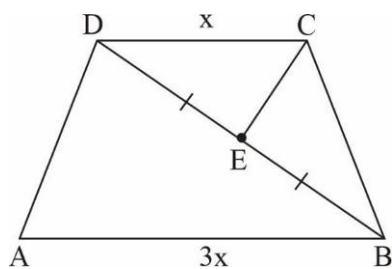


IOQM | SOLUTIONS

1.(8)



$$\frac{\text{ar}(\text{ABCD})}{\text{ar}(\text{CDE})} = n$$

$$\frac{\text{ar}(\triangle ABD)}{\text{ar}(\triangle CBD)} = \frac{\frac{1}{2} \times h \times 3x}{\frac{1}{2} \times h \times x} = 3$$

$$\text{also } \text{ar}(\triangle CBD) = 2(\text{ar}(\triangle CDE))$$

$$\Rightarrow \frac{\text{ar}(\triangle ABD)}{2\text{ar}(\triangle CDE)} = 3 \Rightarrow \text{ar}(\triangle ABD) = 6\text{ar}(\triangle CDE) \therefore \text{ar}(\text{ABCD}) = 8\text{ar}(\triangle CDE) \Rightarrow 8$$

2.(64) $5b^2 + 0b + 3b^0 = 3(b+2)^2 + 0 \times (b+2)^1 + 5 \times b^0$

$$5b^2 + 3 = 3(b^2 + 4b + 4) + 5$$

$$5b^2 + 3 = 3b^2 + 12b + 12 + 5$$

$$2b^2 - 12b - 14 = 0$$

$$b^2 - 6b + 7 = 0$$

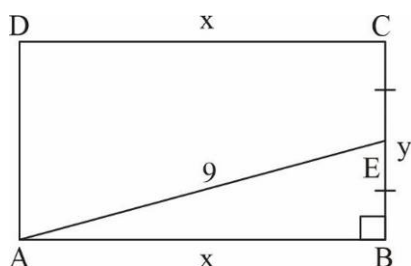
$$(b-7)(b+1) = 0, b = 7$$

$$N = 5 \times 7^2 + 3 = 248 \rightarrow \text{Product of digits} = 2 \times 4 \times 8 = 64$$

3.(99) $\sum_{k=1}^n \frac{2k+1}{k^2(k+1)^2} = 0.9999 \Rightarrow \sum_{k=1}^N \left[\frac{1}{k^2} - \frac{1}{(k+1)^2} \right] = \frac{9999}{10000}$

$$\Rightarrow 1 - \frac{9999}{10000} = \frac{1}{(k+1)^2} \Rightarrow k+1 = 10 \Rightarrow k = 99$$

4.(19)



(1) $2x + y = 20$

(2) $x^2 + \frac{y^2}{4} = 81$

$$\Rightarrow 4x^2 + y^2 = 324$$

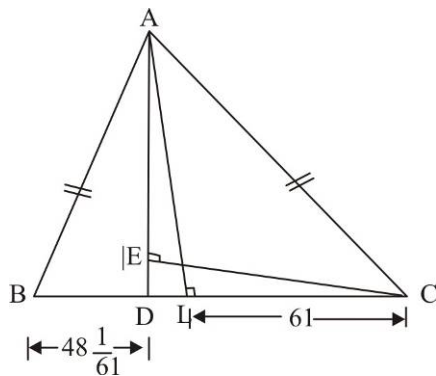
$$\text{Squaring (1)} \quad 4x^2 + y^2 + 4xy = (20)^2 \Rightarrow 324 + 4xy = 400 \Rightarrow xy = 19$$

5.(18) $||x|-2020|<5 \Rightarrow -5<|x|-2020<5$

$$\Rightarrow 2015 < |x| < 2025 \Rightarrow x \in (-2025, 2015) \cup (2015, 2025) \Rightarrow \text{integer values are 18}$$

6.(15) $2^5 \times 3^6 \times 4^3 \times 5^3 \times 6^7 \Rightarrow 2^{18} \times 3^{13} \times 5^3 \Rightarrow$ multiply by $3 \times 5 = 15$

