x-2}{2} = \frac{y-2}{2} = \frac{z-3}{3}$ | | | | | | |
| | A line L_3 having direction ratios $1, -1, -2$, intersection | ets L_1 and L_2 at the points P and Q respectively. | | | | | |
| | Then the length of line segment PQ is: | _ | | | | | |
| | (1) $3\sqrt{2}$ (2) $4\sqrt{3}$ (3) | | | | | | |
| | Let $x, y, z > 1$ and $A = \begin{bmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 2 & \log_y z \end{bmatrix}$. | 2 | | | | | |
| 14. | Let $x, y, z > 1$ and $A = \left \log_y x \right = \left \log_y z \right $. | then $ adj(adj A^2) $ is equal to: | | | | | |
| | $\left\lfloor \log_z x \log_z y 3 \right\rfloor$ | | | | | | |
| | (1) 4^8 (2) 2^4 (3) | | | | | | |
| 15. | Let $f(x) = \int \frac{2x}{(x^2+1)(x^2+3)} dx$. If $f(3) = \frac{1}{2}(\log_e 5)$ | $f - \log_e 6$), then $f(4)$ is equal to: | | | | | |
| | (1) $\log_e 17 - \log_e 18$ (2) | $\frac{1}{2} \left(\log_e 19 - \log_e 17 \right)$ | | | | | |
| | (3) $\frac{1}{2} (\log_e 17 - \log_e 19)$ (4) | $\log_e 19 - \log_e 20$ | | | | | |

| 16. | The d | istance of the | point (| $(6, -2\sqrt{2})$ from | the con | nmon tangent | y = mx | +c,m>0, of the curves |
|-----|----------------------------------|--------------------------------------|---------------|-----------------------------|----------|-----------------------------|------------------------|---|
| | $x = 2y^2$ and $x = 1 + y^2$ is: | | | | | | | |
| | (1) | 5 | (2) | 5√3 | (3) | $\frac{1}{3}$ | (4) | $\frac{14}{3}$ |
| 17. | Let x | =2 be a local | minima | of the function | f(x) = | $=2x^4-18x^2+8$ | 3x+12, x | $x \in (-4, 4)$. If M is local |
| | maxim | um value of the | function | f in (-4,4), th | nen M = | | | |
| | (1) | $12\sqrt{6} - \frac{31}{2}$ | (2) | $18\sqrt{6} - \frac{31}{2}$ | (3) | $18\sqrt{6} - \frac{33}{2}$ | (4) | $12\sqrt{6} - \frac{33}{2}$ |
| 18. | The di | stance of the po | oint P(4, | (6, -2) from the | line pas | sing through the | e point (| (-3, 2, 3) and parallel to a |
| | line wi | th direction ratio | os 3,3,- | 1 is equal to: | | | | |
| | (1) | $\sqrt{14}$ | (2) | $\sqrt{6}$ | (3) | 3 | (4) | $2\sqrt{3}$ |
| 19. | If a_r is | s the coefficient | of x^{10-r} | in the Binomial | expansio | on of $(1+x)^{10}$, t | then $\sum_{r=1}^{10}$ | $r^3 \left(\frac{a_r}{a_{r-1}}\right)^2$ is equal to: |
| | (1) | 4895 | (2) | 5445 | (3) | 3025 | (4) | 1210 |
| 20. | | | | | | | | their sum is 66. Let the |
| | sample | e space $S \left\{ x \in Z \right\}$ | Z: x(66) | $(-x) \ge \frac{5}{9}M$ and | d the ev | ent $A = \{x \in S$ | :xisan | nultiple of 3 . Then P(A) |
| | is equa | ll to: | | | | | | |
| | (1) | $\frac{1}{3}$ | (2) | $\frac{7}{22}$ | (3) | $\frac{1}{5}$ | (4) | $\frac{15}{44}$ |

SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be rounded off to the nearest integer.

- 21. Let A_1, A_2, A_3 be the three A.P. with the same common difference d and having their first terms as A, A+1, A+2, respectively. Let a, b, c be the 7th, 9th, 17th terms of A_1, A_2, A_3 , respectively such that $\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix} + 70 = 0.$ If a = 29, then the sum of first 20 terms of an AP whose first term is c-a-b and common difference is $\frac{d}{12}$, is equal to ______.
- 22. Let the equation of the plane passing through the line x-2y-z-5=0=x+y+3z-5 and parallel to the line x+y+2z-7=0=2x+3y+z-2 be 2x+by+cz=65. Then the distance of the point (a,b,c) from the plane 2x+2y-z+16=0 is ______.
- 23. Let x and y be distinct integers where $1 \le x \le 25$ and $1 \le y \le 125$. Then, the number of ways of choosing x and y, such that x + y is divisible by 5, is _____.

24. The constant term in the expansion of
$$\left(2x + \frac{1}{x^7} + 3x^2\right)^5$$
 is _____.

- 25. If the sum of all the solutions of $\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3}, -1 < x < 1, x \neq 0$, is $a \frac{4}{\sqrt{3}}$, then *a* is equal to _____.
- 26. If the area enclosed by the parabolas $P_1: 2y = 5x^2$ and $P_2: x^2 y + 6 = 0$ is equal to the area enclosed by P_1 and y = ax, a > 0, then a^3 is equal to ______.

27. Let
$$S = \left\{ a : \log_2(9^{2a-4} + 13) - \log_2\left(\frac{5}{3} \cdot 3^{2a-4} + 1\right) = 2 \right\}$$
. Then the maximum value of β for which the equation $x^2 - 2\left(\sum_{a \in S} a\right)^2 x + \sum_{a \in S} (a+1)^2 \beta = 0$ has real roots, is _____.

- 28. Let $S = \{1, 2, 3, 5, 7, 10, 11\}$. The number of non-empty subsets of S that have the sum of all elements a multiple of 3, is ______.
- 29. for some $a, b, c, \in N$, let f(x) = ax 3 and $g(x) = x^b + c, x \in R$. If $(fog)^{-1}(x) = \left(\frac{x 7}{2}\right)^{\frac{1}{3}}$, then (fog)(ac) + (gof)(b) is equal to ______.

30. The vertices of a hyperbola H are $(\pm 6,0)$ and its eccentricity is $\frac{\sqrt{5}}{2}$. Let N be the normal to H at a point in the first quadrant and parallel to the line $\sqrt{2}x + y = 2\sqrt{2}$. If *d* is the length of the line segment of N between H and the y-axis then d^2 is equal to ______.