JEE Main – 2023

1st FEB 2023 (Evening Shift)

General Instructions

1. The test is of **3 hours** duration and the maximum marks is **300.**
2. The question paper consists of **3 Parts** (Part I: **Physics**, Part II: **Chemistry**, Part III: **Mathematics**). Each Part has **two** sections (Section 1 & Section 2).
3. **Section 1** contains **20 Multiple Choice Questions**. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE CHOICE** is correct.
4. **Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. You will NOT be allowed to attempt the sixth question. If you wish to attempt any other question apart from the five already attempted, then you will have to delete any one response from the five previously answered and then proceed to answer the new one.**
   The answer to each question should be **rounded off to the nearest integer**.
5. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. inside the examination room/hall.

Marking Scheme

1. **Section – 1:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.
2. **Section – 2:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.

SUBJECT I: PHYSICS	MARKS: 100

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. A coil is placed in magnetic field such that plane of coil is perpendicular to the direction of magnetic field. The magnetic flux through a coil be changed.

(A) By changing the magnitude of the magnetic field within the coil.

(B) By changing the area of coil within the magnetic field.

(C) By changing the angle between the direction of magnetic field and the plane of the coil.

(D) By reversing the magnetic field direction abruptly without changing its magnitude.

(1) A and C only **(2)** A and B only

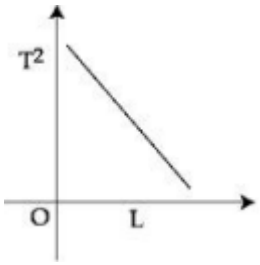
(3) A, B and D only **(4)** A, B and C only

2. Equivalent resistance between the adjacent corners of a regular *n*-sided polygon of uniform wire of resistance *R* would be:

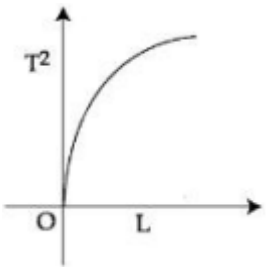
(1) $\frac{n^2R}{n-1}$ **(2)** $\frac{(n-1)R}{n}$ **(3)** $\frac{(n-1)R}{n^2}$ **(4)** $\frac{(n-1)R}{(2n-1)}$

3. Choose the correct length (*L*) versus square of time period (T^2) graph for a simple pendulum executing simple harmonic motion.

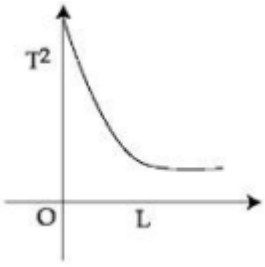
(1)



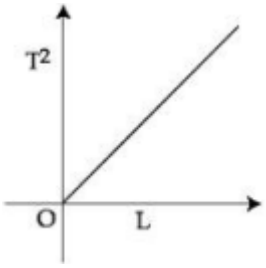
(2)



(3)



(4)



4. The ratio of average electric energy density and total average energy density of electromagnetic wave is:

(1) 1 **(2)** 2 **(3)** 3 **(4)** 1/2

5. The escape velocities of two planets *A* and *B* are in the ratio 1 : 2. If the ratio of their radii respectively is 1 : 3, then the ratio of acceleration due to gravity of planet *A* to the acceleration of gravity of planet *B* will be:

(1) $\frac{3}{4}$ **(2)** $\frac{3}{2}$ **(3)** $\frac{2}{3}$ **(4)** $\frac{4}{3}$

6. Choose the correct statement about Zener diode:

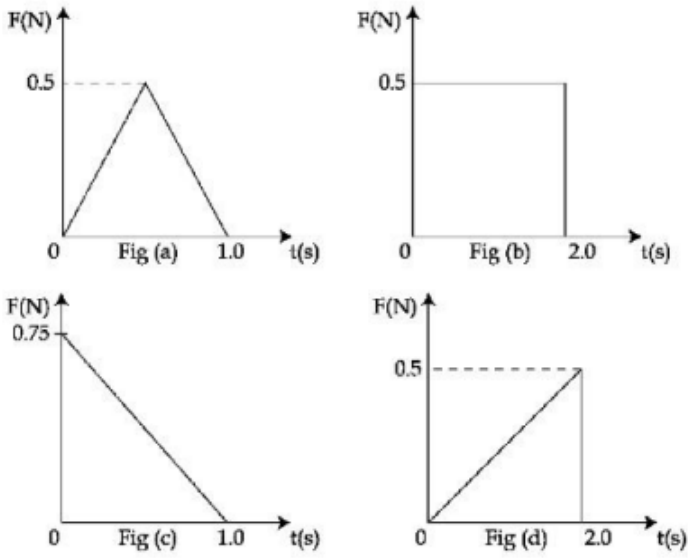
(1) It works as a voltage regulator only in forward bias

(2) It works as a voltage regulator in both forward and reverse bias

(3) It works as a voltage regulator in forward bias and behaves like simple *pn* junction diode in reverse bias

(4) It works as a voltage regulator in reverse bias and behaves like simple *pn* junction in forward bias

7. Figure (a), (b), (c) and (d) show variation of force with time:



The impulse is highest in figure:

(1) Figure (c) **(2)** Figure (a) **(3)** Figure (d) **(4)** Figure (b)

8. Two objects *A* and *B* are placed at 15 *cm* and 25 *cm* from the pole in front of a concave mirror having radius of curvature 40 *cm*. The distance between images formed by the mirror is:

(1) 60 *cm* **(2)** 40 *cm* **(3)** 100 *cm* **(4)** 160 *cm*

9. As shown in the figure, a long straight conductor with semicircular are of radius $\frac{\pi}{10}m$ is carrying current $I = 3A$. The magnitude of the magnetic field, at the centre *O* of the arc is:

(The permeability of the vacuum $= 4\pi \times 10^{-7} NA^{-2}$).

