JEE Main – 2023

1st Feb 2023 (Morning Shift)

General Instructions

1. The test is of **3 hours** duration and the maximum marks is **300.**
2. The question paper consists of **3 Parts** (Part I: **Physics**, Part II: **Chemistry**, Part III: **Mathematics**). Each Part has **two** sections (Section 1 & Section 2).
3. **Section 1** contains **20 Multiple Choice Questions**. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE CHOICE** is correct.
4. **Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. You will NOT be allowed to attempt the sixth question. If you wish to attempt any other question apart from the five already attempted, then you will have to delete any one response from the five previously answered and then proceed to answer the new one.**
   The answer to each question should be **rounded off to the nearest integer**.
5. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. inside the examination room/hall.

Marking Scheme

1. **Section – 1:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.
2. **Section – 2:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.

SUBJECT I: PHYSICS	MARKS: 100

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. The average kinetic energy of a molecule of the gas is:

(1) proportional to absolute temperature **(2)** proportional to pressure

(3) dependent on the nature of the gas **(4)** proportional to volume

2. Given below are two statements:

Statements I: Acceleration due to gravity is different at different places on the surface of earth.

Statements II: Acceleration due to gravity increases as we go down below the earth's surface.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) Both Statement I and Statement II are false

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Statement I is false but Statement II if true

3. $\left(P+\frac{a}{V^2}\right)(V-b)=RT$ represents the equation of state of some gases. Where P is the pressure, V is the volume. T is the temperature and a, b, R are the constants. The physical quantity, which has dimensional formula as that of $\frac{b^2}{a}$, will be:

(1) Modulus of rigidity **(2)** Compressibility

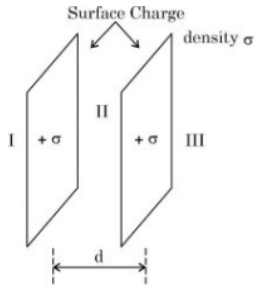
(3) Energy density **(4)** Bulk modulus

4. An object moves with speed v_1, v_2 and v_3 along a line segment AB, BC and CD respectively as shown in figure. Where $AB=BC$ and $AD=3AB,$ then average speed of the object will be:

(1) $\frac{v_1v_2v_3}{3(v_1v_2+v_2v_3+v_3v_1)}$ **(2)** $\frac{(v_1+v_2+v_3)}{3}$

(3) $\frac{(v_1+v_2+v_3)}{3v_1v_2v_3}$ **(4)** $\frac{3v_1v_2v_3}{(v_1v_2+v_2v_3+v_3v_1)}$

5. Let σ be the uniform surface charge density of two infinite thin plane sheets shown in figure. Then the electric fields in three different region E_I, E_H and E_{HI} are:



(1) $\vec{E}_I=0, \vec{E}_{II}=\frac{\sigma}{\epsilon_0}n, \vec{E}_{III}=0$ **(2)** $\vec{E}_I=-\frac{\sigma}{\epsilon_0}n, \vec{E}_{II}=0, \vec{E}_{III}=\frac{\sigma}{2\epsilon_0}n$

(3) $\vec{E}_I=-\frac{\sigma}{\epsilon_0}n, \vec{E}_{II}=0, \vec{E}_{III}=\frac{\sigma}{\epsilon_0}n$ **(4)** $\vec{E}_I=\frac{2\sigma}{\epsilon_0}n, \vec{E}_{II}=0, \vec{E}_{III}=\frac{2\sigma}{\epsilon_0}n$

6. A mercury drop of radius $10^{-3}m$ is broken into 125 equal size droplets. Surface tension of mercury is 0.45 Nm^{-1}. The gain in surface energy is:

(1) $17.5\times10^{-5}J$ **(2)** $5\times10^{-5}J$ **(3)** $28\times10^{-5}J$ **(4)** $2.26\times10^{-5}J$

7. A proton moving with one tenth of velocity of light has certain de Broglie wavelength of λ. An alpha particle having certain kinetic energy has the same de-Broglie wavelength λ. The ratio of kinetic energy of proton and that of alpha particle is:

(1) $1:2$ **(2)** $4:1$ **(3)** $2:1$ **(4)** $1:4$

8. Match List I with List II:

List I	List II
A. Intrinsic semiconductor	**I.** Fermi-level near the valence band
B. *n*-type semiconductor	**II.** Fermi-level in the middle of valence and conduction band.
C. p-type semiconductor	**III.** Fermi-level near the conduction band
D. Metals	**IV.** Fermi-level inside the conduction band

(1) A-III, B-I, C-II, D-IV **(2)** A-I, B-II, C-III, D-IV

(3) A-II, B-III, C-I, D-IV **(4)** A-II, B-I, C-III, D-IV

9. Which of the following frequencies doe not belong to FM broadcast.

(1) 64 MHz **(2)** 89 MHz **(3)** 99 MHz **(4)** 106 MHz

10. A block of mass 5kg is placed at rest on a table of rough surface. Now, if a force of 30N is applied in the direction parallel to surface of the table, the block slides through a distance of 50 m in an interval of time 10 s. Coefficient of kinetic friction is: (given g = 10 m/s^2)