7.(25)



$$BC = \frac{6650}{61}$$

$$LD = CD - LL = 61 - \frac{BC}{2} = 61 - \frac{3325}{61} = \frac{396}{61}$$

$$ALD \sim CED$$

$$\frac{AD}{CD} = \frac{LD}{ED} \Rightarrow AD = 61 \times \frac{396}{61} \times \frac{1}{11}$$

$$AD = 36$$

So, $AE = 36 - 11 = 25$

8.(15) $k, k+1, k+2, 3k, k+3$

k must be either 1, 2 or 3

$\therefore 3k$ must be then 10 so numbers can be

1, 2, 3, 3, 6

Or 2, 3, 4, 6, 5

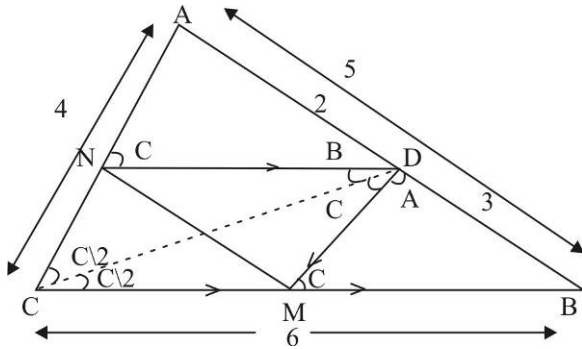
Or 3, 4, 5, 9, 6

Out of these 3 only 34596 (i) square of 186

So $m = 186$

And sum of digits of $m_{13} = 15$

9.(2)



$$\frac{AD}{BD} = \frac{4}{6} = \frac{2}{3} \quad (\text{angle bisector prop})$$

$$AD = 2, BD = 3$$

$$\text{And } BM = \frac{13}{5}, CM = \frac{12}{5} \quad (\because \text{By BPT})$$

$$DBM \sim ABC$$

$$\frac{BM}{BC} = \frac{DM}{AC} \Rightarrow \frac{13/5}{6} = \frac{DM}{4}$$

$$AN = \frac{8}{5}, NC = \frac{12}{5}$$

$$ANO \sim ACB$$

$$\frac{AN}{AC} = \frac{ND}{BC} \Rightarrow \frac{8/5}{4} = \frac{ND}{6}$$

$$ND = \frac{12}{5}$$

In $\triangle DNM$

$$\cos C = \frac{DN^2 + DM^2 - MN^2}{2 \triangle DNM} \quad \dots\dots\dots(i)$$

In $\triangle ACB$

$$\cos C = \frac{BC^2 + AC^2 - AB^2}{2BC.AC} \quad \dots\dots\dots(ii)$$

Equal (i) and (ii)

$$\frac{\left(\frac{12}{5}\right)^2 + \left(\frac{12}{5}\right)^2 - MN^2}{2 \cdot \frac{12}{5} \cdot \frac{12}{5}} = \frac{(6)^2 + (4)^2 - (5)^2}{2 \cdot 6 \cdot 4} \Rightarrow 2\left(\frac{144}{25}\right) - MN^2 = 27 \times \frac{6}{25}$$

$$MN^2 = \frac{286}{25} - \frac{162}{25} = \frac{126}{25} \quad \therefore P = 123, q = 25$$

$$|P - Q| = 101$$

$$\text{Sum of digit } |P - Q| = 2$$

10.(40) say $0 \leq x_i \leq 100$

$$\langle \bar{x} \rangle = \frac{\sum x_i}{5}$$

And $x_1 \leq x_2 \leq x_3 \leq x_4 \leq x_5$

$$\text{Difference} = \left| \frac{\sum x_i}{5} - x_3 \right| = \left| \frac{x_1 + x_2 + x_3 + x_4 + x_5 - 4x_3}{5} \right| = \left| \frac{(x_1 - x_3) + (x_2 - x_3) + (x_4 - x_3) + (x_5 - x_3)}{5} \right|$$