(1) $1\,\mu T$ **(2)** $3\,\mu T$

(3) $4\,\mu T$ **(4)** $6\,\mu T$

10. Given below are two statements: One is labelled as **Assertion A** and the other is labeled as **Reason (R)**.

Assertion A: Two metallic spheres are charged to the same potential. One of them is hollow and another is solid, and both have the same radii. Solid sphere will have lower charge than the hollow one.

Reason R: Capacitance of metallic spheres depend on the radii of spheres.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) Both A and R are true but R is not the correct explanation of A

(2) A is true but R is false

(3) Both A and R are true and R is the correct explanation of A

(4) A is false but R is true

11. Given below are two statements: One is labeled as **Assertion A** and the other is labeled as **Reason R.**

Assertion A: For measuring the potential difference across a resistance of $600\,\Omega$, the voltmeter with resistance $1000\,\Omega$ will be preferred over voltmeter with resistance $4000\,\Omega$.

Reason R: Voltmeter with higher resistance will draw smaller current than voltmeter with lower resistance.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Both A and R are correct but R is not the correct explanation A

(2) A is not correct but R is correct

(3) A is correct but R is not correct

(4) Both A and R are correct and R is the correct explanation of A

12. A Carnot engine operating between two reservoirs has efficiency 1/3. When the temperature of cold reservoir raised by *x*, its efficiency decreases to 1/6. The value of *x*, if the temperature of hot reservoir is 99°C, will be:

(1) 33 *K* **(2)** 62 *K* **(3)** 66 *K* **(4)** 16.5 *K*

13. The threshold frequency of a metal is f_0. When the light of frequency $2f_0$ is incident on the metal plate, the maximum velocity of photoelectrons is v_1. When the frequency of incident radiation is increased to $5f_0$, the maximum velocity of photoelectrons emitted is v_2. The ratio of v_1 and v_2 is:

(1) $\frac{v_1}{v_2}=\frac{1}{4}$ **(2)** $\frac{v_1}{v_2}=\frac{1}{16}$ **(3)** $\frac{v_1}{v_2}=\frac{1}{8}$ **(4)** $\frac{v_1}{v_2}=\frac{1}{2}$

14. For a body projected at an angle with the horizontal from the ground, choose the correct statement.

(1) The kinetic energy (K.E.) is zero at the highest point of projectile motion

(2) Gravitational potential energy is maximum at the highest point

(3) The horizontal component of velocity is zero at the highest point

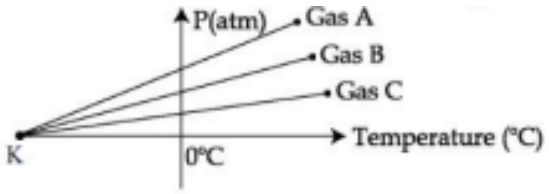
(4) The vertical component of momentum is maximum at the highest point

15. If the velocity of light *c*, universal gravitational constant *G* and Planck's constant *h* are chosen as fundamental quantities. The dimensions of mass in the new system is:

(1) $[h^{1/2}\,c^{1/2}\,G^{-1/2}]$ **(2)** $[h^{1}\,c^{1}\,G^{-1}]$

(3) $[h^{1/2}\,c^{-1/2}\,G^{1}]$ **(4)** $[h^{-1/2}\,c^{1/2}\,G^{1/2}]$

16. For three low density gases *A*, *B*, *C* pressure versus temperature graphs are plotted while keeping them at constant volume, as shown in the figure:

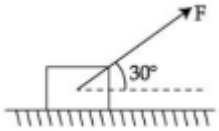


The temperature corresponding to the point '*K*' is:

(1) –100°*C* **(2)** –373°*C* **(3)** –273°*C* **(4)** –40°*C*

17. As shown in the figure a block of mass 10 *kg* lying on a horizontal surface is pulled by a force *F* acting at an angle 30°, with horizontal. For $\mu_s = 0.25,$ the block will just start to move for the value of *F*:

[Given $g = 10\, ms^{-2}$]



(1) 20 *N* **(2)** 35.7 *N* **(3)** 33.3 *N* **(4)** 25.2 *N*

18. In an amplitude modulation, a modulating signal having amplitude of X V is superimposed with a carrier signal of amplitude *YV* in first case. Then, in second case, the same modulating signal is superimposed with different carrier signal of amplitude 2Y V. The ratio of modulation index in the two cases respectively will be:

