Solutions to JEE Main 2024 | Sample Paper

Physics

SINGLE CHOICE

1.(C) Pendulum length $l=(2\pm 0.1)\,cm$

$T=(2.5\pm 0.05)\sec$

Acceleration due to gravity $g=4\pi^2\dfrac{l}{T^2}$; $\dfrac{\Delta g}{g}=\dfrac{\Delta l}{l}+2\left(\dfrac{\Delta T}{T}\right)=\left(\dfrac{0.1}{2}+2\times\dfrac{0.05}{2.5}\right)=0.09$

Percentage error $=0.09\times 100=9\%$

2.(B) Maximum velocity of particle

$(V_P)_{\max}=2\pi bA$

Wave velocity $=\dfrac{b}{a}$

Given $2\pi bA=\dfrac{4b}{a}$; $A=\dfrac{2}{\pi a}$

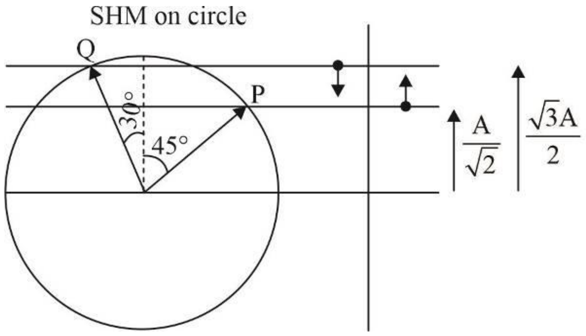
3.(C) We know $g_s=\dfrac{GM}{R_e^2}$

Given $mg_s=900\,N$

g at height $g=\dfrac{GM}{(R_e+h)^2}=\dfrac{4GM}{9R_e^2}=\dfrac{4}{9}\times\left(\dfrac{GM}{R_e^2}\right)$

New weight $=mg=\dfrac{4}{9}mg_s=\dfrac{4}{9}\times 900=400\,N$

4.(C)



$t=\dfrac{5T}{24}$

5.(D) For LR circuit, $\tan\phi=\dfrac{x_L}{R}=2$

Old power factor $=\cos\phi=\dfrac{1}{\sqrt{5}}$

For LCR circuit, $\tan\phi'=\dfrac{X_L-X_C}{R}=\dfrac{2R-R}{R}=1$

New power factor $=\cos\phi'=\dfrac{1}{\sqrt{2}}$ $\therefore$ Required ratio $=\sqrt{\dfrac{5}{2}}$

6.(C) Dimensions of $\mu = [MLT^{-2}A^{-2}]$

Dimensions of $\varepsilon = [M^{-1}L^{-3}T^{4}A^{2}]$

Dimensions of $R = [ML^{2}T^{-3}A^{-2}]$

Dimensions of $\frac{\mu}{\varepsilon} = [M^{2}L^{4}T^{-6}A^{-4}] = [R^{2}]$

7.(D) Given Maximum height of two projectiles are equal

$$\frac{U_A^2 \sin^2 30°}{2g} = \frac{U_B^2 \sin^2 60°}{2g} \;;\quad \frac{U_A}{U_B} = \sqrt{3}$$

Ratio of ranges $$\frac{R_A}{R_B} = \frac{\frac{U_A^2 \sin 60°}{g}}{\frac{U_B^2 \sin 120°}{g}} = \left(\frac{U_A}{U_B}\right)^2 ;\quad \frac{R_A}{R_B} = \frac{3}{1}$$

8.(D) At steady state

Inductor behaves like connecting wire

All of these three resistance are in parallel

So current in

25Ω is 4A

5Ω is 20A

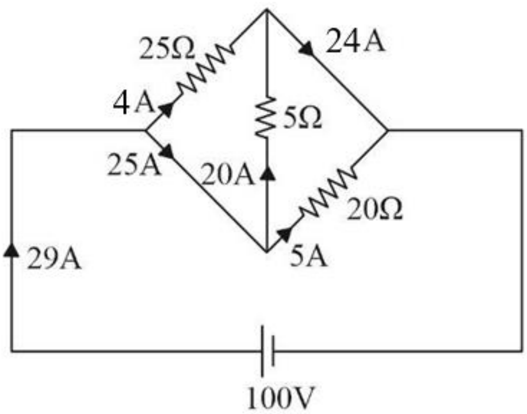
20Ω is 5A

Total current = 29A

By applying KCL at junctions

Current in L_1 is 25A

Current in L_2 is 24A So, ration is $\frac{25}{24}$



9.(C) Given $\vec{E}$ parallel to $\vec{B}$

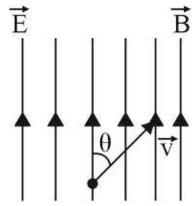
θ : angle velocity vector with field

$F_{net} = q\vec{E} + q\vec{v} \times \vec{B}$

If θ is 0° or 180° the particle follow straight line as only electric force will be acting

For other angles particle will follow helical path with variable pitch

So most appropriate answer is (C)



10.(D) Given $\vec{E} = \frac{2ht}{\lambda_0 q t^2}$

$$F = qE = \frac{2ht}{\lambda_0 T^2}$$

$$\Delta\vec{P} = \int F.dt$$

$$P = \frac{2h}{\lambda_0 T^2}\int_0^t t\,dt \;;\quad P = \frac{ht^2}{\lambda_0 T^2}$$

We know $$\lambda_{debroglie} = \frac{h}{P} = \frac{\lambda_0 T^2}{t^2}$$

11.(C) We know for solid spherical charge distribution

$$V(r)=\begin{Bmatrix} r<R & \dfrac{KQ}{2R^3}\left(3R^2-r^2\right) \\ r\ge R & \dfrac{KQ}{r} \end{Bmatrix}$$

For $r<R$, open downward parabolic

$r>R$, hyperbolic

12.(B) Binding energy per nucleon decides the stability of a nucleus. Higher is the binding energy per nucleon more is the stability.

