JEE Main – 2023

24th JAN 2023 (Evening Shift)

General Instructions

1. The test is of **3 hours** duration and the maximum marks is **300.**
2. The question paper consists of **3 Parts** (Part I: **Physics**, Part II: **Chemistry**, Part III: **Mathematics**). Each Part has **two** sections (Section 1 & Section 2).
3. **Section 1** contains **20 Multiple Choice Questions**. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE CHOICE** is correct.
4. **Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. You will NOT be allowed to attempt the sixth question. If you wish to attempt any other question apart from the five already attempted, then you will have to delete any one response from the five previously answered and then proceed to answer the new one.**
   The answer to each question should be **rounded off to the nearest integer**.
5. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. inside the examination room/hall.

Marking Scheme

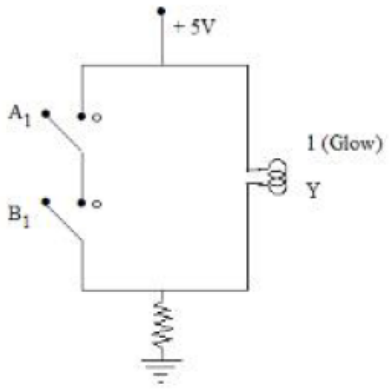
1. **Section – 1:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.
2. **Section – 2:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.

SUBJECT I: PHYSICS	MARKS: 100

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

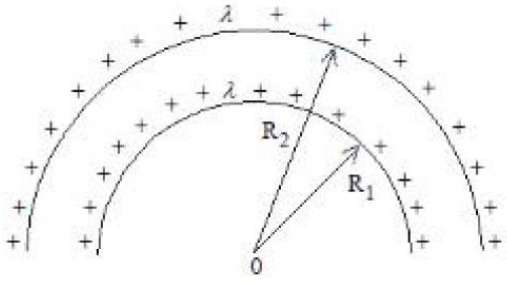
1.



The logic gate equivalent to the given circuit diagram is:

(1) NAND **(2)** AND **(3)** NOR **(4)** OR

2. The electric potential at the centre of two concentric half rings of radii R_1 and R_2, having same linear charge density λ is:



(1) $\frac{2\lambda}{\epsilon_0}$ **(2)** $\frac{\lambda}{2\epsilon_0}$ **(3)** $\frac{\lambda}{4\epsilon_0}$ **(4)** $\frac{\lambda}{\epsilon_0}$

3. The electric field and magnetic field components of an electromagnetic wave going through vacuum is described by:

$E_x = E_0 \sin(kz - \omega t)$

$B_y = B_0 \sin(kz - \omega t)$

Then the correct relation between E_0 and B_0 is given by:

(1) $E_0 B_0 = \omega k$ **(2)** $\omega E_0 = kB_0$ **(3)** $kE_0 = \omega B_0$ **(4)** $E_0 = kB_0$

4. Match List I with List II:

LIST I		LIST II	
A.	AM Broadcast	I.	88 – 108 MHz
B.	FM Broadcast	II.	540 – 1600 kHz
C.	Television	III.	3.7 – 4.2 GHz
D.	Satellite Communication	IV.	54 MHz – 890 MHz

Choose the correct answer from the options given below:

(1) A – I, B – III, C – II, D – IV **(2)** A – II, B – III, C – I, D – IV

(3) A – IV, B – III, C – I, D – II **(4)** A – II, B – I, C – IV, D – III

5. An $\alpha-$particle, a proton and an electron have the same kinetic energy. Which one of the following is correct in case of their de-Broglie wavelength:

(1) $\lambda_a < \lambda_p < \lambda_e$ **(2)** $\lambda_a > \lambda_p < \lambda_e$ **(3)** $\lambda_a = \lambda_p = \lambda_e$ **(4)** $\lambda_a > \lambda_p > \lambda_e$

6. A long solenoid is formed by winding 70 turns cm^{-1}. If 2.0 A current flows, then the magnetic field produced inside the solenoid is _____ $(\mu = 4\pi \times 10^{-7}\ TmA^{-1})$

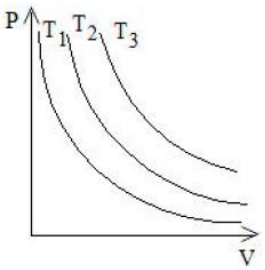
(1) $176 \times 10^{-4}\ T$ **(2)** $88 \times 10^{-4}\ T$ **(3)** $352 \times 10^{-4}\ T$ **(4)** $1232 \times 10^{-4}\ T$

7. A body of mass 200g is tied to a spring of spring constant 12.5 N/m, while the other end of spring is fixed at point O. If the body moves about O in a circular path on a smooth horizontal surface with constant angular speed 5 rad/s. Then the ratio of extension in the spring to its natural length will be:

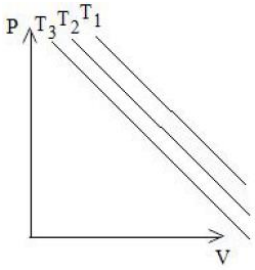
(1) 2 : 5 **(2)** 1 : 1 **(3)** 1 : 2 **(4)** 2 : 3

8. In an Isothermal change, the change in pressure and volume of a gas can be represented for three different temperature; $T_3 > T_2 > T_1$ as:

(1)



(2)



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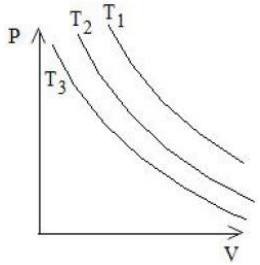

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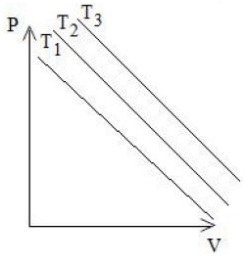


9. Given below are two statements:

Statement I: Acceleration due to earth's gravity decreases as you 'up' or 'down' from earth's surface.

Statement II: Acceleration due to earth's gravity is same at a height '*h*' and depth '*d*' from earth's surface, if *h* = *d*.

In the light of above statements, choose the most appropriate answer form the options given below:

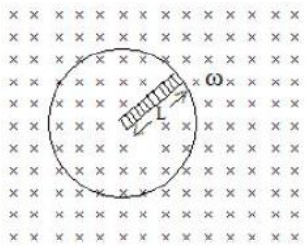
(1) Statement I is incorrect but statement II is correct.

(2) Both Statement I and statement II are correct.

(3) Statement I is correct but statement II is incorrect.

(4) Both Statement I and II are correct.

10. A metallic rod of length 'L' is rotated with an angular speed of 'ω' normal to a uniform magnetic field 'B' about an axis passing through one end of rod as shown in figure. The induced emf will be:



(1) $\frac{1}{4}B^2L\omega$ **(2)** $\frac{1}{4}BL^2\omega$ **(3)** $\frac{1}{2}B^2L^2\omega$ **(4)** $\frac{1}{2}BL^2\omega$

11. If the distance of the earth from Sun is 1.5×10^6 *km*. Then the distance of an imaginary planet from Sun, if its period of revolution is 2.83 years is:

(1) 6×10^7 *km* **(2)** 3×10^6 *km* **(3)** 6×10^6 *km* **(4)** 3×10^7 *km*

12. If two vectors $\vec{P}=\hat{i}+2m\hat{j}+m\hat{k}$ and $\vec{Q}=4\hat{i}-2\hat{j}+m\hat{k}$ are perpendicular to each other. Then, the value of *m* will be:

(1) – 1 **(2)** 2 **(3)** 1 **(4)** 3

13. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: A pendulum clock when taken to Mount Everest becomes fast.

Reason R: The value of g (acceleration due to gravity) is less at Mount Everest than its value on the surface of earth.

In the light of the above statements, choose the most appropriate answer from the options given below:

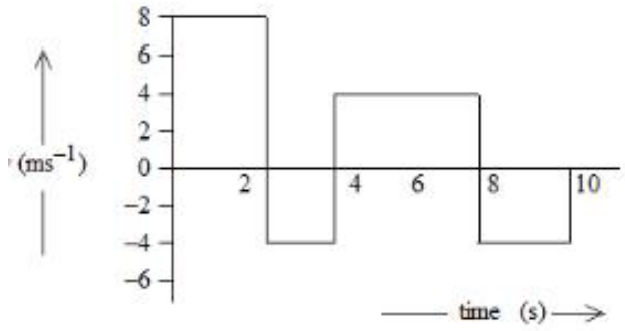
(1) Both A and R are correct and R is the correct explanation of A.

(2) A is correct but R is not correct.

(3) Both A and R are correct but R is not the correct explanation of A.

(4) A is not correct but R is correct.