To maximize $x_1 = x_3$ and $x_2 = x_3$

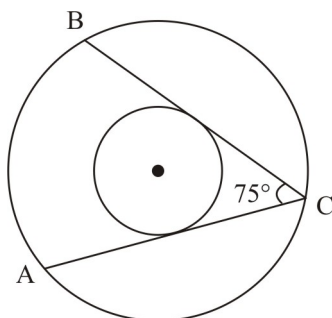
As x_4 and $x_5 = 100$

And $(x_4 - x_3)$ and $(x_5 - x_3)$ should be maximum if $x_3 = 0$

$$\text{Difference} = \frac{200}{5} = 40$$

11.(24)

12.(24)



We will return to point A if $75(k_1) = 360(k_2)$ and we are looking for the least value of k_1

$$\Rightarrow k_1 = 24$$

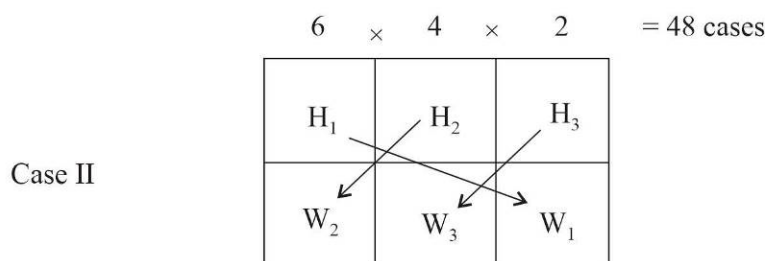
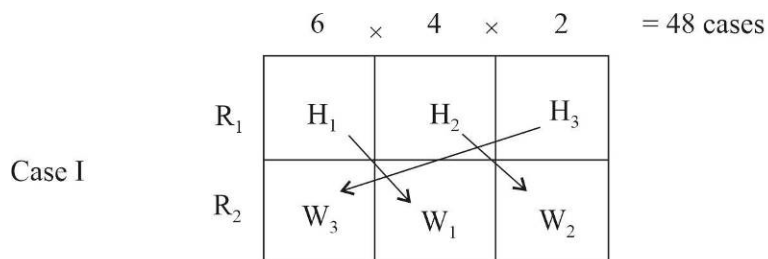
13.(6) $|2^n + 5^n - 65|$ is a perfect square.

The only possible values of n are 2, 4

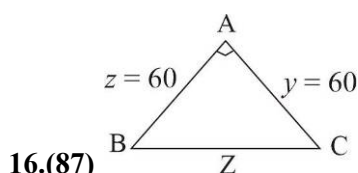
Hence the sum is 6.

14.(20) $a \times b \times c \times d \times e = 2^2 \times 3 \times 5^3 \times 11 \times 13$
 $= 5 \times 11 \times 13 \times 15 \times 20$.

15.(96) $\begin{matrix} & C_1 & \\ H_1 & \diagdown & W_1 \end{matrix} \quad \begin{matrix} & C_2 & \\ H_2 & \diagdown & W_2 \end{matrix} \quad \begin{matrix} & C_3 & \\ H_3 & \diagdown & W_3 \end{matrix}$



∴ Total = 48 + 48 + 96 cases.



$$(y - x) = \frac{2\Delta(y - x)}{xy}$$

$$\Delta = \frac{1}{2}xy. \quad \Rightarrow \quad \angle A = 90^\circ \quad \therefore \text{largest side} = Z = \sqrt{x^2 + y^2} = \sqrt{60^2 + 63^2} = 87$$

17.(30) 4 positive factors

2 digit numbers

$$10a + b = a_1 a_2 a_3 a_4 = 2^{k_1} 3^{k_2} (5)^{k_3} (7)^{k_4}$$

For $k_1 = 3$ (1, 2, 4, 8 ×)

$$k_2 = 3$$
 (1, 3, 9, 27)

$$k_3 = 2$$
 (1, 5, 25) $\nearrow^{50}$ not possible $\searrow_{75}$

$$k_4 = 1, 7, 49 \nearrow^{98, 2}$$
 not possible

$$k_3 = 1, 1, 5$$

$$\rightarrow 7, 35$$

$$\rightarrow 11, 55$$

$$\rightarrow 13, 65$$

$$\rightarrow 17, 85$$

$$k_1 = 1, 1, 2, 3, 6 \times$$

$$5, 10$$

$k_4 = 1$	1, 7	7, 14
	11, 77	11, 22
	13, 91	13, 26
		17, 34
	2, 6 ×	19, 38
$k_2 = 1$	1, 3	5, 15 ×
	7, 21	23, 46
	13, 39	29, 58
	17, 51	31, 62
	19, 57	37, 74
	23, 69	41, 82
	29, 87	43, 86
	31, 93	47, 94
	49, 98	
	1, 3, 11, 33	

30 such numbers possible.

$$18.(80) \sum_{k=1}^{40} \left(\sqrt{1 + \frac{1}{k^2} + \frac{1}{(k+1)^2}} \right) = a + \frac{b}{c}$$

$$T_k = \sqrt{\left(\frac{k^2 + k + 1}{k(k+1)} \right)^2} = \frac{k^2 + k + 1}{k(k+1)} = 1 + \frac{1}{k(k+1)}$$

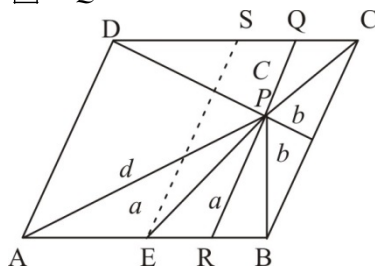
$$t_k = \frac{1}{k} - \frac{1}{k+1}$$

$$s_k = 1 - \frac{1}{41}$$

$$s = 40 + 1 - \frac{1}{41} = 40 + \frac{40}{41}$$

$$a + b = 80$$

19.(40) Area of $\square APQ\Delta$



$= 2(\text{area of } \triangle APD)$ (between some parallel line AD and PQ)

Area of $\square PBCQ = 2(\text{area of } \triangle BPC)$ (between same parallel line PQ and BC)

Area of $\square PBCQ = 2(\text{area of } \triangle BPC)$ (between same parallel line PQ and BC)

$$\text{Area of } \triangle APD + \text{area of } \triangle BPC = \frac{1}{2} (\text{area of } \square ABCD)$$

$$d + 2b = 50 \quad \dots (i) \quad \rightarrow d = 50 - 2b$$

$$\text{Similarly } a + 2b = 25 \quad \dots (ii) \quad \rightarrow a = 25 - 2b$$

$$b + c = 25 \quad \dots (iii) \quad c = 25 - b$$

$$a + d + c = 75 \quad \dots (iv)$$

$$25 - 2b + 50 - 2b + 25 - b = 75$$

$$100 - 5b = 75 \quad 5b = 25$$

$$b = 5$$

$$\text{Area of } \triangle APB = 50 - 2 \times 5 = 40 \text{ unit maximum area.}$$

20.(0) Let n women $\rightarrow$ 45 hours

$$\text{Work done by one woman} = \frac{1}{45n} \text{ per hour}$$

Let work was completed in t hour when working one by one $x =$ intervals

$$\frac{1}{45n} \left[t + \left(t - \frac{t}{n} \right) + \left(t - \frac{2t}{n} \right) + \dots + t - \frac{(n+1)t}{n} \right] = 1$$

$$\frac{t}{45n} \left[n - \frac{1}{n} (1 + 2 + \dots + (n+1)) \right] = 1$$

$$\frac{t}{45n} \left[n - \frac{1}{n} \times \frac{(n+1)}{2} \right] = 1$$

$$\frac{t}{45n} \left[\frac{2n - (n+1)}{2} \right] = 1$$

From given condition

$$t = 5 \left(t - \frac{(n+1)t}{n} \right)$$

$$\frac{1}{5} = 1 - \frac{n+1}{n} = 1 - 1 - \frac{1}{n} \quad \boxed{n = 5}$$

$$t[n+1] = 90n$$

$$t = \frac{90n}{n+1} = \frac{90 \times 5}{6} = 75 \text{ hours}$$

21.(35) N every year

↙ ↓ ↘

A B C

$$\frac{N}{2} b \left(\frac{3N}{7} - 1 \right) < \left(\frac{N}{14} + 1 \right) \quad b + c = \frac{N}{2}$$

$$\frac{3N}{7} \quad b - 1 \left(\frac{3N}{7} - 2 \right) \quad 2C \left(\frac{N}{7} + 2 \right) \quad \frac{3N}{7} + b + 2c - 1 = N$$

$$b + 2c - 1 = \frac{4N}{7}$$

$$b + 2c = \frac{4N}{7} + 1$$

$$c + \frac{N}{2} = \frac{4N}{7} + 1$$

$$c = \frac{4N}{7} - \frac{N}{2} + 1 = \frac{N}{14} + 1$$

$$b + \frac{N}{14} + 1 = \frac{N}{2}$$

$$b = \frac{N}{2} - \frac{N}{14} - 1 = \frac{3N}{7} - 1$$

Diff of age will be constant.

$A \rightarrow B$

$$\frac{N}{2} - \left(\frac{3N}{7} - 1 \right) = \left(\frac{N}{14} + 1 \right) \quad | \quad A - C$$

$$\left(\frac{N}{2} - \frac{N}{14} - 1 \right) = \frac{6N}{14} - 1 = \frac{3N}{7} - 1$$

$$\frac{N}{2} - \left(\frac{3N}{7} - 1 \right) = \left(\frac{N}{14} + 1 \right)$$

$$\frac{3N}{7} - \left(\frac{3N}{7} - 2 \right) = 2$$

$$\frac{\frac{N}{14} + 1}{2} = \frac{\frac{3N}{7} - 1}{2 \left(\frac{N}{7} - 1 \right)} \quad \left| \quad \left(\frac{N}{2} - \frac{N}{14} - 1 \right) = \frac{6N}{14} - 1 = \frac{3N}{7} - 1 \right.$$

$$\frac{3N}{7} - \left(\frac{3N}{7} - 2 \right) = 2 \quad \frac{3N}{7} - \frac{N}{7} - 2 = \frac{2N}{7} - 2$$

$$\frac{\frac{N}{14} + 1}{2} = \frac{\frac{3N}{7} - 1}{2 \left(\frac{N}{7} - 1 \right)}$$

$$\left(\frac{N}{14} + 1 \right) \left(\frac{N}{7} - 1 \right) = \frac{3N}{7} - 1$$

$$\text{Let } \frac{N}{14} = k$$

$$(k + 1)(2k - 1) = 6k - 1$$

$$2k^2 + k - 1 = 6k - 1$$

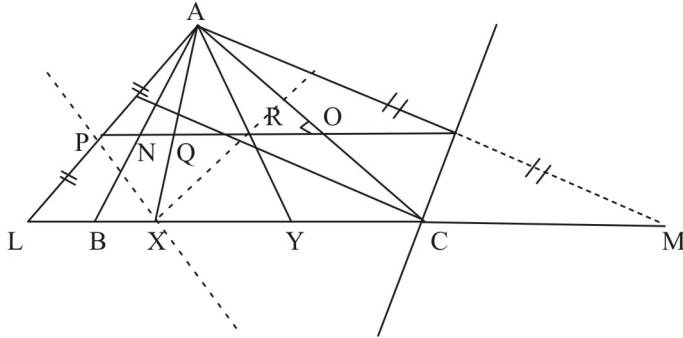
$$2k^2 - 5k = 0$$

$$k(2k - 5) = 0$$

$$k = \frac{5}{2}$$

$$\frac{N}{14} = \frac{5}{2} \rightarrow N = 35$$

22.(84)



P is middle point $P'Q$ AL

Q is middle point AX

R is middle point AY

S is middle point AM

Hence, $PQ \parallel LX, QR \parallel XY, RS \parallel YM$ (By mid-point theorem)

And N is also mid point of AB

O is also mid point of AC

APBR and AQCS are rectangle

So, $PR = AB = 13$ and $AC = QS = 14$

$$NR = \frac{AB}{2} = 6.5$$

$$OQ = \frac{QS}{2} = 7$$

$$NQ = NR - 6$$

$$RO = OQ - 6$$

$$NQ = .5$$

$$RO = 1$$

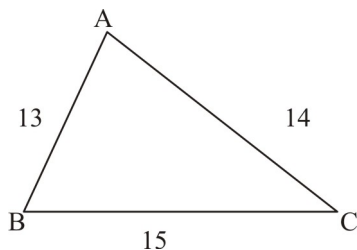
In $\triangle ABX$, $BX = 2NQ = 1$

In $\triangle AYC$, $YC = 2.RO = 2$ {by mid pt theorem}

And In $\triangle AXY$

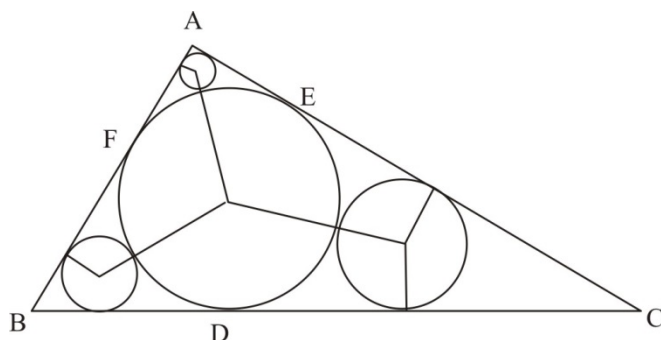
$XY = 2PQ = 12$ {by mid point theorem}

Now $\triangle ABC$ is



$$\text{Area} = \sqrt{21(6)(7)(8)} = 84$$

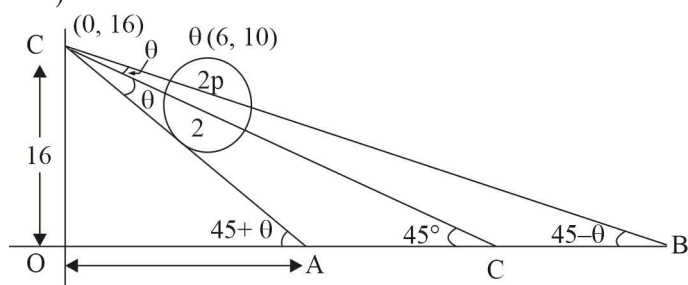
$$23.(74) \quad r = \sqrt{r_A r_B} + \sqrt{r_B r_C} + \sqrt{r_C \cdot r_A} = 20 + 30 + 24 = 74.$$



$$24.(21) \quad \text{Slope of } PC = \frac{-10}{10} = -1$$

$$AB = OB - OA$$

$$= \frac{16}{\tan(45 + \theta)}$$



$$= \frac{16}{\tan(45 + \theta)} - \frac{16}{\tan(45 + \theta)}$$

$$CP = \sqrt{36 + 36} = 6\sqrt{2} \quad \tan \theta = \frac{2}{\sqrt{68}} = \frac{1}{\sqrt{17}}$$

$$AB = 16 \left[\frac{1}{\tan 145 + \theta} - \frac{1}{\tan 145 + \theta} \right]$$

$$AB = 16 \left[\frac{1}{\tan 145 + \theta} - \frac{1}{\tan 145 + \theta} \right]$$

$$\tan(45 - \theta) = \frac{\sqrt{17} - 1}{\sqrt{17} + 1}$$

$$AB = 16 \left[\frac{\sqrt{17} + 1}{\sqrt{17} - 1} - \frac{\sqrt{17} - 1}{\sqrt{17} + 1} \right] = 16 \left[\frac{17 + 1 + 2\sqrt{17} - [17 + 1 - 2\sqrt{17}]}{16} \right]$$

$$= 2\sqrt{17}$$

$$\text{Hence} = 21$$

25.(56) The given problem is

$$< 91 > < 120 > < 143 > < 180 > < N >$$

$$= 91 \cdot 120 \cdot 143 \cdot 180 \cdot N$$

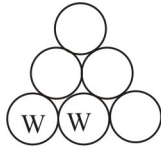
$$\Rightarrow 100.121.144.169 < N > = 91.120.143.180N$$

$$\Rightarrow 22 < N > = 21N \Rightarrow 21 | < N >$$

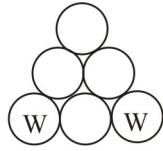
$$\text{But } < N > \text{ is a perfect square so } 441 < N >$$

We had be lucky $\langle N \rangle = 441$ satisfies the equation put $\langle N \rangle = 441$ get $N = 462$ we are lucky indeed sum of squares of digits = 56

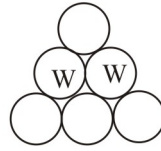
26.(18)



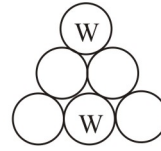
I



II



III



IV

To eliminate the possibility of repetition, let's think of the whole scenario w.r.t. one colour.

$$I \rightarrow \frac{4!}{2!2!} = 6 \text{ cases}$$

In II, III, IV we have 4 ways to colour.

Hence answer = 18

27.(87) The bug can travel along parallel vertical and 1 horizontal lines except for the 12 points. So the total number of points = $11 \times 9 - 12 = 87$

28.(93) All numbers except power of 2.

Say if a number is odd $= 2k = k + k + 1$

Say if a number is even but has an odd divisor

Then $k(k+1) + \dots + (k+r+1) = a$

$$rk + (1 + 2 + 3 + \dots + r - 1) = a \Rightarrow rk + \frac{r(r-1)}{2} = a \Rightarrow r \left(k + \frac{r-1}{2} \right) = a$$

Now $r-1 = \text{even} \Rightarrow r = \text{odd}$.

So, for every a for which there is an odd divisor we have an answer = $100 - (7) = 93$

29.(99) Say $a - b = K > 0$ as $c > 0$

$$a + b + c = b + k + b + \frac{b(b+k)}{k} = 3b + k + \frac{b^2}{k} \geq 5b \text{ (using A.M. - G.M.)}$$

$$\Rightarrow b \leq 19$$

$$\text{as } a + b + c \leq 99$$

Try $b = 19$, the max value of $a + b + c$ is 95

Try $b = 18$, the max value of $a + b + c = 99$ for $a = 27$ as $c = 44$

30. Case - 1: $C - km - 1 = 0$

$$\Rightarrow C - km - 1 \leq 1 \Rightarrow (k-2) = C(c - km - 1) \leq -11 \Rightarrow k \leq -9 \text{ \{impossible\}}$$

Case - 2: $C - km - 1 > 0$

$$\Rightarrow C \geq km + 2 \quad \dots (i)$$

$$\Rightarrow C \geq km + 2 \quad \dots (ii)$$

$$k - 2 \geq C \quad \dots (2)$$

$$\Rightarrow km + 2 \leq k - 2$$

Impossible

$$\Rightarrow C = km + 1$$

Let $C = a + 1$

Given $C \mid b$

$$\Rightarrow b = mc + 1, m \in \mathbb{Z}^+$$

Also $b \mid c^2 - c + 2$

$$\Rightarrow c^2 - c + 2 \geq 100 \Rightarrow c \geq 11$$

$$\text{Also, } c^2 - c + 2 = kb, k \in \mathbb{Z}^+ \Rightarrow c^2 - c + 2 = kmc + k \Rightarrow c[c - km - 1] = k - 2$$

Now, $c - km - 1 \in \mathbb{Z}$

$$\text{AND } c \geq 11 \Rightarrow c = 2m + 1 \Rightarrow a = 2m$$

$$\text{AND } b = mc + 1 = 2m^2 + m + 1$$

$$\text{Also, } b \in [100, 999]$$

Gives us $m \in [7, 22]$

16 possible values.