(1) 1 : 1 **(2)** 1 : 2 **(3)** 2 : 1 **(4)** 4 : 1

19. The Young's modulus of a steel wire of length 6*m* and cross-sectional area $3\, mm^2,$ is $2\times10^{11}\, N/m^2.$ The wire is suspended from its support on a given planet. A block of mass 4*kg* is attached to the free end of the wire. The acceleration due to gravity on the planet is 1/4 of its value on the earth. The elongation of wire is: (Take *g* on the earth $= 10\, m/s^2$)

(1) 1 *mm* **(2)** 0.1 *cm* **(3)** 1 *cm* **(4)** 0.1 *mm*

20. An electron of a hydrogen like atom, having *Z* = 4, jumps from 4th energy state to 2nd energy state. The energy released in this process, will be: (Given *Rch* = 13.6 eV)

Where *R* = Rydberg constant

c = Speed of light in vacuum

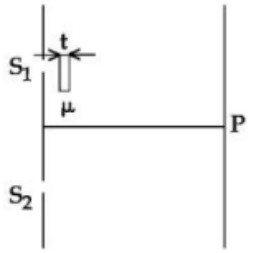
h = Plank's constant

(1) 10.5 *eV* **(2)** 40.8 *eV* **(3)** 3.4 *eV* **(4)** 13.6 *eV*

SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be **rounded off to the nearest integer**.

21. As shown in the figure, in Young's double slit experiment, a thin plate of thickness $t = 10\,\mu m$ and refractive index $\mu = 1.2$ is inserted infront of slit S_1. The experiment is conducted in air $(\mu = 1)$ and uses a monochromatic light of wavelength $\lambda = 500\,nm$. Due to the insertion of the plate, central maxima is shifted by a distance of $x\beta_0, \beta_0$ is the fringe-width before the insertion of plate. The value of the x is _____.



22. Nucleus A having $Z = 17$ and equal number of protons and neutrons has 1.2 *MeV* binding energy per nucleon.

Another nucleus B of $Z = 12$ has total 26 nucleon and 1.8 *MeV* binding energy per nucleons. The difference of binding energy of B and A will be _____ *MeV*.

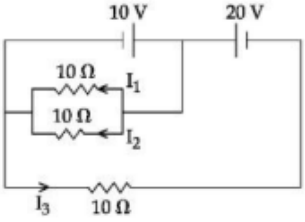
23. For a train engine moving with speed of $20\,ms^{-1}$, the driver must apply brakes at a distance of 500 m before the station for the train to come to rest at the station. If the brakes were applied at half of this distance, the train engine would cross the station with speed $\sqrt{x}\,ms^{-1}$. The value of x is _____. (Assume same retardation is produced by brakes)

24. A force $F = (5 + 3y^2)$ acts on a particle in the y-direction, where F is in newton and y is in meter. The work done by the force during a displacement from $y = 2m$ to $y = 5m$ is _____ J.

25. The surface of water in a water tank of cross section area $750\,cm^2$ on the top of a house is h m above the tap level. The speed of water coming out through the tap of cross section area $500\,mm^2$ is 30 cm/s. At that instant, $\frac{dh}{dt}$ is $x \times 10^{-3} m/s$. The value of x will be _____.

26. Moment of inertia of a disc of mass M and radius 'R' about any of its diameter is $MR^2/4$. The moment of inertia of this disc about an axis normal to the disc and passing through a point on its edge will be, $\frac{x}{2}MR^2$. The value of x is _____.

27. In the given circuit, the value of $\left|\frac{I_1 + I_3}{I_2}\right|$ is _____.



28. A cubical volume is bounded by the surfaces $x = 0$, $x = a$, $y = 0$, $y = a$, $z = 0$, $z = a$. The electric field in the region is given by $\vec{E} = E_0 x \hat{i}$. Where $E_0 = 4 \times 10^4 NC^{-1}m^{-1}$. If $a = 2$ *cm*, the charge contained in the cubical volume is $Q \times 10^{-14} C$. The value of Q is _____.

29. A square shaped coil of area $70\, cm^2$ having 600 turns rotates in a magnetic field of $0.4\, wbm^{-2}$, about an axis which is parallel to one of the side of the coil and perpendicular to the direction of field. If the coil completes 500 revolution in a minute, the instantaneous emf when the plane of the coil is inclined at 60° with the field, will be _____ *V*. (Take : $\pi = 22/7$)

30. A block is fastened to a horizontal spring. The block is pulled to a distance $x = 10$ *cm* from its equilibrium position (at $x = 0$) on a frictionless surface from rest. The energy of the block at $x = 5$ *cm* is 0.25 *J*. The spring constant of the spring is _____ Nm^{-1}.