14. The velocity time graph of a body moving in a straight line is shown in figure.



The ratio of displacement to distance travelled by the body in time 0 to 10s is:

(1) 1 : 1 **(2)** 1 : 2 **(3)** 1 : 3 **(4)** 1 : 4

15. When a beam of while light is allowed to pass through convex lens parallel to principal axis, the different colours of light converge at different point on the principle axis after refraction. This is called:

(1) Scattering **(2)** Chromatic aberration

(3) Spherical aberration **(4)** Polarisation

16. A photon is emitted in transition from n = 4 to n = 1 level in hydrogen atom. The corresponding wavelength for this transition is: (given, $h = 4\times10^{-15}$ eVs)

(1) 94.1 nm **(2)** 974 nm **(3)** 99.3 nm **(4)** 941 nm

17. The frequency (v) of an oscillating liquid drop may depend upon radius (r) of the drop, density (ρ) of liquid and the surface tension (s) of the liquid as: $v=r^a\rho^b s^c$. The values of a, b and c respectively are:

(1) $\left(-\frac{3}{2},\frac{1}{2},\frac{1}{2}\right)$ **(2)** $\left(\frac{3}{2},-\frac{1}{2},\frac{1}{2}\right)$ **(3)** $\left(\frac{3}{2},\frac{1}{2}-\frac{1}{2}\right)$ **(4)** $\left(-\frac{3}{2},-\frac{1}{2},\frac{1}{2}\right)$

18. Let γ_1 be the ratio of molar specific heat at constant pressure and molar specific heat at constant volume of a monoatomic gas and γ_2 be the similar ratio of diatomic gas. Considering the diatomic gas molecule as a rigid rotator, the ratio, $\frac{\gamma_1}{\gamma_2}$ is:

(1) $\frac{35}{27}$ **(2)** $\frac{21}{25}$ **(3)** $\frac{27}{35}$ **(4)** $\frac{25}{21}$

19. A cell of emf 90V is connected across series combination of two resistors each of 100Ω resistance. A voltmeter of resistance 400Ω is used to measure the potential difference across each resistor. The reading of the voltmeter will be:

(1) $40\ V$ **(2)** $90\ V$ **(3)** $45\ V$ **(4)** $80\ V$

20. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.
Assertion A: Steel is used in the construction of building and bridges.
Reason R: Steel is more elastic and its elastic limit is high.
In the light of above statements, choose the most appropriate answer from the options given below

(1) A is not correct but R is correct.
(2) A is correct but R is not correct.
(3) Both A and R are correct but R is not the correct explanation of A.
(4) Both A and R are correct and R is the correct explanation of A.

SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be **rounded off to the nearest integer**.

21. A convex lens of refractive index 1.5 and focal length 18*cm* in air is immersed in water. The change in focal length of the lens will be _____ *cm*.

(Given refractive index of water $=\frac{4}{3}$)

22. A single turn current loop in the shape of a right angle triangle with sides 5 *cm*, 12 *cm*, 13 *cm* is carrying a current of 2A. The loop is in a uniform magnetic field of magnitude 0.75 T whose direction is parallel to the current in the 13 *cm* side of the loop. The magnitude of the magnetic force of the 5 *cm* side will be $\frac{x}{130}N$. The value of *x* is _____ .

23. A mass *m* attached to free end of a spring executes SHM with a period of 1s. If the mass is increased by 3 kg the period of oscillation increases by one second, the value of mass *m* is _____ kg.

24. The energy released per fission of nucleus of ^{240}X is 200 MeV. The energy released if all the atoms in 120g of pure ^{240}X undergo fission is _____ $\times 10^{25}$ MeV.

(Given $N_A = 6\times10^{23}$)

25. A uniform solid cylinder with radius R and length L as moment of inertial I_1, about the axis of the cylinder. A concentric solid cylinder of radius $R'=\frac{R}{2}$ and length $L'=\frac{L}{2}$ is carved out of the original cylinder. If I_2 is the moment of inertia of the carved out portion of the cylinder then $\frac{I_1}{I_2}$ = _____ .

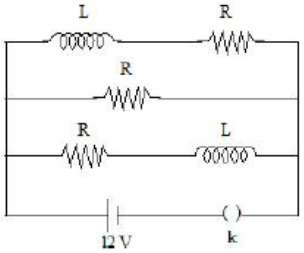
(Both I_1 and I_2 are about the axis of the cylinder)

26. A body of mass 1*kg* begins to move under the action of a time dependent force $\vec{F}=\left(t\hat{i}+3t^2\hat{j}\right)$ N, where $\hat{i}$ and $\hat{j}$ are the unit vectors along x and y axis. The power developed by above force, at the time *t* = 2*s*, will be _____ W.

27. If a copper wire is stretched to increase its length by 20%. The percentage increase in resistance of the wire is _____ %.

28. A spherical ball of radius 1 mm and density 10.5 g/cc is dropped in glycerin of coefficient of viscosity 9.8 poise and density 1.5 g/cc. Viscous force on the ball when it attains constant velocity is: 3696×10^{-x} N. The value of *x* is: (Given, g = 9.8 m/s^2 and $\pi=\frac{22}{7}$)

29. Three identical resistors with resistance R = 12Ω and two identical inductors with self inductance L = 5mH are connected to an ideal battery with emf of 12 V as shown in figure. The current through the battery long after the switch has been closed will be _____ A.