13.(B) Given

$$\vec{r}=10t^2\hat{i}+5t^3\hat{j}$$

$$\vec{v}=20t\hat{i}+15t^2\hat{j}$$

$$m=200\,gm=\frac{1}{5}kg$$

$$\vec{P}=m\vec{v}$$

$$P=4t\,\hat{i}+3t^2\,\hat{j}$$

Given $\quad P_y=12$

$$3t^2=12$$

$$t=2$$

$$P_x=4\times 2=8$$

14.(B) For satellite

$$PE=\frac{-GMm}{r}$$

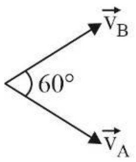
$$KE=\frac{GMm}{2r}$$

$$TE=\frac{-GMm}{2r}\;;\quad TE=-KE$$

Binding energy $=-TE=KE$

15.(C) Given $|\vec{v}_A|=|\vec{v}_B|=v_0$

$$\vec{v}_{red}=\vec{v}_B-\vec{v}_A$$

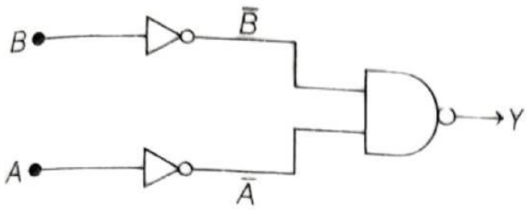


$\theta=60°$

$$|\vec{v}_B-\vec{v}_A|=\sqrt{v_A^2+v_B^2-2v_Av_B\cos\theta}$$

$$=\sqrt{v_0^2+v_0^2-2v_0^2\cos 60°}=v_0$$

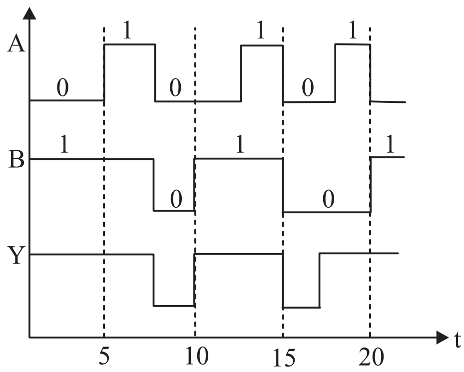
16.(B) Given circuit



Output, $Y = \overline{\overline{A} \cdot \overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$

Truth Table

A	B	$Y = A + B$
0	0	0
0	1	1
1	0	1
1	1	1



So, most appropriate option is (B).

17.(C) $T.E. = \frac{1}{2} stress \times strain \times volume = \frac{1}{2}\frac{T}{A}\left[\frac{\Delta l_{cu}}{l_{cu}} \times V_{cu} + \frac{\Delta l_{steel}}{l_{steel}} \times V_{steel}\right]$

$$= \frac{1}{2}\frac{T}{A}\left[\frac{T}{Y_{cu}A}(Al_{cu}) + \frac{T}{Y_{steel}A}(Al_{steel}\right] = \frac{1}{2}\frac{T^2}{A}\left[\frac{l_{cu}}{Y_{cu}} + \frac{l_{steel}}{Y_{steel}}\right] = 0.25 \text{ J}$$

18.(C) In adiabatic process $Q = 0$

But temperature can change due to work by gas

So statement 1 is incorrect

Statement 2 is correct

19.(B) Pressure of gas inside the balloon is same as the pressure of surrounding. Also gas inside the balloon obeys isothermal process, then :

$$(P_0 + \rho gh)V_1 = P_0 V_2 \quad \therefore \; V_2 = \left(1 + \frac{10^3 \times 10 \times 40}{10^5}\right) \times 0.09 = 0.45 m^3$$

20.(A) For a telescope in normal setting

$f_0 + f_e = L$

(length of the tube of telescope)

and $\frac{f_0}{f_e} = m$ (magnification)

where f_0 & f_e is the focal length of the objective and eyepiece, respectively. According to the given values in the question, we have $f_0 + f_e = 60\ cm$ & $\frac{f_0}{f_e} = 5$; $f_e = 10 cm$

NUMERICAL TYPE

1.(300) We know $KE = \frac{P^2}{2m}$

If KE is increased 1500%

Then new KE is 16 times of initial KE

$P_f = 4P_i$

$\Delta P = 3P_i$

Impulse $= 3P_i$

So impulse is 300% of initial momentum

2.(66) Mass of one atom of U^{235} is 235.121420 amu

Mass of one neutron = 1.008665 amu

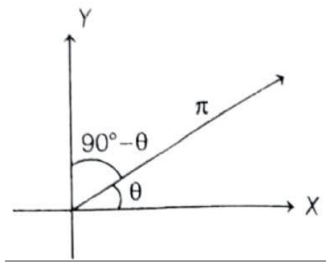
Sum of the masses of U^{235} and neutron = 236.130085 = 236.130 amu

Mass of one atom of U^{236} is 236.123050 amu = 236.123 amu

Mass defect = 236.136 – 236.123 = 0.007 amu

Therefore, energy require to remove one neutron is $0.007 \times 931 MeV = 6.517 MeV = 6.5 MeV$

3.(60) Torque applied on a dipole $\tau = pE\sin\theta$ where $\theta =$ angle between axis of dipole and electric field



For electric field $E_1 = E\hat{i}$

It means field is directed along positive X direction so angle between dipole and field will remain θ therefore torque in this direction

$\tau_1 = pE_1\sin\theta$

In electric field $E_2 = \sqrt{3}E\hat{j}$

It means field is directed along positive Y-axis so angle between dipole and field will be $90° - \theta$

Torque in this direction

$\tau_2 = pE\sin(90° - \theta) = p\sqrt{3}E_1\cos\theta$

According to question

$\tau_2 = -\tau_1 \Rightarrow |\tau_2| = |\tau_1|$ $\quad\therefore\ pE_1\sin\theta = p\sqrt{3}E_1\cos\theta$

$\tan\theta = \sqrt{3} \Rightarrow \tan\theta = \tan 60°$ $\quad\therefore\ \theta = 60°$