SUBJECT II: CHEMISTRY	MARKS: 100

SECTION-1

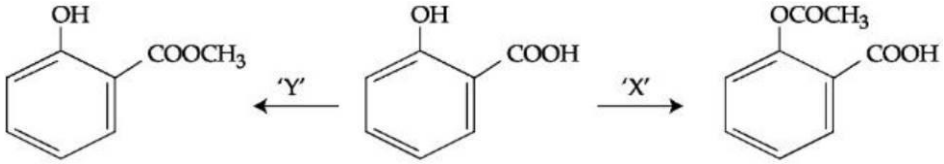
This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. For electron gain enthalpies of the elements denoted as $\Delta_{eg}H$, the incorrect option is:

(1) $\Delta_{eg}H\,(I) < \Delta_{eg}H\,(At)$ **(2)** $\Delta_{eg}H\,(Cl) < \Delta_{eg}H\,(F)$

(3) $\Delta_{eg}H\,(Se) < \Delta_{eg}H\,(S)$ **(4)** $\Delta_{eg}H\,(Te) < \Delta_{eg}H\,(Po)$

2. In a reaction,



reagents 'X' and 'Y' respectively are:

(1) $(CH_3CO)_2O/H^+$ and $(CH_3CO)_2O/H^+$

(2) $CH_3OH/H^+, \Delta$ and $CH_3OH/H^+, \Delta$

(3) $(CH_3CO)_2O/H^+$, and $CH_3OH/H^+, \Delta$

(4) $CH_3OH/H^+, \Delta$ and $(CH_3CO)_2O/H^+$

3. Given below are two statements: one is labeled as **Assertion (A)** and the other is labeled as **Reason (R).**

Assertion (A): Cu^{2+} in water is more stable than Cu^+.

Reason (R): Enthalpy of hydration for Cu^{2+} is much less than that of Cu^+.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) Both A and R are correct and R is the correct explanation of A

(2) Both A and R are correct but R is not the correct explanation of A

(3) A is not correct but R is correct

(4) A is correct but R is not correct

4. The starting material for convenient preparation of deuterated hydrogen peroxide (D_2O_2) in laboratory is:

(1) $K_2S_2O_8$ **(2)** BaO **(3)** BaO_2 **(4)** 2-ethylanthraquinol

5. O — O bond length in H_2O_2 is $\underline{X}$ than the O — O bond length in F_2O_2. The O — H bond length in H_2O_2 is $\underline{Y}$ than that of the O — F bond in F_2O_2.

Choose the correct option for $\underline{X}$ and $\underline{Y}$ from those given below:

(1) X – longer, Y - longer **(2)** X – shorter, Y - longer

(3) X – shorter, Y - shorter **(4)** X – longer, Y - shorter

6. Which one of the following sets of ions represented a collection of isoelectronic species?
(Given: Atomic number: F : 9, Cl : 17, Na = 11, Mg = 12, Al = 13, K = 19, Ca = 20, Sc = 21).

(1) $Li^+, Na^+, Mg^{2+}, Ca^{2+}$ **(2)** $N^{3-}, O^{2-}, F^-, S^{2-}$

(3) $K^+, Cl^-, Ca^{2+}, Sc^{3+}$ **(4)** $Br^{2+}, Sr^{2+}, K^+, Ca^{2+}$

7. The effect of addition of helium gas to the following reaction in equilibrium state is:

$$PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$$

(1) The equilibrium will shift in the forward direction and more of Cl_2 and PCl_3 gases will be produced

(2) Helium will deactivate PCl_5 and reaction will stop

(3) The equilibrium will go backward due to suppression of dissociation of PCl_5

(4) Addition of helium will not affect the equilibrium

8. The correct order of bond enthalpy $(kJ\ mol^{-1})$ is:

(1) Si – Si > C – C > Ge – Ge > Sn – Sn **(2)** Si – Si > C – C > Sn – Sn > Ge – Ge

(3) C – C > Si – Si > Sn – Sn > Ge – Ge **(4)** C – C > Si – Si > Ge – Ge > Sn – Sn

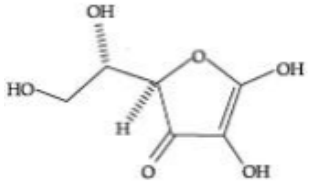
9. The industrial activity held least responsible for global warming is:

(1) Industrial production of urea **(2)** Electricity generation in thermal power plants

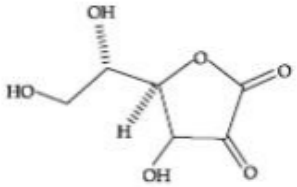
(3) Steel manufacturing **(4)** Manufacturing of cement

10. All structures given below are of vitamin C. Most stable of them is:

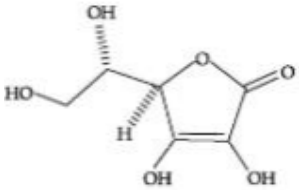
(1)



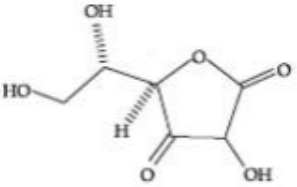
(2)



(3)



(4)



11. Given below are two statements: one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A): An aqueous solution of KOH when used for volumetric analysis, its concentration should be checked before the use.

Reason (R): On adding, KOH solution absorbs atmospheric CO_2.