(1) 0.75 **(2)** 0.25 **(3)** 0.60 **(4)** 0.50

11. A steel wire with mass per unit length $7.0\times10^{-3}kg\,m^{-1}$ is under tension of 70 N. The speed of transverse wave is the wire will be:

(1) 100 m/s **(2)** 10 m/s **(3)** 200 $\pi\, m/s$ **(4)** 50 m/s

12. The mass of proton, neutron and helium nucleus are respectively 1.0073 u, 1.0087 u and 4.0015 u. The binding energy of helium nucleus is:

(1) 28.4 MeV **(2)** 7.1 MeV

(3) 14.2 MeV **(4)** 56.8 MeV

13. A sample of gas at temperature T is adiabatically expanded to double its volume. The work done by the gas in the process is: (given, $\gamma=\frac{3}{2}$)

(1) $W=\frac{T}{R}\left[\sqrt{2}-2\right]$ **(2)** $W=TR\left[\sqrt{2}-2\right]$

(3) $W=\frac{R}{T}\left[2-\sqrt{2}\right]$ **(4)** $W=RT\left[2-\sqrt{2}\right]$

14. A child stands on the edge of the cliff 10 m above the ground and throws a stone horizontally with an initial speed of 5 ms^{-1}. Neglecting the air resistance, the speed with which the stone hits the ground will be _____ ms^{-1}. (given, $g=10\,ms^{-2}$)

(1) 15 **(2)** 25 **(3)** 30 **(4)** 20

15. Match List I with List II:

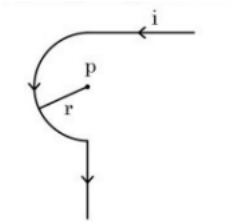
List I		List II	
A.	Microwaves	**I.**	Radio active decay of the nucleus
B.	Gamma rays	**II.**	Rapid acceleration and deceleration of electron in aerials
C.	Radio waves	**III.**	Inner shell electrons
D.	X-rays	**IV.**	Klystron valve

Choose the correct answer from the options given below:

(1) A-I, B-III, C-IV, D-II **(2)** A-I, B-II, C-III, D-IV

(3) A-IV, B-III, C-II, D-I **(4)** A-IV, B-I, C-II, D-III

16. Find the magnetic field at the point P in figure. The curved portion is a semicircle connected to two long straight wires.



(1) $\frac{\mu_0 i}{2r}\left(\frac{1}{2}+\frac{1}{\pi}\right)$ **(2)** $\frac{\mu_0 i}{2r}\left(1+\frac{2}{\pi}\right)$ **(3)** $\frac{\mu_0 i}{2r}\left(\frac{1}{2}+\frac{1}{2\pi}\right)$ **(4)** $\frac{\mu_0 i}{2r}\left(1+\frac{1}{\pi}\right)$

17. Match List I with List II:

List I		List II	
A.	AC generator	**I.**	Presence of both L and C
B.	Transformer	**II.**	Electromagnetic Induction
C.	Resonance phenomenon to occur	**III.**	Quality factor
D.	Sharpness of resonance	**IV.**	Mutual Induction

Choose the ***correct*** answer from the options given below:

(1) A-IV, B-II, C-I, D-III **(2)** A-II, B-IV, C-I, D-III

(3) A-II, B-I, C-III, D-IV **(4)** A-IV, B-III, C-I, D-II

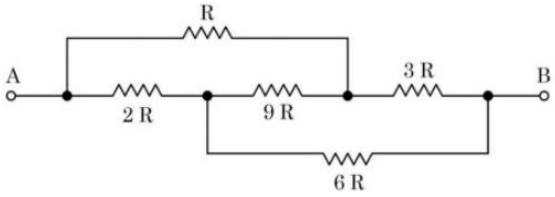
18. '*n*' polarizing sheets are arranged such that each makes an angle $45°$ with the proceeding sheet. An unpolarized light of intensity I is incident into this arrangement. The output intensity is found to be $\frac{I}{64}$. The value of *n* will be:

(1) 4 **(2)** 5 **(3)** 3 **(4)** 6

19. If earth has a mass nine times and radius twice to that of a planet *P*. Then $\frac{v_e}{3}\sqrt{x}\,ms^{-1}$ will be the minimum velocity required by a rocket to pull out of gravitational force of *P*, where v_e is escape velocity on earth. The value of *x* is:

(1) 2 **(2)** 18 **(3)** 1 **(4)** 3

20. The equivalent resistance between A and B of the network shown in figure:



(1) $21\,R$ **(2)** $\frac{8}{3}R$ **(3)** $14R$ **(4)** $11\frac{2}{3}R$

SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be **rounded off to the nearest integer**.

21. Two equal positive point charges are separated by a distance 2*a*. The distance of a point from the centre of the line joining two charges on the equatorial line (perpendicular bisector) at which force experienced by a test charge q_0 becomes maximum in $\frac{a}{\sqrt{x}}$. The value of *x* is _____ .