4.(7) $f_0 - f_c = 2$

$\frac{v}{2l} - \frac{v}{4l} = 2\ or\ \frac{v}{4l} = 2$; $\frac{v}{l} = 8$

When length of OOP is halved and that of COP is doubled, the beat frequency will be

$f'_0 - f'_c = \frac{v}{l} - \frac{v}{8l} = \frac{7}{8}\frac{v}{l} = \frac{7}{8} \times 8 = 7$

5.(6) $I = \frac{2}{3}MR^2$ (hollow – hemisphere)

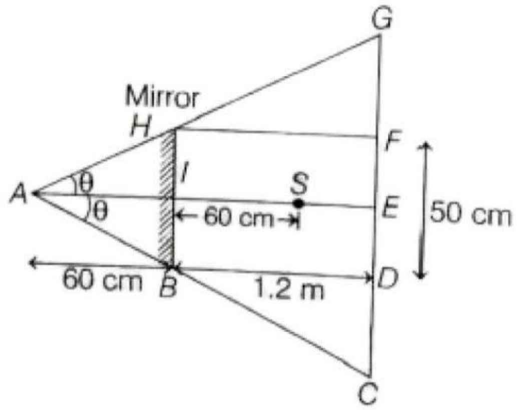
$I = \frac{4}{6}MR^2 \quad \therefore\ x = 6$

6.(150) Given, length of mirror, m = 50 cm $= 50 \times 10^{-2} m$

Distance of source from mirror, d = 60 cm $= 60 \times 10^{-2} m$

Distance of man from mirror, $d_m = 1.2 m$

By using the concept of ray diagram of plane mirror shown below



Now, using the concept of similar triangle

$\Delta HA\ell \sim \Delta GAE$ & $\Delta BA\ell \sim \Delta CAE$

$\therefore\ \frac{A\ell}{AE} = \frac{H\ell}{EG} \quad \Rightarrow \quad \frac{0.60}{1.8} = \frac{0.25}{EG} \qquad (\because A\ell = \ell S)$

$\Rightarrow\ EG = 0.25 \times \frac{1.8}{0.6} = 0.25 \times 3 = 0.75 m$

$CG = 2EG \ \Rightarrow\ CG = 0.75 \times 2 = 1.50 m$

Hence, distance between the extreme points, where he can see image of light source in mirror is 150 cm

7.(21) Magnetic field due to a ling solenoid is given by

$B = \mu_0 n I$

From given data

$6.28 \times 10^{-2} = \mu_0 \times 200 \times 10^2 \times I \qquad \ldots\text{(i)}$

$B = \mu_0 \times 100 \times 10^2 \times \left(\frac{I}{3}\right) \qquad \ldots\text{(ii)}$

Solving equation (i) and (ii), we get

$B \approx 1.05 \times 10^{-2} Wb / m^2 = 10.5 mT$

8.(1) $l = 5A$

$\rho = 1.7 \times 10^{-8} \Omega - m$

$r = 5mm = 5 \times 10^{-3} m$

$v_d = 1.1 \times 10^{-3} m / s$

Mobility of charges in a conductor is given by

$$\mu = \frac{v_d}{E} \qquad \ldots(i)$$

And resistivity is given by

$$\rho = \frac{E}{J} = \frac{E}{l/A} \quad (\because J = \sigma E = \frac{1}{\rho} \times E)$$

$$\Rightarrow \rho = \frac{EA}{l}$$

$$E = \frac{\rho l}{A} \qquad \ldots(ii)$$

From equation (i) and (ii), we get $\mu = \frac{v_d A}{\rho l}$

Substituting the given values we get

$$= \frac{1.1 \times 10^{-3} \times \pi \times (5 \times 10^{-3})^2}{1.7 \times 10^{-8} \times 5} = \frac{86.35 \times 10^{-9}}{8.5 \times 10^{-8}} = 10.1 \times 10^{-1} \Rightarrow \mu \approx 1 m^2 / V - s$$

9.(3) The total energy is given as,

$$U = \frac{1Q^2}{2C}$$

The energy stored in the capacitor is given as,

$$U_C = \frac{1(Q/2)^2}{2C}$$

$$U_C = \frac{1}{4}U$$

The energy stored in the inductor is given as,

$$U_L = U - \frac{1}{4}U$$

$$U_L = \frac{3}{4}U$$

10.(20) The velocity attained by the sphere in falling freely from a height h is

$$v = \sqrt{2gh} \qquad \ldots(i)$$

This is the terminal velocity of the sphere is water. Hence by Stoker's law, we have

$$v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$$

Where r is the radius of the sphere, ρ is the density of the material of the sphere

$\sigma (1.0 \times 10^3 kg / m^3)$ is the density of water and η is coefficient of viscosity of water

$$v = \frac{2 \times (1.0 \times 10^{-3})^2 (1.0 \times 10^4 - 1.0 \times 10^3) \times 10}{9 \times 1.0 \times 10^{-3}} = 20 \text{ m/s}$$

From equation (i) we have $h = \frac{v^2}{2g} = \frac{20 \times 20}{2 \times 10} = 20 m$

Chemistry

SINGLE CHOICE

1.(B) $S_8 + 12OH^- \longrightarrow 4S^{2-} + 2S_2O_3^{2-} + 6H_2O$

x = 4, y = 2

$x^y = 16$

2.(B) To be spontaneous $E_{cell} > 0$

$E_{cell} = -\frac{0.0529}{n}\log\left(\frac{P_1}{P_2}\right) \quad \therefore P_1 < P_2$

3.(B) At cathode $2H_2O + 2e^- \longrightarrow H_2 + 2OH^-$

At anode $2Cl^- \longrightarrow Cl_2 + 2e^-$

Moles of e^- = $\frac{it}{96500} = \frac{5 \times 965}{96500} = 0.05\,mol$

Moles of OH^- formed = 0.05

$[OH^-] = \frac{0.05}{0.5} = 1 \times 10^{-1}$

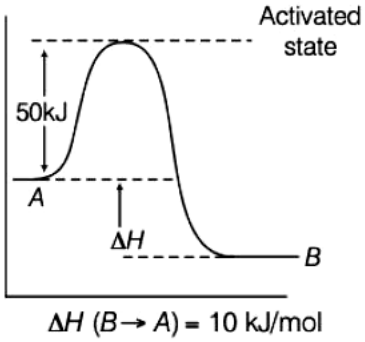
$[H^+] = 1.0 \times 10^{-13}$ and $pH = 13$