In the light of the above statements, choose the **correct** answer from the option given below:

(1) A is not correct but R is correct

(2) Both A and R are correct and R is the correct explanation of A

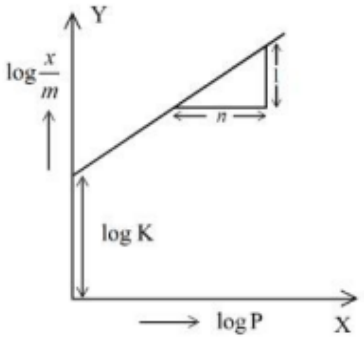
(3) A is correct but R is not correct

(4) Both A and R are correct but R is not correct explanation of A

12. Which element is not present in Nesseler's reagent?

(1) Iodine **(2)** Potassium **(3)** Oxygen **(4)** Mercury

13. In figure, a straight line is given for Freundlich Adsorption ($y = 3x + 2.505$). The value of 1/n and log K are respectively.



(1) 3 and 0.7033 **(2)** 3 and 2.505 **(3)** 0.3 and 0.7033 **(4)** 0.3 and 2.205

14. Given below are two statements:

Statement I: Sulphanilic acid gives esterification test for carboxyl group.

Statement II: Sulphanilic acid gives red colour in Lassaigne's test for extra element detection.

In the light of the above statements, choose the **most appropriate** answer from the option given below:

(1) Statement I is incorrect but statement III is correct

(2) Both statement I and statement II are incorrect

(3) Both statement I and statement II are correct

(4) Statement I is correct but statement II is incorrect

15. Given below are two statements: one is labeled as **Assertion (A)** and the other is labeled as **Reason (R).**

Assertion (A): Gypsum is used for making fireproof wall boards.

Reason (R): Gypsum is unstable at high temperatures.

In the light of the above statement, choose the **correct** from the options given below:

(1) A is not correct but R is correct

(2) Both A and R are correct but R is not the correct explanation of A

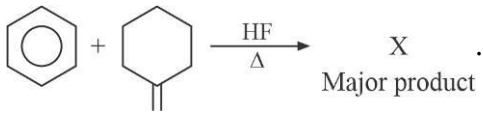
(3) A is correct but R is not correct

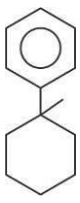
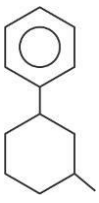
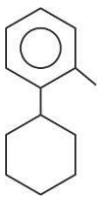
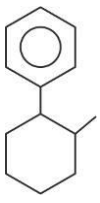
(4) Both A and R are correct and R is the correct explanation of A

16. The complex cation which has two isomers is:

(1) $[Co(NH_3)_5Cl]^{2+}$ **(2)** $[Co(H_2O)_6]^{3+}$

(3) $[Co(NH_3)_5Cl]^{+}$ **(4)** $[Co(NH_3)_5NO_2]^{2+}$

17. 'X' is : 

(1)  **(2)**  **(3)**  **(4)** 

18. Given below are two statements: one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A): α-halocarboxylic acid on reaction with dil NH_3 gives good yield of α-amino carboxylic acid whereas the yield of amines is very low when prepared from alkyl halides.

Reason (R): Amino acids exist in zwitter ion form in aqueous medium.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) A is not correct but R is correct

(2) A is correct but R is not correct

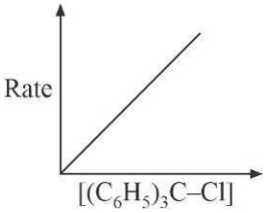
(3) Both A and R are correct and R is the correct explanation of A

(4) Both A and R are correct but R is not the correct explanation A

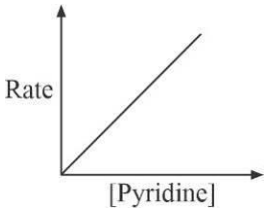
19. The graph which represents the following reaction is:

$$(C_6H_5)_3C-Cl \xrightarrow[\text{Pyridine}]{OH^-} (C_6H_5)_3C-OH$$

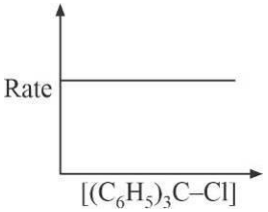
(1)



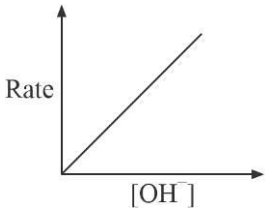
(2)



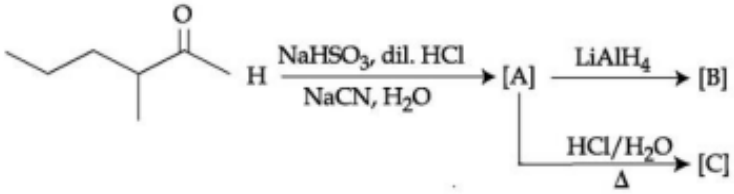
(3)



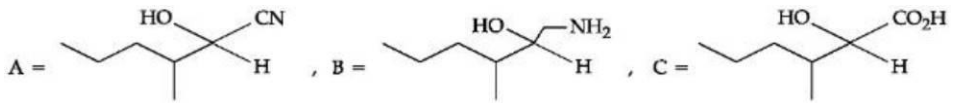
(4)



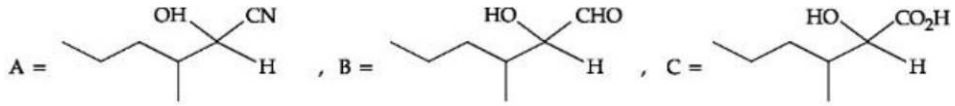
20. The structures of major products A, B and C in the following reaction are sequence.