30. A parallel plate capacitor with air between the plate has a capacitance of 15pF. The separation between the plate becomes twice and the space between them if filled with a medium of dielectric constant 3.5. Then the capacitance becomes $\frac{x}{4}$ pF. The value of x is _____ .

SUBJECT II: CHEMISTRY	MARKS: 100

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. What is the number of unpaired electron(s) in the highest occupied molecular orbital of the following species: N_2 ; N_2^+; O_2 ; O_2^+ ?

(1) 2, 1, 2, 1 **(2)** 0, 1, 0, 1 **(3)** 0, 1, 2, 1 **(4)** 2, 1, 0, 1

2. The number of s-electrons present in an ion with 55 protons in its unipositive state is:

(1) 10 **(2)** 9 **(3)** 12 **(4)** 8

3. $K_2Cr_2O_7$ paper acidified with dilute H_2SO_4 turns green when exposed to:

(1) Sulphur dioxide **(2)** Carbon dioxide

(3) Sulphur trioxide **(4)** Hydrogen sulphide

4. Which of the following cannot be explained by crystal field theory?

(1) Stability of metal complexes

(2) Magnetic properties of transition metal complexes

(3) The order of spectrochemical series

(4) Colour of metal complexes

5. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: Beryllium has less negative value of reduction potential compared to the other alkaline earth metals.

Reason R: Beryllium has large hydration energy due to small size of Be^{2+} but relatively large value of atomization enthalpy.

In the light of above statements, choose the most appropriate answer from the options given below:

(1) A is correct but R is not correct.

(2) Both A and R are correct but R is not the correct explanation of A.

(3) A is not correct but R is correct.

(4) Both A and R are correct and R is the correct explanation of A.

6. In which of the following reactions the hydrogen peroxide acts as a reducing agent?

(1) $Mn_2 + + H_2O_2 \rightarrow Mn^{4+} + 2OH^-$

(2) $HOCI + H_2O_2 \rightarrow H_3O^+ + CI^- + O_2$

(3) $PbS + 4H_2O_2 \rightarrow PbSO_4 + 4H_2O$

(4) $2Fe^{2+} + H_2O_2 \rightarrow 2Fe^{3+} + 2OH^-$

7. The metal which is extracted by oxidation and subsequent reduction from its ore is:

(1) Ag **(2)** AI **(3)** Fe **(4)** Cu

8. Identify the correct statements about alkali metals.

(A) The order of standard reduction potential $(M^+ | M)$ for alkali metal ions is Na > Rb > Li.

(B) CsI is highly soluble in water.

(C) Lithium carbonate is highly stable to heat.

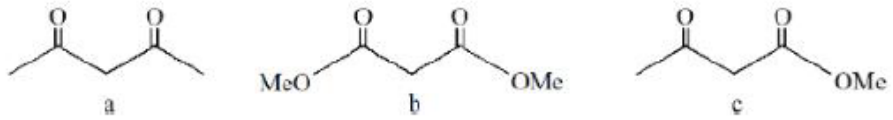
(D) Potassium dissolved in concentrated liquid ammonia is blue in colour and paramagnetic.

(E) All the alkali metal hydrides are ionic solids.

Choose the correct answer from the options given below:

(1) A, B, D only **(2)** C and E only **(3)** A, B and E only **(4)** A and E only

9. Which will undergo deprotonation most readily in basic medium?



(1) c only **(2)** Both a and c **(3)** a only **(4)** b only

10. Which one amongst the following are good oxidizing agents?

(A) Sm^{2+} **(B)** Ce^{2+} **(C)** Ce^{4+} **(D)** Tb^{4+}

Choose the most appropriate answer from the options given below:

(1) A and B only **(2)** D only **(3)** C only **(4)** C and D only

11. Match List I with List II:

LIST I Type		**LIST II Name**	
A.	Antifertility drug	I.	Norethindrone
B.	Tranquilizer	II.	Meprobomate
C.	Antihistamine	III.	Seldane
D.	Abtibiotic	IV.	Ampicillin

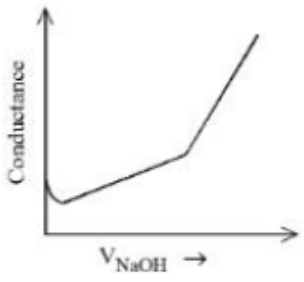
Choose the correct answer from the options given below:

(1) A – I, B – III, C – II, D – IV **(2)** A – IV, B – III, C – II, D – I

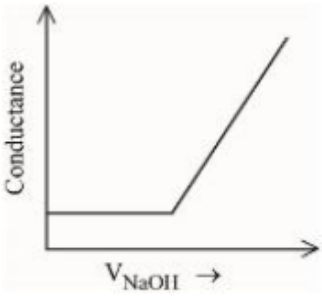
(3) A – II, B – I, C – III, D – IV **(4)** A – I, B – II, C – III, D – IV