22. A light of energy 12.75 eV is incident on a hydrogen atom in its ground state. The atom absorbs the radiation and reaches to one of its excited states. The angular momentum of the atom in the excited state is $\frac{x}{\pi}\times10^{-17}eVs$. The value of x is _____. (use $h=4.14\times10^{-15}eVs, c=3\times10^{8}\,ms^{-1}$).

23. In an experiment to find emf of a cell using potentiometer, the length of null point for a cell of emf 1.5 V is found to be 60 cm. If this cell is replaced by another cell of emf E, the length-of null point increases by 40 cm. The value of E is $\frac{x}{10}V$. The value of *x* is _____.

24. A thin cylindrical rod of length 10 cm is placed horizontally on the principle axis of a concave mirror of focal length 20 cm. The rod is placed in a such a way that mid point of the rod is at 40 cm from the pole of mirror. The length of the image formed by the mirror will be $\frac{x}{3}$ *cm*. The value of *x* is _____ .

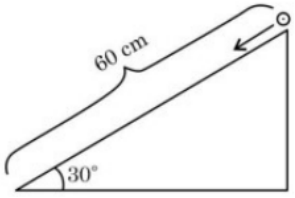
25. A small particle moves to position $5\hat{i}-2j+k$ from its initial position $2\hat{i}+3j-4k$ under the action of force $5\hat{i}+2j+7k$ N. The value of work done will be _____ J.

26. A certain pressure 'P' is applied to 1 litre of water and 2 litre of a liquid separately. Water gets compressed to 0.01% whereas the liquid gets compressed to 0.03%. the ratio of Bulk modulus of water to that of the liquid is $\frac{3}{x}$. The value of *x* is _____ .

27. The series LCR circuit is connected to an ac source of 220 V, 50 Hz. The circuit contain a resistance $R=100\Omega$ and an inductor of indictive reactance $X_L=79.6\Omega$. The capacitance of the capacitor needed to maximize the average rate at which energy is supplied will be _____ μF.

28. The amplitude of a particle executing SHM is 3 cm. The displacement at which its kinetic energy will be 25% more than the potential energy is: _____ cm.

29. A solid cylinder is released from rest from the top of an inclined plane of inclination 30° and length 60 cm. If the cylinder rolls without slipping, its speed upon reaching the bottom of the inclined plane is _____ ms^{-1}. (Given $g=10\,ms^{-2}$)



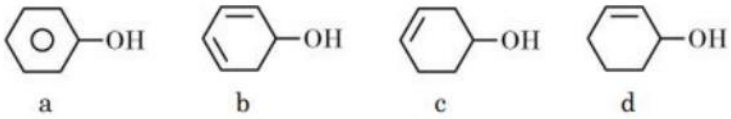
30. A charge particle of $2\mu C$ accelerated by a potential difference of 100 V enters a region of uniform magnetic field of magnitude 4 mT at right angle to the direction of field. The charge particle completes semicircle of radius 3 cm inside magnetic field. The mass of the charge particle is _____ $\times10^{-18}\,kg$.

SUBJECT II: CHEMISTRY	MARKS: 100

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. Decreasing order of dehydration of the following alcohols is:



(1) $a > d > b > c$ **(2)** $d > b > c > a$ **(3)** $b > d > c > a$ **(4)** $b > a > d > c$

2. Match List I with List II:

List I		List II	
Test		Functional group/Class of Compound	
A.	Molisch's Test	**I.**	Peptide
B.	Biuret Test	**II.**	Carbohydrate
C.	Carbylamine Test	**III.**	Primary amine
D.	Schiff's Test	**IV.**	Aldehyde

Choose the correct answer from the options given below:

(1) (A)-I, (B)-II, (C)-III, (D)-IV **(2)** (A)-II, (B)-I, (C)-III, (D)-IV

(3) (A)-III, (B)-IV, (C)-II, (D)-I **(4)** (A)-III, (B)-IV, (C)-I, (D)-II

3. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A: Hydrogen is an environment friendly fuel.

Reason R: Atomic number of hydrogen is 1 and It is very light element.

In the light of the above statements, choose the correct answer from the options given below:

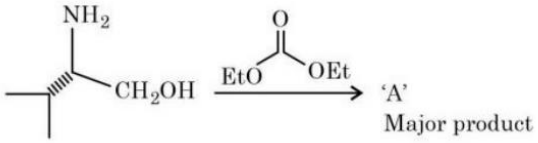
(1) A is false but R is true

(2) Both A and R are true but R is NOT the correct explanation of A

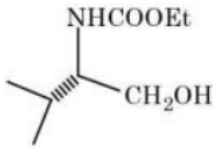
(3) Both A and R are true and R is the correct explanation of A

(4) A is true but R is false

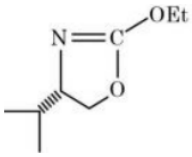
4. In the following reaction, 'A' is:



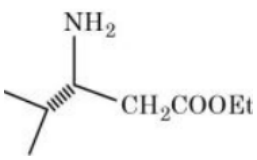
(1)



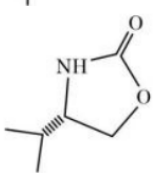
(2)



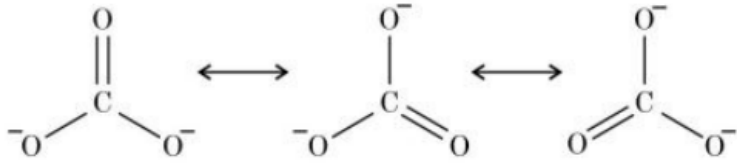
(3)



(4)



5. Resonance in carbonate ion $\left(CO_3^{2-}\right)$ is :



Which of the follwing is true?

(1) All these structures are in dynamic equilibrium with each other.

(2) Each structure exists for equal amount of time

(3) CO_3^{2-} has a single structure i.e, resonance hybrid of the above three structure

(4) It is possible to identify each structure individually by some physical or chemical method.

6. Match List I with List II.

List I	List II
(A) Tranquilizers	(I) Anti blood clotting
(B) Aspirin	(II) Salvarsan
(C) Antibiotic	(III) antidepressant drugs
(D) Antiseptic	(IV) soframicine

Choose the correct answer from the options given below:

(1) (A)-III, (B)-I, (C)-II, (D)-IV **(2)** (A)-II, (B)-I, (C)-III, (D)-IV

(3) (A)-IV, (B)-II, (C)-I, (D)-III **(4)** (A)-II, (B)-IV, (C)-I, (D)-III

7. Match List I with List II.

List I	List II
A. Slaked lime	(I) NaOH
B. Dead burnt plaster	(II) $Ca(OH)_2$
C. Caustic soda	(III) $Na_2CO_3.10H_2O$
D. Washing soda	(IV) $CaSO_4$

Choose the correct answer from the options given below:

(1) (A)-II, (B)-IV, (C)-I, (D)-III **(2)** (A)-III, (B)-II, (C)-IV, (D)-I

(3) (A)-I, (B)-IV, (C)-II, (D)-III **(4)** (A)-III, (B)-IV, (C)-II, (D)-I

8. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: In an Ellingham diagram, the oxidation of carbon to carbon monoxide shows a negative slope with respect to temperature.

Reason R: CO tends to get decomposed at higher temperature.

In the light of the above statements, choose the correct answer from the options given below:

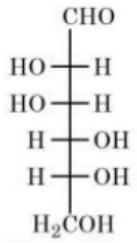
(1) A is correct but R is not correct

(2) Both A and R are correct but R is NOT the correct explanation of A

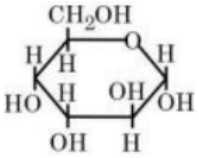
(3) A is not correct but R is correct

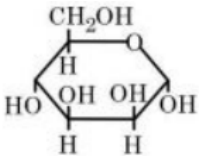
(4) Both A and R are correct and R is the correct explanation of A

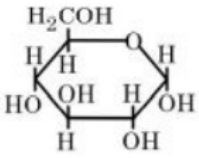
9. The correct representation in six membered pyranose form for the following sugar $[X]$ is

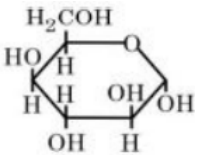


Sugar $[X]$:

(1) 

(2) 

(3) 

(4) 

10. How can photochemical smog be controlled?

(1) By using catalyst convertors in the automobiles/industry

(2) By using catalyst

(3) By complete combustion of fuel

(4) By using tall chimneys

11. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Amongst He, Ne, Ar and Kr.

1 g of activated charcoal adsorbs more of Kr.

Reason R: The critical volume $V_c\left(cm^3\,mol^{-1}\right)$ and critical pressure P_c (atm) is highest for Krypton but the compressibility factor at critical point Z_c is lowest for Krypton.

In the light of the above statements, choose the correct answer from the options given below:

(1) A is true but R is false

(2) Both A and R are true and R is the correct explanation of A

(3) Both A and R are true but R is NOT the correct explanation of A

(4) A is false but R is true

12. Highest oxidation state of Mn is exhibited in Mn_2O_7. The correct statements about Mn_2O_7 are

(A) Mn is tetrahedrally surrounded by oxygen atoms

(B) Mn is octahedrally surrounded by oxygen atoms

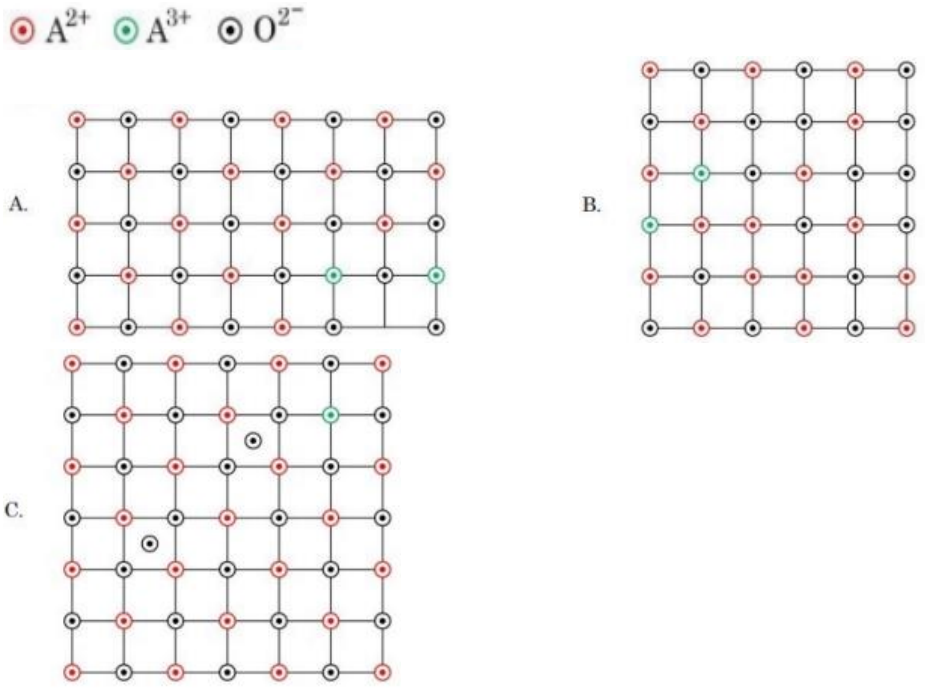
(C) Contains Mn-O-Mn bridge

(D) Contains Mn-Mn bond

Choose the correct answer from the option given below:

(1) B and C only **(2)** A and C only **(3)** A and D only **(4)** B and D only

13. Which of the following represents the lattice structures of $A_{0.95}O$ containing A^{2+}, A^{3+} and O^{2-} ions?



(1) A only **(2)** B only **(3)** A and B only **(4)** B and C only

14. Which of the following are the example of double salt?

A. $FeSO_4.(NH_4)_2.SO_4.6H_2O$ B. $CuSO_4.4NH_3.H_2O$

C. $K_2SO_4.Al_2(SO_4)_3.24H_2O$ D. $Fe(CN)_2.4KCN$

Choose the correct answer:

(1) A and B only **(2)** B and D only **(3)** A and C only **(4)** A, B and D only

15. Choose **the correct statement(s).**

A. Beryllium oxide is purely acidic in nature.

B. Beryllium carbonate is kept in the atmosphere of CO_2

C. Beryllium sulphate is readily soluble in water.

D. Beryllium shows anomalous behavior.

Choose the correct answer from the options given below:

(1) A and B only **(2)** A only

(3) A, B and C only **(4)** B, C and only

16. Given below are two statements:

Statement I: Chlorine can easily combine with oxygen to form oxides: and the product has a tendency to explode.

Statement II: Chemical reactivity of an element can be determined by its reaction with oxygen and halogens.

In the light of the above statements, choose the correct answer from the options given below

(1) Statement I is false but Statement II is true

(2) Statement I if true but Statement II if false

(3) Both the Statements I and II are false

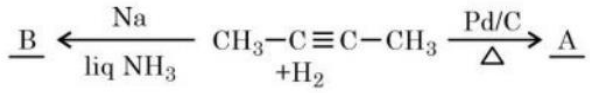
(4) Both the Statements I and II are true

17. A solution of $FeCl_3$ when treated with $K_4\left[Fe(CN)_6\right]$ gives a prussiun blue precipitate due to the formation of :

(1) $Fe_3\left[Fe(CN)_6\right]_2$ **(2)** $Fe\left[Fe(CN)_6\right]$

(3) $K\left[Fe_2(CN)_6\right]$ **(4)** $Fe_4\left[Fe(CN)_6\right]_3$

18. But-2-yne is reacted separately with one mole of Hydrogen as shown below:



A. A is more soluble than B

B. The boiling point and melting point of A are higher and lower than B respectively.

C. A is more polar than B because dipole moment of A is zero.

D. Br_2 adds easily to B than A

Identify the incorrect statements from the options given below:

(1) A and B only **(2)** B and C only

(3) A, C and D only **(4)** B, C and D only

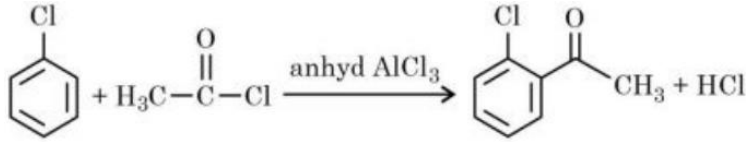
19. Which of the following complex will show largest splitting of d-orbitals?

(1) $\left[Fe(NH_3)_6\right]^{3+}$ **(2)** $\left[Fe(C_2O_4)_3\right]^{3-}$

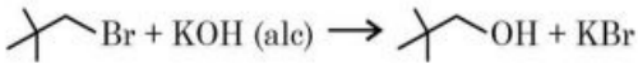
(3) $\left[FeF_6\right]^{3-}$ **(4)** $\left[Fe(CN)_6\right]^{3-}$

20. Identify the incorrect option from the following:

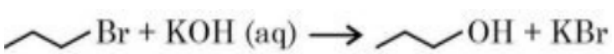
(1)



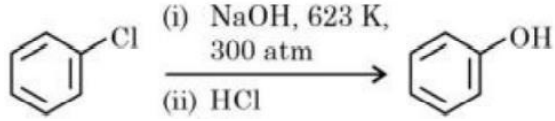
(2)



(3)



(4)



SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be **rounded off to the nearest integer**.

21. Number of isomeric compounds with molecular formula $C_9H_{10}O$ which (i) do not dissolve in NaOH (ii) do not dissolve in HCl. (iii) do not give orange precipitate with 2,4-DNP (iv) on hydrogenation give identical compound with molecular formula $C_9H_{12}O$ is _____.

22. 25 mL of an aqueous solution of KCl was found to require 20 mL of 1 M $AgNO_3$ solution when titrated using K_2CrO_4 as an indicator. What is the depression in freezing point of KCl solution of the given concentration ? _____ (Nearest integer).

(Given : $K_f = 2.0 K\,kg\,mol^{-1}$)

Assume (1) 100% ionization and (2) density of the aqueous solution as 1 g mL^{-1}

23. Electrons in a cathode ray tube have been emitted with a velocity of $1000\,ms^{-1}$. The number of following statements which is/are true about the emitted radiation is _____.

Given : $h = 6\times10^{-34} Js, m_e = 9\times10^{-31} kg$

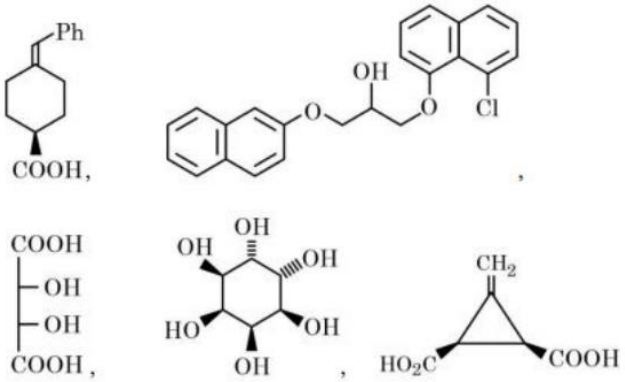
(A) The deBroglie wavelength of the electron emitted is 666.67 nm.

(B) The characteristic of electrons emitted depend upon the material of the electrodes of the cathode ray tube.

(C) The cathode rays start from cathode and move towards anode.

(D) The nature of the emitted electrons depends on the nature of the gas present in cathode ray tube.

24. The total number of chiral compound/s from the following is _____.



25. At what pH, given half cell $MnO_4^-(0.1M)|Mn^{2+}$ (0.001 M) will have electrode potential of 1.282 V? _____. (Nearest Integer)

Given $E^\circ_{MnO_4^-|Mn^{2+}} = 1.54V, \frac{2.303RT}{F} = 0.059V$

26. Sum of oxidation state of bromine in bromic acid and perbromic acid is :

27. A and B are two substances undergoing radioactive decay in a container. The half life of A is 15 min and that of B is 5 min. If the initial concentration of B is 4 times that of A and they both start decaying at the same time, how much time will it take for the concentration of both of them to be same?_____ min.

28. **(i)** $X(g) \rightleftharpoons Y(g) + Z(g) \quad K_{p1} = 3$

(ii) $A(g) \rightleftharpoons 2B(g) \quad K_{p2} = 1$

If the degree of dissociation and initial concentration of both the reactants X(g) and A(g) are equal, then the ratio of the total pressure at equilibrium $\left(\frac{p_1}{p_2}\right)$ is equal to x: 1. The value of x is _____. (Nearest integer)

29. At 25°C, the enthalpy of the following processes are given:

$H_2(g) + O_2(g) \rightarrow 2OH(g) \quad \Delta H° = 78\,kJ\,mol^{-1}$

$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(g) \quad \Delta H° = -242\,kJ\,mol^{-1}$

$H_2(g) \rightarrow 2H(g) \quad \Delta H° = 436\,kJ\,mol^{-1}$

$\frac{1}{2}O_2(g) \rightarrow O(g) \quad \Delta H° = 249\,kJ\,mol^{-1}$

What would be the value of X for the following reaction?_____.
(Nearest integer)

$H_2O(g) \rightarrow H(g) + OH(g) \quad \Delta H° = X\,kJ\,mol^{-1}$

30. The density of 3 M solution of NaCl is 1.0 $g\,mL^{-1}$. Molality of the solution is _____$\times 10^{-2}\,m$. (Nearest integer).

Given : Molar mass of Na and Cl is 23 and 35.5 $g\,mol^{-1}$ respectively.