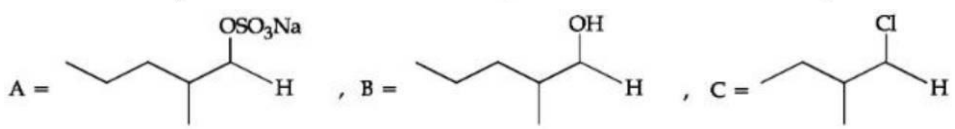
(1)



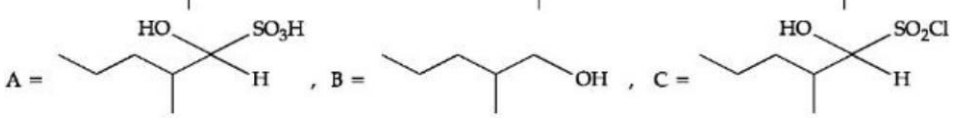
(2)



(3)



(4)



SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be **rounded off to the nearest integer**.

21. The spin only magnetic moment of $[Mn(H_2O)_6]^{2+}$ complexes is _____ B.M. [Nearest integer)]
(Given: Atomic number of Mn is 25).

22. Among following compounds, the number of those present in copper matte is _____.
(A) $CuCO_3$ **(B)** Cu_2S **(C)** Cu_2O **(D)** FeO

23. A metal M crystallizes into two lattices: face centred cubic (fcc) and body centred cubic (bcc) with unit cell edge length of 2.0 and 2.5Å respectively. The ratio of densities of lattices fcc to bcc for the metal M is _____. [Nearest integer]

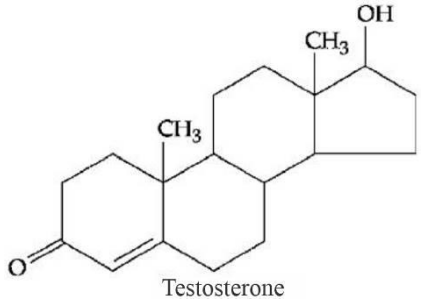
24. 20% of acetic acid is dissociated when its 5g is added to 500 mL of water. The depression in freezing point of such water is _____ $\times 10^{-3}$ °C.
Atomic mass of C, H and O are 12, 1 and 16 a.m.u. respectively.
[Given: Molal depression constant and density of water are $1.86\ K\ mol^{-1}$ and $1\ g\ cm^{-3}$ respectively].

25. The molality of a 10% (v/v) solution of di-bromine solution in CCl_4 (carbon tetrachloride) is 'x'.
x = _____ $\times 10^{-2}$M. [Nearest integer]
(Given: molar mass of $Br_2 = 160\ g\ mol^{-1}$, atomic mass of $C = 12\ g\ mol^{-1}$, atomic mass of $Cl = 35.5\ g\ mol^{-1}$, density of dibromine $= 3.2\ g\ cm^{-3}$, density of $CCl_4 = 1.6\ g\ cm^{-3}$).

26. 0.3 g of ethane undergoes combustion at 27°C in a bomb calorimeter. The temperature of calorimeter system (including the water) is found to rise by 0.5°C. The heat evolved during combustion of ethane of constant pressure is _____ $kJ\ mol^{-1}$. [Nearest integer]
(Givne: The heat capacity of the calorimeter system is $20\ kJ\ K^{-1}$, $R = 8.3\ JK^{-1}\ mol^{-1}$. Assume ideal gas behavior. Atomic mass of C and H are 12 and $1\ g\ mol^{-1}$ respectively).

27. 1×10^{-5} M $AgNO_3$ is added to 1L of saturated solution of AgBr. The conductivity of this solution at 298 K is _____ $\times 10^{-8}\ S\ m^{-1}$. [Given: $K_{sp}(AgBr) = 4.9\times10^{-13}$ at 298 K, $\Lambda^0_{Ag^+} = 6\times10^{-3}\ S\ m^2\ mol^{-1}$, $\Lambda^0_{Br^-} = 8\times10^{-3}\ S\ m^2\ mol^{-1}$, $\Lambda^0_{NO_3^-} = 7\times10^{-3}\ S\ m^2\ mol^{-1}$].

28. Testosterone, which is a steroidal hormone, has the following structure.



Testosterone

The total number of asymmetric carbon atom/s in testosterone is _____.