12. Choose the correct representation of conductometric titration of benzoic acid vs sodium hydroxide.

(1)



(2)



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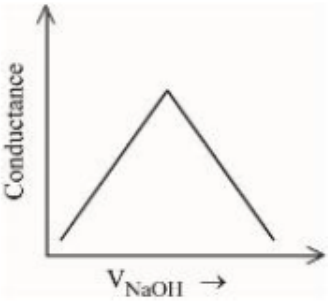

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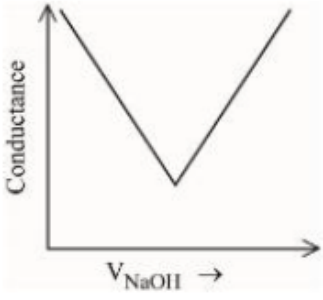


13. The hybridization and magnetic behaviour of cobalt ion in $\left[Co(NH_3)_6\right]^{3+}$ complex, respectively is:

(1) sp^3d^2 and paramagnetic **(2)** sp^3d^2 and diamagnetic

(3) d^2sp^3 and diamagnetic **(4)** d^2sp^3 and paramagnetic

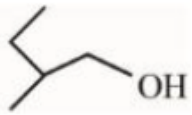
14. A student has studied the decomposition of a gas AB_3 at 25^oC. He obtained the following data.

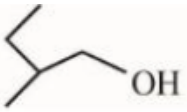
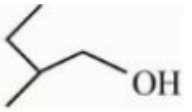
p (mm Hg)	50	100	200	400
Relative $t_{1/2}$ (s)	4	2	1	0.5

The order of the reaction is:

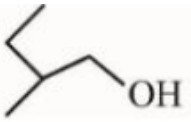
(1) 0.5 **(2)** 0 (zero) **(3)** 1 **(4)** 2

15. Find out the major products from the following reactions.

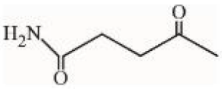
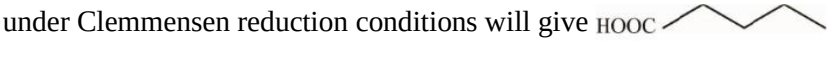
(1) A =  , B = 

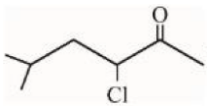
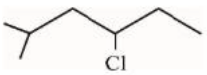
(2) A =  , B = 

(3) A =  , B = 

(4) A =  , B = 

16. Given below are two statements:

Statement I:  under Clemmensen reduction conditions will give 

Statement II:  under Clemmensen reduction conditions will give 

(1) Statement I is false but Statement II is true.

(2) Both Statement I and Statement II are false.

(3) Statement I is true but Statement II is false.

(4) Both statement I and Statement II are true.

17. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: Benzene is more stable than hypothetical cyclohexatriene

Reason R: The delocalized π electron cloud is attracted more strongly by nuclei of carbon atoms.

In the light of above statements, choose the most appropriate answer from the options given below:

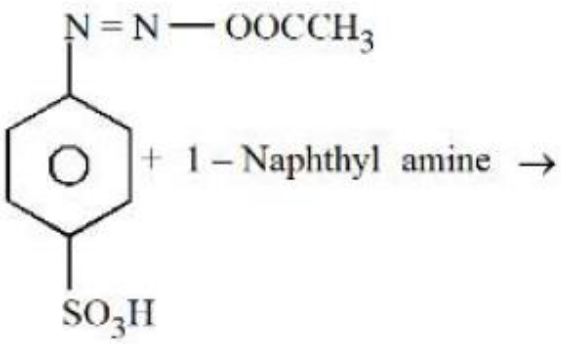
(1) Both A and R are correct but R is not the correct explanation of A.

(2) Both A and R are correct and R is the correct explanation of A.

(3) A is false but R is true.

(4) A is true but R is false.

18. Choose the correct colour of the product for the following reaction.



(1) Yellow **(2)** White **(3)** Red **(4)** Blue

19. Given below are two statements.

Statement I: Pure Aniline and other arylamines are usually colourless.

Statement II: Arylamines get coloured on storage due to atmospheric reduction

In the light of above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are correct

(2) Both Statement I and Statement II are incorrect

(3) Statement I is incorrect but Statement II is correct

(4) Statement I is correct but Statement II is incorrect

20. Correct statement is:

(1) An average human being consumes 100 times more air than food

(2) An average human being consumes equal amount of food and air

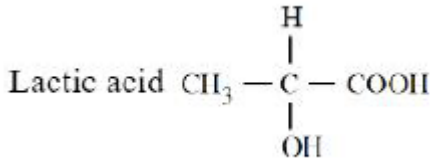
(3) An average human being consumes nearly 15 times more air than food

(4) An average human being consumes more food than air

SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be **rounded off to the nearest integer**.