4.(C) Due to inert pair effect the stability of lower oxidation state gradually increases while stability of higher oxidation state gradually decreases down the group in elements of group 13th to 15th. So correct order are :

$Pb^{2+} > Pb^{4+}, Bi^{3+} > Bi^{5+}$

$Sn^{2+} < Pb^{2+}, Sn^{4+} > Pb^{4+}$

5.(D) $\Delta H(A \rightarrow B) = -10$ kJ / mol, i.e., it is an exothermic reaction



$E_a(A \rightarrow$ activated state) = 50 kJ/mol

$E_a(B \rightarrow$ activated state) = 50 + 10 = 60 kJ/mol

6.(C) NCERT based fact

7.(D) Magnetic moment $= \sqrt{n(n+2)}\,BM$

N : Number of unpaired e^-

As atomic number increases in d-block element number of unpaired e^- first increases upto middle then decreases.

8.(D) If the resultant dipole moment is zero then molecule is non-polar.

9.(B) $CuCl_2$ and CuF_2 is stable hence doesn't show above reaction.

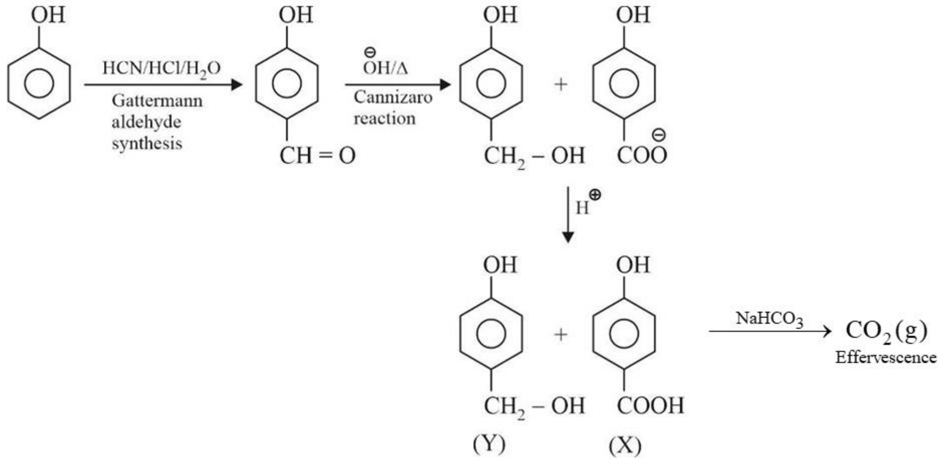
10.(B) Compound I is d^6 with all electrons paired so number of unpaired electrons = 0

Compound II is d^3 number of unpaired electrons = 3

Compound III is d^6 with all electrons paired so number of unpaired electrons = 0

Compound IV is d^8 with all electrons paired so number of unpaired electrons = 0

11.(D)



12.(D) CN^- with Fe^{3+}; H_2O and NO_2^- with Co^{3+} will form low spin complexes with hybridization d^2sp^3

13.(B) $CH_3-\underset{\underset{Br}{|}}{CH}-CH_3 \xrightarrow{\text{Alc. KOH}} CH_3CH=CH_2 \xrightarrow[\text{Peroxide}]{\text{HBr}}$

$CH_3CH_2-\underset{\underset{Br}{|}}{CH_2} \xrightarrow[\text{Williamsons}]{CH_3ONa} \underset{\text{1–methoxy propane}}{CH_3CH_2CH_2OCH_3}$

14.(C) $LiAlH_4$ can reduce ester as well as carbonyl to alcohol.

15.(D) Boiling point of ethylene glycol is more than ethanol.

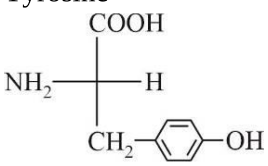
16.(A) $\rightarrow$ 1º Aliphatic amine reacts with Nitrous Acid to give N_2 gas

$\rightarrow$ 1º Aliphatic Amine and Aniline reacts with $CHCl_3$ and KOH to give Carbylamine which has foul odour.

$\rightarrow$ $(C_2H_5)_2NH$ is strongest base, So has lowest value of pK_b

$\rightarrow$ 3º Amines do not reacts with benzenesulphonyl chloride

17.(A) Tyrosine



18.(C)

	$2HI(g)$	$\rightleftharpoons$	$H_2(g)$	+	$I_2(g)$
t = 0	0.2 atm		0		0
change	-2x		+x		+x
$t = t_{eq}$	(0.2 – 2x)		x		x

From question, $0.2-2x=0.04$

$2x=0.2-0.04=0.16$

$K_P=\dfrac{0.08\times0.08}{(0.04)^2}=4$

$K_P=K_C(RT)^{\Delta n}$

$\Delta n=0$

$\therefore\ K_P=K_C$

19.(B) Sodium bisulphite solution show addition reaction with aldehyde and not with alcohols

Sodium bicarbonate solution reacts with acid to release CO_2 gas but not with phenols

Tollen's reagent forms silver salt with terminal alkynes

20.(B) $Na_4[\underset{\substack{\downarrow\\(+2)}}{Fe}(CN)_5NOS]$

$Fe^{2+}: s^0d^6$

NUMERICAL VALUE TYPE

1.(6) $\underset{17}{(-OH)}+CH_3COCl\longrightarrow\left(\underset{59}{-O-\overset{\overset{\displaystyle O}{||}}{C}-CH_3}\right)+HCl$

Mass gain due to incorporation of one acetyl group = 59 -17 = 42
Net mass gain due to acetylation = 518 – 266 = 252
Hence, six hydroxyl groups $(6\times42=252)$ were present

2.(4) As, all lines of P-fund terminate at n = 5 from energy levels above. So, here no line observed in P-fund series

Total number of possible lines observed $=\dfrac{(n_2-n_1)(n_2-n_1+1)}{2}$ $[\because n_1=1;\ n_2=5]$

$=\dfrac{(5-1)(5-1+1)}{2}=10$

4 lines corresponding to following transition observed in UV region

$5\to1;\ 4\to1;\ 3\to1;\ 2\to1$

3 lines observed in Balmer series corresponding to following transition :

$5\to2;\ 4\to2;\ 3\to2$

3.(3) I, II, III are more ionic than $AlCl_3$.