29. $A \rightarrow B$

The above reaction is of zero order. Half-life of this reaction is 50 min. The time taken for the concentration of A to reduce to one-fourth of its initial value is _____min. [Nearest integer]

30. Among the following the number of tranquilizer's is/are _____.

(A) Chloroliazepoxide **(B)** Veronal

(C) Valium **(D)** Salvarsan

SUBJECT III: MATHEMATICS	MARKS: 100

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. Let the plane P pass through the intersection of the planes $2x - 3y - z = 2$ and $x + 2y + 3z = 6$, and be perpendicular to the plane $2x + y - z + 1 = 0$. If d is the distance of P from the point $(-7, 1, 1)$, then d^2 is equal to:

(1) $\frac{15}{53}$ **(2)** $\frac{25}{83}$ **(3)** $\frac{250}{83}$ **(4)** $\frac{250}{82}$

2. Which of the following statements is a tautology?

(1) $p \vee (p \wedge q)$ **(2)** $p \rightarrow (p \wedge (p \rightarrow q))$

(3) $(p \wedge (p \rightarrow q)) \rightarrow q$ **(4)** $(p \wedge q) \rightarrow (\sim (p) \rightarrow q)$

3. The value of the integral $\int_{-\pi/2}^{\pi/2} \frac{x+\frac{\pi}{4}}{2-\cos 2x}\,dx$ is:

(1) $\frac{\pi^2}{3\sqrt{3}}$ **(2)** $\frac{\pi^2}{6\sqrt{3}}$ **(3)** $\frac{\pi^2}{12\sqrt{3}}$ **(4)** $\frac{\pi^2}{6}$

4. Two dice are thrown independently. Let A be the event that the number appeared on the 1st die is less than the number appeared on the 2nd die, B be the event that the number appeared on the 1st die is even and that on the second die is odd, and C be the event that the number appeared on the 1st die is odd and that on the 2nd is even. Then:

(1) B and C are independent

(2) The number of favorable cases of the event $(A \cup B) \cap C$ is 6

(3) The number of favourable cases of the events A, B and C are 15, 6 and 6 respectively

(4) A and B are mutually exclusive

5. Let $\vec{a} = 2\hat{i} - 7j + 5k$, $\vec{b} = \hat{i} + k$ and $\vec{c} = \hat{i} + 2j - 3k$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r} \cdot \vec{b} = 0$, then $|\vec{r}|$ is equal to:

(1) $\frac{11}{7}\sqrt{2}$ **(2)** $\frac{11}{7}$ **(3)** $\frac{\sqrt{914}}{7}$ **(4)** $\frac{11}{5}\sqrt{2}$

6. Let $P(S)$ denote the power set of $S = \{1, 2, 3, \ldots\ldots 10\}$. Define the relations R_1 and R_2 on $P(S)$ as AR_1B if $(A \cap B^c) \cup (B \cap A^c) = \varnothing$ and AR_2B if $A \cup B^c = B \cup A^c, \forall\, A, B \in (PS)$. Then:

(1) Only R_2 is an equivalence relation

(2) Both R_1 and R_2 are equivalence relations

(3) Only R_1 is an equivalence relation

(4) Both R_1 and R_2 are not equivalence relations

7. If $A=\frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$, then:

(1) $A^{30}+A^{25}+A=I$ **(2)** $A^{30}=A^{25}$

(3) $A^{30}+A^{25}-A=I$ **(4)** $A^{30}-A^{25}=2I$

8. Let $\alpha x=\exp(x^{\beta}y^{\gamma})$ be the solution of the differential equation $2x^2y\,dy-(1-xy^2)dx=0,\ x>0,$ $y(2)=\sqrt{\log_e 2}$. Then $\alpha+\beta-\gamma$ equals:

(1) 3 **(2)** 0 **(3)** –1 **(4)** 1

9. Let $P(x_0, y_0)$ be the point on the hyperbola $3x^2-4y^2=36,$ which is nearest to the line $3x+2y=1.$ Then $\sqrt{2}\,(y_0-x_0)$ is equal to:

(1) 3 **(2)** –9 **(3)** 9 **(4)** –3

10. Let $S=\left\{x\in R:0<x<1 \text{ and } 2\tan^{-1}\left(\frac{1-x}{1+x}\right)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}$. If $n(S)$ denotes the number of elements in S then:

(1) $n(S) = 0$

(2) $n(S) = 1$ and the element in S is less than 1/2

(3) $n(S) = 1$ and the elements in S is more than 1/2

(4) $n(S) = 2$ and only one element in S is less than 1/2

11. For the system of linear equations $\alpha x+y+z=1,\ x+\alpha y+z=1,\ x+y+\alpha z=\beta,$ which one of the following statements is NOT correct?