SUBJECT III: MATHEMATICS	MARKS: 100

SECTION-1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. Let $f(x)=2x+\tan^{-1}x$ and $g(x)=\log_e\left(\sqrt{1+x^2}+x\right), x\in[0,3]$.

Then :

(1) $\max f(x)>\max g(x)$

(2) there exists $x\in[0,3]$ such that $f'(x)<g'(x)$

(3) $\min f'(x)=1+\max g'(x)$

(4) there exist $0<x_1<x_2<3$ such that $f(x)<g(x),\ \forall x(x_1,x_2)$

2. The combined equation of the two lines $ax+by+c=0$ and $a'x+b'y+c'=0$ can be written as $(ax+by+c)(a'x+b'y+c')=0$. The equation of the angle bisectors of the lines represented by the equation $2x^2+xy-3y=0$ is :

(1) $3x^2+xy-2y^2=0$ **(2)** $x^2-y^2+10xy=0$

(3) $3x^2+5xy+2y^2=0$ **(4)** $x^2-y^2-10xy=0$

3. The mean and variance of 5 observations are 5 and 8 respectively. If 3 observations are 1, 3,5, then the sum of the cubes of the remaining two observations is :

(1) 1792 **(2)** 1456 **(3)** 1072 **(4)** 1216

4. The value of $\frac{1}{1!50!}+\frac{1}{3!48!}+\frac{1}{5!46!}+....+\frac{1}{49!2!}+\frac{1}{5!1!}$ is :

(1) $\frac{2^{51}}{51!}$ **(2)** $\frac{2^{51}}{50!}$ **(3)** $\frac{2^{50}}{50!}$ **(4)** $\frac{2^{50}}{51!}$

5. If $y=y(x)$ is the solution curve of the differential equation

$\frac{dy}{dx}+y\tan x=x\sec x,\ 0\le x\le\frac{\pi}{3}, y(0)=1$

Then $y\left(\frac{\pi}{6}\right)$ is equal to :

(1) $\frac{\pi}{12}+\frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$ **(2)** $\frac{\pi}{12}-\frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$

(3) $\frac{\pi}{12}+\frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$ **(4)** $\frac{\pi}{12}-\frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

6. $\lim\limits_{n\to\infty}\left[\frac{1}{1+n}+\frac{1}{2+n}+\frac{1}{3+n}+...+\frac{1}{2n}\right]$ is equal to :

(1) $\log_e\left(\frac{3}{2}\right)$ **(2)** 0 **(3)** $\log_e\left(\frac{2}{3}\right)$ **(4)** $\log_e 2$

7. Let the image of the point $P(2,-1,3)$ in the plane $x+2y-z=0$ be *Q*. Then the distance of the plane $3x+2y+z+29=0$ from the point *Q* is :

(1) $\frac{24\sqrt{2}}{7}$ **(2)** $3\sqrt{14}$ **(3)** $2\sqrt{14}$ **(4)** $\frac{22\sqrt{2}}{7}$

8. The negation of the expression $q\vee((\sim q)\wedge p)$ is equivalent to:

(1) $p\wedge(\sim q)$ **(2)** $(\sim p)\wedge(\sim q)$ **(3)** $(\sim p)\vee q$ **(4)** $(\sim p)\vee(\sim q)$

9. If the center and radius of the circle $\left|\frac{z-2}{z-3}\right|=2$ are respectively (α,β) and γ, then $3(\alpha+\beta+\gamma)$ is equal to:

(1) 11 **(2)** 10 **(3)** 9 **(4)** 12

10. Let *R* be a relation on *R*, given by $R=\{(a,b):3a-3b+\sqrt{7}$ is an irrational number $\}$.

Then *R* is :

(1) reflexive and transitive but not symmetric

(2) reflexive and symmetric but not transitive

(3) an equivalence relation

(4) reflexive but neither symmetric nor transitive

11. The area enclosed by the closed curve C given by the differential equation $\frac{dy}{dx}+\frac{x+a}{y-a}=0,\ y(1)=0$ is 4π

Let *P* and *Q* be the points of interaction of the curve *C* and the *y*-axis. If normal at *P* and *Q* on the curve *C* intersect *x*-axis at points R and S respectively, then the length of the line segment *RS* is :

(1) 2 **(2)** $\frac{2\sqrt{3}}{3}$ **(3)** $2\sqrt{3}$ **(4)** $\frac{4\sqrt{3}}{3}$

12. The shortest distance between the lines $\frac{x-5}{1}=\frac{y-2}{2}=\frac{z-4}{-3}$ and $\frac{x+3}{1}=\frac{y+5}{4}=\frac{z-1}{-5}$ is :

(1) $5\sqrt{3}$ **(2)** $6\sqrt{3}$ **(3)** $4\sqrt{3}$ **(4)** $7\sqrt{3}$

13. Let *S* denote the set of all real values of λ such that the system of equations

$\lambda x+y+z=1$

$x+\lambda y+z=1$

$x+y+\lambda z=1$

is inconsistent, then $\sum_{\lambda\in S}\left(|\lambda|^2+|\lambda|\right)$ is equal to :

(1) 2 **(2)** 6 **(3)** 12 **(4)** **4**

14. For a triangle *ABC*, the value of $\cos 2A+\cos 2B+\cos 2C$ is least. If its inradius is 3 and incentre is *M*, then which of the following is NOT correct ?

(1) area of ΔABC is $\frac{27\sqrt{3}}{2}$

(2) $\overrightarrow{MA}.\overrightarrow{MB}=-18$

(3) $\sin 2A+\sin 2B+\sin 2C=\sin A+\sin B+\sin C$

(4) perimeter of ΔABC is $18\sqrt{3}$

15. In a binomial distribution $B(n, p)$, the sum and the product of the mean and the variance are 5 and 6 respectively, then $6(n+p-q)$ is equal to :

(1) 53 **(2)** 50 **(3)** 52 **(4)** 51

16. The sum to 10 terms of the series $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+\ldots\ldots$ is :

(1) $\frac{55}{111}$ **(2)** $\frac{56}{111}$ **(3)** $\frac{59}{111}$ **(4)** $\frac{58}{111}$

17. If the orthocentre of the triangle, whose are $(1,2),(2,3)$ and $(3,1)$ is (α,β), then the quadratic equation whose roots are $\alpha+4\beta$ and $4\alpha+\beta$, is :

(1) $x^2-22x+120=0$ **(2)** $x^2-19x+90=0$

(3) $x^2-18x+80=0$ **(4)** $x^2-20x+99=0$

18. Let $f(x)=\begin{vmatrix}1+\sin^2 x & \cos^2 x & \sin 2x\\ \sin^2 x & 1+\cos^2 x & \sin 2x\\ \sin^2 x & \cos^2 x & 1+\sin 2x\end{vmatrix}, x\in\left[\frac{\pi}{6},\frac{\pi}{3}\right]$. If α and β respectively are the maximum and the minimum value of *f*, then :

(1) $\beta^2-2\sqrt{\alpha}=\frac{19}{4}$ **(2)** $\beta^2+2\sqrt{\alpha}=\frac{19}{4}$

(3) $\alpha^2-\beta^2=4\sqrt{3}$ **(4)** $\alpha^2+\beta^2=\frac{9}{2}$

19. Let *S* be the set of all solutions of the equation $\cos^{-1}(2x)-2\cos^{-1}\left(\sqrt{1-x^2}\right)=\pi, x\in\left[-\frac{1}{2},\frac{1}{2}\right]$.

Then $\sum_{x\in S}2\sin^{-1}\left(x^2-1\right)$ is equal to :

(1) 0 **(2)** $\pi-2\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

(3) $-\frac{2\pi}{3}$ **(4)** $\pi-\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

20. Let $S=\{x: x\in R$ and $\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4}+\left(\sqrt{3}-\sqrt{2}\right)^{x^2-4}=10$. Then $n(S)$ is equal to :

(1) 6 **(2)** 2 **(3)** 0 **(4)** 4

SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be **rounded off to the nearest integer**.

21. Let $a_1 = 8, a_2, a_3, ..., a_n$ be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170, then the product of its middle two terms is _____.

22. If $f(x) = x^2 + g'(1)x + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f''(x)$, then the value of $f(4) - g(4)$ is equal to _____.

23. Let $\vec{v} = \alpha\hat{i} + 2j - 3k, \vec{w} = 2\alpha\hat{i} + j - k$ and $\vec{u}$ be a vector such that $|\vec{u}| = \alpha > 0$. If the minimum value of the scalar triple product $[\vec{u}\,\vec{v}\,\vec{w}]$ is $-\alpha\sqrt{3401}$, and $|\vec{u}.\hat{i}| = \frac{m}{n}$ where *m* and *n* are coprime natural numbers, then $m+n$ is equal to _____.

24. If $\int_0^1 \left(x^{21} + x^{14} + x^7\right)\left(2x^{14} + 3x^7 + 6\right)^{1/7} dx = \frac{1}{l}(11)^{m/n}$ where $l, m, n \in N, m$ and n are coprime then $l+m+n$ is equal to _____.

25. Let A be the area bounded by the curve $y = x|x-3|$, the x-axis and the ordinates $x = -1$ and $x = 2$. Then 12A is equal to _____.

26. If $f : R \to R$ be a differentiable function such that $f'(x) + f(x) = \int_0^2 f(t)dt$. if $f(0) = e^{-2}$, then $2f(0) - f(2)$ is equal to _____.

27. $A(2,6,2), B(-4,0,\lambda), C(2,3,-1)$ and $D(4,5,0), |\lambda| \le 5$ are the vertices of a quadrilateral ABCD. If its area is 18 square units, then $5 - 6\lambda$ is equal to _____.

28. The number of 3-digit numbers, that are divisible by either 2 or 3 but not divisible by 7, is _____.

29. The remainder, when $19^{200} + 23^{200}$ is divided by 49, is _____.

30. The number of words, with the without meaning, that dan be formed using all the letters of the word ASSASSINATION so that the vowels occur together, is _____.