4.(8) $C_p=4R\Rightarrow C_p-C_v=R$ $\quad\because\ C_v=3R$

$\gamma=\dfrac{C_p}{C_v}=\dfrac{4R}{3R}=1.33$

For adiabatic expansion

$\left(\dfrac{V_2}{V_1}\right)^{\gamma-1}=\left(\dfrac{T_1}{T_2}\right)=\left(\dfrac{300}{150}\right)\quad\Rightarrow\left(\dfrac{nV_1}{V_1}\right)^{\gamma-1}=\left(\dfrac{2}{1}\right)\Rightarrow\ (n)^{1.33-1}=(2)$

$\Rightarrow\ (n)^{0.33}=(2)\ \Rightarrow\ (n)=(2)^3\Rightarrow\ n=8$

5.(125) From graph I and II,

$\Delta T_b = 105 - 100 = 5$

$\Delta T_b = mK_b$

$5 = 1 \text{ mol kg}^{-1} \times K_b$

$K_b = 5\text{K kg mol}^{-1}$

From graph I and III

$\Delta T_b = 110 - 100 = 10$

$m = \dfrac{\Delta T_b}{K_b} = \dfrac{10}{5} = 2$

2 moles of solute in 1 kg of solvent

$2M_s$ grams of solute in $(1000 + 2M_s)$ grams of solution

$\dfrac{20}{100} = \dfrac{2M_s}{1000 + 2M_s}$

$M_s = 125$

6.(7) Given pH = 4.63

$[H^+] = 10^{-4.63}$

$= 2.33 \times 10^{-5} M$

$\dfrac{[\ln^-]}{[HIn]} = \dfrac{75}{25} = 3$

$K_{In} = 2.33 \times 10^{-5} \times \dfrac{75}{25} \simeq 7.0 \times 10^{-5} M$

7.(3) I. $K = Ae^{-E_a/RT}$

If $T \to \infty$ or $Ea \to 0$ then K = A

II. Catalyst does not change ΔH of reaction

III. A negative catalyst decrease rate of reaction by increasing activation energy

8.(2) Correct order are :

I. $Cl_2 > Br_2 > F_2 > I_2$ (bond energy)

II. Tl > In > Al > Ga > B (atomic size)

III. C > Pb > Si = Ge = Sn (electronegativity)

IV. Si < C < Ge < Sn < Pb (density)

V. Al < Ga < In < Tl $(E^0_{M^{+3}/M})$

9.(68) As per question,

$H_2C = CH - CH_2 - CH = CH_2$ satisfies the condition

$\therefore$ Molar mass of C_5H_8 is 68.

10.(73) $\% \text{ C} = \dfrac{12}{44} \times \dfrac{\text{wt of } CO_2}{\text{wt. of org. sub}} \times 100 = \dfrac{12}{44} \times \dfrac{0.147}{0.2} \times 100 = 20\%$

$\% \text{H} = \dfrac{2}{18} \times \dfrac{\text{wt of } H_2O}{\text{wt of org. sub}} \times 100 = \dfrac{2}{18} \times \dfrac{0.12}{0.2} \times 100 = 6.66\%$

$\% \text{ oxygen} = 100 - (20 + 6.66) = 73.3\%$

Mathematics

SINGLE CHOICE

1.(C) $f(x) = x + \sin x$ $\quad \therefore \; \frac{dy}{dx} = 1 + \cos x \Rightarrow g'(y) = \frac{dx}{dy} = \frac{1}{1+\cos x}$

$y = \frac{\pi}{4} + \frac{1}{\sqrt{2}} = x + \sin x \Rightarrow x = \frac{\pi}{4}$ $\quad \therefore \; g'\left(\frac{\pi}{4} + \frac{1}{\sqrt{2}}\right) = \frac{1}{1+\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}}{\sqrt{2}+1} = 2 - \sqrt{2}$

2.(B) If l, m, n are direction cosines of two lines are such that

$l + m + n = 0 \qquad \ldots\text{(i)}$

$l^2 + m^2 - n^2 = 0 \qquad \ldots\text{(ii)}$

$\Rightarrow \; l^2 + m^2 - (-l-m)^2 = 0$

$\Rightarrow \; 2lm = 0 \Rightarrow l = 0 \; or \; m = 0$

If $l = 0$, then $n = -m$

$\Rightarrow \; l : m : n = 0 : 1 : -1$

and if $m = 0$, then $n = -l$

$\Rightarrow \; l : m : m = 1 : 0 : -1$

$$\therefore \; \cos\theta = \frac{0+0+1}{\sqrt{0+1+1}\sqrt{0+1+1}} = \frac{1}{2}$$

$$\Rightarrow \; \theta = \frac{\pi}{3}$$

3.(B) Let $I = \int \frac{(1+x)}{x(1+xe^x)^2} dx = \int \frac{(1+x)e^x}{(xe^x)(1+xe^x)^2} dx,$

Put $1 + xe^x = t$

$\therefore \; (1+x)e^x dx = dt = \int \frac{dt}{(t-1)\cdot t^2}$, applying partial fraction

We get, $\frac{1}{(t-1)t^2} = \frac{A}{t-1} + \frac{B}{t} + \frac{C}{t^2}$

$\Rightarrow \; 1 = A(t^2) + Bt(t-1) + C(t-1)$

For $t = 1 \Rightarrow A = 1$

For $t = 0 \Rightarrow C = -1$ and $B = -1$

$$\therefore \; I = \int \left\{ \frac{1}{t-1} - \frac{1}{t} - \frac{1}{t^2} \right\} dt = \log|t-1| - \log|t| + \frac{1}{t} + C$$

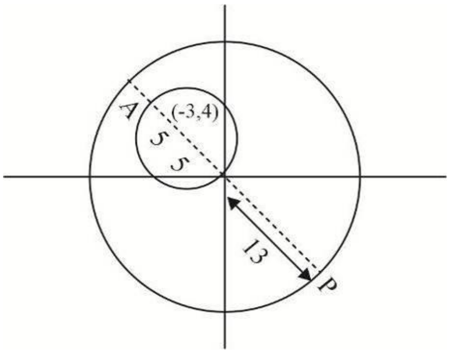
$$= \log|xe^x| - \log|1+xe^x| + \frac{1}{1+xe^x} + C = \log\left|\frac{xe^x}{1+xe^x}\right| + \frac{1}{1+xe^x} + C$$

4.(A) $$\sum_{r=1}^{\infty}\frac{(4r+5)5^{-r}}{r(5r+5)}=\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{(5r+5)-r}{r(5r+5)}\right)\cdot\frac{1}{5^r}$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{5r+5}\right)\frac{1}{5^r}=\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{r\cdot 5^r}-\frac{1}{(r+1)5^{r+1}}\right)$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{5}-\frac{1}{(n+1)5^{n+1}}\right)=\frac{1}{5}-0=\frac{1}{5}$$

5.(A)



$\Rightarrow\quad AP=23$

6.(D) $P(F)=0.90,\ P(M)=0.10$

$$P\left(\frac{R}{F}\right)=0.08,\ P\left(\frac{R}{M}\right)=0.95$$

$$\therefore\ P\left(\frac{M}{R}\right)=\frac{P(M)\cdot P\left(\frac{R}{M}\right)}{P(M)\cdot P\left(\frac{R}{M}\right)+P(F)\cdot P\left(\frac{R}{F}\right)}$$

$$=\frac{0.10\times 0.95}{0.10\times 0.95+0.90\times 0.08}=\frac{0.095}{0.167}=\frac{95}{167}$$

7.(A) Directrix : $x-3=-\dfrac{1}{2},\ x=3-\dfrac{1}{2}=\dfrac{5}{2}$ ...(i)

Slope of tangent at $P(5,2):2y\dfrac{dy}{dx}=2,\ \dfrac{dy}{dx}=\dfrac{1}{2}$

Equation of tangent $y-2=\dfrac{1}{2}(x-5)$ ...(ii)

$$y-2=\frac{1}{2}\left(\frac{5}{2}-5\right)=-\frac{5}{4},\ y=2-\frac{5}{4}=\frac{3}{4}$$

$Q=\left(\dfrac{5}{2},\dfrac{3}{4}\right)$ circumcentre is mid-point of P and $Q\equiv\left(\dfrac{5+\frac{5}{2}}{2},\dfrac{2+\frac{3}{4}}{2}\right)=\left(\dfrac{15}{4},\dfrac{11}{8}\right)$

8.(D) Here, $AB = -2\hat{j},\ BC = (a-1)\hat{i} + (b+1)\hat{j} + c\hat{k}$

The points are collinear, then $AB = k(BC)$

$-2\hat{j} = k\{(a-1)\hat{i} + (b+1)\hat{j} + c\hat{k}\}$

On comparing, $k(a-1) = 0,\ k(b+1) = -2,\ kc = 0$

Hence, $c = 0,\ a = 1$ and b is arbitrary scalar.

9.(B) Let $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = \lambda,\ x = 2\lambda+1,\ y = 3\lambda-1,\ z = 4\lambda+1$ lies on 2nd line $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$

$\frac{2\lambda+1-3}{1} = \frac{3\lambda-1-k}{2} = \frac{4\lambda+1}{1}$, so $2\lambda - 2 = 4\lambda+1, 2\lambda = -3 \Rightarrow \lambda = -\frac{3}{2}$

$\alpha = -2,\ \beta = -\frac{11}{2},\ \gamma = -5,\ \alpha+\gamma-2\beta = -2-5+11 = 4$

10.(D) Let $A = \{a_1, a_2, a_3, \ldots\ldots, a_n\}$

A general element of A must satisfy one of the following possibilities
[here, general element be $a_i (1 \le i \le n)$]

(i) $a_i \in P,\ a_i \in Q$ (ii) $a_i \in P,\ a_i \notin Q$

(iii) $a_i \notin P,\ a_i \in Q$ (iv) $a_i \notin P,\ a_i \notin Q$

Therefore, for one element a_i of A, we have four choices (i), (ii), (iii) and (iv)

Total number of cases for all element $= 4^n$
And for one element a_i of A, such that $a_i \in P \cup Q,$ we have there choices (i), (ii) and (iii)

Number of cases for all elements belong to $P \cup Q = 3^n$
Here, number of ways in which atleast one element of A does not belong to

$P \cup Q = 4^n - 3^n$

11.(C) $(1+x)^{101}(1-x+x^2)^{100} = (1+x)((1+x)(1-x+x^2))^{100}$

$= (1+x)(1+x^3)^{100} = (1+x)(1+{}^{100}C_1x^3 + {}^{100}C_2x^6 + {}^{100}C_3x^9 + \ldots + \ldots + {}^{100}C_{10}x^{300})$

Clearly, in this expression x^3 will present if $n = 3\lambda$ or $n = 3\lambda+1$. So, n cannot be of the form $3\lambda+2$.