(1) It has infinitely many solutions if $\alpha=2$ and $\beta=-1$

(2) It has infinitely many solutions if $\alpha=1$ and $\beta=1$

(3) It has no solution if $\alpha=-2$ and $\beta=1$

(4) $x+y+z=\frac{3}{4}$ if $\alpha=2$ and $\beta=1$

12. The number of integral values of k, for which one root of the equation $2x^2-8x+k=0$ lies in the interval (1, 2) and its other root lies in the interval (2, 3), is:

(1) 1 **(2)** 0 **(3)** 2 **(4)** 3

13. If $y(x)=x^x,\ x>0,$ then $y''(2)-2y'(2)$ is equal to:

(1) $8\log_e 2-2$ **(2)** $4(\log_e 2)^2+2$

(3) $4(\log_e 2)^2-2$ **(4)** $4\log_e 2+2$

14. Let $9=x_1<x_2<\ldots<x_7$ be in an A.P. with common difference d. If the standard deviation of $x_1, x_2\ldots, x_7$ is 4 and the mean is $\bar{x},$ then $\bar{x}+x_6$ is equal to:

(1) 25 **(2)** $2\left(9+\frac{8}{\sqrt{7}}\right)$

(3) $18\left(1+\frac{1}{\sqrt{3}}\right)$ **(4)** 34

15. Let $f:\mathbb{R}-\{0,1\}\to\mathbb{R}$ be a function such that $f(x)+f\left(\frac{1}{1-x}\right)=1+x$. Then $f(2)$ is equal to:

(1) $\frac{7}{3}$ **(2)** $\frac{9}{4}$ **(3)** $\frac{9}{2}$ **(4)** $\frac{7}{4}$

16. The area of the region given by $\{(x, y): xy \le 8, 1 \le y \le x^2\}$ is:

(1) $16\log_e 2+\frac{7}{3}$ **(2)** $8\log_e 2-\frac{13}{3}$

(3) $8\log_e 2+\frac{7}{6}$ **(4)** $16\log_e 2-\frac{14}{3}$

17. Let $\vec{a}=5\hat{i}-j-3k$ and $\vec{b}=\hat{i}+3j+5k$ be two vectors. Then which one of the following statements is TRUE?

(1) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$

(2) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$

(3) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is same as of $\vec{b}$

(4) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is same as of $\vec{b}$

18. The sum $\sum_{n=1}^{\infty}\frac{2n^2+3n+4}{(2n)!}$ is equal to:

(1) $\frac{11e}{2}+\frac{7}{2e}$ **(2)** $\frac{13e}{4}+\frac{5}{4e}$

(3) $\frac{13e}{4}+\frac{5}{4e}-4$ **(4)** $\frac{11e}{2}+\frac{7}{2e}-4$

19. The sum of the absolute maximum and minimum values of the function $f(x)=\left|x^2-5x+6\right|-3x+2$ in the interval [–1, 3] is equal to:

(1) 10 **(2)** 12 **(3)** 13 **(4)** 24

20. Let *a*, *b* be two real numbers such that *ab* < 0. If the complex number $\frac{1+ai}{b+i}$ is of unit modulus and *a* + *ib* lies on the circle $|z-1|=|2z|$, then a possible value of $\frac{1+[a]}{4b}$, where [*t*] is greatest integer function, is:

(1) 1 **(2)** $-\frac{1}{2}$ **(3)** $\frac{1}{2}$ **(4)** –1

SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be **rounded off to the nearest integer**.

21. Number of integral solutions to the equation $x+y+z=21$, where $x\geq 1, y\geq 3, z\geq 4$, is equal to _____.

22. If the x-intercept of a focal chord of the parabola $y^2=8x+4y+4$ is 3, then the length of this chord is equal to _____.

23. The line $x = 8$ is the directrix of the ellipse $E:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with the corresponding focus (2, 0). If the tangent to E at the point P in the first quadrant passes through the point $(0, 4\sqrt{3})$ and intersects the x-axis at Q, then $(3PQ)^2$ is equal to _____.

24. Let the sixth term in the binomial expansion of $\left(\sqrt{2^{\log_2(10-3^x)}}+\sqrt[5]{2^{(x-2)\log_2 3}}\right)^m$, in the increasing powers of $2^{(x-2)\log_2 3}$, be 21. If the binomial coefficients of the second, third and fourth terms in the expansion are respectively the first, third and fifth terms of an A.P., then the sum of the squares of all possible values of x is _____.

25. Let $\alpha x+\beta y+yz=1$ be the equation of the plane passing through the point (3, –2, 5) and perpendicular to the line joining the points (1, 2, 3) and (–2, 3, 5). Then the value of $\alpha\beta y$ is equal to _____.

26. If the term without x in the expansion of $\left(x^{2/3}+\frac{\alpha}{x^3}\right)^{22}$ is 7315, then $|\alpha|$ is equal to _____.

27. The point of intersection C of the plane $8x+y+2z=0$ and the line joining the points A(–3, –6, 1) and B(2, 4, –3) divides the line segment AB internally in the ratio k : 1. If $a,b,c(|a|,|b|,|c|$ are coprime) are the direction ratios of the perpendicular from the point C on the line $\frac{1-x}{1}=\frac{y+4}{2}=\frac{z+2}{3}$, then $|a+b+c|$ is equal to _____.

28. The total number of six digit numbers, formed using the digits 4, 5, 9 only and divisible by 6, is _____.

29. If $\int_0^{\pi}\frac{5^{\cos x}(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x)dx}{1+5^{\cos x}}=\frac{k\pi}{16}$, then k is equal to _____.

30. The sum of the common terms of the following three arithmetic progressions.
3, 7, 11, 15, ……, 399,
2, 5, 8, 11, ……, 359 and
2, 7, 12, 17, ……, 197,
is equal to _____.