21. If the pKa of lactic acid is 5, then the pH of 0.005 M calcium lactate solution at 25° C is _____ $\times 10^{-1}$ (Nearest integer)

Lactic acid 

22. The total pressure observed by mixing two liquids A and B is 350 mm Hg when their mole fractions are 0.7 and 0.3 respectively.

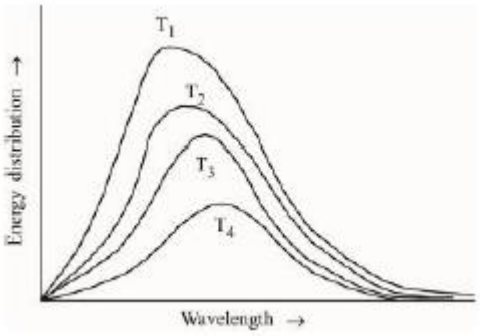
The total pressure becomes 410 mm Hg if the mole fractions are changed to 0.2 and 0.8 respectively for A and B. The vapour pressure of pure A is _____ mm Hg. (Nearest integer)

Consider the liquids and solutions behave ideally.

23. The number of units, which are used to express concentration of solutions from the following is _____

Mass percent, Mole, Mole fraction, Molarity, ppm, Molality

24. Following figure shows spectrum of an ideal black body at four different temperatures. The number of correct statement/s from the following is _____ .



(A) $T_4 > T_3 > T_2 > T_1$

(B) The black body consists of particles performing simple harmonic motion.

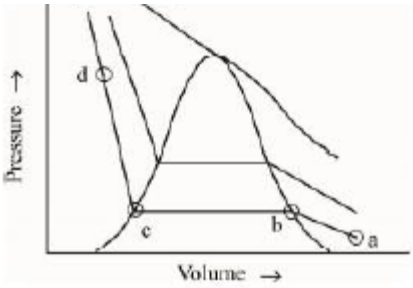
(C) The peak of the spectrum shifts to shorter wavelength as temperature increases.

(D) $\frac{T_1}{\nu_1} = \frac{T_2}{\nu_2} = \frac{T_3}{\nu_3} \neq$ constant

(D) The given spectrum could be explained using quantisation of energy.

25. Maximum number of isomeric monochloro derivatives which can be obtained from 2, 2, 5, 5-tetramethylhexane by chlorination is _____ .

26. The number of statement/s, which are correct with respect to the compression of carbon dioxide from point (a) in the Andrews isotherm from the following is _____ .



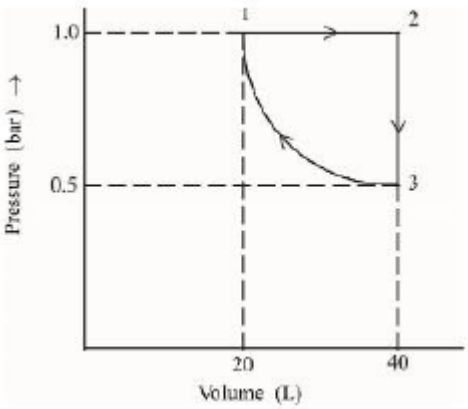
(A) Carbon dioxide remains as a gas upto point (b)

(B) Liquid carbon dioxide appears at point (c)

(C) Liquid and gaseous carbon dioxide coexist between points (b) and (c)

(D) As the volume decreases from (b) to (c), the amount of liquid decreases

27. One mole of an ideal monoatomic gas is subjected to changes as shown in the graph. The magnitude of the work done (by the system or on the system) is _____ J. (nearest integer)



Given: log 2 = 0.3

In 10 = 2.3

28. Total number of tripeptides possible by mixing of valine and proline is _____ .

29. Sum of π – bond present in peroxodisulphuric acid and pyrosulphuric acid is:

30. The number of statement/s which are the characteristics of physisorption is _____ .

(A) It is highly specific in nature

(B) Enthalpy of adsorption is high

(C) It decreases with increase in temperature

(D) It results into unimolecular layer

(E) No activation energy is needed

SUBJECT III: MATHEMATICS	MARKS: 100

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. If $f(x)=\frac{2^{2x}}{2^{2x}+2}$, $x \in R$, then $f\left(\frac{1}{2023}\right)+f\left(\frac{2}{2023}\right)+...+f\left(\frac{2022}{2023}\right)$ is equal to:

(1) 2011 **(2)** 2010 **(3)** 1010 **(4)** 1011

2. $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}dx$ is equal to:

(1) $\frac{\pi}{2}$ **(2)** 2π **(3)** $\frac{\pi}{3}$ **(4)** $\frac{\pi}{6}$

3. The equations of the sides AB and AC of a triangle ABC are $(\lambda+1)x+\lambda y=4$ and $\lambda x+(1-\lambda)y+\lambda=0$ respectively. Its vertex A is on the *y*-axis and its orthocentre is (1, 2). The length of the tangent from the point C to the part of the parabola $y^2=6x$ in the first quadrant is:

(1) 4 **(2)** $2\sqrt{2}$ **(3)** $\sqrt{6}$ **(4)** 2

4. The number of square matrices of order 5 with entries from the set {0, 1}, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is:

(1) 120 **(2)** 125 **(3)** 150 **(4)** 225

5. Let the plane containing the line of intersection of the planes $P1: x+(\lambda+4)y+z=1$ and $P2: 2x+y+z=2$ pass through the points (0, 1, 0) and (1, 0, 1). Then the distance of the point $(2\lambda, \lambda, -\lambda)$ from the plane P2 is:

(1) $3\sqrt{6}$ **(2)** $5\sqrt{6}$ **(3)** $4\sqrt{6}$ **(4)** $2\sqrt{6}$

6. Let $y=y(x)$ be the solution of the differential equation $(x^2-3y^2)\,dx+3xy\,dy=0,\ y(1)=1.$

Then $6y^2(e)$ is equal to:

(1) $\frac{3}{2}e^2$ **(2)** $2e^2$ **(3)** $3e^2$ **(4)** e^2

7. Let A be a 3 × 3 matrix such that $|\text{adj}(\text{adj}(\text{adj A}))|=12^4$ Then $|A^{-1}\,\text{adj A}|$ is equal to:

(1) 1 **(2)** 12 **(3)** $2\sqrt{3}$ **(4)** $\sqrt{6}$

8. The locus of the mid points of the chords of the circle $C_1 : (x-4)^2 + (y-5)^2 = 4$ which subtend an angle θ_i at the centre of the circle C_1, is a circle radius r_i. If $\theta_1 = \frac{\pi}{3}, \theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$, then θ_2 is equal to:

(1) $\frac{\pi}{4}$ **(2)** $\frac{\pi}{6}$ **(3)** $\frac{\pi}{2}$ **(4)** $\frac{3\pi}{4}$

9. Let $\vec{\alpha} = 4\hat{i} + 3\hat{j} + 5\hat{k}$ and $\vec{\beta} = \hat{i} + 2\hat{j} - 4\hat{k}$. Let $\vec{\beta}_1$ be parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ be perpendicular to $\vec{\alpha}$. If $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, then the value of $5\vec{\beta}_2 \cdot (\hat{i}, \hat{j}, \hat{k})$ is:

(1) 7 **(2)** 9 **(3)** 11 **(4)** 6

10. Let $f(x)$ be a function such that $f(x+y) = f(x).f(y)$ for all $x, y \in N$. If $f(1) = 3$ and $\sum_{k=1}^{n} f(k) = 3279$, then the value of *n* is:

(1) 6 **(2)** 7 **(3)** 8 **(4)** 9

11. The number of integers, greater than 7000 that can be formed, using the digits 3, 5, 6, 7, 8 without repetition, is:

(1) 120 **(2)** 48 **(3)** 168 **(4)** 220

12. The set of all values of *a* for which $\lim_{x \to a} ([x-5] - [2x+2]) = 0$, where $[\propto]$ denotes the greatest integer less than or equal to $\propto$ is equal to:

(1) $[-7.5, -6.5]$ **(2)** $[-7.5, -6.5)$

(3) $(-7.5, -6.5)$ **(4)** $[-7.5, -6.5)$

13. If $f(x) = x^3 - x^2 f'(1) + xf''(2) - f'''(3), x \in R$, then:

(1) $f(3) - f(2) = f(1)$ **(2)** $2f(0) - f(1) + f(3) = f(2)$

(3) $f(1) + f(2) + f(3) = f(0)$ **(4)** $3f(1) + f(2) = f(3)$

14. If the foot of the perpendicular drawn from (1, 9, 7) to the line passing through the point (3, 2, 1) and parallel to the planes $x + 2y + z = 0$ and $3y - z = 3$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to:

(1) – 1 **(2)** 5 **(3)** 1 **(4)** 3

15. If the system of equations:

$x + 2y + 3z = 3$

$4x + 3y - 4z = 4$

$8x + 4y - \lambda z = 9 + \mu$

Has infinitely many solutions, then the ordered pair (λ, μ) is equal to:

(1) $\left(-\frac{72}{5}, \frac{21}{5}\right)$ **(2)** $\left(-\frac{72}{5}, -\frac{21}{5}\right)$