12.(B) Let P = Number of ways, 12 boys and 2 girls are seated in a row
$= 14! = 14 \times 13 \times 12! = 182 \times 12!$

$P_1 =$ Number of ways, the girls can sit together

$= (14-2+1) \times 2! \times 12! = 26 \times 12!$

$P_2 =$ Number of ways, one boy sits between the girls

$= (14-3+1) \times 2! \times 12! = 24 \times 12!$

$P_3 =$ Number of ways, two boys sit between the girls

$= (14-4+1) \times 2! \times 12! = 22 \times 12!$

$\therefore$ Required number of ways $= (182-26-24-22) \times 12!$

$= 110 \times 12! = \lambda \times 12!$
$\lambda = 110$

13.(C) There are 6 letters I, I, E, E, T, J

The following cases arise

Case I : All letters are different

$^4P_4 = 4! = 24$

Case II : Two alike and two different

$^2C_1 \times {}^3C_2 \times \dfrac{4!}{2!} = 72$

Case III : Two alike of one kind and two alike of another kind

$^2C_2 \times \dfrac{4!}{2!\,2!} = 6$

Hence, number of words $= 24 + 72 + 6 = 102$

14.(B) Here, $\lim\limits_{x \to 0} \dfrac{\log_e\left[\cot\left(\dfrac{\pi}{4} - K_1 x\right)\right]}{\tan K_2 x} = 1$

$$\Rightarrow \lim_{x \to 0} \frac{\log\left[\cot\left(\dfrac{\pi}{4} - K_1 x\right) - 1 + 1\right]}{\tan K_2 x} = 1$$

$$\Rightarrow \lim_{x \to 0} \frac{\log\left(1 + \dfrac{2\tan K_1 x}{1 - \tan K_1 x}\right)}{\tan K_2 x} = 1$$

$$\Rightarrow \lim_{x \to 0} \frac{\log\left(1 + \dfrac{2\tan K_1 x}{1 - \tan K_1 x}\right)}{\dfrac{2\tan K_1 x}{1 - \tan K_1 x}} \cdot \frac{\dfrac{2\tan K_1 x}{1 - \tan K_1 x}}{\tan K_2 x} = 1$$

15.(A) $\alpha + \beta = -\dfrac{b}{a}$ & $\alpha\beta = \dfrac{c}{a}$

$\therefore\ A_{n+2} = \alpha^{n+2} + \beta^{n+2}$

$= (\alpha + \beta)(\alpha^{n+1} + \beta^{n+1}) - \alpha\beta^{n+1} - \beta\alpha^{n+1}$

$= (\alpha + \beta)(\alpha^{n+1} + \beta^{n+1}) - \alpha\beta(\alpha^n + \beta^n) = \dfrac{b}{a} A_{n+1} - \dfrac{c}{a} A_n$

$\Rightarrow\ aA_{n+2} + bA_{n+1} + cA_n = 0$

16.(B) $2x - 3 > 0 \cap x^2 - 5x - 6 > 0 \cap 2x - 3 \neq 1$

$x > \dfrac{3}{2} \cap (x - 6)(x + 1) > 0 \cap x \neq 2$

$\Rightarrow\ (6, \infty)$

17.(A) We have, $A=\begin{bmatrix}1 & 2 & -1\\ -1 & 1 & 2\\ 2 & -1 & 1\end{bmatrix}$

$\Rightarrow\ |A| = 1(1+2)-2(-1-4)-1(1-2) = 3+10+1 = 14$

We know that, for a square matrix of order *n*,

adj(adjA) $=|A|^{n-2}A$, if $|A|\neq 0$

$\Rightarrow \quad \det(\text{adj}(\text{adj}A)) = \left||A|^{n-2}A\right| \quad \Rightarrow \quad \det(\text{adj}(\text{adj}A)) = \left(|A|^{n-2}\right)^n |A|$

$\Rightarrow \quad \det(\text{adj}(\text{adj}A)) = |A|^{n^2-2n+1}$

Here, $n=3\ \&\ |A|=14$

Therefore, det(adj(adjA)) $=(14)^{3^2-2\times3+1}=14^4$

18.(D) Area bounded by both curves

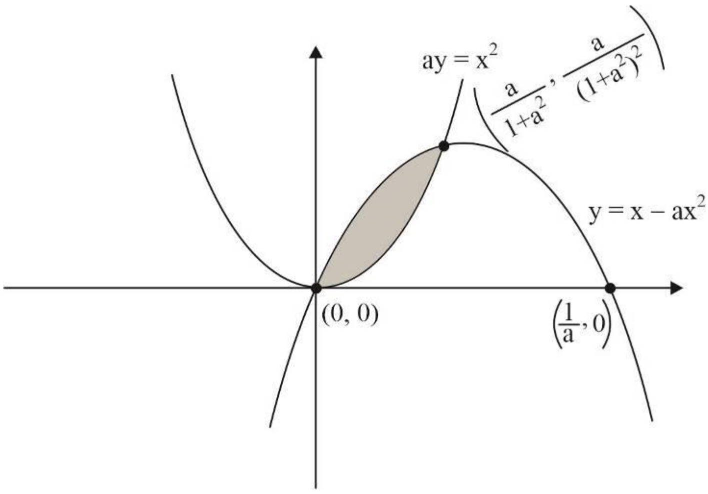
$$=\int_0^{\frac{a}{1+a^2}} (x-ax^2)-\left(\frac{x^2}{a}\right)dx$$

$$=\int_0^{\frac{a}{1+a^2}} x\,dx-\frac{a^2+1}{a}\int_0^{\frac{a}{1+a^2}} x^2dx$$

$$=\left.\frac{x^2}{2}\right]_0^{\frac{a}{1+a^2}}-\frac{a^2+1}{a}\cdot\left.\frac{x^3}{3}\right]_0^{\frac{a}{1+a^2}}$$

$$=\frac{a^2}{2(1+a^2)^2}-\frac{a^2}{3(1+a^2)^2}$$

$$=\frac{a^2}{6(1+a^2)^2}=\frac{1}{6\left(a+\frac{1}{a}\right)^2}$$



Area will be maximum when $a=1$

19.(B) If $A=\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}$, $P=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$, $Q=P^TAP$, we have

$$PQ^{2014}P^T=\frac{P(P^TAP)(P^TAP)....(P^TAP)P^T}{2014\ times}$$

$$(PP^T)A(PP^T)A(PP^T)....(PP^T)A(PP^T)$$

Matrix multiplication is associative

$$PP^T = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2$$

Hence, $PQ^{2014}P^T = A^{2014}$

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \Rightarrow A^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 8 \\ 0 & 1 \end{pmatrix} \qquad \Rightarrow A^n = \begin{pmatrix} 1 & 2n \\ 0 & 1 \end{pmatrix} \;\&\; A^{2014} = \begin{pmatrix} 1 & 4028 \\ 0 & 1 \end{pmatrix}$$

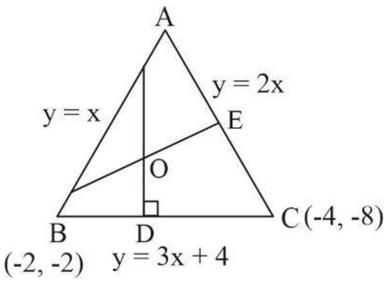
20.(D) AD : $x + 3y + 18 = 0$

BE : $x + 2y + 10 = 0$

on solving we get circumcenter $\equiv (6, -8)$

$\alpha = 6, \beta = -8$

$(\alpha + \beta)^2 + \alpha - \beta = 18$



NUMERICAL TYPE

1.(7) Consider the function $f(x) = \dfrac{x^2}{x^3 + 200}$

$$f'(x) = x\frac{(400 - x^3)}{(x^3 + 200)^2} = 0$$

$\Rightarrow \; x^3 = 400 \quad \Rightarrow \; x = (400)^{1/3}$

When $x < 0,\; f'(x) > 0$

Therefore, $f(x)$ has maximum at $x = (400)^{1/3}$ since, $7 < (400)^{1/3} < 8,$ either a_7 *or* a_8 is the greatest term of the sequence

$\because \; a_7 = \dfrac{49}{543},\; a_8 = \dfrac{8}{89}$ and $\dfrac{49}{543} > \dfrac{8}{89}$

$a_7 = \dfrac{49}{543}$ is the greatest.

2.(1) $y(1 + xy)dx - xdy = 0 \;\Rightarrow\; \dfrac{dy}{dx} = \dfrac{y(1 + xy)}{x} \;\Rightarrow\; xdy - ydx = xy^2dx$

$$\Rightarrow \frac{xdy - ydx}{y^2} = xdx \Rightarrow \int -d\left(\frac{x}{y}\right) = \int xdx \Rightarrow -\frac{x}{y} = \frac{x^2}{2} + C$$

$x = 4,\;\; y = 2$

$$-\frac{4}{2} = \frac{4^2}{2} + C$$

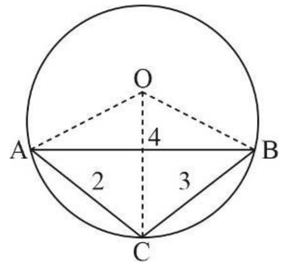
$-2 = 8 + C$

$C = -10$

$$-\frac{x}{y} = \frac{x^2}{2} - 10 \Rightarrow \frac{x}{y} = \frac{20 - x^2}{2} \Rightarrow y = \frac{2x}{20 - x^2}$$

$$f(2) = \frac{4}{16} = \frac{1}{4}$$

3.(425) Using the property that equal chords subtends equal angles at centre of circle, then problem can be converted to the diagram in adjoining figure



AB = 4, AC = 2, BC = 3

$\angle ABC = \alpha / 2$

$$\cos(\alpha/2) = \frac{9+16-4}{2\times 3\times 4} = \frac{7}{8} \Rightarrow \cos\alpha = 2\cos^2(\alpha/2) - 1$$

$$= 2\times\frac{49}{64} - 1 = \frac{98-64}{64} \Rightarrow \cos\alpha = \frac{34}{64} = \frac{17}{32}$$

4.(4) Since $f(x)$ and $g(x)$ are one-one and onto and are also the mirror images of each other with respect to the line $y = 2$. It clearly indicates that $h(x) = f(x) + g(x)$ will be a constant function and will always be equal to 4.

5.(0) Since, these two lines are intersecting so shortest distance between these two lines will be 0.

6.(16) $\left[\frac{70}{5}\right] + \left[\frac{70}{25}\right] = 14 + 2 = 16$

7.(9) $\sigma^2 = \frac{20\times 1 + 40\times 2^2}{60} = 3$ (Because mean of both samples is the same)

8.(6) Let $\sqrt[3]{x^2 + 2x} = y = f(x)$

$$x = -1 + (y^3 + 1)^{1/2}$$

$$I = \int_0^2 (f^{-1}(x) + f(x) + 1)\,dx$$

Consider $\int_0^2 f^{-1}(x) = \int_0^2 tf'(t)\,dt$

Let $f^{-1}(x) = t$; $x = f(t)$; $dx = f'(t)\,dt$ $= tf(t)\Big|_0^2 - \int_0^2 dx = 6$

9.(2) $\left|\vec{a} + \vec{b}\right| = \sqrt{3}$ $\Rightarrow$ Squaring both sides

$\Rightarrow$ $\vec{a}\cdot\vec{b} = \frac{1}{2}$; $\vec{c} = \vec{a} + 2\vec{b} - 3\vec{a}\times\vec{b}$

$\Rightarrow$ $\vec{a}\cdot\vec{c} = 2$ & $\vec{b}\cdot\vec{c} = \frac{5}{2}$

$$p = \left|(\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a}\right|$$

$$p = \sqrt{\left|2\vec{b} - \frac{5}{2}\vec{a}\right|^2}$$

$$p = \frac{\sqrt{21}}{2} \Rightarrow [p] = 2$$

10.(4) $(3^{|x-2|} + (3^{|x-2|-9})^{1/5})^7$

$T_6 = {}^7C_5 \cdot (3^{|x-2|})^2 \cdot 3^{|x-2|-9} = 567$ $\Rightarrow 3^{3|x-2|-9} = 27 \Rightarrow |x-2| = 4 \Rightarrow x = 6, -2$