(3) $\left(\frac{72}{5}, -\frac{21}{5}\right)$ **(4)** $\left(\frac{72}{5}, \frac{21}{5}\right)$

16. The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is:

(1) $-\frac{1}{2}\left(1-i\sqrt{3}\right)$ **(2)** $-\frac{1}{2}\left(\sqrt{3}-i\right)$

(3) $\frac{1}{2}\left(1-i\sqrt{3}\right)$ **(4)** $\frac{1}{2}\left(\sqrt{3}+i\right)$

17. Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6,$ be in A. P. and $a_1 + a_3 = 10.$ If the mean of these six numbers is $\frac{19}{2}$ and their variance is σ^2, then $8\sigma^2$ is equal to:

(1) 200 **(2)** 105 **(3)** 210 **(4)** 220

18. The number of real solutions of the equation $3\left(x^2+\frac{1}{x^2}\right)-2\left(x+\frac{1}{x}\right)+5=0,$ is:

(1) 4 **(2)** 3 **(3)** 2 **(4)** 0

19. If $\left({}^{30}C_1\right)^2+2\left({}^{30}C_2\right)^2+3\left({}^{30}C_3\right)^2+...+30\left({}^{30}C_{30}\right)^2=\frac{\alpha 60!}{(30!)^2}$ then α is equal to:

(1) 10 **(2)** 15 **(3)** 30 **(4)** 60

20. Let *p* and *q* be two statements. The $\sim(P\wedge(p\Rightarrow\sim q))$ is equivalent to:

(1) $p\vee(p\wedge(\sim q))$ **(2)** $p\vee(p\wedge q)$

(3) $p\vee((\sim p)\wedge q)$ **(4)** $(\sim p)\vee q$

SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be **rounded off to the nearest integer**.

21. Let f be a differentiable function defined on $\left[0,\frac{\pi}{2}\right]$ such that $f(x)>0$ and

$f(x)+\int_0^x f(t)\sqrt{1-(\log_e f(t))^2}\,dt=e, \forall x\in\left[0,\frac{\pi}{2}\right]$. Then $\left(6\log_e f\left(\frac{\pi}{6}\right)\right)^2$ is equal to _____ .

22. Let $S=\{\theta\in[0,2\pi):\tan(\pi\cos\theta)+\tan(\pi\sin\theta)=0\}$.

Then $\sum_{\theta\in S}\sin^2\left(\theta+\frac{\pi}{4}\right)$ is equal to _____ .

23. If $\frac{1^3+2^3+3^3+\ldots \text{up to } n \text{ terms}}{1.3+2.5+3.7+\text{up to } n \text{ terms}}=\frac{9}{5}$, then the value of of n is:

24. Let the sum of the coefficients of the first three terms in the expansion of $\left(x-\frac{3}{x^2}\right)^n$, $x\neq 0. n\in N$, be 376. Then the coefficient of x^4 is _____ .

25. The equations of the sides AB, BC and CA of a triangle ABC are: $2x+y=0, x+py=21a\ (a\neq 0)$ and $x-y=3$ respectively. Let $P(2,a)$ be the centroid of ΔABC. The $(BC)^2$ is equal to:

26. The minimum number of elements that must be added to the relation $R=\{(a,b),(b,c),(b,d)\}$ on the set $\{a,b,c,d\}$ so that it is an equivalence relation, is _____ .

27. If the shortest between the lines:

$$\frac{x+\sqrt{6}}{2}=\frac{y-\sqrt{6}}{3}=\frac{z-\sqrt{6}}{4} \text{ and } \frac{x-\lambda}{3}=\frac{y-2\sqrt{6}}{4}=\frac{z+2\sqrt{6}}{5}$$

is 6, then the square of sum of all possible values of λ is:

28. If the area of the region bounded by the curves $y^2-2y=-x, x+y=0$ is A, then 8 A is equal to:

29. Let $\vec{a}=\hat{i}+2\hat{j}+\lambda\hat{k}, \vec{b}=3\hat{i}-5\hat{j}-\lambda\hat{k}, \vec{a}.\vec{c}=7.2\,\vec{b}.\vec{c}+43=0, \vec{a}\times\vec{c}=\vec{b}\times\vec{c}$. Then $|\vec{a}.\vec{b}|$ is equal to:

30. Three urns A, B and C contain 4 red, 6 black; 5 red, 5 black; and λ red, 4 black balls respectively. One of the urns is selected at random and a ball is drawn. If the ball drawn is red and the probability that is drawn from urn C is 0.4 then the square of the length of the side of the largest equilateral triangle, inscribed in the parabola $y^2=\lambda x$ with one vertex at the vertex of the parabola, is: